Scalable Private Signaling

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Abstract

Private messaging systems that use a bulletin board, like privacy-preserving blockchains, have been a popular topic during the last couple of years. In these systems, typically a private message is posted on the board for a recipient and the privacy requirement is that no one can determine the sender and the recipient of the message. Until recently, the efficiency of these recipients was not considered, and the party had to perform a naive scan of the board to retrieve their messages.

More recently, works like Fuzzy Message Detection (FMD), Private Signaling (PS), and Oblivious Message Retrieval (OMR) have studied the problem of protecting recipient privacy by outsourcing the message retrieval process to an untrusted server. However, FMD only provides limited privacy guarantees, and PS and OMR greatly lack scalability.

In this work, we present a new construction for private signaling which is both asymptotically superior and concretely orders of magnitude faster than all prior works while providing full privacy. Our constructions make use of a trusted execution environment (TEE) and an Oblivious RAM to improve the computation complexity of the server. We also improve the privacy guarantees by keeping the recipient hidden even during the retrieval of signals from the server. Our proof-of-concept open-source implementation shows that for a server serving a hundred thousand recipients and ten million messages, it only takes < 6 milliseconds to process a sent message, and < 200 milliseconds to process a retrieval (of 100 signals) request from a recipient.

1 Introduction

The protection of message contents in messaging applications is well-studied, and end-to-end encryption is nowadays widely practiced. Protecting the metadata, such as sender and recipient identity, is essential in anonymous message delivery systems [1, 2, 3, 4, 5], especially when messages are posted on a publicly-visible blockchain or bulletin board. This model of anonymous communication using a bulletin board is not new, in fact, stealth transactions [6] and privacy-preserving transactions[7, 8, 9, 10] both work in the same model. Anonymous messaging services such as Riposte[2], also deploy a bulletin board for parties to post their messages.

However, despite the importance, there is room for improvement in recipient efficiency. With respect to private blockchain transactions, it seems that there is a tradeoff between recipient privacy and efficiency. Namely, if all messages are encrypted on the bulletin board, the only means for a recipient to find their messages would be to try and decrypt each ciphertext on the board. An approach that is popular with stealth addresses is to outsource the search for transactions to a server that attempts to decrypt each transaction with a different viewing key and notifies the recipient when a transaction is posted on the chain for them. In this case, the recipient naturally loses all privacy to the server. Thus there is some trade-off between the efficiency and privacy of the receiver in these systems.

Recently, there have been efforts to improve the privacy of the recipients while maintaining the efficiency of recipients by using an untrusted server. Fuzzy Message Detection [5] by Beck et al. was the first work to address this. The authors propose a decoy-based approach: the server detects messages including both the real messages addressed to the recipient as well as some randomly chosen false positives. The number of false positives can be adjusted depending on the privacy requirements of the recipient. For full privacy, the server would detect all messages on the board to be pertinent to the recipient. This would require the recipient to decrypt all messages, thereby not improving the efficiency of the recipient. On the other hand, the recipient could choose to have negligible false positives, but in this setting, the recipient has (almost) zero privacy since the server knows exactly which messages correspond to the recipient. Therefore, in their work, there is an implicit trade-off between efficiency and privacy that a recipient can hope to achieve.

Two follow-up works by Madathil et al[11] and Liu and Tromer [12] improved this to entirely hide the set of pertinent messages (i.e., the message addressed to the retrieving recipient). Madathil et. al. [11] denote the problem as *private signaling* where a sender sends a signal to a recipient that a message exists on the board through an untrusted server. They propose two solutions, one based on trusted hardware and one based on two non-colluding servers running a garbled-circuit-based protocol. We will describe this construction in more detail in Section 2. Liu and Tromer [12] independently solve a similar problem and called it *oblivious message retrieval*, and propose a solution where a server is able to obliviously detect and retrieve messages for recipients based on leveled homomorphic encryption. While both works have made great progress in proposing schemes providing full privacy, while preserving efficiency for the recipient, they still lack scalability.

In both of the works for a board with N messages, a server serving M recipients needs to spend $\Omega(N \cdot M)$ work to distribute the messages in a privacy-preserving way. Thus, when N and M are huge, one server can hardly handle the computation cost.

The focus of this work is on the following natural question:

> *Can we have a scheme that is fully private and the runtime of the server scales sublinear in M and (almost) linear in N?*

1.1 Our contribution

In this paper, we firmly answer *yes* to this question. We propose a scheme that is *fully private* and the server runtime is only $\tilde{O}(N)$ [1] to process N sent messages and $\tilde{O}(M\bar{\ell})$ [2] to perform M retrievals, where $\bar{\ell}$ is the number of signals retrieved at a time, which are asymptotically almost optimal except for some logarithmic factors (not even poly-log), and concretely, orders of magnitude faster than the prior works.

- **Scalable PS construction.** We propose a private signaling construction such that for N signals and M recipients, a server only needs to do $O(N(\log(M) + \log(N)))$ work to find the signals addressed to each recipient *with full privacy guarantee*. Concretely, with practical parameters (e.g., 10 million messages and 100 thousand recipients), the total cost is only < 6 milliseconds to process a sent signal and < 200 milliseconds to do a retrieval for a recipient (for 100 per retrieval). These are orders of magnitude faster than prior works (the fastest prior work [11] requires $\sim$5 seconds to process a sent signal with 100 thousand recipients, each having at most 100 signals).
  Moreover, our protocol achieves a stronger correctness functionality compared to prior schemes. Schemes in [11] suffer from a simple DoS attack we describe in Section 2. The scheme in [12] output overflow when there are more than $\bar{\ell}$ messages for a recipient (where $\bar{\ell}$ is some upper bound on the number of messages addressed to a recipient, provided by the recipient when doing retrieval [3]). However, our scheme does not have any of the issues and thus provides a stronger correctness guarantee.

[1] $O(N \cdot (\log(M) + \log(N)))$
[2] $O(M(\bar{\ell}\log(N) + \log(M)))$
[3] When this happens, the recipient needs to either download the board themselves or re-outsource the job with a larger $\bar{\ell}$. Either way, it introduces a large overhead.

- **Strengthened Private Signaling problem.** We present a stronger private signaling functionality than the one introduced in [11], where the server cannot link two retrieval requests. This property is denoted as detection key unlinkability in [12]. We define this functionality in the UC framework[13] and show that our protocol UC-realizes this functionality.
- **Open-sourced implementation.** We implement and evaluate our protocol, and our implementation is open-sourced at [14]. We observed that our prototype is significantly more performant and scalable than all prior work.

2 Technical Overview

In this section, we give an overview of the main technique in our construction of an efficient private signaling protocol.

Original protocol review. We start by reviewing the recent TEE-based protocol of Madathil et al[11].

In their protocol, the TEE stores a mapping of parties and their pertinent signals (i.e., the signals addressed to the corresponding party). Since this mapping is stored inside the TEE, all information about the signals is private. In more detail, their construction works as follows: The TEE stores a table T of size $M \times \bar{\ell}$, where M is the total number of participants and $\bar{\ell}$ is the total number of signals the TEE can store for a recipient.

To send a signal, a sender computes an encryption of the recipient's identity and the location on the board that the sender wants to communicate to the recipient. The encryption is done using the public key of the TEE. This ciphertext constitutes the signal and is sent to the server running the TEE. The server invokes the TEE with this ciphertext as input, and TEE decrypts the ciphertext and appends the signal to the recipient's row in the table T.

At any point in time, a party can choose to retrieve its locations by sending an authenticated message to the server. The server verifies the signature and inputs this authenticated retrieval request to the TEE. Upon verification, TEE encrypts all the locations in the corresponding row of the recipient under their public key and outputs it to the server, which then forwards it to the recipient. Upon every retrieval, the TEE flushes the row of the retrieving recipient so they can receive the next set of signals. Security is guaranteed since all the signals are stored privately inside the TEE, and upon retrieval, they are all encrypted under the public key of the recipient. Therefore no entity can link either the senders or the locations of messages on the blockchain to a recipient.

In addition, to mitigate certain side-channel attacks, [11] proposes to update the entire table after each signal arrives. Thus, for each signal sent to TEE, the TEE runtime is $O(M\bar{\ell})$. TEE local storage is also $O(M\bar{\ell})$. Both of these limit the efficiency and scalability of the original protocol.

An inherent issue with [11]. Despite this being a secure solution, the work of [11] has an inherent issue regarding the correctness guarantee. Recall that the constructions presented in [11] only allows for storage of up to $\bar{\ell}$ signals per recipient. Now, what would happen if a particular recipient were to receive more than $\bar{\ell}$ signals before they retrieve their signals from the TEE?

In an earlier version of the same work by Madathil et. al. [15]: the authors suggest that the older signals would be overwritten upon receipt of new signals that have not been retrieved. In the newer version [11], the authors do not deal with this directly, but assume that the recipient retrieves the signals frequently enough, such that there are at most $\bar{\ell}$ signals between two retrievals.

Both of these are clearly unsuitable for cases when a recipient receives more than $\bar{\ell}$ messages in a short period of time. Note that one could set $\bar{\ell}$ to be a very large number, but the size of $\bar{\ell}$ will be constrained by the runtime and the storage space of the TEE since they are both linear in $\bar{\ell}$.

Moreover, their designs could actually lead to a DoS attack where an adversary simply sends $\bar{\ell}$ dummy signals to the recipient via the TEE. This would ensure that an honest recipient cannot retrieve its actual signals: the dummy signals would overwrite all the honest ones.

Our approach in a nutshell. In this work we show how we can improve the scalability of the

TEE-based private signaling protocol of [11], and ensure that the above-mentioned issues with $\bar{\ell}$ are mitigated.

We present a construction that maintains the signals pertinent to a recipient in the form of a linked list. Moreover, these linked lists are not stored inside the TEE but with the server that runs the TEE. Since there are no storage constraints on the server, these linked lists can be of arbitrary length. To prevent the server from learning which signals are retrieved, the signals are stored in an Oblivious RAM[16, 17, 18]. Moreover, each retrieval has a fixed number of signals that can be retrieved, and the recipient's identity is hidden during retrieval. Thus, the server does not know how many signals are received by a single recipient.

We describe below our ideas of changing data structure in Section 2.1, an overview of the ORAM-based construction in Section 2.2, and an overview of our new retrieval mechanism in Section 2.3.

2.1 Changing data structure to a linked list

In this subsection, we will first present how to mitigate the issue of the server's ability to store only $\bar{\ell}$ number of signals per recipient. These changes will have certain privacy and efficiency drawbacks, that we fix in the next subsection.

Recall that in the original construction of [11], the authors suggested a solution where the TEE stores the table of signals locally and updates the entire table for each incoming signal. This implies a computation complexity of $O(M \times \bar{\ell})$ for the TEE, and a local storage of size $O(M \times \bar{\ell})$. [4]

We take a completely different approach as mentioned above: a linked list is used to store the list of signals pertinent to a recipient. Every incoming signal is stored and includes a pointer to the previous pertinent signal for the same recipient. This way, the TEE only needs to keep track of the latest signal pertinent to each recipient. The previous signals are retrieved by using the pointer to the prior relevant signal in the linked list.

In more detail, for each signal TEE receives, TEE stores an encrypted pointer in the server. Each pointer points to the *previous* pertinent signal. Upon processing a new signal, the TEE updates the recipient's last signal and adds an encryption of the pointer to the previous pertinent signal. The TEE also does dummy updates to the last signal of all the other parties as well to prevent any leakage. Thus per each signal, the TEE only needs to perform $O(M)$ computation as compared to $O(M \times \bar{\ell})$ with the original construction of Madathil et al [11].

To retrieve their signals, a receiver sends an authenticated request to the TEE. The TEE looks up the last signal pertinent to this receiver, and also the pointer to the previous signal pertinent to this receiver. The TEE requests the ciphertext corresponding to the previous signal from the server and determines the pointer to the previous one. The TEE requests these ciphertexts until the pointer is a default value (e.g. -1). The TEE then encrypts these corresponding signals under the public key of the recipient and outputs this list of ciphertexts to the recipient.

We present an overview of this technique in Figure 1.

2.2 ORAMs to prevent information leakage

A careful reader might have observed that the approach outlined above is flawed. During the retrieval of signals, the server executing TEE sees exactly which ciphertexts are requested by the TEE. This allows the server to learn which of the signals were pertinent to the recipient. To prevent this leakage we must hide the access patterns of the TEE. To this end, we use an Oblivious RAM to store all the signals as well as the pointers to the previously pertinent signals. An Oblivious RAM (ORAM) ensures that the server cannot know which locations are accessed by the TEE and which locations were written by the TEE. This ensures that the server does not know which signals are pertinent to a user.

[4] In the older version of [11], i.e., [15], the authors suggest storing the table in the server and re-encrypt it after every incoming signal. This also results in a computation complexity of $O(M \times \bar{\ell})$ for the TEE, with a much larger hidden constant, while maintaining the local storage to be $O(1)$.

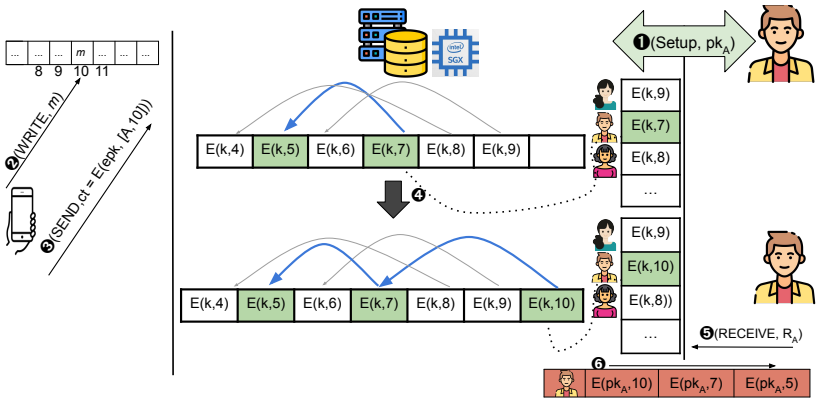


Figure 1: Overview: ❶ Party A registers with his public key pk_A with the TEE. ❷ A sender writes a message m at position 10 on the board for party A. ❸ The sender then sends a signal $(\mathsf{Enc}(\mathsf{epk}, [A, 10]))$ to the server. ❹ The TEE maintains a linked list of encrypted signals and a table with the latest signals of each recipient on the server. The TEE then appends an encryption of 10 and adds a link to party A's previous signal which is an encryption of 7. ❺ The party A requests its signals from the TEE. ❻ The TEE retrieves all the signals by iterating through the linked list (via ORAM) which is 10, 7, and 5, and returns them encrypted under A's public key.

