# A Method Based On Weighted Least Squares For Estimating Voltage Of Distribution Network System Integrated With Distributed Generations Using Remote Measurement Data

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Voltage quality of a distribution network system is one of the most important issues. To evaluate the voltage in real-time, this paper proposes a method of voltage estimation in distribution network systems which are integrated with distributed generations (DGs). In the proposed method, the weighted least squares (WLS) method is used to optimize the objective function of the error measurement which is built by the network configuration, real-time and pseudo measurements at the main substation and DG buses. The proposed method is verified by simulation case studies of the 17-bus radial distribution network integrated with DGs including a solar power plant and two wind power plants. The simulation results in this work confirm that the proposed method is high accuracy for voltage estimation.

**Keywords:** distribution network system, real-time measurement, distributed generation, voltage estimation



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## 1. Introduction

Distribution network systems are conventionally based on radial feeders whose power flow follows by single direction from the main substation to the customers in the feeders. Due to the penetration level of distributed generations (DGs) such as solar power plants, wind power plants, etc. in the distribution network system, the power flow must be varified following bidirectional [1]. Moreover, the uncertainties of these renewable resources have impacts on voltage quality monitoring and controlling of this system [2]. On the other hand, the system supplies directly electric energy for the customers, so its voltage quality needs to be at a high level. This means that the bus voltages in normal operating modes must be within an acceptable range [3].

The distribution network system is limited about the communication information for controlling and monitoring in real-time. The remote measurement data can be only acquired from the main substation, DGs, several specific customers in the system via the communication system. The measurement data is then transmitted to the supervisory control and data acquisition (SCADA) system at the central control system [4, 5]. Therefore, to be able to control and monitor voltage quality in a distribution network system, the set of pseudo measurement is added as an additional data. The voltage estimation is then performed to estimate voltage magnitudes and phase angles at the buses. These estimation parameters are used not only to make decisions related to control and operation of the network, but also to give information of the network status [6, 7] as shown in Fig. 1.

The voltage estimation in a distribution network system is an interesting topic that has been researched by many published works. In [6], an admittance matrix was used to propose the new method for reducing the number of status. The method was then implemented to model the three-phase four-wire distribution network system. Moreover,

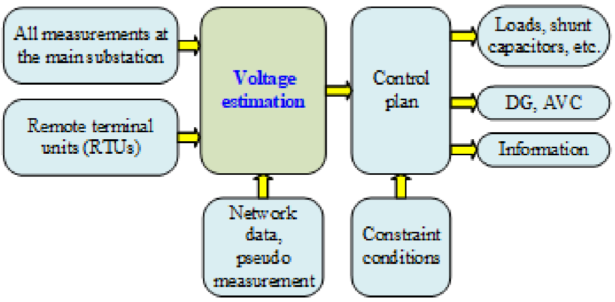



**Fig. 1.** Voltage estimation in distribution network system

the admittance matrix incorporated to the neutral coupling effect on the phase conductors was used to formulate the state estimation of the low voltage distribution network system. The state of active distribution networks was estimated by the proposed procedure in [7]. This procedure did not require any remote measurement and it was used the load modelling techniques and the discrete step communications. Reference [8] presented the fast decoupled state estimation in which the branch ampere measurements was reformulated as active and reactive branch loss measurements. The authors in [9] designed the state estimation algorithm to monitor unbalanced distribution networks. The algorithm investigated many various uncertainties from the measurement devices and the network behavior. To estimate voltage in a radial distribution feeder integrated with DGs, the new method using the high-precision measurements was proposed in [10]. Due to several individual loads in the feeder were not measured in real-time, the bus voltages were estimated through the correction of the section load center. The authors in [11] presented a comprehensive mathematical analysis of voltage estimation based on the weighted least squares technique.

