Engineering methodology for calculation of corrugated beams for bending and torsion. flat bend form stability

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Abstract. An engineering design procedure for calculating bending with torsion in corrugated beams, including finding the critical load of general stability, implemented in the commonly used MS Excel spreadsheet and verified in the LIRA-SAPR software package, has been proposed. The method makes it possible to take into account the beams section variability and corrugation parameters, to estimate the loading eccentricity effect, availability of supporting ribs and reinforcements of the compressed chord from the bending plane. Calculation results of particular design solutions for corrugated beams with definition of critical loads at transverse bending are given. Results of test calculation of a bent channel beam with calculation of stress-strain state parameters and its verification with the LIRA-SAP software package are given.

1 Introduction

A corrugated beam is a welded I-beam with a wall that has a transverse corrugation of any profile (triangular, sinusoidal, etc.). Corrugated beams have been used in construction, shipbuilding, aircraft construction since 30s of XX century [1-6]. During this time, corrugated beams have proven to be an effective element of metal structures [5-6].

To date, the issues of strength estimation and local stability of corrugated beam elements have been studied most fully [7-9]. At the same time the issues of torsion bending and general stability of corrugated beams are not fully disclosed. Thus, the existing normative documents (p. 20.6.3.4, BC 294.1325800.2017), instead of determining the overall stability of the corrugated beam in transverse bending, prescribe the compressed chord for overall stability as a separately taken centrally compressed rod by p. 7.1.3 BC 16.133330.2017. This does not take into account the joint work of the flanges and the girder wall, nor does it take into account the mandatory presence of support ribs, which underestimates the load capacity [10].

In [11] the author considers corrugated beam stability at transverse bending in a wall plane, but only for a case of loading by two oppositely directed bending moments applied

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to beam ends and without considering support ribs. In [12] the author investigates a numerical solution of goffer beam stability problem for transverse bending, however, he considers only one loading variant (uniformly distributed transverse load in the wall plane), and in this case no allowance is made for impact of support ribs and possible buckling of compressed chord in the span from the bending plane.

In this paper, the authors have proposed a procedure for engineering design of cross-section bending with torsion for thin-walled beams of any cross-section (I-beam, channel, omega section, z-profile) loaded with any vertical loads of variable length according to a specified law, including concentrated loads and possibility to calculate the critical load of beam stability in cross-section bending. The methodology is implemented and tested in the generally accessible MS Excel spreadsheet with verification of results in the LIRA CAD software package.

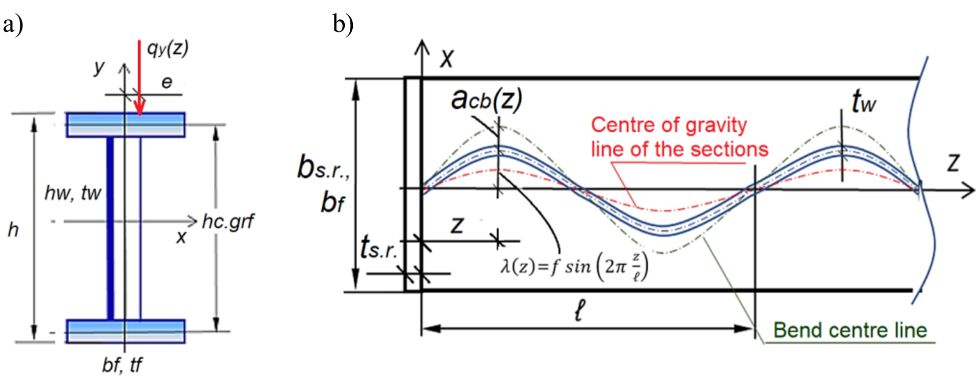
In particular, as applied to goffered beams, calculation results of uniformly distributed critical loads with allowance for goffered parameters, allowance for support ribs, and presence of reinforcement of compression girder from bending plane are given.

