Refinement of fatigue curve parameters using fuzzy set theory and tomography images processing of censored composite samples

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Abstract. The fatigue curve is important in engineering practice. Since the task of constructing a fatigue curve is characterized, among other things, by the presence of qualitative (not numerical), inaccurate and incomplete information, the question arises of the appropriateness of applying the theory of fuzzy sets. To clarify information on censored, namely, removed from testing upon reaching the test base specimens, the use of computed tomography to study the degree of their damage is proposed. A fuzzy regression model has been developed to consider linguistic variable that describes the status of a sample (broken or censored). With fuzzy dependent variables of the of regression model, the coefficients of the equation also turn out to be fuzzy. For the fuzzy dependent variable of the time before failures of censored samples, the degree of damage is additionally evaluated based on the rating of experts by the analysis of tomography images. The model made it possible to estimate the interval of expected life for a given stress amplitude using an uncertainty accounting method that differs from the classical statistical description. An example of constructing a fuzzy fatigue curve for polymer composite materials is considered.

1 Introduction

Linguistic variables or objects of non-numerical nature are elements of a non-linear space. Examples of qualitative scales are ranking, zoning, and experts' review of data. These objects cannot be added and multiplied by numbers without losing their meaningful meaning. In this way they differ from the numerical data used for a long time in applied statistics [1].

The task of considering information related to censored (i.e., not broken before the end of testing) samples when constructing a fatigue curve is directly related to the concept of a linguistic variable, since there are two categories of samples – the broken and censored. The fatigue curve plays important role in the comparative evaluation of the materials

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quality and in service life evaluation using fatigue damage accumulation hypotheses. At the same time, fatigue tests are costly, also in terms of research time. Therefore, upon reaching a predetermined test base, the test sample is usually removed from the testing machine and sent, at best case, for the storage in a container or disposed. In our case we propose the special analysis of the censored samples.

The fatigue curve is most often built using regression analysis only from information about the failed samples. This approach does not seem justified since information about the time to censorship is also important. After all, it indicates that the sample did NOT break before the test base is reached, i.e., this information is informative and must be used to increase the reliability of the estimation.

The problem of estimation by single-censored samples obtained during fatigue testing of PCM (Polymer composite material) specimens using mathematical methods is considered in [2]. The work [3] is devoted to the problems of assessing the quality of PCM. The authors of [4] constructed the prediction intervals for hybrid right censoring with possible left censoring data using suitable pivotal quantities and intervals based on the highest density. In [5] the authors investigated the motor position sensor is one of the monitoring targets in urban air mobility control. They used the sigmoid functions for fault detection and fusion method robust against sudden censoring. The robustness of estimate was demonstrated by simulation and experimental results.

The problem of statistical uncertainty increases many times in the problems of studying composites. For several reasons, these materials are characterized by a significant spread of properties even during static tests. This problem is exacerbated during fatigue testing. The slope coefficient of the fatigue curve m reaches significant values for composites (m= 12 ... 25 for composites and only m = 5 ... 10 for metals). The fatigue curve “flattens out”, i.e., with a slight change in the stress amplitude, the average durability values change significantly. How else can this phenomenon be called if not a scatter? The cloud of points resulting from the construction of the fatigue curve of composites allows (not without reason) some researchers to assert that "composites do not have fatigue."

During operation and fatigue testing of PCM products, a transition from microdefects to macro defects is observed. The most common PCM defect is porosity. The reasons for its occurrence can be very different and can greatly depend on the technology of their production. Due to the practical importance of the problem of the strength of polymer composite materials, extensive literature is devoted to the study, diagnostics, and collection of information about defects in composites. The contemporary method of research is ultrasound (see, for example, [6]). At the same time, research using computed tomography is becoming increasingly important. Ultrasonic and tomographic studies are important both in the control of the production of composites and in the diagnosis of failures. The main types of manufacturing defects are systematized in [7]. There, the main types of micro-, mini- and macro-defects in PCM are considered. The main emphasis in the work is on up-to-date research. It is shown here that tomography made it possible to create a three-dimensional model of an object and evaluate not only the integral porosity of the matrix (as traditional methods do), but also the distribution of pores in size and space.

2 Methodology and materials of the experiment

The object of research in this work is a composite material consisting of carbon fiber, the matrix of which is an epoxy binder. The fatigue curve (Figure 1) for miniature specimens (1x10mm cross section) under compression was built using INSTRON testing machine on the base of 10e5. As it can be noticed from the Figure 1, the scatter is large. There is also a high percentage of censored (not broke during testing) samples. To overcome the problem with uncertainty with censorship and scatter, a special methodology with fuzzy regression model based on processing the tomographic images was developed. In [8-11] the examples of such methodology in varied scientific fields are described. The paper [12] describes the developed model of fuzzy control of the production capacity of a natural gas liquefaction complex based on the theory of fuzzy sets.