The rest of this paper is arranged as follows. Section 2 presents the proposed method based on the weighted least squares (WLS) algorithm. In Section 3, the simulation results for the 17-bus distribution network system integrated with one solar power plant and two wind power plants are analyzed and discussed. The conclusions of this work are included in Section 4.

## 2. Wls-based proposed method

In this work, the main research object is to estimate voltage in distribution network systems with and without DGs (solar and wind power plants). To observer the DGs' buses, the additional set of real-time measurement data, pseudo measurement data at the DGs' buses are utilized. In addition, the DGs' reactive power is calculated by its grid connection method [12–17], the DGs' active power is determined by the DG type as follows:

For wind turbines, their active power depends on the wind speed as follows [18–21]:

$$P_{WT} = P_T = \frac{1}{2}\rho_a A_{WT} c_p(\lambda) v^3 \quad (1)$$

where $\rho_a$ is the air density approximately $1.225\,\mathrm{kg/m^3}$; $A_{WT}$ is the wind picking area of the wind turbine $(\mathrm{m^2})$; $c_p(\lambda)$ is the power factor, which is defined by the set of characteristics and is approximated by the following expression:

$$c_p(\lambda) = \sum_{k=0}^{n} a_k \lambda^k \quad (2)$$

where $a_k$ is the coefficient determined by the least squares method; $\lambda$ is the ratio of the wind speed which is defined as follows:

$$\lambda = \omega_R R / v \quad (3)$$

$\omega_R$ is angle speed of the rotor $(\mathrm{rad/s})$; $R$ is the radius of wind turbine blade $(\mathrm{m})$; $v$ is the wind speed $(\mathrm{m/s})$

For photovoltaic panels, their active power is determined as follows [22, 23] :

$$P_{PV} = P_T = \eta_{PV} \Re \cos\gamma A_{PV} \quad (4)$$

where $\eta_{PV}$ is the efficient of the photovoltaic panel; $\Re$ is the solar radiation $(\mathrm{W/m^2})$; $\gamma$ is the angle of the panel for picking sunlight; and $A_{PV}$ is the photovoltaic panel area $(\mathrm{m^2})$.

The weighted least squares (WLS) method is applied to minimize the objective function of error measurement as follows [24]:

$$\min_x \left\{ J(x) = [z - h(x)]^T R^{-1} [z - h(x)] \right\} (5) \quad (5)$$

subject to

$$c(x) = 0 \quad (6)$$

where $x$ is the state vector of voltage magnitude and phase angle at the buses in the system except the slack bus; $z$ is the measurement vector; $h(x)$ is the function of the state variables; $R = \mathrm{diag}\left\{\sigma_1^2, \sigma_2^2, \ldots \sigma_M^2\right\}$; $\sigma_i$ is the $i_m^{\text{th}}$ standard weighted factor and $M$ is the number of real-time and pseudo measurements.

The WLS-based flowchart as shown in Fig. 2 for estimating voltages in distribution network systems integrated with DGs includes the following steps:

Step 1: Read the bus data, line data, and status of the circuit breakers and reclosers initialize all virtual measurement at the buses without load and generation buses, Read all the real-time measurement from the SCADA system and real all the pseudo measurement based on the typical loading profile or the forecasting result.