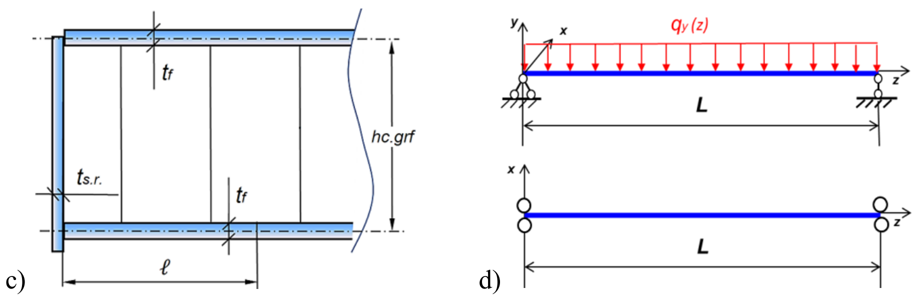
Results of test calculation of purlin made of bent channel at bending with torsion with calculation of parameters of tense-deformed state of section in the middle of the span are also given.

2 Main Part

A corrugated beam is a welded I-beam with a wall having a transverse corrugation of any (sinusoidal, triangular, trapezoidal, etc.) profile. The basic calculations in this material are given in relation to corrugators with a sinusoidal profile corrugated wall.

Fig. 1 shows parameters of such corrugated beams with designation of dimensions, adopted loading scheme with supports, with designation of section centres of gravity and bending centres: L is a beam span, corrugation amplitude - f, corrugation step - l, wall thickness - t_w, wall height - h_w, shelf thickness - t_f, shelf width - b_f, full section height $H = h_w + 2t_f$, distance between the centres of gravity of the chords $H_{c.\ gr.} = h_w + t_f$, $q_y(z)$ – load, generally variable in length, concentrated, e – possible eccentricity of the load given by the calculator, $a_{cb}(z)$ is generally the variable coordinate of the bending centre of the beam section.





c.gr
s.r.
cb
centre of gravity line of the sections
bend centre line

Fig. 1. corrugated beam diagram with dimensions and loading diagram: a) Sectional view of the corrugated beam b) Corrugated beam scheme c) Side view d) Calculation diagram of supports and loading

To solve problems of transverse bending with torsion of thin-walled beams, including solving problems of finding critical loads of general stability in transverse bending, we considered the system of equations by V.Z. Vlasov ([13], p. 375):

$$EJ_x(z)V^{IV}(z) = q_y(z),$$
$$EJ_\omega(z)\varphi_z{}^{IV}(z) - GJ_t(z)\varphi_z{}''(z) = m_x(z)\,, \qquad (1)$$

here $V(z)$ – displacement/deflection along the vertical axis y;

$\varphi_z(z)$ – section angle in relation to the axis z;

$J_x(z)$, $J_\omega(z)$, $J_t(z)$ – in general, the length-variable moment of inertia of the cross-section about the axis x, sectional moment of inertia of the beam section, moment of inertia of the section in pure torsion;

E, G – modulus of elasticity and shear modulus of the beam material;

$m_x(z)$ – the distributed external bending moment about the x-axis, generally calculated according to formula (4) of this article;

$q_y(z)$ – in general, a length-variable distributed external transverse load on the beam with the possibility of incorporating concentrated forces.

The following considerations are given for a beam with a sinusoidal corrugated wall with parallel chords according to Fig. 1.

Moment of inertia J_t is constant along the length of the corrugated beam and is determined according to the formula as for a flat wall I-beam:

$$J_t = \frac{1}{3}\left(2b_f t_f{}^3 + b_w t_w{}^3\right) \qquad (2)$$

As the corrugated wall hardly takes up any longitudinal normal forces, we will assume that the bending moment is taken up only by the corrugated girder chords. Therefore, the moment of inertia is constant along the length:

$$J_x = 2J_{x_f} + 2\left(A_f\left(\frac{h_{c.gr}}{2}\right)^2\right), \qquad (3)$$

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here: $J_{x_f} = \frac{b_f t_f^3}{12}$ – chord moment of inertia about the axis *x*, $A_f = b_f t_f$ is chord area.

