Verification of lira-sapr calculation of i-beams with two symmetry axes for general stability in transverse bending in the plane with maximum stiffness

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Abstract. The paper considers verification of calculation of split I-beams with two symmetry axes on the general Eulerian stability at transverse bending in the plane of maximum stiffness in the software package (PC) LIRA-SAD. Calculations for various loads are given. Numerical calculation results in PC LIRA-SAP are compared with calculation by analytical formulas of S.P. Timoshenko and V.Z. Vlasov. Conclusions about calculation accuracy in PC LIRA-SAD in comparison with analytical formulas analysis are made.

1 Introduction

When calculating building structures in modern software packages using the finite element method (LIRA CAD, SCAD office, etc.) the verification of calculations is an important task. One of the easiest ways of calculation verification is comparison of results obtained by numerical simulation and by analytical expressions.

In "Verification report on LIRA-SAD software" (pp. 78-68, [3]) considered the question of verification of calculation of rods for general stability at transverse bending. Here the authors consider the static calculation of a rod model that does not take into account the sectional moment of inertia of the section, i.e. the stability calculation is performed without taking into account the deplanation of the sections (no support ribs, no constriction torsion at the loss of stability). This means that the LIRA CAD software inadequately reflects the performance of thin-walled rods under torsion that accompanies the loss of beam stability.

It is obvious that the rod model does not fully correspond to the real operation of an I-beam at the stage of stability loss.

In this paper an attempt is made to calculate the overall stability of I-beams in the PC LIRA-SAPR when modelling the flanges, wall, support ribs by shell finite elements which is not considered in the verification report [3].

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Thus, the question of calculation of real I-beams on the general stability in PC "LIRA-SAPR" remains open.

For the first time the question of general stability of I-beams at transverse bending is considered in the work of academician S.P. Timoshenko "Stability of elastic systems" [1]. In this paper the author obtained formulas for determining critical loads on a rod element subjected to compression and bending in two planes. As a special case, [1] gives an expression for the calculation of general stability of a sectioned I-beam, the ends of which are subjected to bending moments in the plane of maximum rigidity.

On the calculation of thin-walled rods and, in particular, I-beams was dealt with by Professor V.Z. Vlasov. In his paper "Thin-walled elastic rods" [2] he developed a bending and torsional model of thin-walled rods, and showed the effect of support and intermediate ribs on the torsional stiffness of the rods. Also, in [2] general conditions of stability of thin-walled rods in transverse bending are considered and expressions for determining the critical force applied in the middle of the span and the critical uniformly distributed load in the plane of maximum stiffness for a sectional I-beam with two symmetry axes are given.

In [8 - 10] authors conducted researches to clarify methods of calculation of I-beams for general stability in transverse bending in the plane of maximum stiffness.

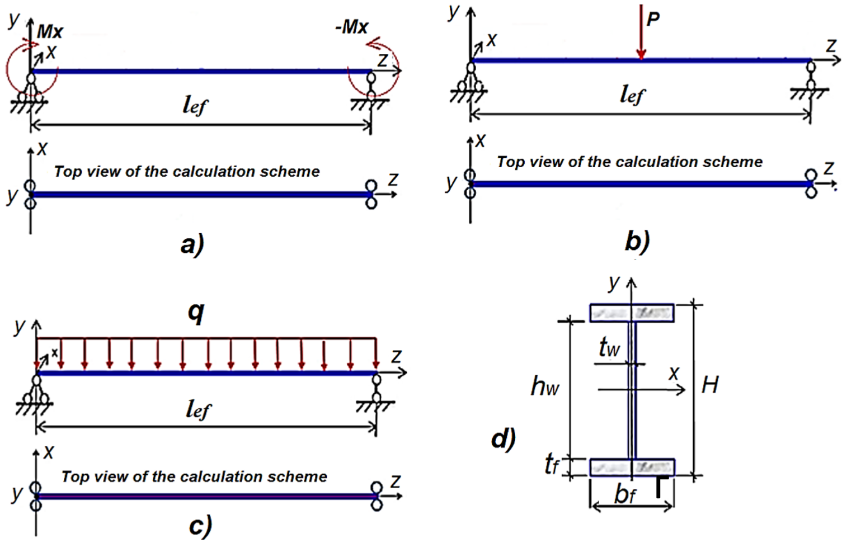
In this paper comparison of results of calculation of sectional I-beams with two symmetry axes on general stability by analytical formulas of S.P. Timoshenko, Z.V. Vlasov and with numerical simulation in PC LIRA-SAPR taking into account deplanation of sections and constrained torsion is made.