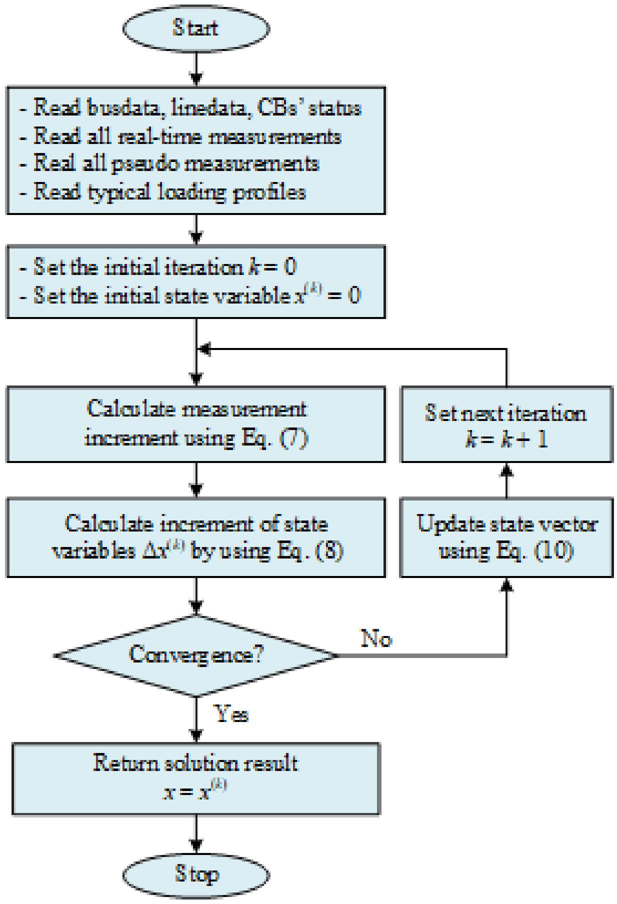



**Fig. 2.** The WLS-based flowchart of voltage estimation

Step 2: Set the initial iteration $k = 0$ and set the initial state variable $x^{(k)} = 0$.

Step 3: Calculate the measurement increment:

$$\Delta z^{(k)} = z^{(k)} - h\left(x^{(k)}\right) \tag{7}$$

Step 4: Calculate the increment of state variables $\Delta x^{(k)}$ by using the following equation:

$$\begin{bmatrix} G\left(x^{(k)}\right) & C\left(x^{(k)}\right) \\ C\left(x^{(k)}\right) & 0 \end{bmatrix} \begin{bmatrix} \Delta x^{(k)} \\ -\lambda^{(k)} \end{bmatrix} = \begin{bmatrix} H\left(x^{(k)}\right)^T R^{-1} \Delta z^{(k)} \\ -c\left(x^{(k)}\right) \end{bmatrix} \tag{8}$$

Step 5: Check the convergence criteria:

$$\max\left|\Delta x_i^{(k)}\right| \leq \varepsilon; \quad i = 1, 2, \ldots, N_s \tag{9}$$

where $N_s$ is the number of state variables and $\varepsilon$ is the convergence error. If the condition (9) is satisfied, the solution result is $x = x^{(k)}$ and the iteration process is ended; else the iteration process is moved to Step 6 for updating the state variables vector.

Step 6: Update the state variables vector.

$$x^{(k+1)} = x^{(k)} + \Delta x^{(k)} \tag{10}$$

Step 7: Set the next iteration $k = k + 1$ and come back to Step 3 .

The standard error $(\sigma)$ of the real-time/pseudo measurement is calculated from the average of the measurement and the accuracy $(Ac)$ [24]:

$$\sigma = \frac{Ac}{300} \cdot \text{Mean} \tag{11}$$

where the accuracy is assumed by $Ac_{xt} = 3\%$ for the accuracy of the real-time measurement and $Ac_p = 20\%$ of the accuracy of the pseudo measurement.

## 3. Simulation results and discussion

### 3.1. Test system description

The test system used in this work is a 17-bus radial distribution network (see Fig. 3). The power source of this network is the main substation at the bus 1. Therefore, the bus 1 plays a swing bus in the network and its voltage magnitude is remained equal 1.00pu and its phase angle is a reference value of 0.00 degrees. The linedata of the system is given in Table 1. Due to the medium voltage level for this system, each line is modeled by an impedance including a resistance and a reactance, so the line parameters in Table 1 consists of the line segment (from the sending bus to the receiving bus), resistance $(\Omega)$, and inductance (mH). For this system, the three buses 7,13 , and 17 are the DG buses as shown in Fig. 3.