Moment of inertia $J_\omega(z)$ is variable along the length of the corrugated beam and is defined as (p. 66, [12]):

$$J_\omega(z) = \frac{t_f b_f h_w^2}{6}\left[\left(\frac{b}{2} + \lambda(z) - a_{cb}(z)\right)^2 + \left(\frac{b}{2} - \lambda(z) + a_{cb}(z)\right)^2 - \left(\frac{b}{2} + \lambda(z) - a_{cb}(z)\right)\cdot\left(\frac{b}{2} - \lambda(z) + a_{cb}(z)\right)\right] + \frac{t_w h_w^3 a_{c.i.}(z)^2}{2}, \quad (4)$$

here $a_{cb}(z) = {t_f b_f \lambda(z) h_w^2}/{2J_x}$ – the current distance between the centre of bend of the section and the centre of the wall (Fig. 1); $\lambda(z) = f \cdot sin\left(2\pi * \frac{z}{\ell}\right)$ – the current distance from the centre axis of the girder to the centre of the wall.

The system of differential equations with nonlinear coefficients (1) has no analytical solutions.

It should be noted that the second differential equation taken for consideration in system (1) has a complete analogy with the differential equation of bending of straight rods with tension.

Having experience in developing a technique for numerical integration of the bending differential equation of straight rods with tension in the MS Excel spreadsheet [14], we break down the system of two fourth order differential equations (1) into eight first order differential equations with a consistent differential relationship between them:

$$\begin{aligned}
&Q_y'(z) = -q_y(z),\\
&M_x'(z) = Q_y(z),\\
&\varphi_x'(z) = \frac{M_x(z)}{EJ_x(z)},\\
&V'(z) = -\varphi_x(z),\\
&H'(z) = m(z),\\
&B'(z) = H(z) + GJ_t(z)\cdot D(z),\\
&D'(z) = \frac{B(z)}{EJ_\omega(z)},\\
&\varphi_z'(z) = -D(z),
\end{aligned} \quad (5)$$

here: $Q_y(z)$ – internal transverse force in beam section;

$q_y(z)$ – load distributed along the length, generally variable along the length, with the possibility of incorporating concentrated forces;

$M_x(z)$ – internal moment relative to the axis *x*;

$\varphi_x(z)$ – section angle of the girder cross-section in relation to the axis *x*;

$V(z)$ – beam deflection along the axis *y;*

$H(z)$ – internal torque;

$m(z)$ – the distributed external torque, calculated according to the formula (6);

$B(z)$ – internal bimoment in the girder;

$D(z)$ – sectional deplanations;

$\varphi_z(z)$ – section angle of the girder cross-section in relation to the axis *z.*

Using the well-known Euler scheme for numerical integration of ordinary differential equations of the first order and assuming sufficient accuracy of integration, system (5) is transformed from a differential form to a finite-difference one.

The order of the transformation is shown in Table 1.

