Calculation of stresses initiated by electrical explosion of conductors in a composite thick-walled cylinder

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Abstract. In various fields of engineering, such structural elements are widely used that, from the point of view of their strength calculation, can be referred to thin shells (tanks, reservoirs, cylinders, etc.). When calculating thin-walled shells, a number of hypotheses are adopted to simplify the solution of the problem. The problem is most simply solved within the framework of the momentless theory of shells, according to which only the force normal to the section (the membrane force) is different from zero, and all moments and transverse forces are equal to zero. The paper presents an analysis of dynamic loading of the material carried out by the method of electric explosion of conductors (EEC). A composite cylinder, which is a thick-walled cylinder made of polymethylmethacrylate with an axial hole pressed into an aluminum shell, was chosen as an experimental sample. The sample is loaded by applying an electric voltage to a conductor in the form of a copper wire placed inside the axial opening of the sample. The equation of state at electric explosion of the conductor is obtained; experimental and calculated graphs of dependence of pressure in the air channel on the expansion coefficient of EEC products are constructed. A theoretical model describing the change in the radial stresses arising in the process of loading with regard for the boundary and initial conditions is built. The accuracy of the built model is justified by comparing the obtained results with the experimental data. The circumferential stress, at which the rupture of aluminum shell directly occurs, is estimated.

1 Introduction

At present, the methods of high-speed loading, deformation and fracture of materials are being intensively investigated. Of particular interest are electrophysical methods based on electric explosion of conductors (EEC) [1-2]. Practical applications of these

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methods include obtaining pulse pressures to initiate high-speed collision of bodies, production of fine particles, deformation and fracture of materials. Intensive development of research on the analysis of fracture characteristics of materials at different loading rates imposes more and more stringent requirements for a detailed understanding of crack formation processes. There is a need to investigate the microscopic relief of the fractured material surface, linking the fracture processes with the passage of the shock wave inside the specimen. It is required to develop mathematical models adequately describing such processes.

In the present work, the EEC method is used to investigate experimentally the high-speed loading of cylindrical composite samples made of polymethylmethacrylate (PMMA) with an outer shell made of aluminum. The aim of the work is to develop a mathematical model for constructing the stress fields taking into account the boundary and initial conditions.

2 Materials and Methods

The sample under study is a thick-walled PMMA cylinder pressed into an aluminum shell. The outer radius of the cylinder is 3 mm, the inner radius is 0.5 mm, and the thickness of the shell is 0.5 mm. A conductor in the form of a copper wire with a radius of 60 μm and length of 18.5 mm was placed inside the axial hole of the sample.

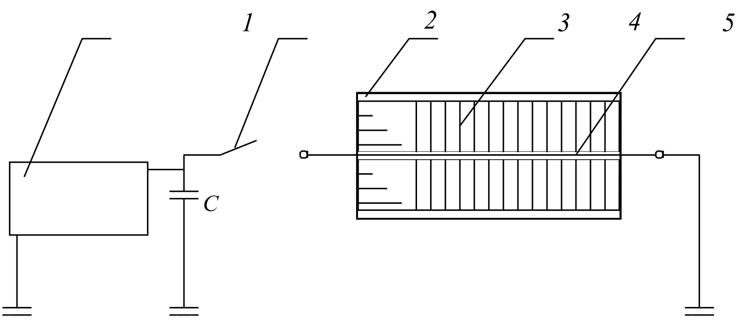

Fig. 1. scheme of the experiment: 1 - charger, 2 - arrester, 3 - aluminum shell, 4 - PMMA cylinder, 5 - conductor explosion channel.

The sample is loaded by applying an electric voltage to the conductor, resulting in an electrical explosion. Schematic diagram of the experiment is shown in Fig. 1.

The energy of the capacitor charge equals 100 J. The pressure from the exploding conductor in the air channel is transferred by the shock wave to the PMMA material and then to the aluminum shell. The PMMA cylinder is destroyed in this case [3] (Fig. 2).

