

Natural and forced oscillations of pipelines in contact with the Winkler medium

II Safarov^{1*}, *M Kh* Teshayev^{2,3,4}, *Z I* Boltayev^{2,3}, *I M* Karimov¹, and *N Q* Esanov⁵

¹Tashkent Institute of Chemical Technology, 32, Navoi str., Tashkent, 100011, Uzbekistan

²Bukhara branch of the Institute of Mathematics named after V.I. Romanovsky, 11, M.Iqbol str., Bukhara, 105017, Uzbekistan

³Bukhara Institute of Engineering and Technology, 15, K. Murtazayev str., Bukhara, 105017, Uzbekistan

⁴Asia International University, 74, Gijduvan str, Bukhara, 105026, Uzbekistan

⁵Bukhara State University, 11, M.Iqbol str., Bukhara, 105017, Uzbekistan

Abstract. In this paper, the pipeline is modeled as a curved rod in contact with the Winkler medium. Linear oscillations of a curved viscoelastic rod lying on the Winkler base are considered. The general formulation of the problem of free oscillations of a spatially curved viscoelastic rod with variable parameters is reduced to a boundary value problem for a system of ordinary integro-differential equations of the 12th order with variable coefficients relative to eigenstates; it can be solved by the method of successive approximations. The relations allowing to present the solution of the boundary value problem for the rod in an analytical form are formulated. It is established that the dimensionless complex frequencies of natural oscillations of a spatially curved rod, while maintaining the elongation of the rod constant, do not depend on it. The Poisson's ratio has little effect on the dimensionless real and imaginary parts of the natural frequencies.

1 Introduction

In recent times, aeromechanics has become one of the developing applied areas in the field of mechanics, within which great attention is paid to the study of the strength of elements under vibration, shock and other types of external influences [1-2]. Biomechanical structures, in general, have a very complex structure and shape [3-4]. Their mechanical properties depend on the individual characteristics of the organism, age, functional state, external factors and are largely determined by the stress-strain state, since the biomechanical system adapts to external influences. An organism, as an object of mechanics, is a complex system in which a hierarchical organization is viewed [5]. Considering the general methods of studying complex systems, it can be argued that their mathematical modeling requires the compilation of models of elements of the lowest level of the hierarchy, that is, in relation to this case, bones, muscles and internal organs. From the given examples of structuring it follows that the elements of the skeleton, i.e. bones,

* Corresponding author: muhsin_5@mail.ru

which are the main supporting elements of the structure of the organism, are an indispensable element of modeling [6]. In the vast majority of cases, bones can be represented as spatially curved rods of variable cross-section. In addition, the elements of the skeleton have pronounced viscoelastic properties [7-8], varying both along the axis and in cross-section.

The method of modal analysis (modal decomposition) is currently used to solve problems on the dynamic behavior of viscoelastic bodies [9-10]. Its advantage is the possibility of using both analytical and discrete models, in which there is a weak dependence on the nature of external influence. To implement the method, the decomposition of the motion of a viscoelastic body according to the modes of vibrations - the functional basis - according to the forms of free vibrations of an elastic body is used. The convenience of this basis is that it is a complete orthogonal system of functions, which simplifies the decomposition technique [11-12].

Therefore, in this paper we consider the question of representing solutions to dynamic problems through combinations of elementary transcendental functions, which in the limit give strict solutions to a system of equations for forms of free oscillations. The complex eigenfrequencies are determined by the Muller-Gauss method. Also, when the curvilinear pipeline lying on the base of the Winkler oscillates, the present work is considered.