Table 1. Conversion of the system of equations (5) into finite-difference form

no.	Differential equation	Finite-difference equation
1	$Q_y{}'(z) = -q_y(z)$	$Q_{y_{(i+1)}} = Q_{y_i} - q_{y_i}\Delta l - P_i$
2	$M_x{}'(z) = Q_y(z)$	$M_{x(i+1)} = M_i + Q_{y_i}\Delta l - M_{in.i}$
3	$\varphi_x{}'(z) = {M_x(z)}/{EJ_x(z)}$	$\varphi_{x(i+1)} = \varphi_{x_i} + {M_{x_i}\Delta l}/{EJ_{xi}}$
4	$V'(z) = -\varphi_x(z)$	$V_{i+1} = V_i - \varphi_{x_i}\Delta l$
5	$H'(z) = -m(z)$	$H_{(i+1)} = H_i - m_i\Delta l - H_{in_i}$
6	$B'(z) = H(z) + GJ_t(z)D(z)$	$B_{i+1} = B_i + [H_i + GJ_{t_i}D_i]\Delta l - B_{in.i}$
7	$D'(z) = {B(z)}/{EJ_\omega(z)}$	$D_{(i+1)} = D_i + {B_i\Delta l}/{EJ_{\omega_i}}$
8	$\varphi_z{}'(z) = -D(z)$	$\varphi_{z_{i+1}} = \varphi_{z_i} - D_i\Delta l$

Table 1 introduces the following designations:

i – current integration step ($i = 0 \ldots n$ - According to the diagram in fig. 1 the value $i = 0$ corresponds to the origin $z = 0$, the value $i = n$ corresponds to the right-hand end of the beam $z = L$);

$i+1$ – next step of integration;

$\Delta l = L/n$ – the value of the integration step, n is the number of divisions of the span value L or the number of integration steps;

P_i – is the value of the external concentrated load acting at the i-th integration step;

B_i – is the value of the bi-moment acting at the i-th integration step (presence of support ribs, stiffening ribs).

The external torque for the current coordinate z is determined by the following formula:

$$m(z) = q_y(z) \cdot [e + a_x(z) + (\pm y) \cdot \varphi_z(z)] \tag{6}$$

here e – the random eccentricity of the external load, specified by the calculator if necessary;

$a_x(z) = a_{cb}(z) + \lambda(z)$ is the distance between the bend centre of the section and the vertical axis of the girder/load application axis;

$(\pm y) \cdot \varphi_z(z)$ – additional eccentricity of the load created by rotating the girder section by an angle $\varphi_z(z)$ when the load is applied to the upper chord $y = +h/2$, to the lower chord $y = -h/2$, and in the middle of the section height $y = 0$.

The adopted scheme for writing down eight differential equations in finite-difference form (Table 1) is conveniently formalized in a commonly available MS Excel spreadsheet processor.

In this case, in accordance with the computational scheme and fixing conditions (Fig. 1), the following initial parameters are taken as known $M_x(0) = V(0) = \varphi_z(0) = B(0) =$

0, the unknown/variable parameters are the values $Q_y(0)$, $\varphi_x(0)$, $D(0)$, $H(0)$, which should deliver zero at the end of the girder span $Q_y(L) = 0, V(L) = 0; B(L) = 0;\ \varphi_z(L) = 0$.

The described algorithm for finding the unknown initial parameters in MS Excel is performed automatically using the built-in "Solution Search" procedure.

The developed technique of integration of the system of differential equations of cross bending of beams with torsion (1) allows one to solve the following problems:

- calculation of stresses and deformations of beams of any cross-section variable in length, including corrugated beams, taking into account section deplanation;
- calculation of critical loads on the overall stability of beams.

It is also possible to calculate beams without support ribs, with support ribs and compressed chord spreads from the bending plane.

In particular, if there are no support ribs, the bimensions at the beginning and at the end of the beam are zero, and if there are support ribs, the bimensions at the ends are determined by the formula (p. 236, [13]):

$$B(0) = B(L) = \frac{E t_{sr}{}^3 \times 2 b_{sr} h_{c.gr}}{12(1+\mu)}\, \varphi_z{}'(0), \tag{7}$$

where t_{sr} is support rib thickness, b_{sr} – support rib width, $h_{c.gr}$ – distance between the centres of gravity of the chords, E, μ - modulus of elasticity and Poisson's ratio of the applied steel.

Presence of compressed chord bracing at the coordinate z_i is modelled in the "Solution Search" procedure by additionally searching for reactive torque in this section H_{in_i}, The authors have developed an Excel template for automated integration of the system of equations in Table 1, where all the boundary conditions for the search procedure "Find Solution" are already included.