Upon receiving a signal, instead of directly writing the encrypted pointer, the TEE writes it with ORAM access. We note that the total cost to process a signal is just $O(M + \log(N))$ as opposed to the previous $O(M \times \bar{\ell})$, as each ORAM operation takes $O(\log(N))$ [18].

Getting rid of the linear dependency on M. Recall that in the approach described above the server maintains a table of size M that stores the last pertinent signal to each recipient. The TEE would need to do a dummy update to the entire table for every incoming new signal to ensure that the server does not learn the recipient of the signal. We, therefore, propose to use another ORAM to store the last pertinent signal for each recipient. The TEE therefore only needs to access the corresponding ciphertext from the ORAM, update it, and write back to the ORAM. This ensures that the runtime cost per signal sending simply becomes $O(\log(M) + \log(N))$ and the local storage remains $O(1)$ [18].

2.3 Private retrieval

As described earlier, to retrieve their signals, a recipient sends a signed request message. This signed message includes an incrementing counter value to prevent replay attacks. If the signature is correct, TEE reads all the pertinent signals from ORAM and sends them back encrypted under the recipient's public key (using a key-private PKE scheme).

Hiding the number of pertinent signals. Note that the above-described solution leaks the *number* of pertinent signals to a particular recipient. While we believe whether this information needs to be hidden is application-dependent, for achieving the strongest functionality, we still prevent the leakage using the following modification to our scheme. Add a bound $\bar{\ell}$ that each recipient retrieves each time, and if a recipient R has more than $\bar{\ell}$ signals stored using the linked list, simply stop at the $\bar{\ell}$-th latest signal and send these $\bar{\ell}$ signals back to the recipient. Then, TEE updates the ORAM database to store the latest signal that is not yet retrieved (such that TEE can resume at this place next time to avoid repetitive work). If the recipient has less than $\bar{\ell}$ signal, the TEE pads the response to have $\bar{\ell}$ ciphertexts in total. In the former case where there are more than $\bar{\ell}$ ciphertexts, the TEE indicates that there are more signals by sending an encryption of -2 along with the encryptions of the pertinent signals, and otherwise sends -1. If there are still more signals left, the recipient simply requests again.

Retrieval privacy. In the discussion, so far we have not shown how the privacy of the recipient is maintained. For example, the server will always learn when a recipient is attempting to retrieve its signals. This could also be an issue when a recipient has more than $\bar{\ell}$ signals. Using some timing information, if a recipient requests to retrieve signals within a short interval, the server could infer that the recipient has greater than $\bar{\ell}$ pertinent signals.

An easy way to resolve this issue is to have the recipient use the TEE's public key to encrypt the signed request. By CPA security, the recipient's identity remains private [5].

To verify the signature, we could have all the verification keys inside the TEE, but since this leads to $O(M)$ overhead in local storage, we store these verification keys in yet another ORAM on the server. The TEE can then privately retrieve the verification key corresponding to the recipient and therefore verify its identity.

By putting everything together, we have a privacy-preserving signal retrieval method, with a cost of $O(\bar{\ell}\log(N) + \log(M))$ per retrieval.

3 Related work

Private Signaling. Private Signaling (PS) [15] tries to address the problem of recipient privacy when retrieving from a privacy-preserving message system. They present their definition of private signaling in the UC framework and present an ideal functionality $\mathcal{F}_{\mathsf{privSignal}}$, that captures the property that all the messages addressed to a recipient can be retrieved and no one except for the sender and the recipient learns the recipient of a message.

The paper provides two solutions, one based on a server with trusted hardware and one based on the garbled circuit with two servers. A later version [11] of the paper modified the construction based on trusted hardware, where the table of recipients mapped to their messages is stored inside the TEE. Both versions of the trusted-hardware-based solution lack some scalability, in different ways, as we discussed earlier in Section 2. Moreover, the constructions in [15, 11] also only achieve a slightly weaker ideal functionality (that some old messages may not be received by the recipient in some circumstances). In comparison, in this work, all messages can be retrieved by a recipient. Furthermore, we strengthen the ideal functionality (by capturing an unlinkability property, details in Section 5), and show a construction that fully realizes the ideal functionality.

Fuzzy Message Detection. FMD [5] uses a decoy-based privacy notion for the recipients. In more detail, they introduce some false positives together with the true positives, such that adversaries cannot distinguish the false positives from the true positives. This is a weaker privacy guarantee susceptible to attacks [21, 22]. Thus, FMD provides a tradeoff between privacy and efficiency for the recipient , where-in for optimal efficiency, the recipient would decrypt only its signals but its full anonymity to the server. On the other hand, to achieve full privacy the recipient needs to decrypt all signals.

Oblivious Message Retrieval. Oblivious Message Retrieval (OMR) [12] also tries to address the problem of recipient privacy when retrieving from a privacy-preserving message system. Similar to PS, they achieve full privacy. The major OMR constructions are based on leveled homomorphic encryptions (leveled-HE). The construction is later improved in [19] and achieves better server runtime efficiency, but still lacks scalability. However, their concrete cost is still relatively high, and asymptotically, their cost is linear in both the total number of messages and the number of recipients doing the retrieval with the server.

An extended group version Group OMR was recently introduced by [19]. We also briefly discuss how we can extend to the group setting (i.e., a message may be addressed to more than one recipient).

ZLiTE. ZLiTE [20] also tries to deal with a similar problem. Their result is also based on a trusted

[5] Of course, there might be some network-level leakage. This can be resolved by, for example, using TOR. Since this is beyond the scope of this paper, we assume this is already addressed appropriately.

	Server runtime per send	Server runtime per retrieve	Total server runtime N sends + M retrievals	Recipient Time	Security Assumption	Environment Assumption	TEE Local Storage	Privacy
PS1 [15]	$O(\bar{\ell}M)$	$O(\bar{\ell})$	$O(N\bar{\ell}M)$	$O(\bar{\ell})$	PKE	Trusted Hardware	$O(1)$	No unlinkability
PS1N [11]	$N\bar{\ell}M$	$O(\bar{\ell})$	$O(N\bar{\ell}M)$	$O(\bar{\ell})$	PKE	Trusted Hardware	$O(\bar{\ell}M)$	No unlinkability
PS2 [15, 11]	$O(N\bar{\ell}M)$	$O(\bar{\ell})$	$O(N\bar{\ell}M)$	$O(\bar{\ell})$	GC	Two non-colluding servers	N/A	No unlinkability
OMR [12, 19]	$O(1)$	$O(N\mathsf{polylog}(\bar{\ell}))$	$O((N\mathsf{polylog}(\bar{\ell})M)$	$O(\bar{\ell}^3)$	LWE	N/A	N/A	Full
ZLiTE [20]	$O(1)$	$O(N\log(\bar{\ell}))$	$O((N\log(\bar{\ell})M)$	$O(\bar{\ell})$	PKE	TEE	O(1)	No unlinkability
Our work	$O(\log(N)+\log(M))$	$O(\bar{\ell}\log(N)+\log(M))$	$O(N\log(M)+M\bar{\ell}\log(N))$	$O(\bar{\ell})$	PKE	Trusted Hardware	$O(1)$	Full

Table 1: Asymptotics and privacy comparisons with prior works. N is the total number of messages on the board. M is the total number of recipients registered with a server. $\bar{\ell}$ is the number of messages that can be retrieved by the recipient per retrieval. Server runtime is calculated for N messages with M recipients, each having $\bar{\ell}$ as the upper bound of the number of messages. Unlinkability refers to the sender unlinkability and recipient unlinkability defined in Section 5.

execution environment and oblivious RAM. However, their approach is to let the recipient send a secret key (viewing key) to the TEE and then ask the TEE to scan the entire chain and send back the signals addressed to the recipient. This results in a server time linear in $M \cdot N$. Furthermore, the model is not entirely the same: all the works above and our work focus on having the servers as a separate component from the blockchain or the other parts of the system, while for ZLiTE, the server is also a full node in Zcash. In contrast, our model is more generic and therefore applicable in systems other than cryptocurrency as well.

3.1 Comparison with prior works

In Table 1, we compare our construction asymptotically with other works. PS1 and PS2 are introduced in [15], and PS1N is a modified version of PS1 introduced in [11]. The central idea remains the same, but for PS1, the table is stored outside the TEE and the TEE re-encrypts the table each time when it updates the table; while for PS1N, they store this table inside TEE, thus trading off local storage for runtime. Since they provide trade-offs and both have some scalability issues in different ways, we compare our scheme with both.

As shown in the table, our scheme gives the second-best asymptotic server runtime to process a sent signal and a retrieve operation and is only a log factor worse than the best ones. In addition, for the total runtime for N sends and M retrievals, our scheme is undoubtedly the best. Moreover, our scheme can keep the TEE local storage to be constant. Thus, our scheme offers the best scalability compared to all the other works.

Privacy-wise, our construction also offers the strongest privacy. In addition to that the adversary does not learn which messages are addressed to a recipient, they also do not learn who is the sender and who is the recipient (which is the unlinkability property, details in Section 5). Note that while OMR also achieves full privacy, our privacy guarantee is defined by UC, and thus provides better composability and extendability compared to OMR.

4 Preliminaries

Notation Let λ be the security parameter, $\mathsf{poly}(\cdot)$ be a polynomial function, $\mathsf{polylog}(\cdot)$ be poly-log function, and let $\mathsf{negl}(\cdot)$ be a negligible function. M denotes the total number of recipients and N denotes the number of signals on the board. $\tilde{O}(x)$ denotes $x\log(x)$ in our paper (as opposed to $x\mathsf{polylog}(x)$).

Public board: $\mathcal{G}_{\mathsf{ledger}}$. We assume that all parties have read and write access to a public board,

which we abstract via a public ledger ideal functionality $\mathcal{G}_{\mathsf{ledger}}$ (Fig 12) functionality introduced in [23]. $\mathcal{G}_{\mathsf{ledger}}$ maintains a global variable called `state` and parties can read from and write to this global state through the commands `READ` and `SUBMIT`.

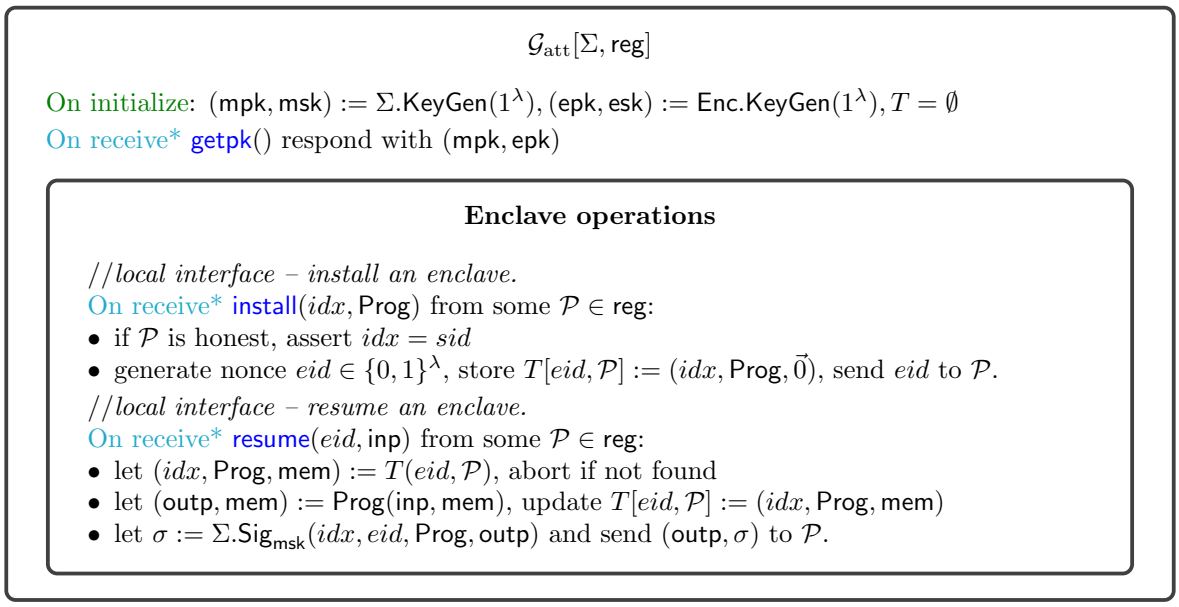


Figure 2: Global functionality modeling a TEE [24]

In this section, we present the crucial definitions and security guarantees of the primitives used in our protocols.

Trusted Execution Environment: $\mathcal{G}_{\text{att}}$. A Trusted Execution Environment (TEE) is a hardware-enforced primitive that protects the confidentiality and integrity of privacy-sensitive software and data from untrusted software including OS/hypervisor, and other applications residing on the same computer. In this paper, we model TEE as a single, globally-shared ideal functionality that is denoted as $\mathcal{G}_{\text{att}}$ following the definition of [24]. The $\mathcal{G}_{\text{att}}$ functionality is depicted in Fig. 2. There are two types of invocations to the TEE - *installation*, which allows installing a piece of software, and a *stateful resume*, which allows executing it on a given input.

There are a few TEE solutions available commercially, hosted on public clouds such as AWS, Azure, and GCP. AWS has AWS Nitro Enclaves [25], Azure offers Confidential Compute machines using Intel SGX [26], and GCP provides AMD SEV [27] machines for trusted execution. We use Intel SGX to implement our protocols – because of two reasons: (1) SGX has a minimal Trusted Computing Base (TCB) compared to others, and (2) our construction uses ORAMs to store huge databases outside the TEE and since SGX is an application-level enclave, it incurs lower memory-access overhead to read/write from untrusted host memory. However, we note that any TEE adhering to Fig. 2 can be used to realize our protocols.

Threat model of SGX. SGX enclaves are strongly isolated from the untrusted host and OS by encrypting all their memory using a Memory Encryption Engine (MEE). Apart from providing confidentiality and integrity of computations inside the enclave; SGX also offers an authentication mechanism via *attestation* using which a recipient can verify the correctness of the enclave. After verification, recipients can encrypt their data towards the enclave using the key provided in the *attestation* report.

However, there are a few limitations for SGX. In particular, side-channel attacks and limited in-enclave memory. SGX is not resilient against page table-based[28], cache-based[29], branch-prediction-

based[30], and physical[31] side channels. An adversary can launch these side channels either via a compromised OS or host application and extract secrets using timing or control-flow analysis. To ensure confidentiality against most of these side channels, the enclaves must implement constant-time programming and oblivious memory-access patterns. To this end, several techniques have been proposed recently and we refer the reader to [32, 33, 34, 35] for a detailed description. Regarding the enclave memory limit, the previous processors – Intel Coffee Lake, 2017 – running SGXv1 can load up to 256 MB of enclave memory. However, the newer processors – Intel Ice Lake, 2020 – running SGXv2 can load up to 1 TB of enclave memory, hence, the memory limit is no more a concern for in-enclave storage-intensive applications.