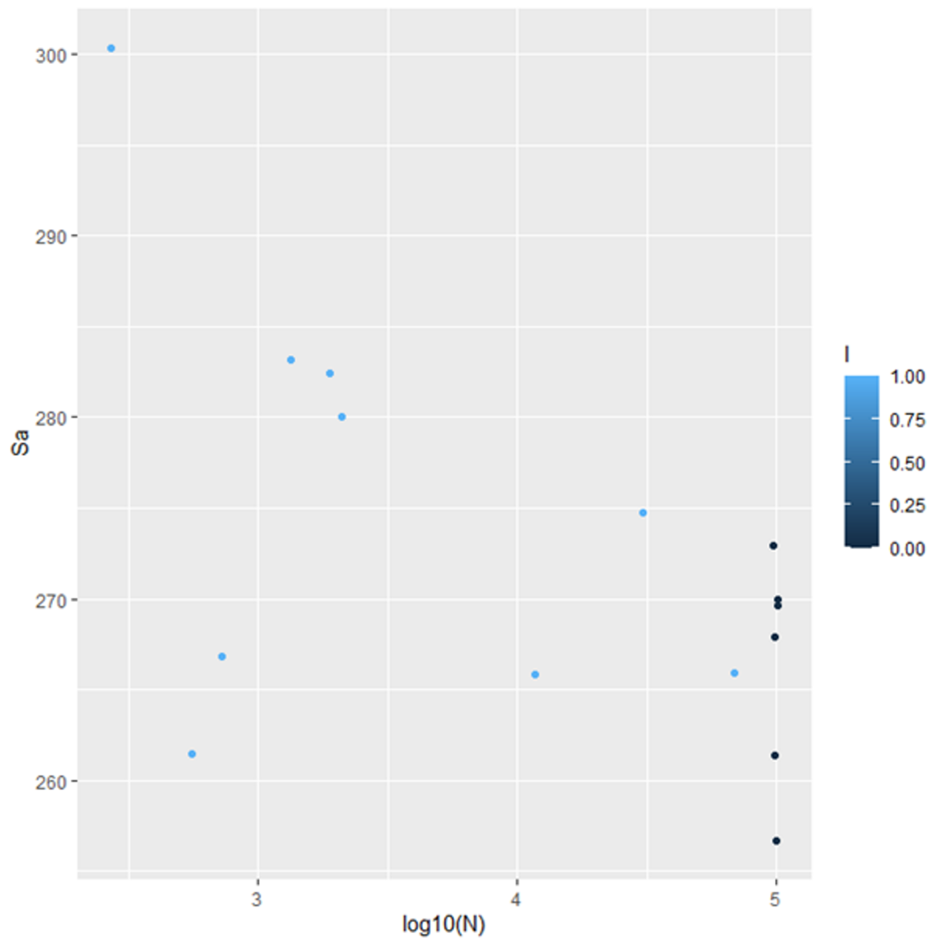

Fig. 1. The fatigue data for PCM specimens. Brite dots are breacking, dark dots are censoring.

Fuzzy regression model. To consider uncertainty when constructing a fatigue curve with censored outcomes, we developed a fuzzy regression model. The task of estimation on censored samples is supplemented by the study of damage to censored samples, recorded on computed tomography images. It is assumed that damage is due to the development of microdefects under cyclic loading and is a measure of the degradation of the properties of censored samples. The experts assigned ratings (R = 0, 1 ... 10) to censored samples as a measure of their damage based on the analysis of tomographic images. Since ratings values were used in the model (i.e., non-numerical information [1]), a fuzzy regression modelling model, like in [13], was developed. The fuzzy regression model included fuzzy outputs and equation parameters, but crisp inputs. A model of this type allows you to set limits on the change of random variables, which are somewhat alike the confidence intervals used in statistics but have their own characteristics and the different scope.