2 Materials and methods

2.1 Problem statement and solution methods

To derive the differential equations of motion of a curved rod, we select the elementary part of the curved rod (Figure 1). Consider the element of a curved rod of length ds and all the acting internal and external forces, which is shown in Figure 1. In Figure 1, the following designations are adopted: $\vec{Q} = Q_1\vec{t} + Q_2\vec{n} + Q_3\vec{b}$ where $Q_1 = N$ axial force $Q_2 = Q_n$ - cutting force or normal force, $Q_3 = Q_b$ - shearing force or components of the shearing force vector by binormals; $\vec{M} = M_t\vec{t} + M_n\vec{n} + M_b\vec{b}$ - vector of internal moments, where M_t - torque moment, M_n and M_b - bending moments; $\vec{q} = q_t\vec{t} + q_n\vec{n} + q_b\vec{b}$ - vector of external loads, q_t, q_n, q_b - components of the vector of external loads.

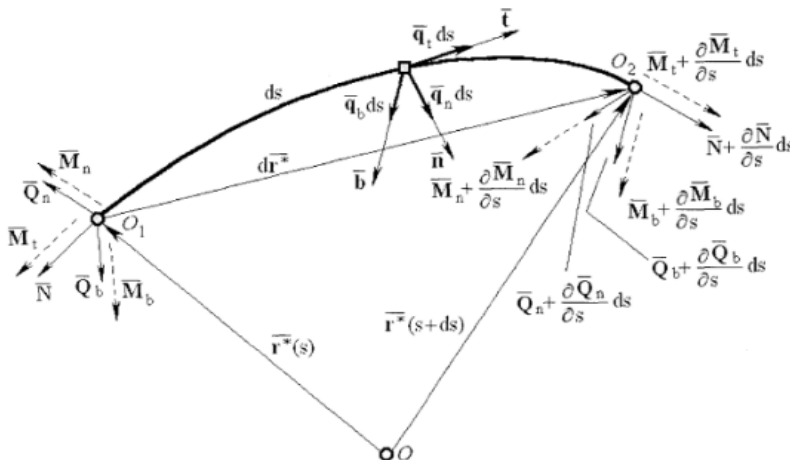


Fig. 1. The element of the rod of infinitely small length.

The selected element is in equilibrium only when the sum of all forces and the sum of moments are equal to zero, which gives two vector equations:

$$\begin{aligned} d\vec{Q} + \vec{q}ds - \rho \frac{\partial^2 \dot{U}}{\partial t^2} ds &= 0, \\ d\vec{M} + ds \vec{t} \times \vec{Q} + \vec{M}_I ds &= 0. \end{aligned} \tag{1}$$

Here- \vec{M}_I is the main moment of inertia forces. The system of equations (1) can be written in the following form

$$\begin{aligned} \frac{\partial \dot{N}}{\partial s} + \frac{\partial \dot{Q}_n}{\partial s} + \frac{\partial \dot{Q}_b}{\partial s} - \rho F \frac{\partial^2 \dot{U}}{\partial t^2} + \vec{q} &= 0, \\ d\vec{M} + d\vec{M}_Q + d\vec{M}_q - d\vec{M}_I &= 0. \end{aligned} \tag{2}$$

Here- $d\vec{M} = \frac{\partial \dot{M}_t}{\partial s} ds + \frac{\partial \dot{M}_n}{\partial s} ds + \frac{\partial \dot{M}_b}{\partial s} ds$ is the main moment arising in the sections of a curved rod; $d\vec{M}_p$ -- the main factor of the forces of external loads. Taking into account the Frensay-Serre formula [13], it is possible to write through projections of tangent vectors \vec{t} , normal \vec{n} and binormals \vec{b} in the form of a system of three differential equations:

$$\begin{aligned} \frac{\partial N}{\partial s} - kQ_n - \rho F \frac{\partial^2 u}{\partial t^2} + q_t &= 0, \\ \frac{\partial Q_n}{\partial s} + kN - Q_b \tau - \rho F \frac{\partial^2 v}{\partial t^2} + q_n &= 0, \\ \frac{\partial Q_b}{\partial s} + Q_n \tau - \rho F \frac{\partial^2 w}{\partial t^2} + q_b &= 0. \end{aligned} \tag{3}$$

Now we write the second equation (3) in expanded form. To do this, we use the expression of moments through displacements:

$$dM_I = (J_t \frac{\partial^2 u_t}{\partial t^2} \vec{t} + J_n \frac{\partial^2 u_n}{\partial t^2} \vec{n} + J_b \frac{\partial^2 u_b}{\partial t^2} \vec{b}) \rho ds \tag{4}$$

Where J_t, J_n, J_b are moments of inertia of the rod relative to the axes.