However, since our protocols do not store a lot of data within the enclave (due to ORAM), in our implementation, we stick with SGXv1. Our experiments in section 7.2 show that the enclave requires less than 50 MB of heap beside the enclave code and stack.

Key-private Public Key Encryption scheme. We also require a correct and semantically secure PKE scheme $(\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ that is also key-private. Correctness and semantic security are both standard. Key-privacy, introduced in [36], basically means that if a ciphertext is encrypted under a public key, one cannot tell which public key was used. This is achieved by most of the commonly used encryption schemes including El Gamal[37], Cramer-Shoup[38], a variant of RSA-OAEP suggested in [36], LWE [39] and so on. We formally capture this property as follows, adapted from [36]:

Definition 1 (Key-privacy)**.** *A PKE scheme* $(\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ *is key-private if for any PPT adversary* $\mathcal{A} = (\mathcal{A}_0, \mathcal{A}_1)$*, it can only win the following game with probability* $\leq 1/2 + \mathsf{negl}(\lambda)$*:* $(\mathsf{pk}_0, \cdot) \leftarrow \mathsf{KeyGen}(\lambda)$*,* $(\mathsf{pk}_1, \cdot) \leftarrow \mathsf{KeyGen}(\lambda)$*,* $(x, s) \leftarrow \mathcal{A}_0(\mathsf{pk}_0, \mathsf{pk}_1), y \leftarrow \mathsf{Enc}(\mathsf{pk}_b, x)$ *where* $b \leftarrow_{\$} \{0, 1\}$*,* $b' \leftarrow \mathcal{A}_1(y, s)$*, and the adversary wins iff* $b' = b$*.*

Oblivious RAM (ORAM). We adapt the ORAM security definition to TEE and the server [17].

At a high level, the goal of ORAM is to hide the data access pattern (which blocks are accessed and whether it's for a read or a write) from the server. Therefore, we need two sequences of read/write operations with the same length to be (computationally) indistinguishable to anyone except for the TEE.

Definition 2. *An ORAM construction contains one PPT algorithm:*

- $\mathsf{data}' \leftarrow \mathsf{Access}(\mathsf{DB}, \mathsf{op}, \mathsf{a}, \mathsf{data})$*: on the input of an identifier* DB *to some database, an operation* op *that can be either* Read *or* Write*, an identifier of the data block (e.g., memory location of the data block), and a data block* data*, returns a data block* data*.*

Let $\mathsf{ORAM.Read}(\mathsf{DB}, \mathsf{a})$ *denote* $\mathsf{Access}(\mathsf{DB}, \mathsf{Read}, \mathsf{a}, \cdot)$ *(here* $\cdot$ *is a dummy input) and let* $\mathsf{ORAM.Write}(\mathsf{DB}, \mathsf{a}, \mathsf{data})$ *denote* $\mathsf{Access}(\mathsf{DB}, \mathsf{Write}, \mathsf{a}, \mathsf{data})$*.*

Let $\vec{x} = ((\mathsf{DB}, \mathsf{op}_1, \mathsf{a}_1, \mathsf{data}_1), \ldots, (\mathsf{DB}, \mathsf{op}_Q, \mathsf{a}_Q, \mathsf{data}_Q))$ *denote a data request sequence of length* Q*, where* op_i *is the operation that can be either* Read *OR* Write*,* a_i *is the identifier of the data block, and* data_i *is the data being written. We define ORAM construction's correctness and privacy as follows:*

- *(Correctness) On the input* $\vec{x}$*, it returns data that is consistent with* $\vec{x}$ *with probability* $\geq 1 - \mathsf{negl}(|\vec{x}|)$*.*
- *(Computational Privacy) Let* $A(\vec{x})$ *sequence of accesses to the remote storage given the sequence of data requests* $\vec{x}$*. An ORAM construction is said to be computationally private if for any two data request sequences* $\vec{y}$ *and* $\vec{z}$ *of the same length, their access patterns* $A(\vec{y})$ *and* $A(\vec{z})$ *are computationally indistinguishable by PPT algorithm except for the TEE.*

State-of-the-art ORAM has achieved the asymptotics of $O(\log(N))$ server time per access in the worst case, with $O(1)$ local memory (i.e., memory required in the TEE) [18].

In addition, for better flexibility, we require the ORAM to have an interface of $\mathsf{ORAM.resize}(\mathsf{DB}, N')$ that takes an ORAM database DB of size N and resize it to an ORAM database of size $N' > N$. This is naively achievable by starting a new ORAM database DB' of size N', and then read every

entry from DB and write it to DB' and set $\mathsf{DB} \leftarrow \mathsf{DB}'$. Even with this naive method, the amortized efficiency per entry is simple $O(\log(N))$. One can use more efficient techniques, e.g. [40], to achieve resize more efficiently.

5 UC-Definition of Private Signaling

Threat model. We assume all the adversaries are computationally bounded, and they can read all public information, including all signals on the bulletin board, all the public keys, all accesses to the bulletin board, and all the communication between the server and the recipients. The adversaries can also act as a sender, a recipient, or a server.

For correctness, we require the server to be semi-honest. For privacy, all parties in the systems may be malicious and colluding. We assume that there is only one server, but can be arbitrarily many senders and recipients. Note that we can easily extend our model to have multiple servers, and we discuss this in more detail in Appendix C.1.

Formal definition. We define the problem of private signaling in the UC-framework [13]. In this framework, the security properties expected by a system are defined through the description of an ideal functionality. The ideal functionality is an ideal trusted party that performs the task expected by the system in a trustworthy manner. When devising an ideal functionality, one describes the ideal properties that the system should achieve, as well as the information that the system will inherently leak.

For the task of private signaling, we want to capture two properties: correctness and privacy, and we adapt the definition from [11] and improve upon it.

- (Correctness) Correctness means that a recipient R_i should be able to learn all signals that are intended for them.
- (Privacy) Privacy contains more complicated components:
  - (Sender unlinkability) An adversary cannot identify the sender of a signal.
  - (Signal recipient hiding) An adversary cannot learn the recipient of a signal.
  - (Recipient unlinkability) An adversary cannot learn the identity of a recipient when they retrieve their signals.
  - (Number of signals a recipient receives) An adversary cannot learn the number of pertinent signals for each recipient.

Note that *Sender unlinkability* and *Recipient unlinkability* are not captured by the prior definition in [11].

Also note that recipient unlinkability is quite essential, as our model assumes the servers to be semi-honest for the correctness of the retrieval process. Thus, if a server does not know who is doing the retrieval, they have fewer incentives to behave maliciously.

Furthermore, we want to capture the following inherent leakage. An observer of the system can always learn that a signal was posted for "someone" (for instance, just by observing the board). Second, a protocol participant can learn that some recipient is trying to retrieve their own signals (for instance, in the serverless case, this can be detected by observing that a node is downloading a big chunk of the board, or in the server-aided case, it is just possible to observe that R_i connected to the server).

All these properties and inherent leakage are captured by the following ideal functionality.

Private Signaling ideal functionality. The functionality $\mathcal{F}_{\mathsf{privSignal}}$ provides the following interface - `SEND` and `RECEIVE`. Furthermore, the functionality maintains a table denoted $\mathcal{T}$. This table maintains a mapping of a recipient R_j to their pertinent locations.

- `SEND` allows a sender to send a signal by specifying the recipient and location of the message on the board. The functionality leaks to the adversary that a `SEND` request has been received and then appends the location to the recipient's row in table $\mathcal{T}$

Functionality $\mathcal{F}_{\mathsf{privSignal}}$
The functionality maintains a table denoted $\mathcal{T}$ indexed by recipient R_j, that contains information on the locations of signals for the corresponding recipient.
Sending a signal (`SEND`). Upon receiving $(\texttt{SEND}, R_j, \mathsf{loc})$ from a sender S_i, send $(\texttt{SEND})$ to the adversary. Upon receiving $(\texttt{SEND}, \texttt{ok})$ from the adversary, append loc to $\mathcal{T}[R_j]$.
Retrieving signals (`RECEIVE`). Upon receiving $(\texttt{RECEIVE})$ from some R_j, send $(\texttt{RECEIVE})$ to the adversary. Upon receiving $(\texttt{RECEIVE}, \texttt{ok})$ from the adversary, send $(\texttt{RECEIVE}, \mathcal{T}[R_j])$ to the recipient R_j and update $\mathcal{T}[R_j] = []$.

Figure 3: Private Signaling functionality

- `RECEIVE` allows a recipient to request their signals from the functionality. The functionality leaks to the adversary that a party is trying to retrieve their messages, and then returns the corresponding row of the table to the party. Finally, the functionality flushes the same row.

Since the only information leaked to the adversary is that a sender has posted a signal and that a recipient has retrieved its signals we capture the privacy requirement of private signaling.

6 A Scalable Private Signaling Protocol With TEE (Π_{TEE})

Π_{TEE} is based on the following building blocks:
1. A $\mathcal{G}_{\text{att}}$ functionality [24] as defined in Section 4. This functionality models the TEE. The program run by the TEE is described in Fig 4.
2. An Oblivious RAM scheme - $(\mathsf{ORAM.Read}, \mathsf{ORAM.Write}, \mathsf{ORAM.resize})$
3. A signature scheme $(\mathsf{Sig}, \mathsf{Ver})$
4. A key-private encryption scheme $(\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$

$\mathcal{G}_{\text{att}}$	Secure processor functionality Figure 2
loc	Location of message on the board
N	Total number of messages on the board
M	Total number of recipients registered in TEE
ℓ	Number of signals per retrieval
RID	Identifier of the recipient
$\mathsf{ct}_{\mathsf{keys}}$	Ciphertext of encryption key and & verification key
ctr_i	Counter to prevent replayability of signatures for party i
$\mathsf{ct}_{\mathsf{Signal}}$	Encryption of $(\mathsf{RID}, \mathsf{loc})$
$\mathsf{mpk}, \mathsf{msk}$	Attestation keys of $\mathcal{G}_{\text{att}}$ functionality
$\mathsf{epk}, \mathsf{esk}$	Encryption keys of $\mathcal{G}_{\text{att}}$
DB	ORAM database of N encrypted pointers
$\mathsf{DBrecip}$	ORAM database of the latest signal of the M recipients
DBkeys	ORAM database of the M recipients' keys
CurIdx	A counter locally kept by TEE on how many signals have been received.
$\mathsf{ctr}_{\mathsf{recip}}$	A counter locally kept by TEE on how many recipients have been registered.
$\mathsf{PrevIdx}$	The entry index in DB of the previous pertinent signal.
$\mathsf{PrevLoc}$	The location of the previous pertinent signal.

Table 2: Notations for Π_{TEE}

In Table 2 we describe all the notations and variables used in our construction.

Overview We present a high-level overview of our construction, and the formal description is presented in Figure 5.

Enclave Setup. The server Srv installs the program for the TEE and initializes it with the following public parameters: N, an upper-bound on the total number of signals that can be stored; M, an upper-bound on the total number of parties that can register with the server; and $\bar{\ell}$, the number of signals that are retrieved by a recipient upon each request.

Upon running this command, the TEE initializes three databases with an ORAM scheme. The first ORAM denoted DB, is of size N and stores all the incoming signals to the server. The second ORAM denoted DBkeys, is of size M and stores the verification keys of the registered recipients. The third ORAM denoted $\mathsf{DBrecip}$, is of size M and stores information on the last received signal per registered recipient. TEE also locally initializes two counters CurIdx and $\mathsf{ctr}_{\mathsf{recip}}$ as zeros. These counters keep track of the number of incoming signals and the number of registered recipients respectively.

Note that the three ORAM databases are upper-bounded by N and M. If the above two counter values reach N and M respectively, we use resize described in Section 4 to resize the databases to $2N, 2M$ respectively and set $N \leftarrow 2N, M \leftarrow 2M$. Note that we can start with large N, M (e.g., 2^{40}) with only limited effects on performance (as our performance scales only with $\log(N)$ and $\log(M)$), and thus in practice, resize may not need to be used.

Note that all the ORAM data entries are encrypted under TEE secret key: esk.

Registration. To register with the server, a party R_i first retrieves the encryption key epk and the verification key mpk of the TEE. R_i then generates its own encryption keys $(\mathsf{pk}_i, \mathsf{sk}_i)$ (recall that this encryption scheme is key private) and signature keys $(\Sigma.\mathsf{vk}_i, \Sigma.\mathsf{sk}_i)$. R_i then encrypts its public key and verification key under the public key of the TEE and sends it to the server indicating that they want to register with the system.

The server then executes the TEE with input "setup" and the ciphertext containing the encryption key (pk_i) and the verification key $(\Sigma.\mathsf{vk}_i)$ of R_i. The TEE first decrypts the ciphertext to compute $(\mathsf{pk}_i, \Sigma.\mathsf{vk}_i)$. The TEE then encrypts the $(\mathsf{pk}_i, \Sigma.\mathsf{vk}_i)$ along with a counter initialized to 1 under the key esk.

These keys are then written (encrypted under esk) to the DBkeys at location RID, where $\mathsf{RID} = \mathsf{ctr}_{\mathsf{recip}}$ is simply a unique index of the recipient. Since the write accesses are hidden using an ORAM and encrypted under esk, the server does not learn the RID or the keys that were written to the ORAM. Next, the TEE initializes the last received message for R_i as a default [-1,-1]. The TEE then encrypts these initialized values under its secret key and writes them to location RID in the $\mathsf{DBrecip}$. The TEE finally increments $\mathsf{ctr}_{\mathsf{recip}}$ by 1, and returns $(\mathsf{pk}, \mathsf{RID}, \mathsf{ctr}_{\mathsf{recip}})$. Note that by the definition of $\mathcal{G}_{\mathrm{att}}$ functionality, all messages that are output by the program are signed by the TEE. This signed output $(\mathsf{pk}, \mathsf{RID}, \mathsf{ctr}_{\mathsf{recip}})$ is then sent by the server to the party R_i. The recipient verifies pk and the signature and publishes $\mathsf{pk}, \mathsf{RID}$ [6].

When the counter $\mathsf{ctr}_{\mathsf{recip}}$ is equal to M, the server executes the TEE program to resize the DBkeys to size $2M$. Again, this can be avoided in practice by setting M to a large enough number at first.

Send. To send a signal to a party R_i with identity RID that a message exists for them at position loc on the board, a sender S does the following. They first retrieve the keys of the TEE - epk and mpk. The sender then encrypts $(\mathsf{RID}, \mathsf{loc})$ under epk and posts the resulting ciphertext $\mathsf{ct}_{\mathsf{Signal}}$ to the board (denoted $\mathcal{G}_{\mathsf{ledger}}$).