2 Main Part

Critical loads of the first form of I-beam stability loss from wall plane according to analytical formulas of S.P. Timoshenko, V.Z. Vlasov and PC LIRA-SAPR for sectional scheme without bracing in the span (calculated span l_{ef}) with a cross-section having two axes of symmetry (fig. 1).

Calculations have been carried out for the following loads:

- Multidirectional bending moments applied at the ends of the beam (Fig.1(a));
- concentrated load in the middle of the span (Fig.1 b));
- uniformly distributed load (Fig. 1(c)).

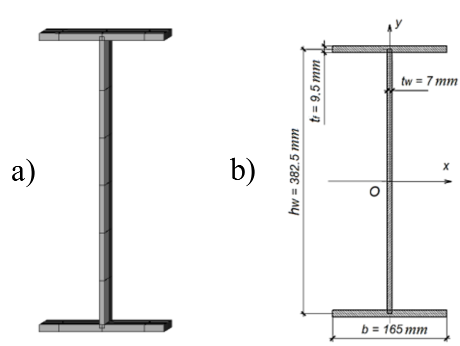


top view of the calculation scheme
a
b
c
d

Fig. 1. design diagrams of the beams under study: a) load with bending moments in the bearing sections of the beam b) concentrated transverse load in the middle of the span c) evenly distributed transverse load d) sectional design

A sectional I-beam rolled beam of size 40B1 according to STO ASChM 20-93 (Figures 2(a, b)) was taken as an object of study. When the examined beam is built with shell elements in the PC LIRA-SAPR radius transitional ribs between the flanges and the beam wall are not reflected and are not taken into account in the calculation. This means that the sectional characteristics of the investigated beam do not correspond to the values given in the product range.

This required a separate calculation of the moments of inertia of the cross-section of the beam in question according to Fig. 2.



mm

Fig. 2. Cross-section of the investigated beam: a) 3D model of the section of the investigated beam in PC LIRA-SAPR b) sectional model of the investigated beam (sectional dimensions are shown)

Sectional moment of inertia of the beam with respect to the axis y:

$$I_y = \frac{h_w s^3}{12} + \frac{B^3 t}{12} = 712.35\ cm^4$$

Sectoral moment of inertia I_w and moment of inertia of the section in pure torsion I_t have been determined in the SCAD office software package when modelling the section according to Fig. 2.

The values of moments of inertia of the investigated beam are presented in Table 1.

Table 1. Geometric characteristics of the cross-section of the beam in question

	Parameter	Value	Measurement units
I_y	Moment of inertia about the central axis Y	712.4	cm^4
I_t	Moment of inertia in free torsion	13.7	cm^4
I_w	Sectoral moment of inertia	259762.6	cm^6
EI_y	Flexural stiffness in the plane XZ	1481688.0	$N*m^2$
GI_t	Pure torsional stiffness	11093.8	$N*m^2$
EI_w	Stiffness at deplanation	54030.0	$N*m^4$

It is shown in [2] that the presence of support ribs slightly increases the overall stability of the beam at transverse bending in the wall plane.

At the same time, it is impossible to carry out a calculation in PC LIRA-SAPR for the overall stability of a beam without support ribs, because in the absence of support ribs the local loss of wall stability in the area of support sections occurs under a smaller load than the loss of overall stability of the beam. Therefore, the minimum thickness of support ribs was selected to ensure the stability of the girder wall with manifestation of the first form of general stability loss.