The fuzzy regression model was previously applied by a few authors in applied problems of various profiles. In [14], a distributed fuzzy detector of the maximum average level with censorings is considered - a detector of a constant frequency of false positives. In [15], a fuzzy regression model is used to evaluate functional relationships between dependent and independent variables in a fuzzy environment. In [16], a fuzzy logic controller was developed in MATLAB software with the Fuzzy Logic Toolbox package. The controller has been developed based on expert knowledge about the corrosion processes of on-board systems. The specified controller made it possible to determine the reliability of air weapons based on input signals in the form of equipment operation time (number of flying hours), the number of cannon shots fired, and the state of equipment corrosion. The paper [17] proposes the use of fuzzy probability theory to account for the uncertainty and the prior knowledge of the process in the parameters′ estimation in the case when only censored data are presented. In [18] the authors proposed to learn rules by maximizing the likelihood of the classifier. In this way they extended the methodology to interval-censored data and propose using upper and lower bounds of the likelihood to evolve rule bases. The authors of [19] present an exploratory analysis methodology for multiple series of multivariate temporal data subject to censoring, and thus requiring the introduction of a coding technique: fuzzy coding preserving a large amount of the distributional information is fully adapted.

The article [13] posed the problem of multiple fuzzy linear regression as applied to the analysis of agricultural production management processes. The author of [13] used n results of observations of the dependent variable y_j, $j=1...n$, as well as $m\times n$ values of independent crisp variables x_{ij}, $i=1...m; j=1...n$ (generally). The resulting variable was searched in the form of a fuzzy multiple linear regression equation:

$$y_{\text{fuzzy i}} = A_{\text{fuzzy 0}} + A_{\text{fuzzy 1}}\, x_{1i} + \ldots + A_{\text{fuzzy m}}\, x_{mi} \qquad (1)$$

where x_i — are the crisp, and $y_{\text{fuzzy i}}$ — are fuzzy functions.

The group of fuzzy parameters $A_{\text{fuzzy i}}$, $i=0, 1 \ldots. m$ was searched based on several conditions. The problem was solved by the linear programming method with assigning several values of the reliability threshold h [13].

Method. In the proposed method we use the two-parametric form of the fatigue curve (Basquin) without a horizontal part:

$$Y= A- m\, x \qquad (2)$$

where A and m – are the factors of the regression curve estimated by the testing data, $x= lg (Sa)$; Sa – amplitudes of alternating stresses in fatigue tests: x - independent variable; $y = lg(N)$, N- number of cycles to sample failure, response, dependent variable. The result of fatigue testing is shown in Figure 1.

To refine the estimate of the fatigue curve, we proposed to introduce data about the censored samples in a special way into equation (2) and supplement the equation with fuzzy logic concepts formed based on tomography analysis. We will look for the resulting variable y in the form of a fuzzy ("fuzzy") linear two-parameter regression equation

$$lg\ N_i = A_{\text{fuzzy}} - m_{\text{fuzzy}}\ lg\ S_{ai} \quad (3)$$

where A_{fuzzy}, m_{fuzzy} – are the equation coefficients to be determined. Like in [13], the independent variables $x= lgSa$ are deterministic. In our case, some of the responses y are crisp (corresponding to failures during fatigue tests), and for censored samples responses are fuzzy, since the true durability of the samples is not known for censored data. It happened because the tests for these samples were stopped upon reaching the test base. In the example below, the censoring threshold is $N_b= 10^5$ and $y_b= lg\ (N_b) = 5$.

So, the equation (3) splits into two, depending on the value of the linguistic variable I, which determines the status of the sample. If I_i =1, then a failure occurs and equation (3) takes the form:

$$lg\ N_{\text{crizp},i} = A_{\text{fuzzy}} - m_{\text{fuzzy}}\ lg\ S_{ai} \text{ and } I_i = 1 \text{ for } i = 1, 2 \ldots. k \quad (4)$$

where N_{crizp} – number of cycles to failure, crisp value; k – number of break sample while building the fatigue curve.
if I_i =0, then censoring (suspension) takes place and equation (3) takes the form:

$$lg\ N_{\text{fuzzy},i} = A_{\text{fuzzy}} - m_{\text{fuzzy}}\ lg\ S_{I} \text{ and } I_i = 0 \text{ for } i = k+1, \ldots.\ n \quad (5)$$

where n – is the total amount of specimens in the test.
$N_{\text{fuzzy},i}$ – is a fuzzy value in interval:

$$N_{\text{fuzzy},i} \in (N^*_{,i}\,;\ \infty] \quad (6)$$

and N^*, 1_i – is the cycle number before censoring for specimens with indexes $i = k+1, \ldots.\ n$.
We define the membership function as in [1,13]:

$$\mu_i \in [0,1] \quad (7)$$

for censored samples with indices $i = k+1, \ldots.\ n$ as one-sided triangular fuzzy number, Figure 2 [20].