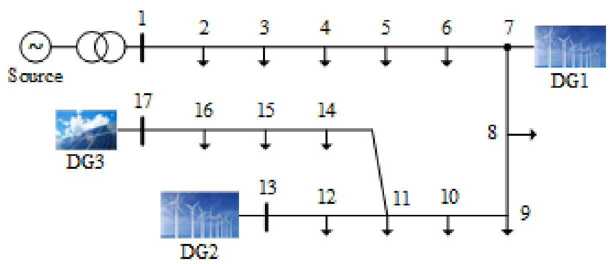


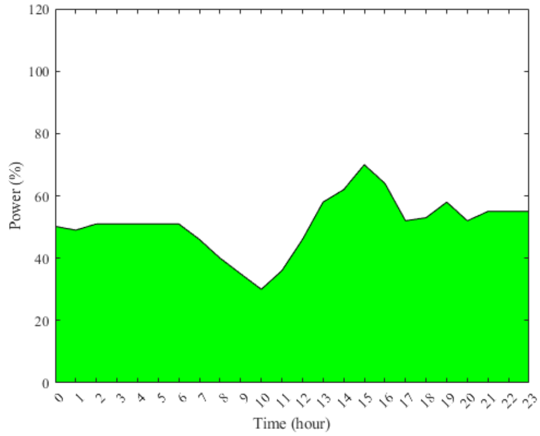

**Fig. 3.** One-line diagram of the test system

It is assumed that all loads of the system have the power rating of 400kVA and the power factor of 0.85. Besides, their typical daily load profile is represented as Fig. 4. In the system, three DGs which are connected to the system

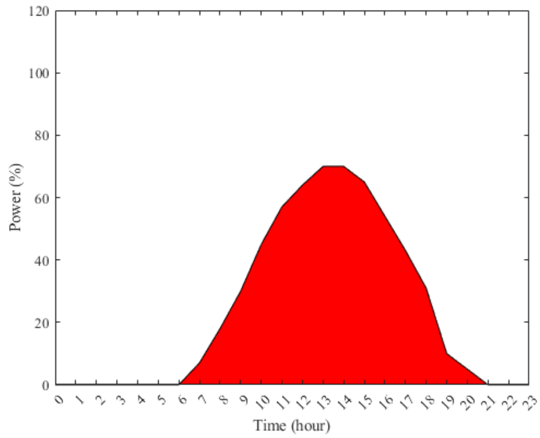
**Table 1.** The linedata of the test distribution network system

| Line segment | R (Ω) | L (mH) |
|---|---|---|
| 1 − 2 | 0.1178 | 0.4644 |
| 2 − 3 | 0.1013 | 0.3993 |
| 3 − 4 | 0.0333 | 0.1311 |
| 4 − 5 | 0.0803 | 0.3165 |
| 5 − 6 | 0.1667 | 0.4919 |
| 6 − 7 | 0.0548 | 0.1617 |
| 7 − 8 | 0.1250 | 0.4930 |
| 8 − 9 | 0.1500 | 0.5916 |
| 9 − 10 | 0.2320 | 0.6846 |
| 10 − 11 | 0.0473 | 0.1864 |
| 11 − 12 | 0.4853 | 1.8298 |
| 12 − 13 | 0.3088 | 1.1644 |
| 11 − 14 | 0.1318 | 0.3891 |
| 14 − 15 | 0.0824 | 0.2432 |
| 15 − 16 | 0.7785 | 2.9355 |
| 16 − 17 | 1.0380 | 3.9140 |

at buses 7, 11, and 17 have the power ratings as given in Table 2. These DGs are operated at the unity power factor and their generation power profiles as shown in Fig. 5, whereas Fig. 5a represents the generation power characteristic of the wind power plants and Fig. 5b represents for the generation power characteristic of the solar power plant in the distribution network. It can be seen clearly that the powers on the $y$-axis in these two figures are calculated in percent of the power rating of each DG.