The server reads the board and retrieves the ciphertext $\mathsf{ct}_{\mathsf{Signal}}$ and executes the TEE with the "send" command and inputs $\mathsf{ct}_{\mathsf{Signal}}$. The TEE first decrypts the ciphertext to receive RID and loc. The TEE then obtains the ciphertext encrypting the last received location for RID from $\mathsf{DBrecip}$, decrypts it, and parses it as $[\mathsf{PrevIdx}, \mathsf{PrevLoc}]$ (which is $[-1, -1]$ if it were the first message) for RID.

[6] Note that in our construction, we need the TEE to pick a unique RID for the recipient, as the ORAM read needs a unique location for each recipient to process. Thus, we cannot simply let the recipient randomly pick $\mathsf{RID} \in [M]$ unless M is exponentially large. This is not an issue with our single server model, but for multiple servers, this becomes more subtle to resolve. We discuss how to resolve this in more detail in Appendix C.1.

On input* ("initialization", pp)

Store $\mathsf{pp} = (N, M, \bar{\ell})$ and locally keep a counter $\mathsf{CurIdx} \leftarrow 0, \mathsf{ctr}_{\mathsf{recip}} \leftarrow 0$.
Initialize an empty database of size N with ORAM and store its identifier DB.
Initialize an empty database of size M with ORAM and store its identifier DBkeys.
Initialize an empty database of size M with ORAM and store its identifier DBrecip.

On input* ("setup", $\mathsf{ct}_{\mathsf{keys}}$)

$(\mathsf{pk}, \Sigma.\mathsf{vk}) \leftarrow \mathsf{Dec}(\mathsf{esk}, \mathsf{ct}_{\mathsf{keys}})$, set $\mathsf{RID} \leftarrow \mathsf{ctr}_{\mathsf{recip}}$.
$\cdot \leftarrow \mathsf{ORAM.Write}(\mathsf{DBkeys}, \mathsf{RID}, \mathsf{Enc}(\mathsf{esk}, (\Sigma.\mathsf{vk}, \mathsf{pk}, 1)))$
$\cdot \leftarrow \mathsf{ORAM.Write}(\mathsf{DBrecip}, \mathsf{RID}, \mathsf{Enc}(\mathsf{esk}, [-1, -1]))$
$\mathsf{ctr}_{\mathsf{recip}} \leftarrow \mathsf{ctr}_{\mathsf{recip}} + 1$
return $\mathsf{pk}, \mathsf{RID}, \mathsf{ctr}_{\mathsf{recip}}$

On input* ("send", $\mathsf{ct}_{\mathsf{Signal}}$)

$[\mathsf{RID}, \mathsf{loc}] \leftarrow \mathsf{Dec}(\mathsf{esk}, \mathsf{ct}_{\mathsf{Signal}})$
$\mathsf{data} \leftarrow \mathsf{ORAM.Read}(\mathsf{DBrecip}, \mathsf{RID}, \cdot)$ ▷ $\cdot$ is a dummy data variable
$[\mathsf{PrevIdx}, \mathsf{PrevLoc}] \leftarrow \mathsf{Dec}(\mathsf{esk}, \mathsf{data})$
$\cdot \leftarrow \mathsf{ORAM.Write}(\mathsf{DBrecip}, \mathsf{RID}, \mathsf{Enc}(\mathsf{esk}, [\mathsf{CurIdx}, \mathsf{loc}]))$
$\cdot \leftarrow \mathsf{ORAM.Write}(\mathsf{DB}, \mathsf{CurIdx}, \mathsf{Enc}(\mathsf{esk}, [\mathsf{PrevIdx}, \mathsf{PrevLoc}]))$
$\mathsf{CurIdx} \leftarrow \mathsf{CurIdx} + 1$
return CurIdx.

On input* ("receive", σ)

$\sigma' \leftarrow \mathsf{Dec}(\mathsf{esk}, \sigma)$
Parse $\sigma' = \mathsf{Sig}(\mathsf{RID}, \mathsf{ctr})$
$\mathsf{data} \leftarrow \mathsf{ORAM.Read}(\mathsf{DBkeys}, \mathsf{RID}, \cdot)$
$(\Sigma.\mathsf{vk}, \mathsf{pk}, \mathsf{ctr}_{\mathsf{RID}}) \leftarrow \mathsf{Dec}(\mathsf{esk}, \mathsf{data})$
if $\Sigma.\mathsf{Ver}(\Sigma.\mathsf{vk}, \sigma') = 1$ and $\mathsf{ctr} = \mathsf{ctr}_{\mathsf{RID}}$ **then**
  Initialize an empty vector $\vec{R}$
  $[\mathsf{index}, \mathsf{loc}] \leftarrow \mathsf{Dec}(\mathsf{esk}, \mathsf{ORAM.Read}(\mathsf{DBrecip}, \mathsf{RID}, \cdot))$
  while $|\vec{R}| \leq \bar{\ell}$ **do**
    if $|\vec{R}| = \bar{\ell}$ **then**
      $\cdot \leftarrow \mathsf{ORAM.Write}(\mathsf{DBrecip}, \mathsf{RID}, \mathsf{Enc}(\mathsf{esk}, [\mathsf{index}, \mathsf{loc}]))$
      Append -1 to $\vec{R}$ if index from previous iteration is -1, else append -2
      Break the loop
    Append loc to $\vec{R}$
    if index from the previous iteration is -1 **then**
      Do a dummy ORAM read and set $[\mathsf{index}, \mathsf{loc}] = [-1, -1]$
    else
      $[\mathsf{index}, \mathsf{loc}] \leftarrow \mathsf{Dec}(\mathsf{esk}, \mathsf{ORAM.Read}(\mathsf{DB}, \mathsf{index}, \cdot))$
  Update $\mathsf{ctr}_{\mathsf{RID}} = \mathsf{ctr}_{\mathsf{RID}} + 1$
  $\cdot \leftarrow \mathsf{ORAM.Write}(\mathsf{DBkeys}, \mathsf{RID}, \mathsf{Enc}(\mathsf{esk}, (\Sigma.\mathsf{vk}, \mathsf{pk}, \mathsf{ctr}_{\mathsf{RID}}))$
  return $\mathsf{Enc}(\mathsf{pk}, \vec{R})$
else
  return $\perp$

On input* ("extend", N')

$\mathsf{DB}' \leftarrow \mathsf{ORAM.resize}(\mathsf{DB}, N')$
Clear DB and set $\mathsf{DB} \leftarrow \mathsf{DB}'$
Update $\mathsf{pp}.N \leftarrow N'$

On input* ("extendrecip", M')

$\mathsf{DBkeys}' \leftarrow \mathsf{ORAM.resize}(\mathsf{DBkeys}, M')$
Clear DBkeys and set $\mathsf{DBkeys} \leftarrow \mathsf{DBkeys}'$
$\mathsf{DBrecip}' \leftarrow \mathsf{ORAM.resize}(\mathsf{DBrecip}, M')$
Clear DBrecip and set $\mathsf{DBrecip} \leftarrow \mathsf{DBrecip}'$
Update $\mathsf{pp}.M \leftarrow M'$

Figure 4: Program Prog run by $\mathcal{G}_{\text{att}}$

The TEE then writes $[\mathsf{CurIdx}, \mathsf{loc}]$ to $\mathsf{DBrecip}$ (again, encrypted under esk). Note that this correctly updates the last received signal for RID in $\mathsf{DBrecip}$.

Next, the TEE writes encryption of $[\mathsf{PrevIdx}, \mathsf{PrevLoc}]$ at index CurIdx in DB . Note that the integer CurIdx links the latest received signal in $\mathsf{DBrecip}$ with the previous signal associated with RID in DB. Finally, the TEE increments CurIdx and returns CurIdx to the server. As above when $\mathsf{CurIdx} = N$, the server executes the TEE program to resize the DB to $2N$.

Retrieving signals. To retrieve their signals, a party R_i first computes a signature on the RID and ctr_i, where ctr_i is the number of times a request has been made to retrieve (including this current one). Recall that the TEE stores the same counter with the encryption and verification keys in DBkeys. The recipient then encrypts this signed request under the public key of the TEE and sends it to the server. The CPA security of the encryption scheme ensures that the server does not know the requesting party's identity.

The server invokes the TEE with "receive" command with received ciphertext as input. The TEE decrypts the ciphertext to retrieve the RID and ctr_i. Then it queries DBkeys to retrieve the keys and counter value associated with the RID. It checks that the received signature verifies and that the ctr value matches the one stored in DBkeys. Upon each retrieval, the TEE increments the ctr value stored inside DBkeys and this ensures that no malicious party is able to simply replay a previous request and learn signals associated with an honest recipient.

The TEE then initializes an empty output vector $\vec{R}$ of length $\bar{\ell}$. This vector will include encryptions of the locations pertinent to the recipient. The TEE first retrieves the latest signal received by R_i from $\mathsf{DBrecip}$. Recall that this is of the form $[\mathsf{index}, \mathsf{loc}]$ (after decryption using esk). The TEE then writes the loc to $\vec{R}$. The TEE then reads the signal stored at position index from DB, which is again of the form $[\mathsf{index}', \mathsf{loc}']$. The TEE then writes loc' to $\vec{R}$. The TEE repeats this operation now with index' until filling $\vec{R}$ with $\bar{\ell}$ locations. If a recipient has less than $\bar{\ell}$ signals, perform dummy ORAM reads and get $[\mathsf{index}', \mathsf{loc}'] = [-1, -1]$.

After that, the TEE writes the last $[\mathsf{index}', \mathsf{loc}']$ to $\mathsf{DBrecip}$, maintaining the invariant that the last non-retrieved signal is stored in $\mathsf{DBrecip}$ (which is $[-1, -1]$ if there are $\leq \bar{\ell}$ messages). If the last index' retrieved is -1, indicating that all messages of the recipient have been retrieved in $\vec{R}$, the TEE appends -1 to $\vec{R}$. On the other hand, if index' is not -1 indicating that there exist more signals for R_i, the TEE appends -2 to $\vec{R}$.

As mentioned above the TEE updates the request counter $\mathsf{ctr}_{\mathsf{RID}}$ that corresponds to R_i and performs a write to DBkeys. Finally, the TEE encrypts the vector $\vec{R}$ under the public key of R_i and sends it to the recipient. Recall that the encryption scheme is key private and thus the identity of the recipient is not leaked.

Asymptotic analysis. We now analyze the asymptotic costs for the TEE to process a signal upon execution of the "send" command, and the cost of retrieval of locations upon execution of the "receive" command. Recall that each ORAM access costs $O(\log N)$ computation for an ORAM of size N. The "send" command consists of one read to $\mathsf{DBrecip}$ and a write to $\mathsf{DBrecip}$ and another write to DB. Thus the total computational complexity per "send" invocation is only $O(\log N + \log M)$. Likewise, for "receive" command, the TEE performs one read to DBkeys, $\bar{\ell}$ reads to DB and one write to $\mathsf{DBrecip}$ and DBkeys. Thus the total computational complexity per "receive" is only $O(\bar{\ell} \log N + \log M)$.

Intuition of privacy. We present an intuition of why privacy is guaranteed for the recipients below.

- (Sender Unlinkability) This is inherent as a signal does not contain any identity-related information about the sender. A signal is simply $\mathsf{Enc}(\mathsf{epk}, (\mathsf{RID}, \mathsf{loc}))$, where RID is the identity of the recipient and loc is the location of the message on the board [7].
- (Signal recipient hiding) This is guaranteed by the nature of TEE and ORAM used in our protocol.
- (Recipient unlinkability) As explained above, we encrypt the signed message and use ORAM to fetch the verification key to verify the signature inside TEE. In addition, our encryption scheme is key private. Thus our construction achieves this property.

[7] Again, we assume the network level leakage is already addressed properly, for example, by using TOR.

Enclave setup

Srv:

1. Run $\mathcal{G}_{\text{att}}.\mathsf{install}(\mathsf{Prog})$ to get eid.
2. $\mathcal{G}_{\text{att}}.\mathsf{resume}(eid, (\text{“initialization”}, \mathsf{pp} = (N, M, \bar{\ell})))$
   ▷ pp are parameters passed to initialize the protocol

Registration

Recipient R_i:

1. Let $(\mathsf{mpk}, \mathsf{epk}) := \mathcal{G}_{\text{att}}.\mathsf{getpk}()$
2. Compute $(\mathsf{pk}_i, \mathsf{sk}_i) \leftarrow \mathsf{Enc.KeyGen}(1^\lambda)$, $(\Sigma.\mathsf{sk}_i, \Sigma.\mathsf{vk}_i) \leftarrow \Sigma.\mathsf{KeyGen}(1^\lambda)$.
3. Set $\mathsf{ct}_{\mathsf{keys},i} = \mathsf{Enc}(\mathsf{epk}, (\mathsf{pk}_i, \Sigma.\mathsf{vk}_i))$ and send $(\text{“setup”}, \mathsf{ct}_{\mathsf{keys},i})$ to Srv. ▷
4. Await $((eid, \mathsf{pk}_i, \mathsf{RID}), \sigma_T)$ from Srv.
5. Assert $\Sigma.\mathsf{Ver}_{\mathsf{mpk}}((eid, (\mathsf{pk}, \mathsf{RID}, \mathsf{ctr}_{\mathsf{recip}})), \sigma_T) = 1$ and publish $(\mathsf{pk}_i, \mathsf{RID})$. Initialize $\mathsf{ctr}_i = 0$.
   ▷ Note that if the returned pk_i is not the pk_i sent, abort.

Srv:

1. Upon receiving $(\text{“setup”}, \mathsf{ct}_{\mathsf{keys},i})$ from R_i, let $((\mathsf{pk}_i), \sigma_T) = \mathcal{G}_{\text{att}}.\mathsf{resume}(eid, (\text{“setup”}, \mathsf{ct}_{\mathsf{keys},i}))$. Upon receiving ORAM request $\mathsf{data} \leftarrow \mathsf{ORAM.Access}(\cdot,\cdot,\cdot,\cdot)$ from TEE when executing the $\mathcal{G}_{\text{att}}.\mathsf{resume}(\cdot,\cdot)$ operation: process and return data to $\mathcal{G}_{\text{att}}$.
2. Send $((eid, (\mathsf{pk}, \mathsf{RID}, \mathsf{ctr}_{\mathsf{recip}})), \sigma_T)$ to R_i.
3. Upon return of $\mathsf{ctr}_{\mathsf{recip}}$ from TEE, if $\mathsf{ctr}_{\mathsf{recip}} = M$, call procedure $\mathcal{G}_{\text{att}}.\mathsf{resume}(eid, (\text{“extendrecip”}, 2M))$ and update $M \leftarrow 2M$.