To calculate the moments from the forces, we use the well-known ratio from vector analysis

$$dM_Q = d\vec{r}^s \times \vec{Q} = \begin{vmatrix} \vec{t} & \vec{n} & \vec{b} \\ N & Q_n & Q_b \\ u & v & w \end{vmatrix} \tag{5}$$

If we use the Fresne trihedron [13], then we will have:

$$dM_Q = \left(\vec{t} + \frac{\partial u}{\partial s} \vec{t} + \frac{\partial v}{\partial s} \vec{n} + \frac{\partial w}{\partial s} \vec{b} \right) \times (Q_t \vec{t} + Q_n \vec{n} + Q_b \vec{b}) \tag{6}$$

Vector multiplication (6) can be written through the components of vectors in scalar form. Now using equations (5) and (6) we obtain three more differential equations in scalar form:

$$\begin{aligned}
 \frac{\partial N}{\partial s} - kQ_n - \rho F \frac{\partial^2 u}{\partial t^2} + q_t &= 0, \\
 \frac{\partial Q_n}{\partial s} + kN - Q_b \tau - \rho F \frac{\partial^2 v}{\partial t^2} + q_n &= 0, \\
 \frac{\partial Q_b}{\partial s} + Q_n \tau - \rho F \frac{\partial^2 w}{\partial t^2} + q_b &= 0, \\
 \frac{\partial M_t}{\partial s} + M_n k - J_t \rho \frac{\partial^2 u_t}{\partial t^2} &= 0, \\
 \frac{\partial M_n}{\partial s} + M_t k - M_b \tau - Q_b l - J_n \rho \frac{\partial^2 u_n}{\partial t^2} &= 0, \\
 \frac{\partial M_b}{\partial s} + M_n \tau - Q_n l - J_b \rho \frac{\partial^2 u_b}{\partial t^2} &= 0,
 \end{aligned} \tag{7}$$

Thus, we obtained six differential equations with 12 unknowns: $N, Q_n, Q_b, M_t, M_n, M_b, u, v, w, u_t, u_n, u_b$.

To obtain a closed system of equations, additional geometric and physical equations are needed. Normal stresses at any point of the rod, taking into account the viscoelastic properties of the rod material, are represented by Hooke's law [14-16]:

$$\sigma_s = E_{0n} \left[\varepsilon_s(t, s) - \int_0^t R_{0E}(t - \tau) \varepsilon_s(\tau, s) d\tau \right] \tag{8}$$

Where E_{0n} is instantaneous values of the modulus of elasticity, $R_{0E}(t - \tau)$ - the relaxation core of the material, σ_s - the tension of the rod along the axial line. Deformations along the axis:

$$\varepsilon_s = \varepsilon - \frac{\partial u_b}{\partial s} y - \frac{\partial u_n}{\partial s} z \tag{9}$$

Here- ε is deformation of the rod axis; u_b is movement of the rod particle along the binormal; u_n is movement of the rod particle along the normal. The deformations of the axis of the rod are determined by (9) and takes the following form:

$$\varepsilon = u' - kv \tag{10}$$

If we use Hooke's law (8), then for internal points we get known formulas of power factors:

$$\begin{aligned}
 N &= \int_{F_s} \sigma_s dF, M_n = \int_{F_s} (\sigma_s dF) z, \\
 M_b &= \int_{F_s} (\sigma_s dF) y, M_t = G^0 \frac{\partial u_t}{\Delta s \partial s} \int_{F_s} R^2 dF.
 \end{aligned} \tag{11}$$