(a) Wind power




(b) solar power

**Fig. 5.** The generation power characteristics

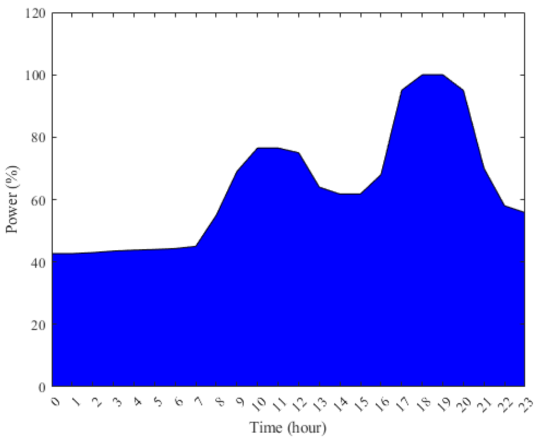



**Fig. 4.** The typical loading profile

### 3.2. Numerical simulation results

The PSS/Adept software, a power system simulator as well as an advanced distribution engineering productivity tool, is applied to model and simulate the 17-bus radial distribution network system. All load and generation power characteristics are used to establish many different operation scenarios of the system. The bus 1 is assumed as the swing bus which has its constant voltage magnitude of 1.00pu and its constant phase angle of 0.00 degrees. As a result, the simulation results of bus voltages at the time 14:00 hour are shown in Fig. 6. It can be clearly seen that the voltage magnitudes are reduced alongside the length of the feeder in the case of without any DGs and the bus-17 voltage magnitude is the lowest value in this case. On the other hand, with integration of the DGs, the voltage magnitudes are increased as shown by the red line in Fig. 6. This means that the DGs supports voltage control in the system.

**Table 2.** The data of the DGs on the test system

| DG | Type of DG | Bus no. | Capacity (MW) |
|---|---|---|---|
| DG1 | Wind turbine | 7 | 2.0 |
| DG2 | Wind turbine | 13 | 3.0 |
| DG3 | Photovoltaic panels | 17 | 3.0 |