The authors have developed an Excel template for automated integration of the system of equations according to Table 1, where all boundary conditions and search conditions of "Solution Search" procedure are already included. To find a solution, all you have to do is enter the raw data and run the "Solution Search" procedure.

The solution will automatically generate transverse force diagrams in the template $Q_y(z)$, of bending moments $M_x(z)$, the angle of rotation of the girder sections in relation to the axis x $\varphi_x(z)$, deflection $V(z)$, of torque in the section $H(z)$, bimoment in the cross section $B(z)$, section deplanation parameter $D(z)$, section angle in relation to the axis z $\varphi_z(z)$.

The technology of calculation of parameters of the stress-strain state of a beam under bending with torsion with a given load in the developed Excel templates consists in a direct integration of the system of equations (1) according to the developed technique.

The solution of a problem of searching the critical load of loss of overall stability of a beam requires the additional condition of searching the maximum load giving an uncertainty of a small rotation of the beam relative to the axis z given a small perturbation, for example, in a form of initial eccentricity e or presence of unpaired waves of a sinusoidal wall of a corrugated beam (a method of nonidealities) by analogy with the solution of problems of searching the critical load of longitudinal stability of beams in central compression.

3 Calculation results

Calculations of critical uniformly distributed load for corrugated beam according to scheme in Fig. 1 with parameters according to Table 2 have been carried out according to methodology described above:
No.1 - without support ribs when load is applied to upper chord of beam, central axis, lower chord;
No. 2 - when the load is applied to the upper chord without support ribs and with support ribs of different thickness;
No. 3 - when the load is applied to the compressed chord with support ribs of different thickness t_{sr} = 8 mm with compression girder bracing in the middle of the span *L/2*.

When calculating the critical loads, the eccentricity of the load application was taken as a small disturbance in the calculations e = 0.1…1.5 mm.

For all calculations, the critical load calculation was verified in PC LIRA CAD with the corrugated beam modelled using Table 2 shell elements.

Table 2. Dimensions of the design corrugated beam

Size	Value
Beam span L, mm	6000
Wall height h_w, mm	249
Wall thickness t_w, mm	5,1
Shelf width b_f, mm	100
Panel thickness t_f, mm	6
Amplitude of corrugation f, mm	20
Corrugation wave spacing$l$, mm	145

The results of the calculations are presented in Table 3.

Table 3. Calculation results

no.	Loading conditions		Critical load in an Excel templatecr, tf/m	Critical load in PC LIRA-SAPR q_{cr}, тс/м	Calculation error compared to LIRA CAD data, %
1	Load to the upper chord		0.2586	0.2692	4.03
	Load towards the centre axis		0.3324	0.3509	5.29
	Load to the lower chord		0.4300	0.4506	4.57
2	Load to the upper chord	t_{sr} = 8 mm	0.2823	0.2880	1.98
		t_{sr} = 20 mm	0.3543	0.3685	3.85
		t_{sr} = 50 mm	0.4313	0.442	2.41
3	Load to the upper chord, t_{sr} = 8 mm with compressed		0.9751	0.9833	0.83