Procedure $(\texttt{SEND}, R_i, \mathsf{loc})$ Snder S:

1. Sender S gets $(\mathsf{mpk}, \mathsf{epk}) := \mathcal{G}_{\text{att}}.\mathsf{getpk}()$ and computes $\mathsf{ct}_{\mathsf{Signal}} = \mathsf{Enc}(\mathsf{epk}, [\mathsf{RID}, \mathsf{loc}])$. S sends $(\texttt{SUBMIT}, (\texttt{SEND}, \mathsf{ct}_{\mathsf{Signal}}))$ to $\mathcal{G}_{\mathsf{ledger}}$

Srv:

1. Send READ to $\mathcal{G}_{\mathsf{ledger}}$
2. Upon receiving $(\texttt{SEND}, \mathsf{ct}_{\mathsf{Signal}})$: Call $\mathcal{G}_{\text{att}}.\mathsf{resume}(eid, (\text{“send”}, \mathsf{ct}_{\mathsf{Signal}}))$. Upon receiving ORAM requests $\mathsf{data} \leftarrow \mathsf{ORAM.Access}(\cdot,\cdot,\cdot,\cdot)$: process and return data to $\mathcal{G}_{\text{att}}$.
3. Upon return of CurIdx from TEE. If $\mathsf{CurIdx} = N$, call procedure $\mathcal{G}_{\text{att}}.\mathsf{resume}(eid, (\text{“extend”}, 2N))$ and update $N \leftarrow 2N$.

Procedure RECEIVE

Recipient R_i:

1. Compute $\sigma_i = \mathsf{Enc}(\mathsf{epk}, \mathsf{Sig}(\mathsf{RID}, \mathsf{ctr}_i))$ and send $(\texttt{RECEIVE}, \sigma_i)$ to Srv. Await $((eid, \mathsf{ct}), \sigma_T)$ from Srv
2. **return** $\vec{R} \leftarrow \mathsf{Dec}(\mathsf{sk}, \mathsf{ct})$.

Srv:

1. Upon receiving $(\texttt{RECEIVE}, \sigma_i)$ from R_i, let $((eid, \mathsf{ct}), \sigma_T) = \mathcal{G}_{\text{att}}.\mathsf{resume}(eid, (\text{“receive”}, \sigma_i))$. Upon receiving ORAM requests $\mathsf{data} \leftarrow \mathsf{ORAM.Access}(\cdot,\cdot,\cdot,\cdot)$: process and return data to $\mathcal{G}_{\text{att}}$.
2. Send $((eid, \mathsf{ct}), \sigma_T)$ to R_i

Procedure ExtendDB

Srv:

1. $N \leftarrow 2N$.
2. $\mathcal{G}_{\text{att}}.\mathsf{resume}(eid, (\text{“extend”}, N))$. Upon receiving ORAM request $\mathsf{data} \leftarrow \mathsf{ORAM.Access}(\cdot,\cdot,\cdot,\cdot)$ from TEE when executing the $\mathcal{G}_{\text{att}}.\mathsf{resume}(\cdot,\cdot)$ operation: process and return data to $\mathcal{G}_{\text{att}}$
3. Free the space of the original databases DB

Procedure ExtendRecip

Srv:

1. $M \leftarrow 2M$.
2. $\mathcal{G}_{\text{att}}.\mathsf{resume}(eid, (\text{“extendrecip”}, M))$. Upon receiving ORAM request $\mathsf{data} \leftarrow \mathsf{ORAM.Access}(\cdot,\cdot,\cdot,\cdot)$ from TEE when executing the $\mathcal{G}_{\text{att}}.\mathsf{resume}(\cdot,\cdot)$ operation: process and return data to $\mathcal{G}_{\text{att}}$
3. Free the space of the original databases $\mathsf{DB}_{\mathsf{recip}}, \mathsf{DB}_{\mathsf{keys}}$

Figure 5: The protocol for private signaling Π_{TEE} in the $\mathcal{G}_{\text{att}}$ hybrid world.

- (Number of signals a recipient receives) As mentioned, this requirement may not be needed by many applications. However, by using $\bar{\ell}$, we do achieve this property.

Theorem 1. *Assume that the signature scheme Σ is existentially unforgeable under chosen message attacks, the encryption scheme* Enc *is CPA secure and key-private, and the underlying ORAM is correct and private. Then the protocol Π_{TEE} in the ($\mathcal{G}_{att}$, $\mathcal{G}_{\mathsf{ledger}}$)-hybrid world UC-realizes the $\mathcal{F}_{\mathsf{privSignal}}$ functionality.*

Proof. (Sketch) To prove UC security, we need to show that there exists a PPT simulator interacting with $\mathcal{F}_{\mathsf{privSignal}}$ that generates a transcript that is indistinguishable from the transcript generated in the real world where the adversary interacts with ($\mathcal{G}_{\mathrm{att}}$, $\mathcal{G}_{\mathsf{ledger}}$) ideal functionalities. The simulator internally simulates the $\mathcal{G}_{\mathrm{att}}$ and the $\mathcal{G}_{\mathsf{ledger}}$ functionalities to the adversary. We consider two cases of corruption here and in both cases, we show that the simulator can simulate without learning the locations of honest recipients. We briefly describe the main idea in the simulation of the aforementioned corruption cases:

- *Sender and server are corrupt*: The simulator receives a ("send", $\mathsf{ct}_{\mathsf{Signal}}$) command via the $\mathcal{G}_{\mathrm{att}}$ interface from the adversary. The simulator decrypts $\mathsf{ct}_{\mathsf{Signal}}$ using the secret key of the simulated TEE functionality esk and learns the recipient (R_i) and the location (loc). The simulator sends ($\mathtt{SEND}, R_i, \mathsf{loc}$) to the $\mathcal{F}_{\mathsf{privSignal}}$ ideal functionality on behalf of the adversary.
- *Receiver and server are corrupt*: The simulator receives a ("receive", ctr, σ) command via the $\mathcal{G}_{\mathrm{att}}$ interface from the adversary. The simulator decrypts σ and verifies the signature and sends `RECEIVE` to the $\mathcal{F}_{\mathsf{privSignal}}$ functionality on behalf of the adversary. The simulator receives a vector of locations that correspond to the adversary. The simulator encrypts these locations under the public key of the receiver and returns the vector of encryptions.

Full proofs can be found at Appendix A.2 □

Finally as noted in [11], it is desirable to not rely on just one server for private signaling. To this end we describe how we can extend our protocol to the multi-server setting in Appendix C.1.

6.1 Extensions

We discuss several extensions in Appendix C, including extending to the multi-server model, replacing some ORAM use with PIR use, and extending to a group messaging setting. Since all the extensions are very natural, we include them in the Appendix.

7 Implementation and Evaluation

This section describes the implementation details of our prototype (proof-of-concept) and its evaluation. All our source code is publicly available at [14].

7.1 Implementation details

We use Intel SGX [26] to instantiate the $\mathcal{G}_{\mathrm{att}}$ functionality of Π_{TEE}, and we implement the program Prog running inside the SGX enclave using Intel SGX Linux SDK [41]. Currently, our prototype implements "initialization", "setup", "send", and "receive" commands of the protocol.

For encryption and authentication, we use OpenSSL [42] on the recipient side and SGX SSL [43] on the enclave side. In the prototype, the recipient uses RSA-OAEP [44] to submit encrypted send/receive requests to the enclave and the enclave uses Elliptic Curve Integrated Encryption Scheme (ECIES) to respond with encrypted replies to the recipient. Using RSA for encrypting responses to the recipients is not storage efficient because it requires storing longer keys (527 byte-keys using $2048 - bit$ RSA modulus) for every recipient. Instead, we use shorter EC keys (65 byte-keys using Prime256v1 curve) for encrypting responses toward the recipients. This reduces the storage required per recipient by an

	Baseline	Π_{TEE}	PS1 [15]	PS1,N [11]	PS2[11]	OMR [12, 19]	FMD [5]
Server computation (per signal per recipient)	NA	0.0077 ms	60 ms	0.228 ms	292 ms	21 ms	0.02 ms
Recipient computation	400 s	< 0.1 ms	< 0.1 ms	< 0.1 ms	< 0.1 ms	5 ms	3.13 s
Signal size	NA	300 bytes	300 bytes	300 bytes	600 bytes	956 bytes	68 bytes
Security	Full	Full	No recipient unlinkability	No recipient unlinkability	No recipient unlinkability	Full	pN-msg-anonymity $p = 2^{-5}$
Environment Assumption	NA	TEE	TEE	TEE	Non-colluding servers	NA	NA

Table 3: Performance and security comparisons between prior work PS1, PS1,N, PS2, FMD, OMR, and our protocol Π_{TEE}, assuming a recipient connect once a day. These results are measured for parameters: $N = 500,000$, $M = 500$, and an upper bound of the number of signals retrieved per recipient $\bar{\ell} = 50$. Runtime is calculated using: (one `SEND` time / M recipients) + (one `RECEIVE` time / N signals).

order of magnitude (given each recipient requires two keys, pk and $\Sigma.\mathsf{vk}$). For authentication, the prototype uses ECDSA signature scheme with $\mathsf{Prime256v1}$ curve.

For ORAM, the prototype uses PathORAM [17] to instantiate two ORAMs in the enclave (one for messages DB and the other for recipient information including DBkeys and $\mathsf{DBrecip}$), where the actual storage is implemented outside the enclave, within the server's untrusted memory. Note that PathORAM has a runtime of $O(\log^2(N))$ for a database with size N. We choose PathORAM instead of the asymptotically better [18], since concretely, PathORAM is much faster. All ORAM storage in the server's untrusted memory is encrypted using AES GCM $128-bit$ encryption scheme with a key only known to the enclave. The enclave samples different keys for each ORAM at instantiation. Currently, the prototype uses the server's main memory (RAM) to store the ORAM database, however, we envision that, in practical deployments, the database must be implemented using a database management system such as SQLite, etc.

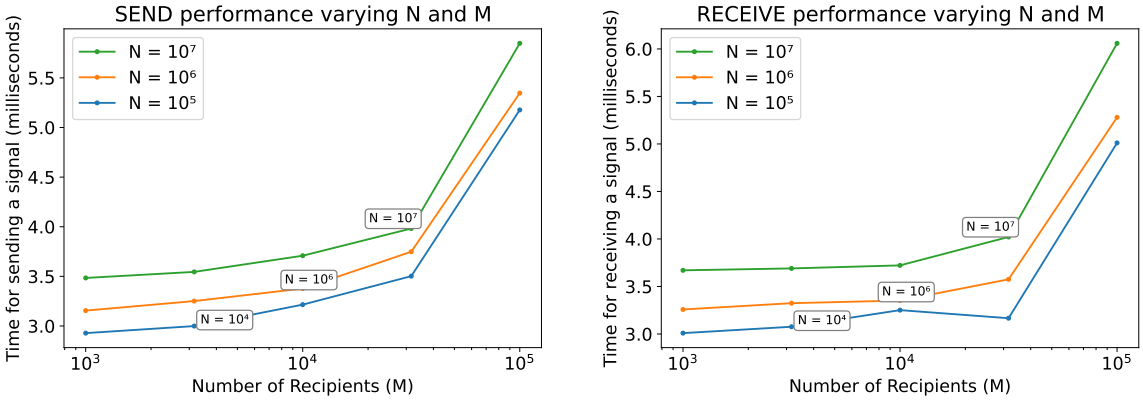
For benchmarking, we deploy the prototype on Azure DC2s v2 (Confidential Compute, 3.7 GHz Intel® Xeon E-2288G Coffee Lake) instance with 2 vCPUs and 8GB RAM. Our prototype runs on Ubuntu 20.04 LTS OS and in total has ≈ 3250 lines of C++ code (≈ 1750 for the enclave and ≈ 1250 for the recipient application).

7.2 Evaluation

Server Computation. Figures 6a and 6b illustrate the server runtime for to process a "send" request and to process a "receive" request while varying the number of recipients M and number of messages N (as a multiplier of M), with $\bar{\ell} = 1$, implying only one message is retrieved. The plots represent enclave processing time after being invoked by the server during procedure `SEND` and `RECEIVE` respectively. This enclave processing time includes one RSA decryption, three ORAM accesses, and one ECDSA signature computation; additionally an ECDSA verification during `RECEIVE`. As shown in the figures, this server runtime for both procedures increases poly-logarithmically as a function of N and M due to ORAM accesses. Concretely, Π_{TEE} takes at most $\sim$6 ms for processing one signal (during both procedures) for 100 thousand recipients and 10 million messages.

Figure 7 depicts the server runtime while varying the number of messages retrieved $\bar{\ell}$. As $\bar{\ell}$ increases, the time taken to process `RECEIVE` increases linearly, while the time for `SEND` is constant, as expected. This is because, for every message retrieved from the message database DB, the enclave must perform one ORAM Read. Compared to `SEND`, `RECEIVE` performs better when there are multiple signals to be processed, which is due to the fact that during `RECEIVE` the enclave can retrieve all of the signals within the same request, unlike during `SEND` where it has to process them one at a time. That said, the baseline performance for both procedures when $\bar{\ell} = 1$ is almost the same. Concretely, Π_{TEE} takes less than 0.5s to receive 100 messages when there are 1 million recipients and 10 million messages in the database.

Recipient Computation. The recipient is only required to perform encryption, decryption, signa-



(a) The average time taken to execute "send" command of the Π_{TEE}.

(b) The average time taken to execute "receive" command of the Π_{TEE}.

Figure 6: Here Y-axis represents the time taken in milliseconds, while the X-axis represents the number of recipients M in log-10 scale. Each line plot depicts the number of signals N as a multiplier of M. $\bar{\ell} = 1$.

ture generation, and verification. In our experiments, these operations took less than 1 ms during `RECEIVE` procedures, even when processing as many as 100 signals during `RECEIVE`. This low latency for the recipient during `RECEIVE` is a consequence of using ECIES encryption instead of RSA.

7.3 Storage and Memory Required

On the server side, the storage required to store ORAM database scales linearly with M and N. In the prototype, we assume the index (index), the counter (ctr), and the location of the message (loc) as 4 byte integers and the recipient EC keys are 65 bytes (as mentioned earlier). Hence, we require a block size of about $3*4+2*65 = 142$ bytes, plus $12+16 = 28$ bytes for AES-GCM encryption metadata (12 bytes for initialization vector and 16 bytes for tag). With 170 byte blocks, message database DB and recipient database (DBkeys and DBrecip) in the untrusted storage require $O(170(N+M))$ space. In our experiments, for 10 million messages and 1 million recipients, the server used 1.8 GB of storage. However, this can be significantly reduced if different block sizes are used for each database because the DB database requires only 36 byte blocks (8 bytes of index and loc, plus 28 bytes of encryption metadata).