Here- $\mathcal{G}[\varphi(t)] = \frac{E_0}{2(1+\nu)} \left[\varphi(t) - \int_0^t R_{0E}(t-\tau)\varphi(\tau)d\tau \right]$, $\varphi(t)$ - an arbitrary function of time. The force factors (11) can be brought into a form convenient for calculation using the above ratios (8)- (10):

$$\begin{aligned}
 N &= E_0 \left[(F_s \varepsilon - F_n \frac{\partial u_n}{\partial s} - F_b \frac{\partial u_b}{\partial s}) - \int_0^t R_{0nb}(t-\tau)(F_s \varepsilon(s,\tau) - F_n \frac{\partial u_n(s,\tau)}{\partial s} - F_b \frac{\partial u_b(s,\tau)}{\partial s})d\tau \right] \\
 M_b &= E_0 \left[(F_b \varepsilon - J_{bn} \frac{\partial u_n}{\partial s} - J_b \frac{\partial u_b}{\partial s}) - \int_0^t R'_{0nb}(t-\tau)(F_b \varepsilon - J_{bn} \frac{\partial u_n}{\partial s} - J_b \frac{\partial u_b}{\partial s})d\tau \right] \\
 M_n &= E_0 \left[(F_n \varepsilon - J_n \frac{\partial u_n}{\partial s} - J_{bn} \frac{\partial u_b}{\partial s}) - \int_0^t R''_{0nb}(t-\tau)(F_n \varepsilon - J_n \frac{\partial u_n}{\partial s} - J_{bn} \frac{\partial u_b}{\partial s})d\tau \right] \\
 M_t &= \frac{E_0 J_t}{2\Delta s(1+\nu)} \left[\frac{\partial u_t(s,t)}{\partial s} - \int_0^t R_{0E}(t-\tau) \frac{\partial u_t(s,\tau)}{\partial s} d\tau \right]
 \end{aligned} \tag{12}$$

Here $F_n = \int_{F_s} z dF_s, F_b = \int_{F_s} y dF_s,$

$J_b = \int_{F_s} y^2 dF_s, J_n = \int_{F_s} z^2 dF_s, J_{bn} = \int_{F_s} xy dF_s, J_t = \int_{F_s} (y^2 + z^2) dF_s$

axial, centrifugal and polar

moment of inertia of the section.

If we use (1), (7) and (12), then we get the following system of integro-differential equations

$$\begin{aligned}
 \frac{\partial N}{\partial s} - kQ_n - \rho F \frac{\partial^2 u}{\partial t^2} + q_t &= 0, \\
 \frac{\partial Q_n}{\partial s} + kN - Q_b \tau - \rho F \frac{\partial^2 v}{\partial t^2} + q_n &= 0, \\
 \frac{\partial Q_b}{\partial s} + Q_n \tau - \rho F \frac{\partial^2 w}{\partial t^2} + q_b &= 0, \\
 \frac{\partial M_t}{\partial s} + M_n k - J_t \rho \frac{\partial^2 u_t}{\partial t^2} &= 0, \\
 \frac{\partial M_n}{\partial s} + M_t k - M_b \tau - Q_b l - J_n \rho \frac{\partial^2 u_n}{\partial t^2} &= 0, \\
 \frac{\partial M_b}{\partial s} + M_n \tau - Q_n l_1 - J_b \rho \frac{\partial^2 u_b}{\partial t^2} &= 0, \\
 u_n - w' + \tau v &= 0, \\
 u_b - v' + ku - \tau w &= 0, \\
 F_s(u' - kv) - F_n \frac{\partial u_n}{\partial s} - F_b \frac{\partial u_b}{\partial s} - \int_0^t R_{0nb}(t-\tau)(F_s(u' - kv) - F_n \frac{\partial u_n(s,\tau)}{\partial s} - F_b \frac{\partial u_b(s,\tau)}{\partial s})d\tau - \frac{N}{E_0} &= 0,
 \end{aligned} \tag{13}$$