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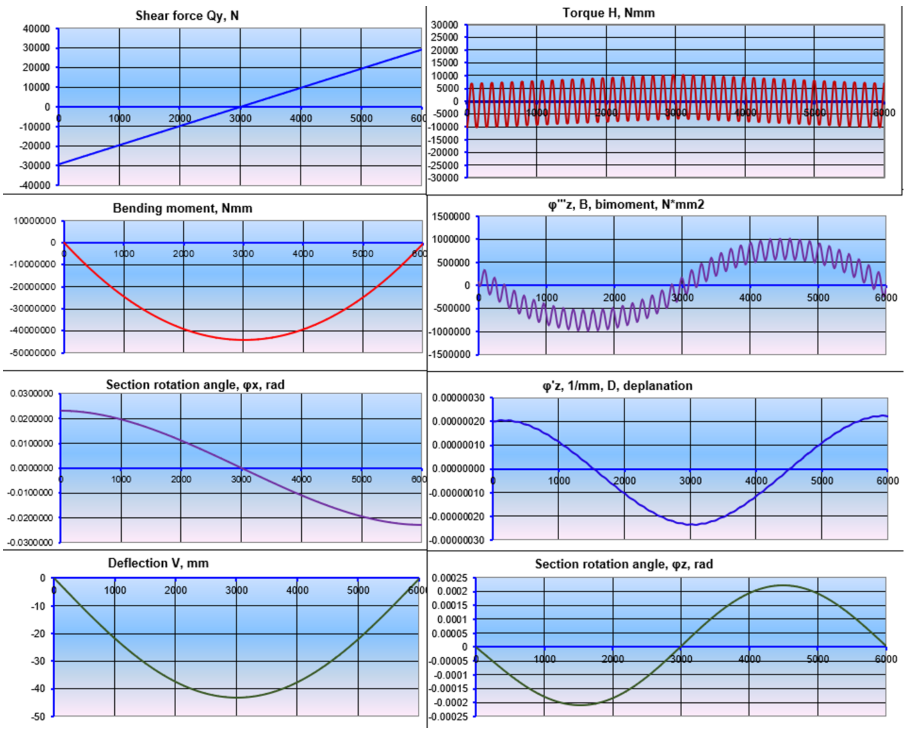
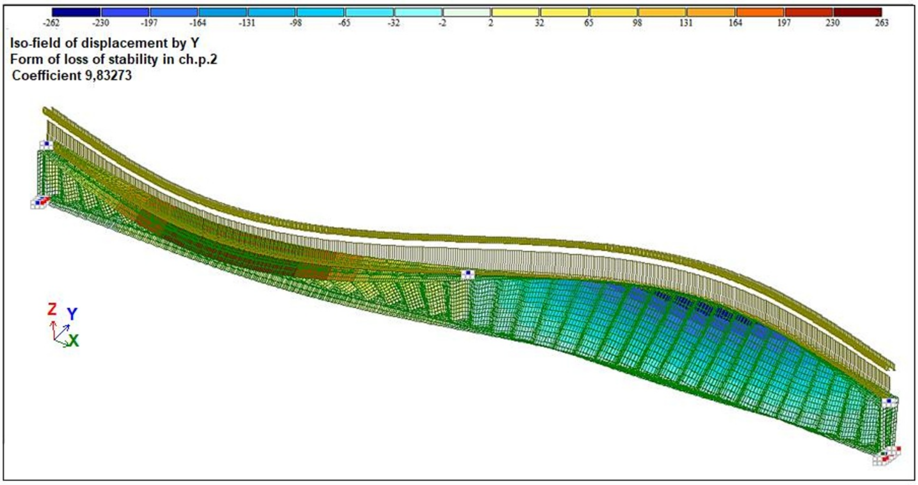
	chord bracing in mid-span			

The graphical results of calculation no. 3 are shown in Fig. 2.



shear force
torque
bending moment, Nmm
bimoment, mm
section rotation angle, rad
mm, deplanation
deflection, mm
section rotation angle, rad

Fig. 2. internal transverse force diagrams in the beam sections $Q_y(z)$, internal bending moment $M_x(z)$, section angles $\varphi_x(z)$, axle deflection $V(z)$, internal torque in the sections $H(z)$, bimoment in corrugated beam $B(z)$, section deplanations $\varphi_z'(z)$, section rotation angles $\varphi_z(z)$



upload
form of loss of stability in ch.p.2
iso-field of displacement by
coefficient

Fig. 3. Graphic result of finding the critical load in PC LIRA-SAPR for calculation no. 3 according to Table 3 q_{cr} = 0.1*9.833 = 0.9833 tf/m

The results of the calculations in Table 3 show that the difference between the values of the critical forces q_{cr}, The calculation of the total number of the required values determined by the proposed method compared to those of the LIRA-SAPR software does not exceed 5.3 %. It is considered that the results obtained are within the accuracy limits of the engineering calculations.