Furthermore, the blocks should ideally include a monotonic counter for integrity purposes against malicious servers. Given the SGX threat model, a malicious server can tamper with the blocks by corrupting them, modifying their order, or replaying old blocks. Our prototype uses AES-GCM, which is an authenticated encryption scheme that prevents corruption attacks. For mitigating order misalignment attacks, the enclave embeds the bucket index in every bucket it writes to the untrusted storage. However, for integrity against replay attacks, the enclave must use hardware monotonic counters (which we currently do not support in our prototype).

Enclave Memory. For each ORAM, the enclave stores a position map (an array that maps a block to its position in ORAM tree) and a stash in its trusted memory. Position map is an integer array of length N and M; and stash, in its worst case complexity, requires $O(log(N))$ and $O(log(M))$ number of blocks, for message and recipient database respectively. Note that we do not implement the recursive position map to reduce the enclave memory, as we believe the current usage is already small. Therefore, the enclave trusted memory in total requires $O((N+log(N))+(M+log(M))+k)$, where

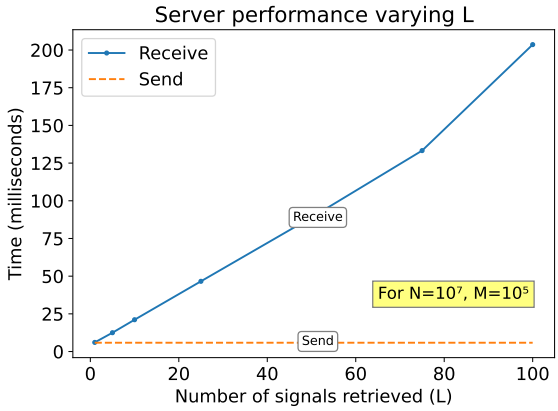


Figure 7: This plot measures Π_{TEE} server's performance with respect to the number of signals retrieved, while keeping its ORAM capacity constant, i.e., $N = 10,000,000$ and $M = 100,000$. Here Y-axis represents the time taken in milliseconds and X-axis represents the number of signals retrieved $\bar{\ell}$.

k is the stack and the enclave binary size (which is constant). In our experiments, for 10 million messages and 100 thousand recipients, the enclave used 41 MB of heap memory to store position maps and stashes for two ORAMs.

Enclave communication The enclave issues ORAM Read and Write. Both accesses require reading/writing an ORAM tree path consisting of $O(Zlog(N))$ and $O(Zlog(M))$ number of blocks for messages and recipients database respectively, where Z is the number of blocks in each bucket, a parameter of the underlying ORAM. In our experiments, using $Z = 4$ and 170 byte blocks, for 10 million messages and 100 thousand recipients, each access reads/writes 15 KB and 11 KB of data respectively.

7.4 Bandwidth

There are two types of bandwidth: (a) for communication between the recipients/blockchain and the server, and (b) for communication between the server and the enclave.

For (a), where the recipient frequently uses two procedures `SEND`, and `RECEIVE`. `SEND` requires 8 bytes for RID and loc, while `RECEIVE` requires 88 bytes for RID and ctr plus an ECDSA signature (of 80 bytes) for authenticating themself to the enclave. Additionally, in these procedures, the recipient encrypts its data using RSA encryption which adds around 292 bytes of metadata making the final ciphertext around 300 and 380 bytes respectively. Therefore, the recipient needs at least 380 bytes of bandwidth for upload. Whereas for download, it is dependent on the number of messages being retrieved during the `RECEIVE` procedure, which is $O(4L)$ plus an ECDSA signature to authenticate that the reply is from the enclave.

For (b), where the enclave issues ORAM Read and Write. Both accesses require reading/writing an ORAM tree path consisting of $O(Zlog(N))$ and $O(Zlog(M))$ number of blocks for messages and recipients database respectively. In our experiments, using $Z = 4$ and 170 byte blocks, for 10 million messages and 1 million recipients, each access reads/writes 15 KB and 13.6 KB of data respectively.

7.5 Comparison with related work

We compare our protocols with the plain baseline solution and the state-of-the-art related work: PS1, PS1,N, PS2, FMD, and OMR in table Table 3. By baseline solution, we mean: to simply use trial decryption from [45] to decrypt all the signals on the board for finding relevant ones. Hence, for

this solution, there are no servers, and thus, there is only recipient computation. For the prior work, we took the results directly from PS, OMR, and FMD papers. For PS1, we generously estimate the results for parameters $\bar{\ell} = 20$, $M = 100$. Note that signal size is based on RSA encryption. From the table, we observe that our results have significantly lower server runtime per signal per recipient, compared to any other scheme. Moreover, all the other schemes scale linearly in N and M, while our scheme scales sublinearly.

Security-wise, we provide equal guarantees as the baseline scheme and OMR scheme, assuming TEE is trusted. Thus, from the table, it is clear that our scheme performs significantly better than the state-of-the-art even for small parameters. And for larger parameters, such as $N = 10^7$ and $M = 10^5$, it is evident from the figures above that only our scheme can support such scalability.

8 Conclusion

In this work, we have presented a scalable private signaling protocol that is asymptotically superior and concretely orders of magnitude faster compared to prior works. More specifically, to process N sent signals and M retrievals of each retrieving $\bar{\ell}$ signals, the cost of the TEE is $\tilde{O}(N + M\ell)$, which is almost optimal except for some logarithmic factor. Moreover, our construction also enjoys the property of retrieval privacy which was not considered in previous works.

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A Proof

A.1 Universal Composability

In UC security we consider the execution of the protocol in a special setting involving an environment machine $\mathcal{Z}$, in addition to the honest parties and adversary. In UC, ideal and real models are considered where a trusted party carries out the computation in the ideal model while the actual protocol runs in the real model. The trusted party is also called the ideal functionality. For example the ideal functionality $\mathcal{F}_{\mathsf{privSignal}}$ is a trusted party that provides the functionality of private signaling. In the UC setting, there is a global environment (the distinguisher) that chooses the inputs for the honest parties, and interacts with an adversary who is the party that participates in the protocol on behalf of dishonest parties. At the end of the protocol execution, the environment receives the output of the honest parties as well as the output of the adversary which one can assume to contain the entire transcript of the protocol. When the environment activates the honest parties and the adversary, it does not know whether the parties and the adversary are running the real protocol –they are in the real world, or they are simply interacting with the trusted ideal functionality, in which case the adversary is not interacting with any honest party, but is simply "simulating" to engage in the protocol. In the ideal world the adversary is therefore called simulator, that we denote by $\mathcal{S}$.

In the UC-setting, we say that a protocol securely realizes an ideal functionality, if there exist no environment that can distinguish whether the output he received comes from a real execution of the

protocol between the honest parties and a real adversary $\mathcal{A}$, or from a simulated execution of the protocol produced by the simulator, where the honest parties only forward date to and from the ideal functionality.

The transcript of the ideal world execution is denoted $\mathsf{IDEAL}_{\mathcal{F},\mathcal{S},\mathcal{Z}}(\lambda, z)$ and the transcript of the real world execution is denoted $_{\Pi,\mathcal{A},\mathcal{Z}}(\lambda, z)$. A protocol is secure if the ideal world transcript and the real world transcripts are indistinguishable. That is, $\{\mathsf{IDEAL}_{\mathcal{F},\mathcal{S},\mathcal{Z}}(\lambda, z\}_{\lambda\in\mathbb{N},z\in\{0,1\}^*} \equiv \{_{\Pi,\mathcal{A},\mathcal{Z}}(\lambda, z)\}_{\lambda\in\mathbb{N},z\in\{0,1\}^*}$.

A.2 Proof of Security of Theorem 1

Theorem. *Assume that the signature scheme Σ is existentially unforgeable under chosen message attacks, the encryption scheme* Enc *is CPA secure and key-private, and the underlying ORAM is correct and private. Then the protocol Π_{TEE} in the ($\mathcal{G}_{att}$, $\mathcal{G}_{\mathsf{ledger}}$)-hybrid world UC-realizes the $\mathcal{F}_{\mathsf{privSignal}}$ functionality.*

Proof. To prove that Π_{TEE} UC-realizes $\mathcal{F}_{\mathsf{privSignal}}$ we show that there exists a simulator $\mathcal{S}$ that interacts with $\mathcal{F}_{\mathsf{privSignal}}$ and the adversary $\mathcal{A}$ to generate a transcript that is indistinguishable from the real world protocol. We consider the following cases of corruption:

- Simulator $\mathcal{S}_N$ for the case when only the server is corrupt.
- Simulator $\mathcal{S}_s$ for the case when a subset of the senders and the server are corrupt.
- Simulator $\mathcal{S}_r$ for the case when a subset of the recipients and the server are corrupt.
- Simulator $\mathcal{S}_{rs}$ for the case when a subset of the recipients, a subset of the senders and the server is corrupt.

We discuss these simulators in more detail in the next subsections. □

A.2.1 Case 1: Neither S nor R is corrupt and only $\mathcal{S}_N$ is corrupt

Simulator overview When neither the sender nor the recipients are corrupt, then the only corrupt entity is the server. In this case, the simulator interacts with the Srv and the $\mathcal{F}_{\mathsf{privSignal}}$ to simulate a transcript that is indistinguishable from the real world. Note that the simulator also simulates $\mathcal{G}_{\mathrm{att}}$ and $\mathcal{G}_{\mathsf{ledger}}$ towards $\mathcal{A}$.

Proof by hybrids We prove security via a sequence of hybrids where we start from the real world and move to the ideal world.

- $\mathbf{Hyb}_0$ The real world protocol.
- $\mathbf{Hyb}_1$ is the same as $\mathbf{Hyb}_0$ except that upon receiving a `SEND` command, the $\mathsf{ct}_{\mathsf{Signal}}$ is replaced with encryption to 0 instead of the actual location. By the CPA security of the underlying encryption scheme the two hybrids are indistinguishable.
- $\mathbf{Hyb}_2$ is the same as $\mathbf{Hyb}_1$ except that all the ORAM calls are now replaced by dummy ORAM calls as in the simulation. By the computational privacy property of the ORAM scheme, the two hybrids are indistinguishable.
- $\mathbf{Hyb}_3$ is the same as $\mathbf{Hyb}_2$ except that in the `RECEIVE` command, the simulator samples a new random public key pk^* and encrypts the locations under this public key. By the key-privacy property of the underlying encryption scheme, these two hybrids are indistinguishable.
- $\mathbf{Hyb}_4$ is the same as $\mathbf{Hyb}_5$ except that in the `RECEIVE` command, the simulator returns encryptions of 0 as $\mathsf{ct}_{\mathsf{loc},i}$ to the server on behalf of $\mathcal{G}_{\mathrm{att}}$. By the CPA security of the underlying encryption scheme, the two hybrids are indistinguishable.
- $\mathbf{Hyb}_5$ is the same as $\mathbf{Hyb}_4$ except that in the **Setup** procedure, the simulator aborts with $\mathsf{sigFailure}_1$. Since we use EUF-CMA signatures, this occurs with negligible probability.
- $\mathbf{Hyb}_6$ is the same as $\mathbf{Hyb}_5$ except that in the `RECEIVE` command, the simulator may abort with $\mathsf{sigFailure}_2$. This occurs with negligible probability since we use EUC-CMA signatures.

Note that $\mathbf{Hyb}_6$ is exactly the same as the ideal world. Therefore through a sequence of hybrids we have proved that the real and the ideal worlds are indistinguishable.

The simulator $\mathcal{S}$ internally simulates $\mathcal{G}_{\text{att}}$ and $\mathcal{G}_{\mathsf{ledger}}$ towards the adversary $\mathcal{A}$

Registration For each recipient R_i:

1. Compute $(\mathsf{pk}_i, \mathsf{sk}_i) \leftarrow \mathsf{Enc.KeyGen}(1^\lambda)$, $(\Sigma.\mathsf{sk}_i, \Sigma.\mathsf{vk}_i) \leftarrow \Sigma.\mathsf{KeyGen}(1^\lambda)$.
2. Set $\mathsf{ct}_{\mathsf{keys},i} = \mathsf{Enc}(\mathsf{epk}_i, (\mathsf{pk}_i, \Sigma.\mathsf{vk}_i))$ and send $(\text{“setup”}, \mathsf{ct}_{\mathsf{keys},i})$ to $\mathcal{A}$ (on behalf of Srv).
3. Upon receiving $(\text{“setup”}, \mathsf{ct}_{\mathsf{keys}})$ on behalf of $\mathcal{G}_{\text{att}}$, send $\mathsf{ORAM.Write}(\mathsf{DBkeys}, \cdot, \cdot, \cdot)$ and $\mathsf{ORAM.Write}(\mathsf{DBrecip}, \cdot, \cdot, \cdot)$ on behalf of $\mathcal{G}_{\text{att}}$ to $\mathcal{A}$.
4. Upon receiving $((eid, \mathsf{pk}_i, \mathsf{RID}), \sigma)$ abort with $\mathsf{sigFailure}_1$ if σ would be validated but the following communication was not recorded: $((\mathsf{pk}_i), \sigma) = \mathcal{G}_{\text{att}}.\mathsf{resume}(eid, (\text{“setup”}, \mathsf{ct}_{\mathsf{keys},i})$.
5. Else publish pk_i and set $\mathsf{index}_i = 0, \mathsf{ctr}_i = 0$ and $\vec{L} = \{0\}_{j=0}^{\bar{\ell}}$.

`SEND`: Upon receiving $(\texttt{SEND})$ from $\mathcal{F}_{\mathsf{privSignal}}$

1. Compute $\mathsf{ct}_{\mathsf{Signal}} = \mathsf{Enc}(\mathsf{epk}, 0)$ and send $(\texttt{SUBMIT}, (\texttt{SEND}, \mathsf{ct}_{\mathsf{Signal}}))$ to $\mathcal{G}_{\mathsf{ledger}}$.
2. Upon receiving $(\text{“send”}, \mathsf{ct}_{\mathsf{Signal}})$ on behalf of $\mathcal{G}_{\text{att}}$ from $\mathcal{A}$:
   (a) Send $\mathsf{ORAM.Read}(\mathsf{DBrecip}, \cdot, \cdot)$ to $\mathcal{A}$
   (b) Send $\mathsf{ORAM.Write}(\mathsf{DBrecip}, \cdot, \cdot, \cdot)$ to $\mathcal{A}$
   (c) Send $\mathsf{ORAM.Write}(\mathsf{DB}, \cdot, \cdot, \cdot)$ to $\mathcal{A}$
3. Send $(\texttt{SEND}, \texttt{ok})$ to $\mathcal{F}_{\mathsf{privSignal}}$.