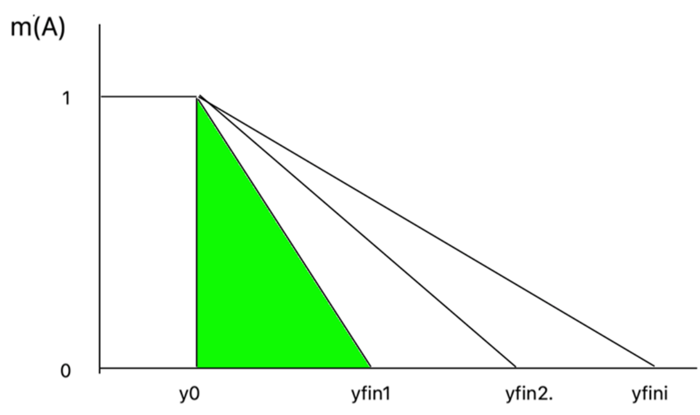

Fig. 2. Fuzzy one-sided triangular number m(A) (the space under triangular fuzzy number shaded green for hypothetical sample 1).

The equation of triangular number is written in the form (8):

$$\mu_A(y) = \begin{cases} \frac{y_{fin}-y}{y_{fin}-y_0} & ,if\ y > y_0 \\ 0, & overwise \end{cases} \quad (8)$$

where y_0 – is the censoring threshold, that is the logarithm of testing base N_b. In our example $N_b = 10^5$ and $y_0 = \lg 10^5 = 5$.

The fuzzy sets Y obtained on the basis of fuzzy parameters must contain the observed values y_i with a degree of confidence not less than a certain degree h (μ_i (y_i) $\geq h$). Since infinite durability is not noticed for composites even under the condition of testing with ultrasonic frequencies [21], then in expression (6) the infinity is replaced by a finite number of N+ = 10^y_{fin} cycles:

$$N_{fuzzy,i} \in (N^*_{,i}\,;\, N+] \quad (9)$$

where $N+ > N_b$, are the fuzzy number in interval.

3 The results of experiments

As the task of this paper was to incorporate the data of relative damage of censored samples into fatigue curve estimation, the tomographic study of censored specimens was performed. We use the industrial tomograph WG v|tome|x m300.

Let us supplement the membership function (8) with information obtained from the processing of images from industrial tomography. Figures 3 and 4 demonstrate some tomographic images of censored samples. Their ratings were assigned by experts according to their damage degree. When assigning ratings, the fact that a crack is the most dangerous option from the point of view of the possibility of destruction was considered, and the presence of micro pores does not necessarily lead to the formation of macro defects [22].

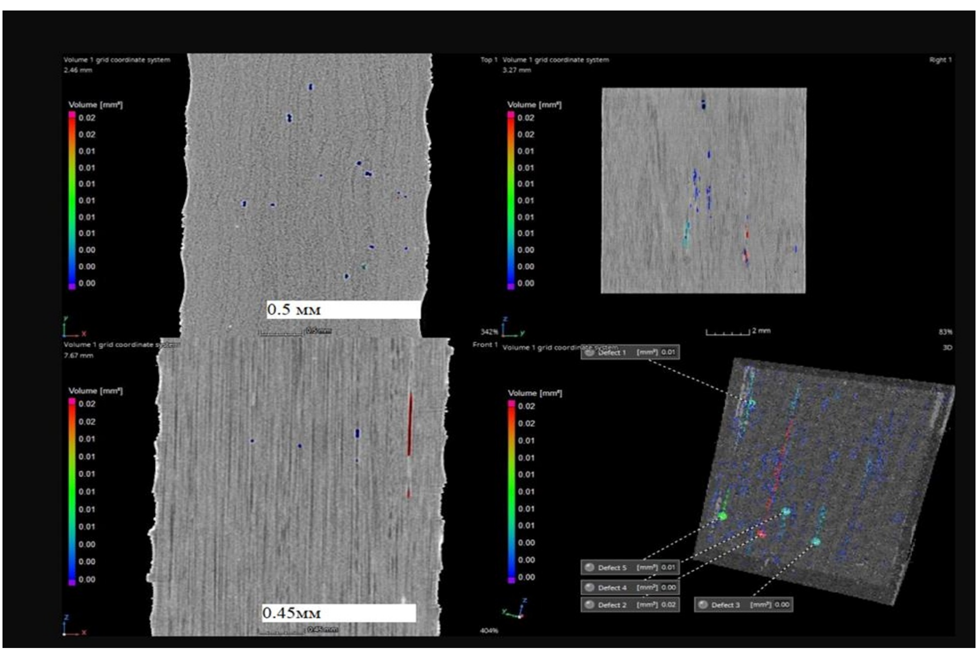

Fig. 3. Relatively "favorable" censored sample No. 9.

