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Lactation curve modelling using Legendre Polynomial in Sahiwal cattle

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The usefulness of any lactation curve model depends on how well it can mimic the biological process of milk production. Several parametric models, such as exponential decline function, inverse polynomial function, gamma type function, exponential function, mixed log function have been developed to describe the lactation curve. These models have been used for modelling the lactation curve of Sahiwal cattle (Dongre *et al*. 2012, Gupta 2013). The increased availability of records per individual lactations and the genetic evaluation based on test day records has shifted the interest towards more flexible and general linear functions. As possible alternatives, general functions like Legendre orthogonal polynomials (Kirkpatrick *et al*. 1990) or regression splines have been proposed. Models with Legendre polynomials as regressions are orthogonal, therefore they show better convergence than parametric models and have remarkable capacity to fit a great range of shapes. Many researchers have reported their advantages in comparison with more traditional models (Strabel *et al*. 2005, Macciotta *et al*. 2010, Otwinowska-Mindur *et al*. 2013). The lactation curve modelling using Legendre polynomial has not been reported in Sahiwal cattle. Hence,the main objective of the study was to find suitable Legendre polynomial model for modelling the lactation curve of Sahiwal cattle and to study lactation curve variation in different lactation.

The first three lactation monthly test day milk yield data of Sahiwal cattle maintained at ICAR-NDRI, Karnal were considered for study. Ten test day milk yield records at monthly interval, viz. first test day (Day 6), second test day (Day 36), third test day (Day 66), fourth test day (Day 96)… and tenth test day (Day 276) were taken. A total of 5890 first lactation, 3884 second lactation and 2852 third lactation monthly test day records of cows calved between year 1961–2012 were taken in the present study. Culling in the middle of lactation, abortion, still-birth or any other pathological causes which affected the lactation yield were considered as abnormalities and such records were not taken for the study. Only those animals were considered for study which had produced milk for at least 100 days and more than 500 kg.

Legendre Polynomials Model: The Legendre polynomials (LEG) are polynomial functions of *n* degree and the equation describing a single observation can be written as

$$y_t = \sum_{i=0}^{n} \alpha_i \, \phi_i(x)$$

where, *x* is standardized unit of time ranging from -1 to +1 and t_{min} (6^{th} day) is the earliest test day and t_{max} (276^{th} day) is the latest test day.

$$x = 2\left(\frac{t_i - t_{min}}{t_{max} - t_{min}}\right) - 1$$

These quantities were normalized using

$$\phi_n(x) = \left(\frac{2n+1}{2}\right)^{0.5} P_n(x)$$

where, $P_n(x)$ is a polynomial of degree *n* and ²”$_n$(x) is the normalized polynomial. Normalized Legendre polynomials functions of standardized units of time *x* and coefficients α_i with degree1, 2, 3, 4, 5 and 6 (LEG1, LEG2, LEG3, LEG4, LEG5 and LEG6) were used in this study for modelling lactation curve in Sahiwal cattle.

The first few standardized Legendre polynomials have the form

$$P_0(x) = 1,\ P_1(x) = x,\ P_2(x) = \frac{3x^2-1}{2},$$

$$P_3(x) = \frac{5x^3-3x}{2},\ P_4(x) = \frac{35x^4-30x^2+3}{8},$$

$$P_5(x) = \frac{63x^5-70x^3+15x}{16},$$

$$P_6(x) = \frac{231x^6-315x^4+105x^2-5}{16}$$

The regression equation was fitted using WOMBAT software (Meyer 2007). The most suitable model were

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identified based on coefficient of determination (R^2), Root Mean Square Error (RMSE), Akaike Information Criterion (AIC) value and Bayesian Information Criterion (BIC) value. The model which gave highest R^2 value and lowest RMSE value was considered best model. Similarly, the model which gave the lowest AIC and BIC values were chosen as the better approximating model.