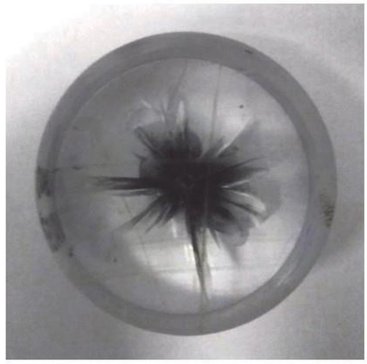

Fig. 2. Fracture of a PMMA sample.

In general, the methodology of the experiment is described in detail in [3]. On the basis of the experimental data on EEC [4] it is possible to construct the equation of state in the form:

$$P\left(\frac{\rho_0}{\rho}\right) = A_e^{-\alpha\frac{\rho_0}{\rho}}$$

here P is pressure, ρ_0/ρ define the expansion coefficient of EEC products, ρ_0 - initial conductor density, ρ — EEC product density; A, α - parameters based on experimental EEC data.

The assumption is made that, at the moment of a conductor explosion, the radius of the explosion channel r is equal to zero. According to [4], immediately after the EEC the density of explosion products ρ is 0.15*103 kg/m3, and the density of invested energy ε — $13*10^6$ J/kg. The expansion coefficient of EEC products in this case will be ρ0/ρ = 59.5. Also the listed parameters were determined for radii 0.2, 0.4, 0.5 mm. The pressure in the air channel was calculated by the Grüneisen formula:

$$P = \gamma\rho\varepsilon \tag{2}$$

here γ — Grüneisen parameter (for air γ = 0.75).

Based on experimental data [4] coefficients in equation of state were determined: $A = 6.38*10^9$ Pa, α = 0.024. Calculation of pressure in the channel was carried out according to equation (1), as well as using experimental results according to equation (2). Experimental and calculated data are presented in Table 1, the calculated (solid line) and experimental (dots) plots P (ρ0/ρ) are shown in Fig. 3.

Table 1. Experimental and calculated data

$r, 10^{-3}$m	$V, 10^{-9}$m^3	$\rho, 10^3$ kg/m^3	ρ_0/ρ	$\varepsilon, 10^6$ J/kg	$P, 10^9$Pa(exp)	$P, 10^9$Pa(calc)
0	0	0.15	59.5	13.00	1.46	1.53
0.2	3.14	0.12	74.4	11.50	1.04	1.07
0.4	12.56	0.10	89.3	10.00	0.75	0.75
0.5	19.62	0.09	99.2	8.50	0.57	0.59

Energy transfer of the exploded conductor to the surrounding gas material in the explosion channel occurs to a greater extent due to the formation of a shock wave in the gas [4]. To determine the pressure in the shock wave that passed through the boundary between the channel of the exploded wire and PMMA, we use the relation

[5],

$$P_{pas} = \frac{2\rho_2 c_2}{\rho_2 c_2 + \rho_1 c_1} \quad (3)$$

here ρ_2, c_2 — density and speed of sound in PMMA, ρ1, c1 - density and speed of the wave in the channel, P_{inc} — pressure in the incident wave in the channel. Pressure of P_{pas} has the meaning of radial normal stress or contact pressure.

Falling wave pressure in the channel P_{inc} = 574 MPa we take from Table 1 for radius r = 0.5 mm. Using relation (3), we determine the contact pressure in PMMA: P_{pas} = 1067 MPa.

Experiments were performed to measure the radial normal stress profiles in PMMA samples of different radii: 3, 17, 29.5, 37.5, 51.5 mm. The length of the samples was 30-40 mm. The radial stress profile in the specimen was recorded using a broadband piezoelectric transducer fixed on the side surface of the specimen. The amplitudes of radial normal stresses at different radii of PMMA specimens obtained from the experiment are shown in Table 2; the experimental dependence of radial normal stress amplitude on radius plotted from these data is shown in Fig. 4.