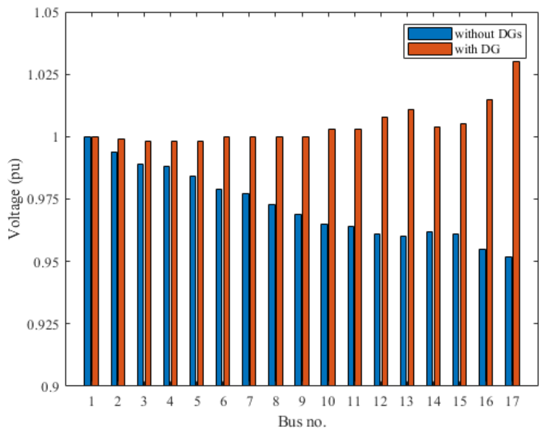


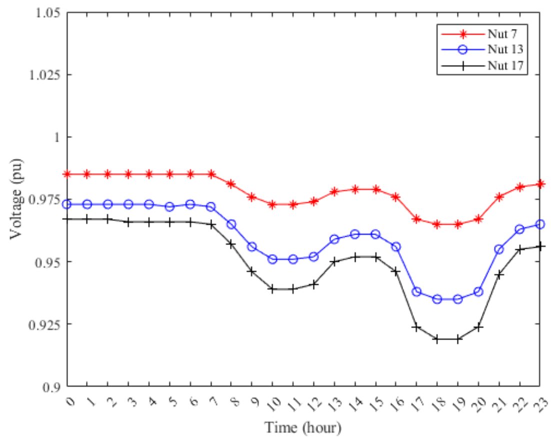

**Fig. 6.** The bus voltage profile at 14:00 hour

To evaluate the impact of the DGs on the voltage profile of the system, the typical daily loading profile, the generation power characteristics of the DGs are applied to simulate and calculate the power flow at the steady state of the network. As a result, the voltages of the buses 7, 13, and 17 are presented in Fig. 7. The obtained simulation results in Fig. 7a represents for the voltages of the buses in the case of without the DGs while the results in Fig. 7b shows the voltages of the buses in the case of with DGs penetration. Based on the simulation results in Fig. 7a, the voltage of the bus 17 is the lowest value among the ones of three buses 7, 13, and 17 during a typical day; however, for the case with DG penetration as shown in Fig. 7b the voltage of the bus 17 is the highest value from 10:00 hour to 17:00 hour.

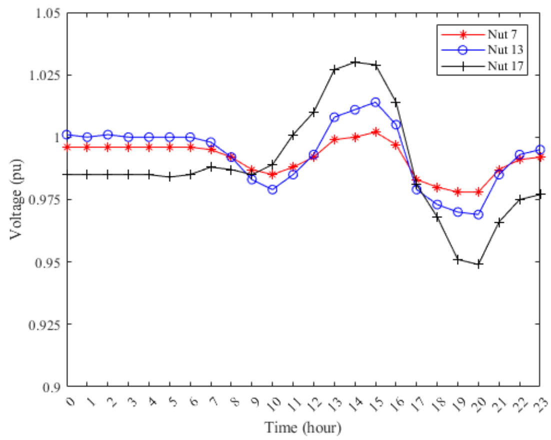
From the above simulation results, the DGs have effects on the voltage quality of the distribution network. The bus voltages will increase when the DGs are connected to the network and they generate their power to respond to loads of the network.

### 3.3. Estimation results

For the 17-bus distribution network, the proposed WLS-based method is applied to estimate the bus voltages. The simulation results are then used to evaluate the performance of the proposed method. The real-time measurement data consists of the voltage, active and reactive powers of the slack bus and the DGs' buses. Furthermore, the pseudo measurement data which represents for the injection active and reactive powers at every bus of the network is added to conduct the proposed method. Moreover, the pseudo measurement data is determined based on the predicted results of the load and DG powers with the accuracy 20%. The real-time and pseudo measurement data of the




(a) Without DG




(b) With DGs

**Fig. 7.** The voltage profile at the buses 7, 13, and 17

active and reactive powers at the buses are shown in Table 3. According to Table 3, the p type represents for the pseudo measurement data for the load buses and the rt type represents for the real-time measurement data for the DG buses (the bus 17 for the solar power plant, the buses 7 and 13 for the wind power plants). It can be clearly seen that the active powers of the DGs are negative while the ones of the other loads are positive. This means that the DGs are generating active powers and the loads are consuming active powers.

The performance of the proposed method for estimating voltages of the 17-bus radial distribution network system is shown in Table 4. As shown in Table 4, the voltage magnitudes and phase angles at the buses are estimated by using the WLS-based method. The actual values of voltages and angles are calculated by the PSS/Adept software with the steady state at the time 10:00 hour while the estimated values of voltages and angles are the output result of the

**Table 3.** The real-time and pseudo measurement data

| Bus no. | Measurement type | P (kW) | Q (kVAR) |
|---|---|---|---|
| 2 | p | 407.986 | 183.19 |
| 3 | p | 313.594 | 199.16 |
| 4 | p | 356.551 | 224.198 |
| 5 | p | 305.165 | 179.999 |
| 6 | p | 373.826 | 192.316 |
| 7 | rt | -1547.52 | 205.85 |
| 8 | p | 294.371 | 168.014 |
| 9 | p | 403.121 | 228.847 |
| 10 | p | 358.478 | 219.769 |
| 11 | p | 387.298 | 211.987 |
| 12 | p | 280.852 | 232.961 |
| 13 | rt | -3154.27 | 210.661 |
| 14 | p | 404.906 | 230.977 |
| 15 | p | 279.719 | 224.148 |
| 16 | p | 320.175 | 228.376 |
| 17 | rt | -3157.46 | 211.816 |

proposed method.