As a result of the simulation, it was not possible to completely eliminate the supporting effect of the support ribs on the overall stability of the beam. From this it follows logically that the critical load determined in the LIRA CAD software must be somewhat greater than the values determined from the analytical expressions.

Figure 3 shows the calculated beam models in LIRA CAD for the three considered loading types.

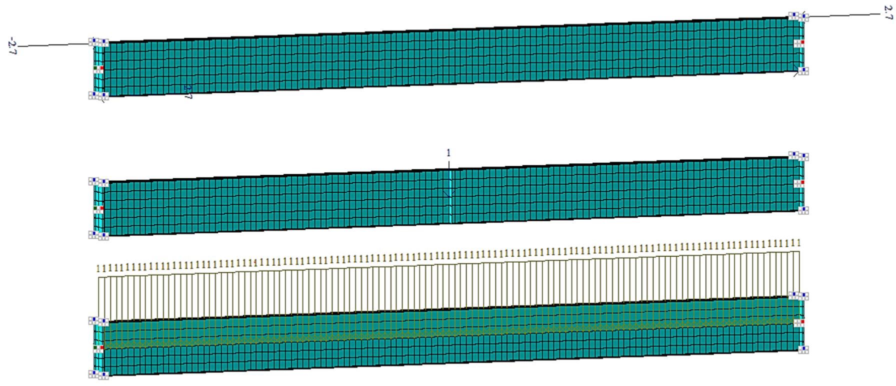


Fig. 3. Shell models of design beams of the three load types for determining the critical load according to the "overall stability" criterion in PC LIRA-SAPR

Based on the results of numerical calculations in PC LIRA-SAD, the critical loads for the adopted loading types take the values shown in Table 2:

Table 2. Calculation results for beams for overall stability in LIRA CAD

Diagram of Fig. 1 (a) (bending moments at girder ends), critical load M_{cr}, tcf*m	Figure 1 (b) (concentrated load), critical load P_{cr}, tcf	Figure 1 (c) (uniformly distributed load), critical load q_{cr}, tcf/m
10.64	9.74	2.71

Further, for three types of loading according to Fig. 1, critical loads were calculated using analytical formulas of S.P. Timoshenko and V.Z. Vlasov.

At differently directed bending moments applied at beam ends (Fig.1 (a)) critical moment calculation M_{1cr} is made according to the formula given by S.P. Timoshenko (p. 560 [1] (formula 91)):

$$\mathrm{M}_{cr} = \frac{\pi}{l_{ef}}\sqrt{E\, I_y\left(C_1\frac{\pi^2}{(l_{ef})^2} + C\right)} = 102.507\, kNm = 10.449\, t*m, \qquad (1)$$

here l_{ef} – girder span without bracing in the span, E- modulus of elasticity of beam material, $C = GI_t$ - torsional stiffness, $C_1 = EI_w$ – deplanation rigidity.

With a concentrated load in the middle of the span (Fig. 1(b)), the analytical expression for the critical load is P_{cr} is given in V.Z. Vlasov's paper on p. 378 [2] (formula 4.8):

$$P_{cr} = k\,\frac{\sqrt{E\, I_y * GI_t}}{l_{ef}}, \qquad (2)$$

here k - coefficient determined according to Table 3 depending on the parameter m^2 calculated according to the formula:

$$m^2 = \frac{GI_t}{EI_w}(l_{ef})^2 = \frac{11093.76}{54030.6}6^2 = 7.392 \qquad (3)$$

Table 3. Dependence of the K coefficient on the factor m^2

m^2	0.4	4	8	16	24	32	48	64	80	96	160	240	320	400
k	86.4	31.9	25.6	21.8	20.3	19.6	18.8	18.3	18.1	17.9	17.5	17.4	17.2	17.2

For a better approximation from the data in Table 3, the dependence is plotted $k(m^2)$ (Fig. 4) and the value of the coefficient $k = 26.1$.