`RECEIVE`: Upon receiving $(\texttt{RECEIVE})$ from $\mathcal{F}_{\mathsf{privSignal}}$

1. Compute $\sigma_i = \mathsf{Enc}(\mathsf{epk}, 0)$ and send $(\texttt{RECEIVE}, \sigma_i)$ to $\mathcal{A}$.
2. Upon receiving $\mathcal{G}_{\text{att}}.\mathsf{resume}(eid, (\text{“receive”}, \sigma_i))$ on behalf of $\mathcal{G}_{\text{att}}$:
   (a) Send $\mathsf{ORAM.Read}(\mathsf{DBkeys}, \cdot, \cdot)$ to $\mathcal{A}$
   (b) Send $\mathsf{ORAM.Read}(\mathsf{DBrecip}, \cdot, \cdot)$
   (c) For $k \in [\bar{\ell}]$: Send $\mathsf{ORAM.Read}(\mathsf{DBrecip}, \cdot, \cdot, \cdot)$ to $\mathcal{A}$
   (d) Send $\mathsf{ORAM.Write}(\mathsf{DBrecip}, \cdot, \cdot)$ to $\mathcal{A}$.
   (e) Sample a random pk^* and compute $\vec{\mathsf{ct}}_{\mathsf{loc},i} = \{\mathsf{Enc}(\mathsf{pk}^*, 0)\}_{k \in [\bar{\ell}]}$
   (f) Compute $\sigma_T = \mathsf{Sig}(\mathsf{msk}, (eid, \vec{\mathsf{ct}}_{\mathsf{loc},i}))$
   (g) Return $(\vec{\mathsf{ct}}_{\mathsf{loc},i}, \sigma_T)$ to $\mathcal{A}$
3. Receive $(eid, \vec{\mathsf{ct}}_{\mathsf{loc},i}, \sigma_T)$ from $\mathcal{A}$. If this was received without the communication with $\mathcal{G}_{\text{att}}$, abort with $\mathsf{sigFailure}_2$.
4. Else send $(\texttt{RECEIVE}, \texttt{ok})$ to $\mathcal{F}_{\mathsf{privSignal}}$.

`READ`: Upon receiving $(\texttt{READ})$ from $\mathcal{A}$, forward $(\texttt{READ})$ to $\mathcal{G}_{\mathsf{ledger}}$ and return whatever is returned.

`ExtendDB`: Upon receiving `ExtendDB` from $\mathcal{A}$, initialize a database DB', repeat N times: $\mathsf{ORAM.Write}(\mathsf{DB}', \cdot, \cdot)$, set $\mathsf{DB} \leftarrow \mathsf{DB}'$ and simply set $\mathsf{pp}.N$ as the received N.

`ExtendRecip`: Upon receiving `ExtendRecip` from $\mathcal{A}$, initialize empty databases $\mathsf{DBrecip}'$ and DBkeys' and call $\mathsf{ORAM.Write}(\mathsf{DBrecip}', \cdot, \cdot)$ and $\mathsf{ORAM.Write}(\mathsf{DBkeys}', \cdot, \cdot)$, N times.

Figure 8: Simulator $\mathcal{S}_N$ for the case of only one corrupt server

The simulator $\mathcal{S}$ internally simulates $\mathcal{G}_{\text{att}}$ towards the adversary $\mathcal{A}$

Setup For each recipient R_i: Same as in Fig 8

`SEND`: Upon receiving $(\texttt{SEND}, S_i)$ from $\mathcal{F}_{\mathsf{privSignal}}$, same as in Fig 8. `//(Honest send)`
Upon receiving $\mathcal{G}_{\text{att}}.\mathsf{resume}(eid_j, (\text{“send”}, \mathsf{ct}_{\mathsf{Signal}}))$ on behalf of $\mathcal{G}_{\text{att}}$ for a $\mathsf{ct}_{\mathsf{Signal}}$ that was not created by the simulator. `//(Malicious send)`

Let $(\mathsf{RID}, \mathsf{loc}) = \mathsf{Dec}(\mathsf{esk}, \mathsf{ct}_{\mathsf{Signal}})$.
if RID corresponds to some R_j **then**
Send $(\texttt{SEND}, R_j, \mathsf{loc})$ to $\mathcal{F}_{\mathsf{privSignal}}$ on behalf of $\mathcal{A}$ and receive $(\texttt{SEND})$ and send back $(\texttt{SEND}, \texttt{ok})$.
Send $\mathsf{data} \leftarrow \mathsf{ORAM.Read}(\mathsf{DBrecip}, \mathsf{RID}, \cdot)$ to $\mathcal{A}$
Send $\cdot \leftarrow \mathsf{ORAM.Write}(\mathsf{DBrecip}, \cdot, \cdot))$ to $\mathcal{A}$
Send $\cdot \leftarrow \mathsf{ORAM.Write}(\mathsf{DB}, \cdot, \cdot))$ to $\mathcal{A}$

`RECEIVE` Same as in Fig 8

`SUBMIT` : Upon receiving a `SUBMIT` request from $\mathcal{A}$, forward to $\mathcal{G}_{\mathsf{ledger}}$. `//(Malicious write to` $\mathcal{G}_{\mathsf{ledger}}$`)`

`READ` Same as in Fig 8

Figure 9: Simulator $\mathcal{S}_s$ for the case corrupt server and sender

A.2.2 Case 2: S and Srv are corrupt

Simulator Overview In this case a subset of the senders are corrupt along with the server. To prove security we need to construct a simulator that interacts with the adversary and the $\mathcal{F}_{\mathsf{privSignal}}$ functionality that is indistinguishable from the real world.

Proof by hybrids Through hybrid arguments we move from the real world to the ideal world and prove that each pair of intermediate hybrids are indistinguishable.

- $\mathbf{Hyb}_0$ is the real world.
- $\mathbf{Hyb}_1$ is the same as $\mathbf{Hyb}_0$ except that for an honest sender, encryptions of 0 are sent instead of the actual locations in the `SEND` command. By the CPA security of the underlying encryption scheme, $\mathbf{Hyb}_1$ and $\mathbf{Hyb}_0$ are indistinguishable.
- $\mathbf{Hyb}_2$ is the same as $\mathbf{Hyb}_1$ except that all the ORAM calls are now replaced by dummy ORAM calls as in the simulation. By the computational privacy property of the ORAM scheme, the two hybrids are indistinguishable.
- $\mathbf{Hyb}_3$ is the same as $\mathbf{Hyb}_2$ except that for `RECEIVE` command, the simulator returns encryptions of 0 to the server as $\mathcal{G}_{\text{att}}$. By the CPA security of the underlying encryption scheme, the two hybrids are indistinguishable.
- $\mathbf{Hyb}_4$ is the same as $\mathbf{Hyb}_3$ except that the “setup” of **Setup** procedure is done as in the simulation and the simulator might abort with $\mathsf{sigFailure}_1$. By the EUF-CMA property of the signature scheme, the two hybrids are indistinguishable.
- $\mathbf{Hyb}_5$ is the same as $\mathbf{Hyb}_4$ except that the `RECEIVE` is done as in the simulation and the simulator might abort with $\mathsf{sigFailure}_2$. By the EUF-CMA property of the signature scheme, the two hybrids are indistinguishable.

The simulator $\mathcal{S}$ internally simulates $\mathcal{G}_{\text{att}}$ towards the adversary $\mathcal{A}$

Setup For each honest recipient R_i: Same as in Fig 8
//(`Malicious receiver`):
Upon receiving $\mathcal{G}_{\text{att}}.\text{resume}(eid, (\text{“setup”}, \mathsf{ct}_{\mathsf{keys},i})$ from Srv on behalf of $\mathcal{G}_{\text{att}}$:
1. Compute $(\mathsf{pk}_i, \Sigma.\mathsf{vk}_i) = \mathsf{Dec}(\mathsf{esk}, \mathsf{ct}_{\mathsf{keys},i})$ and compute pk_i from sk_i.
2. Set $\vec{L} = \{0\}_{j=0}^{\bar{\ell}}$
3. Set $\mathsf{index} = 0$ and $\mathsf{ctr} = 0$
4. Compute $\sigma = \Sigma.\mathsf{Sig}(\mathsf{msk}, (eid, (\mathsf{pk})))$ and send (pk, σ) to $\mathcal{A}$ and store $(\mathsf{pk}_i, \Sigma.\mathsf{vk}_i, \mathsf{index}_i, \mathsf{ctr}_i)$.

`SEND`: Same as in Fig 8

`RECEIVE` For honest recipients, same as in Fig 8.
// `(For malicious recipients)`
1. Receive $\mathcal{G}_{\text{att}}.\text{resume}(eid, (\text{“receive”}, \mathsf{ctr}_i, \sigma_i))$ from Srv on behalf of $\mathcal{G}_{\text{att}}$. If $\mathsf{Sig.Ver}(\Sigma.\mathsf{vk}_i, \mathsf{ctr}_i, \sigma_i) = 0$, return $\perp$. Else if i corresponds to that of an honest recipient, abort with $\mathsf{sigFailure}$. Else:
2. Send $(\texttt{RECEIVE}, R_i)$ to $\mathcal{F}_{\mathsf{privSignal}}$ on behalf of R_i and get back $(\texttt{RECEIVE})$ from $\mathcal{F}_{\mathsf{privSignal}}$. Send $(\texttt{RECEIVE}, \mathsf{ok})$ to $\mathcal{F}_{\mathsf{privSignal}}$ and get back $\vec{R} = [\mathsf{loc}_1 \dots \mathsf{loc}_k]$.
3. Send $\mathsf{ORAM.Read}(\mathsf{DBrecip}, \mathsf{RID}, \cdot)$ to the adversary.
4. Send $\bar{\ell}$ number of calls of $\mathsf{ORAM.Read}(\mathsf{DB}, \mathsf{RID}, \cdot)$ to the server.
5. If $k > \bar{\ell}$: append -2 to $\vec{R}$, else if $k = \bar{\ell}$, append -1 to $\vec{R}$, else for $j \in [k+1, \bar{\ell}]$, write 0 to $\vec{R}[j]$.
6. Run $\cdot \leftarrow \mathsf{ORAM.Write}(\mathsf{DBkeys}, \mathsf{RID}, (\Sigma.\mathsf{vk}, \mathsf{pk}, \mathsf{ctr}_{\mathsf{RID}}))$
7. Compute $\mathsf{ct}_{\mathsf{loc},i} = (\mathsf{Enc}(\mathsf{pk}_i, \mathsf{loc}_1) \dots \mathsf{Enc}(\mathsf{pk}_i, \mathsf{loc}_{\bar{\ell}}))$
8. Compute $\sigma_T = \Sigma.\mathsf{Sig}(\mathsf{msk}, (\mathsf{ct}_{\mathsf{loc},i}))$ and send $(\mathsf{ct}_{\mathsf{loc},i}, \sigma_T)$ to $\mathcal{A}$.

`READ` Same as in Fig 8

Figure 10: Simulator $\mathcal{S}_r$ for the case corrupt server and recipients

A.2.3 Case 3: Srv and R are corrupt

Simulator overview In this case, the simulator needs to simulate a view towards a malicious recipient that is indistinguishable from the real world interaction. For malicious recipients, the simulator is notified of their `RECEIVE` request since the Srv must invoke the $\mathcal{G}_{\text{att}}$ functionality to get . Next the simulator needs to to send the correct locations to the recipient, even though the simulated $\vec{L}$ are just an encryption of 0s. To this end, the simulator simply sends the `RECEIVE` command to the $\mathcal{F}_{\mathsf{privSignal}}$ functionality and learns the locations that correspond to the malicious recipient. It then simulates the $\mathcal{G}_{\text{att}}$ functionality to compute an encryption of the locations and sends it back to the Srv.

Proof by hybrids
- $\mathbf{Hyb}_0$ The real world protocol.
- $\mathbf{Hyb}_1$ is the same as $\mathbf{Hyb}_0$ except that upon receiving a `SEND` command, the $\mathsf{ct}_{\mathsf{Signal}}$ is replaced with an encryption to 0 instead of the actual location. By the CPA security of the underlying encryption scheme the two hybrids are indistinguishable.
- $\mathbf{Hyb}_2$ is the same as $\mathbf{Hyb}_1$ except that all the ORAM calls are now replaced by dummy ORAM calls as in the simulation. By the computational privacy property of the ORAM scheme, the two hybrids are indistinguishable.
- $\mathbf{Hyb}_3$ is the same as $\mathbf{Hyb}_2$ except that in the `RECEIVE` command for an honest recipient, the simulator returns encryptions of 0 as $\mathsf{ct}_{\mathsf{loc},i}$ to the server on behalf of $\mathcal{G}_{\text{att}}$. By the CPA security of the underlying encryption scheme, the two hybrids are indistinguishable.

The simulator $\mathcal{S}$ maintains a public **board** and internally simulates $\mathcal{G}_{\text{att}}$ towards the adversary $\mathcal{A}$

Setup
For each recipient R_i: Same as in Fig 10

`SUBMIT` : Same as in Fig 9

`SEND`: Same as in Fig 9

`RECEIVE` Same as in Fig 10

`READ` Same as in Fig 8

Figure 11: Simulator $\mathcal{S}_{sr}$ for the case corrupt server, senders and recipients

- $\mathbf{Hyb}_4$ is the same as $\mathbf{Hyb}_3$, except that for malicious recipients, the simulator may abort with sigFailure. Since we assume EUF-CMA signatures these two hybrids are indistinguishable.
- $\mathbf{Hyb}_5$ is the same as $\mathbf{Hyb}_4$ except that in the **Setup** procedure, the simulator aborts with $\mathsf{sigFailure}_1$. Because of EUF-CMA signatures this occurs with negl probability.
- $\mathbf{Hyb}_6$ is the same as $\mathbf{Hyb}_5$ except that in the `RECEIVE` command, the simulator may abort with $\mathsf{sigFailure}_2$. Because of EUF-CMA signatures this occurs with negl probability.

A.2.4 Case 4: Corrupt Srv, S and R

Simulator overview This simulator is a combination of the previous simulators, where the simulator simulates the `SEND` command as in the case when the Srv and the sender S are corrupt, for the **Setup** and `RECEIVE` commands the simulator simulates as in the case when the Srv and the recipient R are corrupt.