As an example of implementation of the proposed methodology for calculating stresses, section angles at bending and torsion of thin-walled beams, the results of calculation of a sectional steel beam made of a bent channel (Fig. 4) with a span of L = 8.0 m, loaded with a uniformly distributed load q_y = 1.5 tf/m, applied in the middle of the width of the compressed flange with eccentricity relative to the centre of bend e = 419 mm [15].

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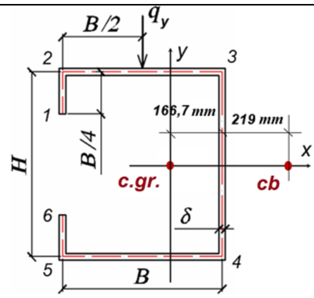
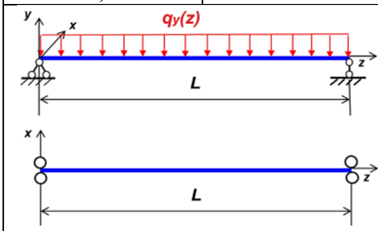


B, mm	400
H, mm	440
δ, mm	16
q, tf/m	1.5
E, MPa	200000



mm	
c.gr.	
cb	

Fig. 4. Calculation diagram and purlin parameters

As a result of the calculation of the purlin in the developed Excel template, the internal transverse forces in the sections shown in Fig. 5 are obtained. The internal transverse forces in the cross-sections of the purlin $Q_y(z)$, internal bending moment $M_x(z)$, section rotation angles $\varphi_x(z)$, axle deflection $V(z)$, internal torque in the sections $H(z)$, bimoment in the run $B(z)$, section deplanations $\varphi_z'(z)$, section rotation angles $\varphi_z(z)$.

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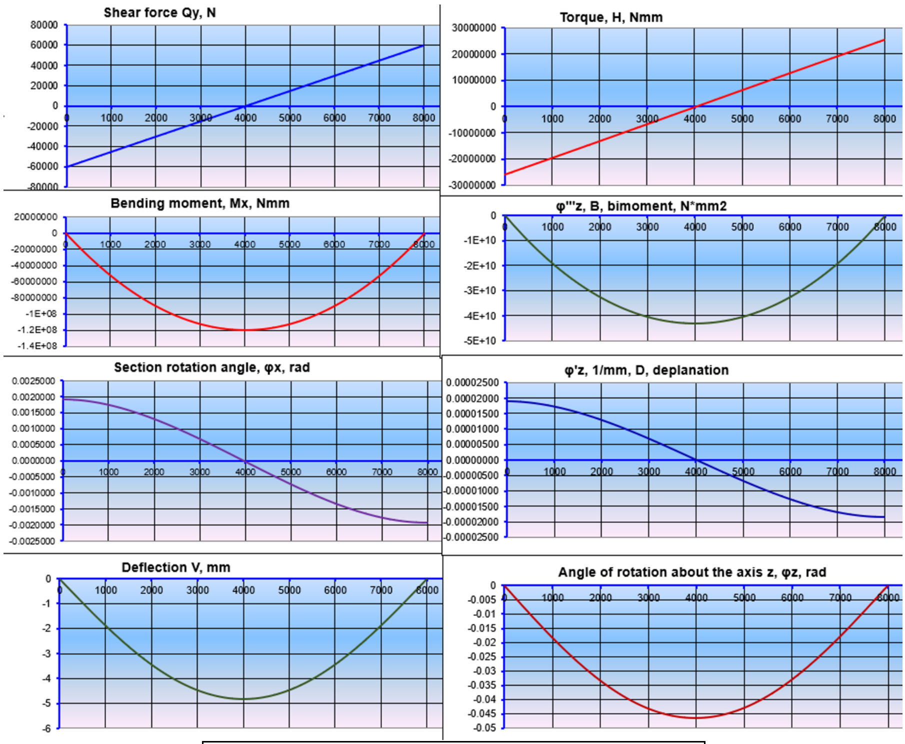

shear force
torque
bending moment, Nmm
bimoment, mm
section rotation angle, rad
mm, deplanation
deflection, mm
angle of rotation about the axis, rad