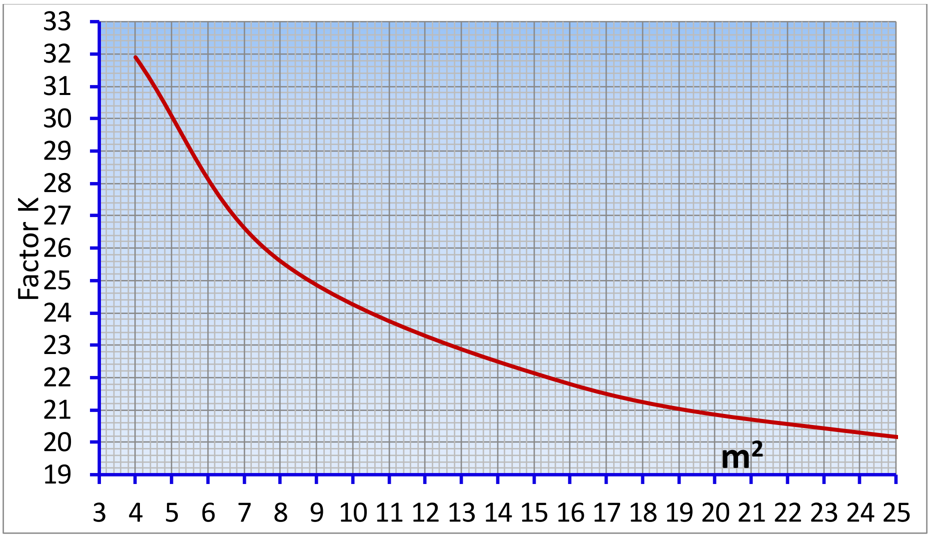


Fig. 4. dependence of the coefficient k on m² for concentrated load

The critical concentrated load P_{cr} per beam was then determined according to the formula (2):

$$P_{cr} = 92811.2\,N = 9.47\,t.$$

For a split beam with a uniformly distributed load (Fig. 1 (c)) acting in the wall plane, the analytical expression for the critical value of $(ql)_{cr}$, is given in V.Z. Vlasov's paper on p. 379 [2]:

$$(ql)_{cr} = k\,\frac{\sqrt{E\,I_y * GI_t}}{(l_{ef})^2}, \tag{4}$$

here k is the coefficient determined according to Table 4 depending on the parameter m^2.

Table 4. Dependence of the k-factor on the coefficient m^2

m^2	0.4	4	8	16	24	32	48	64	80	96	160	240	320	400
k	143	53	42.6	36.3	33.8	32.6	31.5	30.5	30.1	29.4	29	28.6	28.6	28.6

Using the data in Table 4, a graph is plotted $k(m^2)$ (Fig. 5) and with the previously calculated $m^2 = 7.392$ (3), the coefficient value $k = 43.5$ is determined for the formula (4).