$$\begin{aligned}
 &F_b(u' - kv) - J_{bn} \frac{\partial u_n}{\partial s} - J_b \frac{\partial u_b}{\partial s} - \\
 &-\int_0^l R'_{0nb}(t - \tau)(F_b(u' - kv) - J_{bn} \frac{\partial u_n}{\partial s} - J_b \frac{\partial u_b}{\partial s})d\tau - \frac{M_b}{E_0} = 0, \\
 &F_n(u' - kv) - J_n \frac{\partial u_n}{\partial s} - J_{bn} \frac{\partial u_b}{\partial s} - \\
 &-\int_0^l R''_{0nb}(t - \tau)(F_n(u' - kv) - J_n \frac{\partial u_n}{\partial s} - J_{bn} \frac{\partial u_b}{\partial s})d\tau - \frac{M_n}{E_0} = 0, \\
 &\frac{\partial u_t(s, t)}{\partial s} - \int_0^l R_{0E}(t - \tau) \frac{\partial u_t(s, \tau)}{\partial s} d\tau - \frac{2M_t \Delta s(1 + \nu)}{E_0 J_t} = 0.
 \end{aligned}$$

Boundary and initial conditions must be applied to the system of integro- differential equations (13). Six conditions are placed on each end of the rod. Initial conditions are set for displacements and velocities at $t=0$.

Assume that the binormal and normal vectors are directed along the central axes of inertia of the cross-section of a curved rod. Then the static moments and the centrifugal moment of the cross-section of the curved rod are zero.

Then the system of integro-differential equations (13) is simplified and takes the following form

$$\begin{aligned}
 &\frac{\partial N}{\partial s} - kQ_n - \rho F \frac{\partial^2 u}{\partial t^2} + q_t = 0, \\
 &\frac{\partial Q_n}{\partial s} + kN - Q_b \tau - \rho F \frac{\partial^2 v}{\partial t^2} + q_n = 0, \\
 &\frac{\partial Q_b}{\partial s} + Q_n \tau - \rho F \frac{\partial^2 w}{\partial t^2} + q_b = 0, \\
 &\frac{\partial M_t}{\partial s} + M_n k - J_t \rho \frac{\partial^2 u_t}{\partial t^2} = 0, \\
 &\frac{\partial M_n}{\partial s} + M_t k - M_b \tau - Q_b l - J_n \rho \frac{\partial^2 u_n}{\partial t^2} = 0, \\
 &\frac{\partial M_b}{\partial s} + M_n \tau - Q_n l - J_b \rho \frac{\partial^2 u_b}{\partial t^2} = 0, \\
 &u_n - w' + \tau v = 0, \\
 &u_b - v' + ku - \tau w = 0, \\
 &\frac{\partial u}{\partial s} = \frac{N}{E_0 F_s} + kv(s, t) - \int_0^l R_{0nb}(t - \tau)(kv(s, \tau) - \frac{\partial u(s, \tau)}{\partial s})d\tau = 0, \\
 &\frac{\partial u_t(s, t)}{\partial s} = \int_0^l R_{0E}(t - \tau) \frac{\partial u_t(s, \tau)}{\partial s} d\tau + \frac{2M_t \Delta s(1 + \nu)}{E_0 J_t} = 0, \\
 &\frac{\partial u_n}{\partial s} = -\int_0^l R''_{0nb}(t - \tau) \left(\frac{\partial u_n}{\partial s}\right) d\tau - \frac{M_n}{E_0 J_n}, \\
 &\frac{\partial u_b}{\partial s} = -\int_0^l R'_{0nb}(t - \tau) \left(\frac{\partial u_b}{\partial s}\right) d\tau - \frac{M_b}{E_0 J_b}
 \end{aligned} \tag{14}$$

Boundary and initial conditions are set to solve the problem.

Boundary (boundary) conditions. The possible boundary conditions for solving the problem can be divided into two classes: homogeneous and inhomogeneous. For a spatial curved rod, the total number of boundary conditions is 12 (6 conditions at the right end and 6 at the left end of the rod).

