Cosmological Constant As Hyper Geometric Function In Fivedimensional Space-time

Satish T. Rathod and Kalpana Pawar*

Department of Mathematics, Shri R. R. Lahoti Science College, Morshi-444905, Dist. Amravati, M. S., India

Corresponding author. E-mail: drkalpanapawar75@gmail.com

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In this paper, we have studied the Einstein Field equation in Kaluza-Klein space-time in five dimension with the metric $dS^2 = dt^2 - A^2\left(dx^2 + dy^2 + dz^2\right) - B^2 d\psi^2$ under the assumption that $B = \alpha t$, where $\alpha =$ Constant and scale factor satisfying the relation $R^4 = A^3 B$ with perfect fluid having energy momentum tensor $T_{ij} = (\rho + p)v_i v_j - pg_{ij}$

In this paper, we have assumed that $G = \frac{1}{t}$ and we have found the value of cosmological constant Λ is a function time t in terms of hyper geometric function.

Keywords: Kaluza-Klein space-time in five-dimension, perfect fluid, Hypergeometric Function, variable cosmological constant $\Lambda = \Lambda(t)$ and gravitational constant $G = G(t)$

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1. Introduction

Kaluza and Klein consider extra dimension apart from space and time in order to unify the gravitation and electromagnetism [1–3]. Many researchers of theoretical physicist and mathematician tried to the unified theory of gravity and electromagnetism before Kaluza and Klein [1–8]. Alan Chados and steven Detweiler in his paper where has the fifth dimension gone? consider the five-dimensional space-time model in vacuum [9]. Let t_0 is the reference cosmological time. The Kasner solution in five-dimensional space time is given by $dS^2 = -dt^2 + \left(\frac{t}{t_0}\right)\left[\sum_{i=1}^{3}\left(dx^i\right)^2\right] + \left(\frac{t_0}{t}\right)\left(dx^5\right)^2$. If $t = t_0$ then one can observed that the universe is spatially flats and isotropic. If $t \ll t_0$ then the value of $\left(\frac{t}{t_0}\right)$ goes nearer to zero and the value of $\left(\frac{t_0}{t}\right)$ goes aways from zero sufficiently. From this fact, we can observe that $dS^2 = -dt^2 + \left(\frac{t_0}{t}\right)\left(dx^5\right)^2$ and as t approaches to infinity then the cosmos has only one dimensions. If $t \gg t_0$ then the value of $\left(\frac{t}{t_0}\right)$ goes away from zero and the value of $\left(\frac{t_0}{t}\right)$ goes towards zero sufficiently. If we consider $0 \leq x^i \leq L$, where L is the length. Then the distance in fifth dimensions shrunk to $\sqrt{\left(\frac{t_0}{t}\right)}$ L and the space dimensions increase to $\sqrt{\left(\frac{t}{t_0}\right)}$ L. As the t approaches to infinity i.e. universe is sufficiently old, the fifth dimension not observe [6]. J.Demart &J.-L.Hanquin used the results of Fees $G_7 G_8$ and G_{11} Lie Algebra to study the homogeneous and anisotropic cosmological models satisfying five dimension Einstein's field equations [7]. The above explanation motivates us to study the five-dimensional cosmological model.

P.A.M Dirac in his letter Cosmological Constant in 1937, study the behavior of cosmological constant. He observed that the cosmological constant is depend on cosmological time [10].

Sanjay oli studied the five-dimensional cosmological model with variable cosmological constant and Gravitational constant [11].

In [11], Sanjay oli considered the metric $dS^2 = -dt^2 + X^2\left(dx^2 + dy^2 + dz^2\right) + A^2 d\Psi^2$ with the relation $AX^3 = U$. He found the value of Λ and G for $U = c, U = t, U = t^n$

and conclude that the value of Λ and G decreases and increases with time respectively [11]. In [12], R.K. Tiwari et.al studied the five-dimensional cosmological model with the metric $dS^2 = dt^2 - \mathrm{A}^2\left(dx^2 + dy^2 + dz^2\right) - \mathrm{B}^2 d\Psi^2$ with relation $R^4 = A^3B$, where R is the scale factor. In their investigation, they observed that G proportional to $t^{-(1-\omega)}$, where ω is term in equation of states $p = \omega\rho$ with $0 \le \omega \le 1$ [12].

In this paper, we have studied the Einstein Field equation in Kaluza-Klein space-time in five dimension with the metric $dS^2 = dt^2 - A^2\left(dx^2 + dy^2 + dz^2\right) - B^2 d\psi^2$ under the assumption that $B = \alpha t$, where $\alpha =$ Constant and scale factor satisfying the relation $R^4 = A^3B$ with perfect fluid having energy momentum tensor $T_{ij} = (\rho + p)v_i v_j - pg_{ij}$

In this paper, we have assumed that $G = \frac{1}{t}$ and we have found the value of cosmological constant Λ is a function time t in terms of Hyper geometric function.