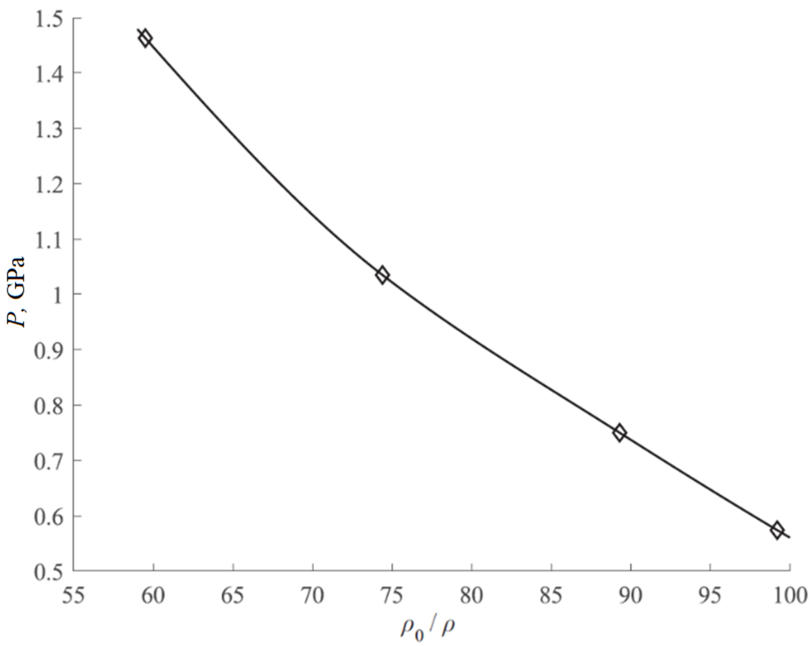

Fig. 3. Dependence graphs P (ρ0/ρ).

Table 2. Amplitudes of radial normal stresses (P) at different sample radii (r)

r,mm	0.5	3	17	29.5	37.5	51.5
P,MPa	1067	970	485	242	194	155

3 Results

Let us consider the problem of dynamic deformation of a thick-walled cylinder under the action of internal pressure. Let a be the inner radius of the cylinder and b be its outer radius. With the notations used in the theory of quasistatic elastic field in axisymmetric geometry and under flat deformation conditions, as well as taking into account the assumption of infinitely long configuration under the condition of no external force, the defining equation of motion of the thick-walled cylinder in the polar coordinate system has the form [5-6]

$$\frac{\partial\sigma_r}{\partial r}+\frac{\sigma_r-\sigma_\theta}{r}=\rho\frac{\partial^2 U}{\partial t^2} \tag{4}$$

here $U = u_r$ is radial displacement, σ_r is radial stress, σ_θ — circumferential tension, r — radius of the cylinder, ρ - density, t - time of development of the deformation process. According to the Saint-Venant principle, the deviation of the stress field from the radially symmetric one should be observed only near the ends of the cylinder, the influence of which is neglected. Consequently, we will assume that the cross sections of the cylinder will remain flat and the stress state in them will be the same. Thus, the required functions will depend only on the cylinder radius r.

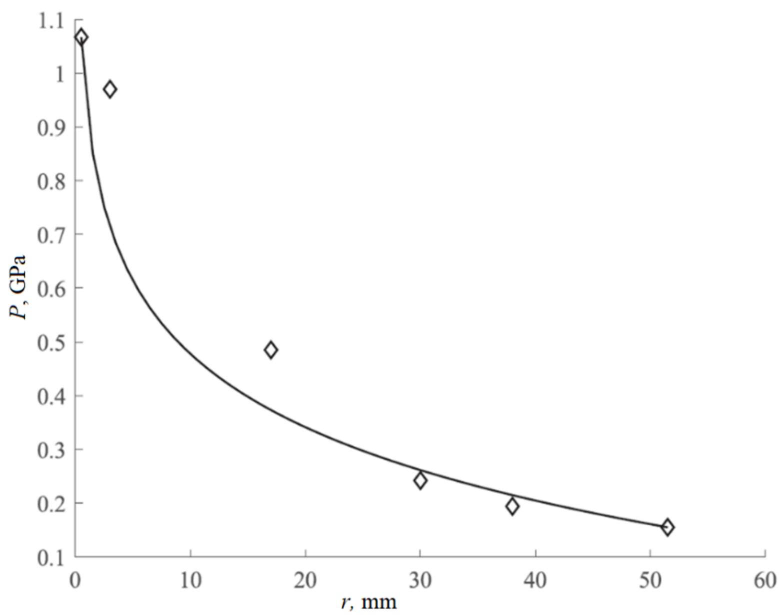

Fig. 4. Radial normal stress versus radius in PMMA.