For the cantilever rod we have the following boundary conditions: $\dot{u}(u, v, w) = 0, \dot{u}_s(u_n, u_b, u_t) = 0$ for a fixed edge; a load is applied to the other end of the rod $\dot{Q} = \dot{P}, \dot{M} = \dot{T}$. Here \dot{P}, \dot{T} - the specified load.

3 Results and Discussion

As an example, consider the natural oscillations of a curved rod, shown in Figure 2. To do this, 6 boundary conditions are placed at the two ends of the rod. The above equations (15) consist of 12 equations. To solve the system (15), the freezing method is first applied. Then we get a system of differential equations with variable coefficients. With natural oscillations, equations (15) in matrix form take the following form

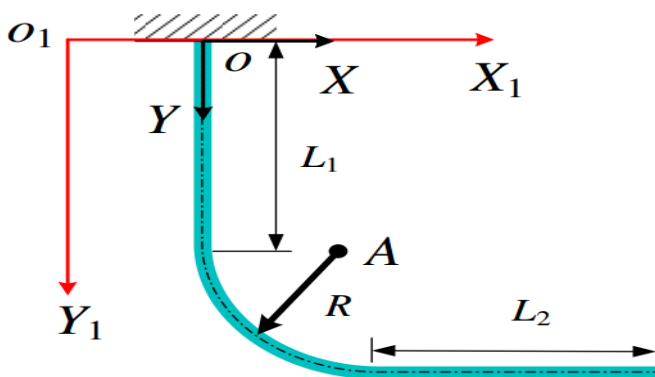


Fig. 2. Elementary part of a curved rod.

$$\frac{d\mathbf{z}}{d\zeta} - ([B^0 - C(\zeta)] - \omega^2 [M^0 - M^1(\zeta)]) [R_m]^{-1} \mathbf{z} = 0$$

The solution of this equation in vector form has the following form

$$\mathbf{z} = \mathbf{C} e^{-\int_{\zeta_0}^{\zeta} \frac{([B^0 - C(\zeta)] - \omega^2 [M^0 - M^1(\zeta)]) \zeta}{[R_m]} d\zeta}$$

As an example of a viscoelastic material, we take the three-parametric Koltunov - Rzhanitsyn relaxation kernel: $R_{0E}(t) = A_{\alpha} e^{-\beta t} / t^{1-\alpha}$ with parameters $A_{\alpha} = 0,048; \beta_{\alpha} = 0,05; \alpha_{\alpha} = 0,1$. Geometric and physico-mechanical parameters take the following values: $L_1 = L_2 = 0,9M; R = 0,127M; E = 210GPa; \rho = 7800 \text{ kg/m}^3$. To solve the problem, the methods of freezing, orthogonalizing, the Muir method, the Gauss method and the method of complex amplitudes are used. The results of calculations of the first four complex natural frequencies, for cantilever rods, are shown in Figure 2 and in Table 1.

The second line shows the results of the frequencies of viscoelastic curved rods obtained by the proposed method. In the larger and fourth lines, the results of De Jong [15], Wu J.H. [16] and [17-20] for elastic curvilinear rods. It has been established that taking into account the viscosity properties of the material of medium frequency frequencies up to 15%.

Table 1. Comparison of the first four natural frequencies.

	Fashion Number	1	2	3	4
2	Proposed methodology	31.5243+i0.2315	68.2819+i0.8942	405.4442+i7.8864	501.3458+i9.7194
3	De Jong [15]	28.4581	62.1266	375.1074	480.4582
4	Wu J.H.[16]	30.2065	65.1195	387.1324	496.5243

4 Conclusion

Thus, the paper has developed a solution technique and an algorithm for studying the natural oscillations of curved deformable rods. With the growth of its own motion, the attenuation decrements increase in the presence of the viscosity of the rod and decrease in the presence of the external viscosity of the Winkler base. Moreover, with an increase in intensive dissipation, aperiodic modes (purely imaginary eigenvalues) arise, starting with the highest eigenforms, in the case of taking into account the viscosity of the rod. By taking into account energy dissipation, the viscoelastic rod model makes it possible to study forced steady-state oscillations at resonances.

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