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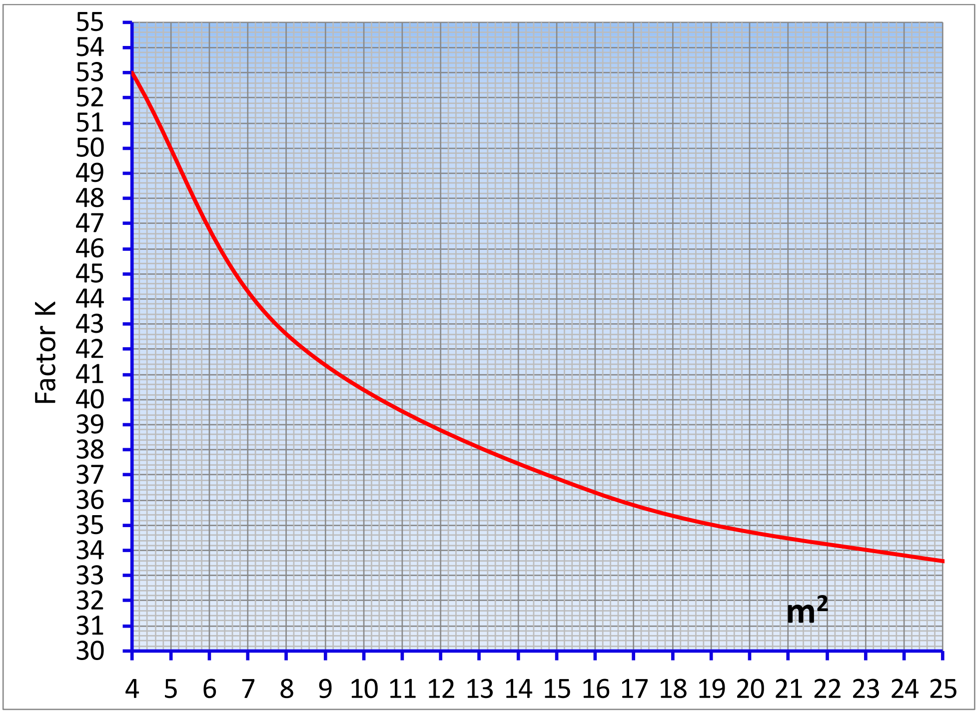

Fig. 5. dependence of K-factor on m^2 for uniformly distributed load

The value of the critical uniformly distributed load on the girder itself was then determined using the formula (4):

$$q_{cr} l_{ef} = 4154801.99\ N$$

$$q_{cr} = 154801.99\ N / l_{ef} = 154801.99\ N / 6\ m = 25.8 \frac{kN}{m} = 2.63\ t/m.$$

The results of the calculations for the three loading options are shown in the Table 5.

Table 5. critical values of girder load on overall stability

Diagram of Fig. 1 (a) (bending moments at the ends of the beam)			Schematic diagram Fig. 1 (b) (concentrated load)			Diagram Fig. 1 (c) (uniformly distributed load)		
Numerical solution, t*m	Analytical solution, t*m	Error relative to analytical solution, %	Numerical solution, t	Analytical solution, t	Error relative to analytical solution, %	Numerical solution, t/m	Analytical solution, t/m	Error relative to analytical solution, %
10.64	10.45	1.8	9.74	9.47	2.9	2.71	2.63	3.0

According to calculation results, the difference between numerically obtained values of critical loads in PC LIRA-SAPR and calculation by analytical models of S.P. Timoshenko and V.Z. Vlasov does not exceed 3%. We consider the calculation results to be satisfactory.

PC LIRA-SAD with sufficient accuracy for engineering calculations calculates critical loads of general stability loss of I-beams at bending in the plane of the wall when modelling beams by shell finite elements taking into account constraint by torsion due to the presence of support ribs.

3 Conclusion

According to the results of the study, the difference between numerical and analytical solutions for the considered I-beam loading schemes does not exceed 3%. In all calculated cases, the critical load determined numerically in LIRA-SAP is greater than the values determined analytically.

The difference is caused by the fact that the cross ribs in the support sections of the beams contribute to the preservation of the flat bending shape of the beam. At the same time, the analytical formulas developed by S.P. Timoshenko and Z.V. Vlasov for the rod model do not take into account the influence of support ribs on the overall stability of beams.

As shown in S.P. Timoshenko [1], [4] and V.Z. Vlasov [2], [5], the critical load depends on the static moment of inertia of the section relative to the axis y I_y, section moment of inertia in pure torsion J_t and sectorial moment of inertia of the section J_w. However, in the presence of supporting ribs, pure torsion is not possible, since in the supporting sections the flanges of the beam cannot freely rotate with respect to the vertical axis. Due to the cramped torsion of the beam, the critical load on the beam in transverse bending, determined numerically, is somewhat higher than the analytical formulas.

Thus, the accuracy of beams calculation for general stability in LIRA-SAPR corresponds to the calculation according to the known analytical expressions with the accuracy acceptable in engineering calculations, and the calculation error not exceeding 3% is explained by the influence of supporting ribs.

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