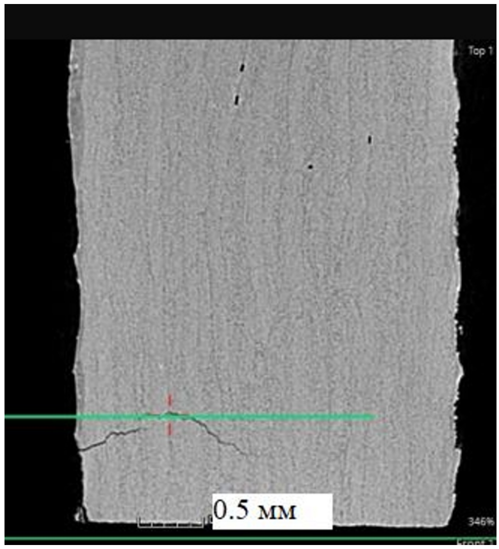

Fig. 4. **"**Unfavorable" censored specimen No. 16. A transverse crack was found. Expert rating is "0". The state of the specimen is critical.

In Table 1 the summary of expert's rating for all censored samples under investigation are presented.

In Figures 3 and 4 the computation proceeded tomography images of specimens are shown. 2D sections of the specimen 9 are shown in Figure 4. Porosity analysis was performed. The color scale shows the size of the pores. Not all the pores develop into macro defects [22]. The volume of the red pore is 0.02 mm3. The 3D model is in the lower right corner. A woven sacrificial (technological) layer is visible. Expert rating for specimen 9 is "6". Figure 4 demonstrates **"**Unfavorable" censored specimen No. 16. A transverse crack was found. Expert rating is "0". The state of the specimen 16 is critical.

Table 1. The main detected defects and ratings for censored samples according to the experts.

Censored sample index	9	14	15	16	18	20	21	22
Main defect	Pores	Transverse crack	Pores and inclusion in the form of a hair	Transverse crack, delamination and metallic inclusion	Layering	Pores	Big pores	Multiple layering
Damage rating (according to experts)	6	0	3 … 4	0	3	2	1	1

4 Results of experimet and discussion

To find fuzzy coefficients m_{fuzzy} and C_{fuzzy} of fuzzy regression equation (4, 5), we use the least squares method to minimize the deviation:

$$f = \frac{1}{n}\sum_{i=1}^{n}[y_i - \mathrm{def}(\tilde{Y}_i)]^2 \quad (10)$$

where y_i are partly the fuzzy responses for crisp factors x_i evaluated by equations (4, 5) and $\tilde{Y}_i$ are the defuzzificated [20] (the closest) crisp value of y_i. Since the system (10) contains more unknown when equations, the decision is being looked for by annealing method [23] or by genetic algorithm [19] for numerical calculation.

The ratings obtained based on expert assessments on tomographic images were used to evaluate the right boundaries of triangular fuzzy numbers (Fig. 2). In Table 2 the most probable values for N+ (equation (8)) of triangular number are shown.

Table 2. Upper bound of the triangular fuzzy number y_{fin} (corresponding to the maximum possible durability for censored samples with rating R)

R	0	1	2	3	4	5	6	7	8	9	10
y_{fin}	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10	10.5

While assigning y_{fin} the data from literature were used, for example [16].

Further, in the R programming software environment, two regression lines were constructed along the left and right sides of the triangle (LOW) and (HIGH). Table 3 shows the durability intervals for three values of stress amplitudes Sa, MPa. When evaluating the boundaries of fuzzy durability numbers, the LOW and HIGH regression lines were used.

Table 3. Fuzzy limits for the fatigue domain

Sa, MPa	*N low*	*N high*
281.8	4168	7413
269.1	2.57e4	1.73e5
257.0	8.51e4	1.41e6

The obtained values of *N low* and *N high* (Table 3) can be considered as like the confidence intervals in the reliability analysis. The main difference lies in the interpretation of the original data and the mathematical formulation.

Comparison of the results of digital radiography [24] of aircraft objects made with the help of additive technologies showed that the performance of control increases in comparison with traditional radiography. Since digital radiography is a more productive method compared to tomography, in the future it is planned to compare these two methods to solve the problem with censoring.

5 Conclusions

- For the first time, a method has been developed that allows using physical research methods (studies of computed tomography images) to achieve a more accurate assessment of the fatigue curve under conditions of censoring and significant data scatter.
- A fuzzy regression model has been developed that considers the presence of censorings (suspensions) and additional information about the state of censored samples.
- For the example of fatigue testing of PCM specimens, the boundaries of a fuzzy number of durability (analogous to a confidence interval) were estimated.

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