Fig. 5. internal transverse force diagrams in the purlin sections $Q_y(z)$, internal bending moment $M_x(z)$, section angles $\varphi_x(z)$, axle deflection $V(z)$, internal torque in the sections $H(z)$, bimoment in the run $B(z)$, section deplanations $\varphi_z'(z)$, section angles $\varphi_z(z)$

Table 4 compares the main design parameters of the run obtained by analytical calculations [17] and the proposed methodology.

Table 4. Calculation results for the run

Calculated parameter	Analytical solution [17]	Solution to the proposed methodology	Calculation error compared to the analytical solution [17], %
$Mx_{max}(L/2)$, Nmm	120000000	120000000	0.00
$V_{max}(L/2)$, mm	4.829	4.829	0.00
$B_{max}(L/2)$, Nmm 2	-41827000000	-43165170128	3.20
$\varphi z_{max}(L/2)$, rad	-0.0480	-0.0466	2.89

Knowing the maximum values of internal forces, deflections and rotation angles of the purlin sections (Fig. 5, Table 4) it is possible to calculate the most loaded sections for strength and stiffness.

For example, the strength test at points 1÷6 (Fig. 4) of the most loaded section in the middle of the span in terms of normal stresses is performed according to the known formula:

$$\sigma_{max} = \frac{M_{xmax}}{J_x} \cdot y + \frac{B_{max}}{J_\omega} \cdot \omega_{cb} \leq R_y \cdot \gamma_c$$

here y – is coordinate of points along the section height;

ω_{cb} determines the values of the sectional coordinates at the calculated points of the section in relation to the bending centre (sectional areas) [17];

J_x, J_ω – moment of inertia of the section relative to the x-axis, sectorial moment of inertia of the section [17];

R_y, γ_c – is design resistance of purlin steel and a working condition factor.

4 Conclusions

The authors have developed and implemented a technique for calculating the stress-strain state of thin-walled beams, in particular, corrugated beams under transverse bending with torsion, in a generally available MS Excel spreadsheet processor. The proposed technique makes it possible to calculate the critical loads of the general stability of beams in cross-section bending taking into account the deplanation of the sections.

The practical value of the work consists in the fact that the Excel template of automatic integration of the system of differential equations of bending with torsion for beams including the thin-walled allows to calculate the parameters of the stress-strain state and the calculation of the critical loads of the general stability loss for beams including corrugated beams with sufficient accuracy for engineering calculations without turning to finite-element complexes.

The developed algorithm, for the first time, takes into account such features as:
- introduction of variable under the given law distributed load along the length of beam $q_y(z)$ with possibility of including the concentrated forces *P* in any section of the beam;
- to apply the loads at any level of the beam height including upper and lower chords with any given eccentricity *e*;
- specifying a beam cross-section variable by a given law along its length, including the geometric parameters of the corrugated beam wall, the parameters of beams with perforated walls;
- the possibility of installing support ribs, intermediate stiffeners of given dimensions with assessment of their impact on the stress-strain state and critical loads of the beams overall stability loss has been taken into account;
- the possibility of installing the reinforcement of the compressed chord in the span is taken into account.

For all the calculations given in the article, verification calculations in PC "LIRA-SAPR" are performed: the difference between the parameters of the stress-strain state, critical loads determined by the proposed method and in PC LIRA-SAPR does not exceed 5 %.

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