Similarily, Fig. 8 shows the other criteria to evaluate the proposed method. This criteria is the magnitude and angle error between the actual voltage and the estimated voltage. As shown in Fig. 8, these errors when applying the proposed to the 17-bus distribution network at the time of 20:00 hour are plotted in bar charts. On the y-axis of these figures, the magnitude errors are presented in per unit and the angle errors are presented in degrees. On the other hand, the x-axis represents for the bus numbers of the network. The highest magnitude error is at the bus 17 as shown in Fig. 8, and the highest angle error is at the bus 15 as shown in Fig. 8.

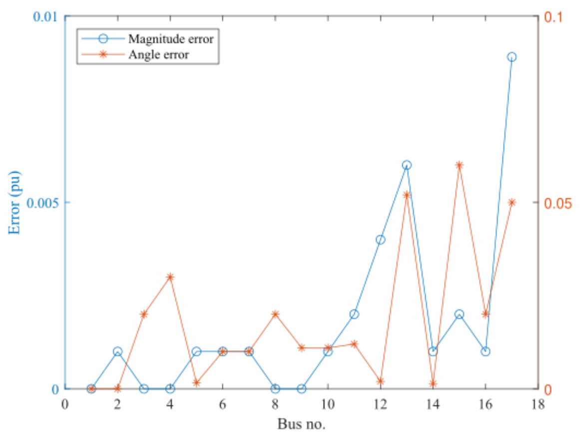



**Fig. 8.** The error between the actual and estimation voltages at the time 20:00 hour

## 4. Conclusions

This paper has proposed a WLS-based method for estimating voltages in distribution network systems integrated with distributed generations (DGs). In the proposed method, the real-time and pseudo measurement data are used to minimize the objective function of error measurement. The simulation results of the 17-bus distribution network system integrated with the one solar power plant and the two wind power plants are simulated and evaluated in the PSS/Adept software. The typical daily profiles of the loads and DGs of the system are carried out to validate voltage quality in the steady state as well as to evaluate the effectiveness of the proposed method. Moreover, the estimated voltage magnitudes and phase angles, known as the output results of the proposed method, are compared with the actual ones, respectively. The simulation results show that the proposed method ensure the necessary accuracy to implement in the distribution network system.

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**Table 4.** The voltage estimation result at 10:00 hour

| Bus no. | Actual voltage | | Estimated voltage | |
|---|---|---|---|---|
| | Magnitude (pu) | Phase angle (degrees) | Magnitude (pu) | Phase angle (degrees) |
| 1 | 1.000 | 0.00 | 1.000 | 0.00 |
| 2 | 0.995 | 0.05 | 0.996 | 0.05 |
| 3 | 0.992 | 0.11 | 0.992 | 0.13 |
| 4 | 0.991 | 0.13 | 0.991 | 0.16 |
| 5 | 0.989 | 0.18 | 0.990 | 0.27 |
| 6 | 0.986 | 0.32 | 0.985 | 0.33 |
| 7 | 0.985 | 0.37 | 0.984 | 0.36 |
| 8 | 0.982 | 0.44 | 0.982 | 0.46 |
| 9 | 0.980 | 0.52 | 0.980 | 0.62 |
| 10 | 0.979 | 0.67 | 0.980 | 0.89 |
| 11 | 0.978 | 0.71 | 0.980 | 0.96 |
| 12 | 0.978 | 0.89 | 0.974 | 0.71 |
| 13 | 0.979 | 1.03 | 0.973 | 0.68 |
| 14 | 0.978 | 0.77 | 0.979 | 1.16 |
| 15 | 0.978 | 0.81 | 0.980 | 1.23 |
| 16 | 0.981 | 1.48 | 0.991 | 1.46 |
| 17 | 0.989 | 2.22 | 1.011 | 2.17 |


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