The monthly test day yield ranged from 5.12 to 8.14 kg, 5.28 to 10.70 kg and 5.17 to 11.65 kg in first, second and third lactation,respectively (Table 1). For all lactation, highest test milk yield was observed on second test day. The highest test day milk yield showed increasing trend with increase in lactation number. The lowest test day milk yield was recorded on first test day in first lactation. However, for second and third lactation lowest test day milk yield was recorded on tenth test day.The first lactation curve was flatter compared to second and third lactation curve. As first parturition occurs in the process of continuous body and udder development as a result of which primiparous cows have limited milk production capacity compared to

Table 1. Actual and predicted monthly test day milk yield (kg) from different Legendre polynomial lactation curve model

Days in milk	No of records (N)	Actual test day yield (kg)	First lactation					
			Predicted test day yield (kg) from different LP model					
			LEG1	LEG2	LEG3	LEG4	LEG5	LEG6
6	635	5.12	7.34	6.49	5.70	5.27	5.15	5.12
36	635	8.14	7.18	6.91	7.19	7.72	8.01	8.12
66	635	8.08	7.01	7.18	7.85	8.24	8.22	8.12
96	634	7.52	6.85	7.30	7.87	7.77	7.55	7.49
126	614	6.99	6.68	7.27	7.46	7.01	6.89	6.97
156	598	6.57	6.52	7.09	6.81	6.38	6.52	6.59
186	583	6.22	6.35	6.76	6.12	6.08	6.31	6.25
216	557	6.00	6.19	6.28	5.60	6.05	6.06	5.96
246	535	5.76	6.03	5.65	5.44	5.98	5.67	5.79
276	464	5.42	5.86	4.87	5.84	5.30	5.45	5.42
		AIC	16656.37	16385.09	16050.11	15866.52	15834.66	15833.74
		BIC	16663.04	16391.77	16056.79	15873.20	15841.34	15840.42
		RMSE	0.786	0.528	0.214	0.041	0.007	0.001
		R^2	24.04	49.07	79.28	96.01	99.30	99.94
			Second lactation					
6	425	7.99	10.18	9.54	8.67	8.18	8.05	8.00
36	425	10.70	9.66	9.43	9.74	10.22	10.55	10.66
66	425	10.01	9.14	9.21	9.95	10.27	10.24	10.08
96	424	9.01	8.61	8.88	9.51	9.32	9.06	8.96
126	417	8.09	8.09	8.45	8.65	8.09	7.97	8.05
156	405	7.25	7.57	7.91	7.58	7.06	7.22	7.30
186	392	6.55	7.05	7.26	6.55	6.43	6.69	6.57
216	361	6.04	6.53	6.51	5.76	6.14	6.14	5.98
246	332	5.64	6.00	5.64	5.44	5.91	5.54	5.68
276	278	5.28	5.48	4.67	5.82	5.15	5.34	5.27
		AIC	12186.72	12124.44	11932.56	11842.38	11824.96	11824.20
		BIC	12192.98	12130.71	11938.84	11848.64	11831.22	11830.47
		RMSE	0.275	0.251	0.157	0.076	0.038	0.013
		R^2	76.39	79.70	92.37	98.10	99.56	99.94
			Third lactation					
6	306	8.54	11.00	10.23	9.33	8.79	8.60	8.54
36	306	11.65	10.38	10.10	10.42	10.94	11.41	11.55
66	306	10.64	9.77	9.85	10.60	10.96	10.92	10.72
96	306	9.51	9.15	9.46	10.11	9.90	9.54	9.42
126	304	8.58	8.54	8.96	9.16	8.56	8.38	8.48
156	298	7.68	7.92	8.32	8.00	7.42	7.64	7.74
186	287	6.96	7.31	7.56	6.84	6.70	7.07	6.93
216	275	6.24	6.69	6.68	5.91	6.33	6.33	6.14
246	255	5.62	6.08	5.67	5.45	5.96	5.45	5.61
276	209	5.17	5.46	4.53	5.68	4.96	5.23	5.14
		AIC	9047.32	8985.72	8844.66	8768.19	8740.22	8738.83
		BIC	9053.28	8991.68	8850.63	8774.15	8746.18	8744.78
		RMSE	0.304	0.272	0.183	0.106	0.048	0.023
		R^2	78.28	82.12	92.06	97.26	99.44	99.90

pluriparous cows. This, combined with cow selection that is also practised in the herds, could be the reason for test day milk yield variations in different lactation.