2. Five-dimensional einstein field equation

In Kaluza-Klein five-dimensional cosmological model, consider the line element

$$dS^2 = dt^2 - A^2\left(dx^2 + dy^2 + dz^2\right) - B^2 d\psi^2 \tag{1}$$

where A, B is the function of cosmological time t and the spatial average scale factor $R(t)$ satisfies the relation $R^4 = A^3B$

Suppose universe contain perfect fluid with energy momentum tensor

$$T_{ij} = (\rho + p)v_i v_j - pg_{ij} \tag{2}$$

Let assume that the matter satisfies the equation of states

$$p = \omega\rho, 0 \le \omega \le 1 \tag{3}$$

where p is the pressures and ρ is the energy density of cosmic matters.

The five-dimensional Einstein Field equation which contain Λ and G as a function of time is given as follows [12]:

$$R^{ij} - \frac{1}{2}Rg^{ij} = -8\pi G T^{ij} - \Lambda g^{ij} \tag{4}$$

By using line element (1), one can express five-dimensional Einstein Field equation (4) as

$$\frac{3\dot{A}^2AB + 3\dot{A}\dot{B}A^2}{R^4} = 8\pi G\rho + \Lambda \tag{5}$$

$$\frac{2\ddot{A}}{A} + \frac{B}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} = -8\pi Gp + \Lambda \tag{6}$$

$$\frac{3\ddot{A}A + 3\dot{A}^2}{A^2} = -8\pi Gp + \Lambda \tag{7}$$

By assuming the energy conservation in general relativity, the covariant derivative in Einstein Field equation gives

$$\dot{\rho} + (\rho + p)\left(\frac{3\dot{A}B + \dot{B}A}{AB}\right) + \frac{\dot{G}}{G}\rho = -\frac{\Lambda}{8\pi G} \tag{8}$$

Equation (8) gives [13]

$$\dot{\rho} + (\rho + p)\left(\frac{3\dot{A}B + \dot{B}A}{AB}\right) = 0 \tag{9}$$

and

$$\dot{\Lambda} = -8\pi\rho\dot{G} \tag{10}$$

3. General solution of five-dimensional einstein field equation

Equation (3), (5)-(7) and (9) contain five independent equations with six unknowns , B, p, ρ, G, Λ. To find the solution, we have assumed that

$$B = \alpha t \tag{11}$$

where α is the constant. From equation (6) and (7), we get

$$\frac{\dot{A}}{A} - \frac{B}{B} = \frac{k_1}{A^3B} \tag{12}$$

where k_1 is the constant of integration.

$$A = \frac{\left(-k_1 + e^{3k_1ct^3}\right)^{\frac{1}{3}}}{\alpha^{\frac{1}{3}}} \tag{13}$$

From equation (11) and (12), we get following line element

$$ds^2 = dt^2 - \left[\frac{\left(-k_1 + e^{3k_1ct^3}\right)^{\frac{1}{3}}}{\alpha^{\frac{1}{3}}}\right]\left(dx^2 + dy^2 + dz^2\right) - (\alpha t)^2 d\psi^2 \tag{14}$$

where c is the constant.

For the model (14), the spatial volume V, matter density ρ, pressure p, gravitational parameter G, cosmological parameter Λ are given by

$$\mathrm{V} = \left[\frac{\left(-k_1 + e^{3k_1ct^3}\right)^{\frac{1}{3}}}{\alpha^{\frac{1}{3}}}\right]^3 (\alpha t) \tag{15}$$

$$\rho = e^{\frac{(1+\omega)\left(\log(t) - \frac{1}{3}\alpha\log\left(-k_1 + e^{3ck_1t^3}\right)\right)}{\alpha}} c_1 \tag{16}$$

$$p = \omega e^{\frac{(1+\omega)\left(\log(t) - \frac{1}{3}\alpha\log\left(-k_1 + e^{3ck_1t^3}\right)\right)}{\alpha}} c_1 \tag{17}$$

P.A.M Dirac say that G is proportional to t^{-1}. Assume that

$$G = \frac{1}{t} \tag{18}$$

$$S_R = \text{Hypergeometric2 F1}\left[\frac{1+\omega}{3}, -\frac{1+\omega+\alpha}{3\alpha}, -\frac{1+\omega-2\alpha}{3\alpha}, \frac{e^{3ck1}t^3}{k1}\right] \tag{19}$$

$$\Lambda = c_2 - \frac{8\pi t^{-\frac{1+\omega+\alpha}{\alpha}}\left(-k_1+e^{3ck_1t^3}\right)^{\frac{1}{3}(-1-w)}\left(1-\frac{e^{3ck1t^3}}{k_1}\right)^{\frac{1+\omega}{3}}\alpha c_1 S_R}{1+w} \tag{20}$$

where c_2, k_1, c_1 are constants.

4. Conclusion and discussion

P.A.M Dirac say that G is proportional to t^{-1}. In this paper, we can observe that when we consider five dimensional Kaluza-Klein cosmological models, then we found that Λ is not constant and it involve Hypergeometric function.

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