Proof by hybrids
- $\mathbf{Hyb}_0$ is the real world.
- $\mathbf{Hyb}_1$ is the same as $\mathbf{Hyb}_0$ except that for an honest sender, encryptions of 0 are sent instead of the actual locations in the `SEND` command. By the CPA security of the underlying encryption scheme, $\mathbf{Hyb}_1$ and $\mathbf{Hyb}_0$ are indistinguishable.
- $\mathbf{Hyb}_2$ is the same as $\mathbf{Hyb}_1$ except that all the ORAM calls are now replaced by dummy ORAM calls as in the simulation. By the computational privacy property of the ORAM scheme, the two hybrids are indistinguishable.
- $\mathbf{Hyb}_3$ is the same as $\mathbf{Hyb}_2$ except that for `RECEIVE` command, the simulator returns encryptions of 0 to the server as $\mathcal{G}_{\text{att}}$. By the CPA security of the underlying encryption scheme, the two hybrids are indistinguishable.
- $\mathbf{Hyb}_4$ is the same as $\mathbf{Hyb}_3$, except that for malicious recipients, the simulator may abort with sigFailure. Since we assume EUF-CMA signatures these two hybrids are indistinguishable.
- $\mathbf{Hyb}_5$ is the same as $\mathbf{Hyb}_4$ except that the "setup" of **Setup** procedure is done as in the simulation and the simulator might abort with $\mathsf{sigFailure}_1$. By the EUF-CMA property of the signature scheme, the two hybrids are indistinguishable.
- $\mathbf{Hyb}_6$ is the same as $\mathbf{Hyb}_5$ except that the `RECEIVE` is done as in the simulation and the simulator might abort with $\mathsf{sigFailure}_2$. By the EUF-CMA property of the signature scheme, the two hybrids are indistinguishable.

- Upon receiving $(\texttt{SUBMIT}, \mathsf{tx})$ from a party P_i:
  1. Choose a unique transaction ID txid and set $\texttt{BTX} := (\mathsf{tx}, \text{txid}, \tau_L, P_i)$
  2. If $\mathsf{Validate}(\texttt{BTX}, \texttt{state}, \texttt{buffer}) = 1$ then $\texttt{buffer} := \texttt{buffer} \cup \texttt{BTX}$
  3. Send $(\texttt{SUBMIT}, \texttt{BTX})$ to $\mathcal{A}$.
- Upon receiving READ from a party P_i, send $\texttt{state}_{P_i}$ to P_i. If received from $\mathcal{A}$, send $(\texttt{state}, \texttt{buffer})$ to $\mathcal{A}$.

Figure 12: Abridged $\mathcal{G}_{\mathsf{ledger}}$ functionality

B Abridged Ledger functionality

In our protocols we model the public board for reads and writes in the form of a $\mathcal{G}_{\mathsf{ledger}}$ ideal functionality presented here. We present an abridged version of the functionality where we present the READ and SUBMIT commands in Fig. 12. For the complete description of the functionality, we refer the reader to Pages 339-340 of [23].

C Extensions

C.1 Extension to multiple servers

In real-world applications, especially in applications like cryptocurrencies, it may be desirable to have multiple servers serving the parties.

To achieve full privacy in the multi-server model, we need two additional properties for our PKE scheme used by the TEE: *wrong-key decryption detection* and *key-private encryption*. At a high level, wrong-key decryption detection basically means that when decrypting a message with the wrong key, TEE can tell that the key used is incorrect and can thus behave accordingly.

We formally capture the property with the following definition.

Definition 3 (Wrong-key Decryption Detection)**.** *A PKE scheme* $(\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ *is wrong-key-decryption-detectable if any* $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}$ *and* $\mathsf{pk}', \mathsf{sk}' \leftarrow \mathsf{KeyGen}$ *and any message* x*, it holds that* $\Pr[\mathsf{Dec}(\mathsf{sk}', \mathsf{Enc}(\mathsf{pk}, x)) = \perp] \geq 1 - \mathsf{negl}(\lambda)$.

Note that this property are very easy to achieve and are achieved by lots of schemes (e.g., trial decryption with key-exchange used in Zcash [45]), without only a very tiny overhead, if any.

Overview. In the multi-server setting, we enable recipients to register with the TEE of their choice, and each recipient announces the TEE public key as well as their public key. Thus if any entity is able to link a signal to a TEE, then the anonymity set for the recipient is reduced to only the set of parties registered with the TEE. To prevent this, we use key-private encryption to encrypt the location and RID. This ensures that any entity observing the signal cannot identify the public key of the TEE that ciphertext is associated to. In fact, any server running the system cannot tell this either. Therefore every signal on the board will be processed by every TEE. While decrypting the ciphertext, the wrong-key decryption detection property above ensures that a TEE is able to determine correctly when a signal is pertinent to one of the parties that are registered with the TEE. The TEE does dummy operations if the signal ciphertext is not encrypted under its key, and does the correct operations otherwise. We present the formal updates to the program in Fig 13 and 14.

C.1.1 Proof Overview

Theorem 2. *Assume that the signature scheme* Σ *is existentially unforgeable under chosen message attacks, the encryption scheme* Enc *is CPA secure, key private, and satisfies wrong-key-decryption*

On input* ("initialization", pp)

Store $\mathsf{pp} = (N, M, \bar{\ell})$ and locally keep a counter $\mathsf{CurIdx} \leftarrow 0, \mathsf{ctr}_{\mathsf{recip}} \leftarrow 0$.
Initialize an empty database of size $(N+1)$ with ORAM and store its identifier DB.
Initialize an empty database of size $(M+1)$ with ORAM and store its identifier DBkeys.
Initialize an empty database of size $(M+1)$ with ORAM and store its identifier DBrecip.
▷ "+1" is for dummy writes. That slot is never touched for non-dummy reads/writes.

On input* ("send", $\mathsf{ct}_{\mathsf{Signal}}$)

$X \leftarrow \mathsf{Dec}(\mathsf{esk}, \mathsf{ct}_{\mathsf{Signal}})$
if $X = \perp$ **then** ▷ Not correct key, do dummy reads and writes.
  $\mathsf{data} \leftarrow \mathsf{ORAM.Read}(\mathsf{DBrecip}, \cdot, \cdot)$
  ▷ Read to whatever value.
  $\cdot \leftarrow \mathsf{ORAM.Write}(\mathsf{DBrecip}, \cdot, \cdot)$
  $\cdot \leftarrow \mathsf{ORAM.Write}(\mathsf{DB}, \cdot, \cdot)$
  ▷ Dummy writes can be writing garbage value to some dummy slot.
else
  Parse X as $[\mathsf{RID}, \mathsf{loc}]$
  $\mathsf{data} \leftarrow \mathsf{ORAM.Read}(\mathsf{DBrecip}, \mathsf{RID}, \cdot)$ ▷ $\cdot$ is a dummy data variable
  $[\mathsf{PrevIdx}, \mathsf{PrevLoc}] \leftarrow \mathsf{Dec}(\mathsf{esk}, \mathsf{data})$
  $\cdot \leftarrow \mathsf{ORAM.Write}(\mathsf{DBrecip}, \mathsf{RID}, \mathsf{Enc}(\mathsf{esk}, [\mathsf{CurIdx}, \mathsf{loc}]))$
  $\cdot \leftarrow \mathsf{ORAM.Write}(\mathsf{DB}, \mathsf{CurIdx}, \mathsf{Enc}(\mathsf{esk}, [\mathsf{PrevIdx}, \mathsf{PrevLoc}]))$
$\mathsf{CurIdx} \leftarrow \mathsf{CurIdx} + 1$
return CurIdx.

Figure 13: Program Prog run by $\mathcal{G}_{\mathrm{att}}$ for multi-server setting

(Definition 3), the ORAM scheme is secure and the encryption scheme Enc_1 *is key-private. Then the protocol* Π_{TEE} *in the* $(\mathcal{G}_{att}, \mathcal{G}_{\mathsf{ledger}})$*-hybrid world UC-realizes the* $\mathcal{F}_{\mathsf{privSignal}}$ *functionality.*

Proof. (Sketch) To prove UC security, we need to show that there exists a PPT simulator interacting with $\mathcal{F}_{\mathsf{privSignal}}$ that generates a transcript that is indistinguishable from the transcript generated in the real world where the adversary interacts with $(\mathcal{G}_{\mathrm{att}}, \mathcal{G}_{\mathsf{ledger}})$ ideal functionalities. The simulator internally simulates the $\mathcal{G}_{\mathrm{att}}$ and the $\mathcal{G}_{\mathsf{ledger}}$ functionalities to the adversary. The simulator works similarly to the simulator for the single server case, except that the simulator announces a key-private encryption key in the setup. Each time the simulator simulates an honest `SEND`, it generates a new encryption key and encrypts 0 under this key. By the key-privacy property of our encryption, this simulated world is indistinguishable from the real world. Moreover, we require that the simulator will abort, if on behalf of any of the TEEs, the decryption under the wrong key gives a valid plaintext. Since we use the property of wrong-key decryption detection in our scheme, this event will occur with negligible probability. Hence the simulated world and the real world are indistinguishable.

□

C.2 Variants and other extensions

PIR-based Variant. We observe that for DB, DBkeys, we can simply replace the ORAM Write with a non-privacy preserving write (e.g., simply append the latest signal or key to the databases). Thus, one alternative is to replace the underlying ORAM with a Private Information Retreival (PIR) scheme [46] for DB, DBkeys, and replace the ORAM.Read's to DB, DBkeys with PIR.query.

At a high-level, PIR is essentially a read-only-ORAM, and PIR.query hides the database entry that TEE is trying to read. Thus, all the privacy analysis still holds.

Registration

Recipient R_i:

1. Let $(\mathsf{mpk}, \mathsf{epk}) := \mathcal{G}_{\mathrm{att}}.\mathsf{getpk}()$
2. Compute $(\mathsf{pk}_i, \mathsf{sk}_i) \leftarrow \mathsf{Enc.KeyGen}(1^\lambda)$, $(\Sigma.\mathsf{sk}_i, \Sigma.\mathsf{vk}_i) \leftarrow \Sigma.\mathsf{KeyGen}(1^\lambda)$.
3. Set $\mathsf{ct}_{\mathsf{keys},i} = \mathsf{Enc}(\mathsf{epk}, (\mathsf{pk}_i, \Sigma.\mathsf{vk}_i))$ and send $(\text{“setup”}, \mathsf{ct}_{\mathsf{keys},i})$ to Srv. ▷
4. Await $((eid, \mathsf{pk}_i), \sigma_T)$ from Srv.
5. Assert $\Sigma.\mathsf{Ver}_{\mathsf{mpk}}((eid, (\mathsf{pk}, \mathsf{RID}, \mathsf{ctr}_{\mathsf{recip}})), \sigma_T) = 1$ and publish $(\mathsf{pk}_i, \mathsf{RID}, eid)$. Initialize $\mathsf{ctr}_i = 0$.

   ▷ Note that if the returned pk_i is not the pk_i sent, abort.

Srv:

1. Upon receiving $(\text{“setup”}, \mathsf{ct}_{\mathsf{keys},i})$ from R_i, let $((\mathsf{pk}_i), \sigma_T) = \mathcal{G}_{\mathrm{att}}.\mathsf{resume}(eid, (\text{“setup”}, \mathsf{ct}_{\mathsf{keys},i}))$. Upon receiving ORAM request $\mathsf{data} \leftarrow \mathsf{ORAM.Access}(\cdot,\cdot,\cdot,\cdot)$ from TEE when executing the $\mathcal{G}_{\mathrm{att}}.\mathsf{resume}(\cdot,\cdot)$ operation: process and return data to $\mathcal{G}_{\mathrm{att}}$.
2. Send $((eid, (\mathsf{pk}, \mathsf{RID}, \mathsf{ctr}_{\mathsf{recip}})), \sigma_T)$ to R_i.
3. Upon return of $\mathsf{ctr}_{\mathsf{recip}}$ from TEE, if $\mathsf{ctr}_{\mathsf{recip}} = M$, call procedure $\mathcal{G}_{\mathrm{att}}.\mathsf{resume}(eid, (\text{“extendrecip”}, 2M)$ and update $M \leftarrow 2M$.

Procedure $(\mathtt{SEND}, R_i, \mathsf{loc})$ Sender S:

1. Sender S obtains eid from recipient R_i (which is published)
2. Sender S gets $(\mathsf{mpk}, \mathsf{epk}) := \mathcal{G}_{\mathrm{att}}.\mathsf{getpk}()$ from TEE with correct eid and computes $\mathsf{ct}_{\mathsf{Signal}} = \mathsf{Enc}(\mathsf{epk}, [\mathsf{RID}, \mathsf{loc}])$. S sends $(\mathtt{SUBMIT}, (\mathtt{SEND}, \mathsf{ct}_{\mathsf{Signal}}))$ to $\mathcal{G}_{\mathsf{ledger}}$

Srv:

1. Send $\mathtt{READ}$ to $\mathcal{G}_{\mathsf{ledger}}$
2. Upon receiving $(\mathtt{SEND}, \mathsf{ct}_{\mathsf{Signal}})$: Call $\mathcal{G}_{\mathrm{att}}.\mathsf{resume}(eid, (\text{“send”}, \mathsf{ct}_{\mathsf{Signal}}))$. Upon receiving ORAM requests $\mathsf{data} \leftarrow \mathsf{ORAM.Access}(\cdot,\cdot,\cdot,\cdot)$: process and return data to $\mathcal{G}_{\mathrm{att}}$.
3. Upon return of CurIdx from TEE. If $\mathsf{CurIdx} = N$, call procedure $\mathcal{G}_{\mathrm{att}}.\mathsf{resume}(eid, (\text{“extend”}, 2N)$ and update $N \leftarrow 2N$.

Figure 14: The protocol for private signaling in the $\mathcal{G}_{\mathrm{att}}$ hybrid world for multi-server setting.

However, one major issue comes from the efficiency of PIR. As PIR is aimed that different users can use the same database to do privacy-preserving-reads (in contrast, ORAM is just for a single user, but the user can do reads and writes to that database). Most of the PIR constructions has query runtime linear in the size of the database. This results in a $O(N)$ runtime in our case. However, recently, doubly-efficient PIR [47, 48] and offline/online PIR [49] have made some great progress allowing query time to be sublinear (e.g., $\tilde{O}(\sqrt{N})$ [49] or even $\mathsf{polylog}(N)$ [48]).

Thus, using these new schemes, we can achieve a similar efficiency as our ORAM-based construction. However, all these works are more theoretical, providing great asymptotic costs, but may not be very practical. Of course, there are some concretely efficient PIR (with preprocessing) constructions [50] that can be used. Although the query time is linear in N, the concrete runtime is still acceptable.

In this paper, we do not implement or benchmark this variant, as we need ORAM for our $\mathsf{DBrecip}$ anyway (unless we keep $\mathsf{DBrecip}$ in TEE), and it is also not yet clear that the PIR protocol can improve the overall runtime. However, we would like to point out that this is a potentially interesting and practical direction to further explore.

Extension to the group setting. In [19], the authors introduce an extension of OMR, applying to the group setting (i.e., a sender may want to send to more than one recipient). This can be similarly extended to our functionality. Our scheme can also be easily extended to the group setting: the sender simply sends all the recipients' identities and TEE simply decrypts and updates all the recipients' records accordingly.

This results in a cost of $O(G(\log(N)+\log(M)))$ per send, when a signal is addressed to G recipients. It may be possible to reduce this cost further by modifying the underlying ORAM to take advantage of the batch setting (i.e., read or write to multiple addresses at the same time). We leave that to future works to explore in more detail.