The actual and predicted value of test day milk yield is given in (Table1) for different lactations. On comparison of predictive ability of different degree of Legendre polynomial model it was seen that as the degree of polynomial increased the predictive ability improved and RMSE value decreased. Thus, most measures of the goodness of fit used in this study indicated that a larger number of parameters ensures its better fit to the data. The R^2 value and RMSE value of the LEG5 model (99.30% and 0.007) and LEG6 model (99.94% and 0.001) were almost similar and thus LEG5 model was sufficient to explain the lactation curve of first lactation of Sahiwal cows. For second lactation LEG6 model gave lowest AIC and BIC values, lowest RMSE value (0.013) and highest R^2 value (99.94%). The LEG5 model was slightly inferior (R^2 = 99.56%, RMSE =0.037) compared to LEG6 model. For third lactation LEG6 model has lowest AIC and BIC value, highest R^2 (99.90%) and lowest RMSE value (0.023). It was also evident that LEG5 model gave almost similar prediction (R^2=99.44%) and RMSE value (0.048). For all three lactations LEG1, LEG2 and LEG3 models are not suitable for modelling the lactation curve in Sahiwal cattle. They were not able to predict initial ascending milk yield, peak yield and declining phase of lactation curve (Fig 1). For all lactation fourth degree Legendre polynomial model (LEG4) has R^2 value greater than 96% but it could not fit the peak yield properly. However, higher degree Legendre polynomial (fifth and sixth degree) mimicked the actual curve with great accuracy. The increase in accuracy is mainly attributed to more number of parameter fitted which take into account all the lactation curve variations. Silvestre *et al.* (2006) also found Legendre polynomial model with five parameters has superior fit. Otwinowska-Mindur *et al.*(2013) also found that to fit the shape of lactation curve correctly, higher-order Legendre polynomials are required. They reported R^2 value of 96.7, 97.1 and 96.9% for first, second and third lactation respectively, using fourth order Legendre polynomial in Polish Holstein-Friesian cattle.

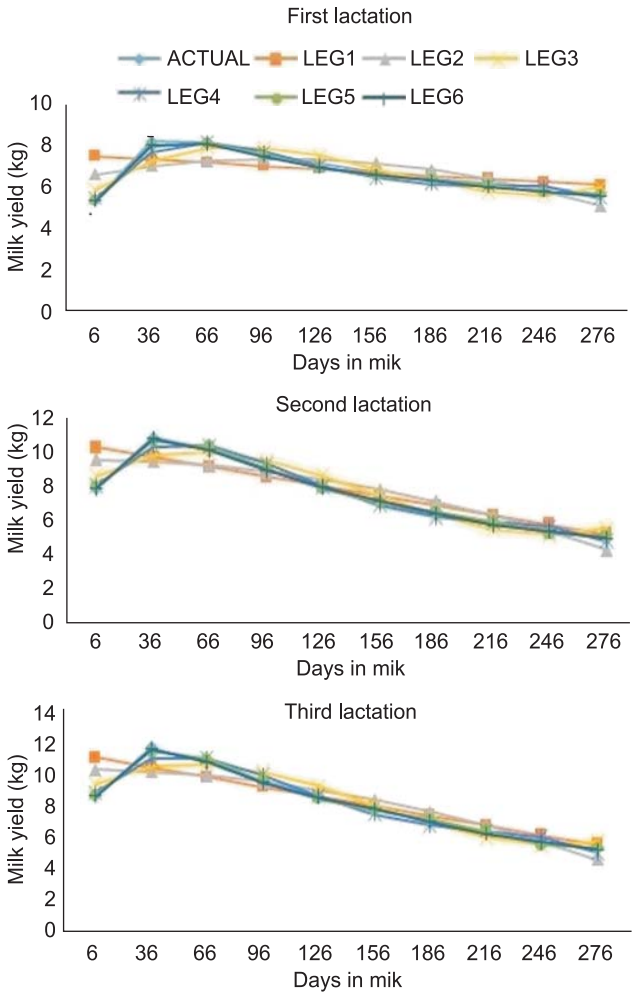

Fig. 1. Average lactation curve of Sahiwal cattle based on different Legendre polynomial model.

SUMMARY

Present study aimed at modelling lactation curve in Sahiwal cattle using normalized Legendre polynomials functions of standardized units of time and understanding the variation of lactation curve in different lactations. Among the first three lactations, third lactation recorded highest peak yield as well other test day yields except the last test day yield. Further, first lactation curve was flatter compared to second and third lactation curve. Among the six different Legendre polynomial model considered higher degree Legendre polynomial model LEG5 and LEG6 fitted lactation curve better. Overall fifth degree Legendre polynomial lactation curve model (LEG5) can be used for accurate fitting for lactation curve in Sahiwal cattle as it gave almost similar accuracy like LEG6 with less parameter fitted.

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