We solve the problem for an incompressible material. The incompressibility condition has the form [7]

$$\frac{\partial U}{\partial r}+\frac{U}{r}=0 \tag{5}$$

We write down the expressions for the deformation $\varepsilon_r=\frac{\partial U}{\partial r}$

$$\varepsilon_\theta=\frac{U}{r}$$

Generalized Hooke's law for the considered isotropic medium is

$$\sigma_{ij}=\lambda I_1(\varepsilon)\delta_{ij}+2\mu\varepsilon_{ij}, \tag{6}$$

here $I_1(\varepsilon) = \varepsilon_{11}$ - the first invariant of the strain tensor, λ, μ are Lamé constants, and δij are Kronecker symbols. Due to the assumption of incompressibility of the medium, only the deviatoric part of the stress tensor needs to be determined, hence the formula (6) will have the following form

$$\sigma_{ij} = 2\mu\varepsilon_{ij}. \tag{7}$$

In a cylindrical coordinate system, we have

$$\sigma_r = 2\mu\varepsilon_r \qquad \sigma_\theta = 2\mu\varepsilon_\theta, \tag{8}$$

It follows that σ_r-σ_θ = s_r-s_θ = 2μ (ε_r-ε_θ), where s_r and s_θ are the stress deviators.

Let us introduce dimensionless parameters

$$\bar{r} = \frac{r}{a}\ , \quad \bar{t} = \frac{t}{t^*} = \frac{tC_0}{a}\ ,$$

here $t^* = a/C_0$, $C_0 = \sqrt{\mu/\rho}$ — shear wave velocity.

The equation of motion with a dimensionless radius will be written as follows:

$$\frac{\partial\sigma_r}{\partial\bar{r}} + \frac{\sigma_r - \sigma_\theta}{\bar{r}} = \rho a \frac{\partial^2 U}{\partial t^2} \tag{9}$$

The condition of incompressibility at a dimensionless radius $\bar{r}$ will look like:

$$\frac{\partial\bar{U}}{\partial\bar{r}} + \frac{\bar{U}}{\bar{r}} = 0. \tag{10}$$

The solution to this equation is:

$$\bar{U} = \frac{U(r,t)}{a} = \frac{C(t)}{\bar{r}}, \tag{11}$$

where C(t) is a function independent of the coordinate r, which we will discuss below. Here we note that we are solving the problem in order to find only the amplitude of the radial stress depending on the radius of the cylinder. Given this solution of the incompressibility equation, let us write down the expressions for deformations:

$$\varepsilon_r = \frac{\partial U}{\partial r} = \frac{\partial U}{a\partial\bar{r}} = -\frac{C(t)}{\bar{r}^2},$$

$$\varepsilon_\theta = \frac{U}{r} = \frac{\partial U}{a\bar{r}} = -\frac{C(t)}{\bar{r}^2}.$$

The stress difference will be:

$$\sigma_r - \sigma_\theta = 2\mu(\varepsilon_r - \varepsilon_\theta) = -4\mu\frac{C(t)}{\bar{r}^2}$$

The inertial term in the equation of motion with regard to dimensionless time is converted as follows:

$$\rho a\frac{\partial^2 U}{\partial t^2} = \rho a^2\frac{C''(\bar{t})}{\bar{r}}\frac{C_0^2}{a^2} = \mu\frac{C''(\bar{t})}{\bar{r}}$$

The equation of motion can then be represented as

$$\frac{\partial\sigma_r}{\partial\bar{r}} = \mu\left(\frac{C''(\bar{t})}{\bar{r}} + 4\frac{C(\bar{t})}{\bar{r}^3}\right) \tag{12}$$

We define the boundary and initial conditions as:

$$\bar{r} = \bar{r}_* = \frac{b}{a}, \quad \sigma_r = -P_2,$$

$$\bar{r} = 1, \ \ \sigma_r = -P_1,$$

here P_1 = 1067 MPa - contact pressure at the channel-PMMA interface (taken from experiment), P_2 = 155 MPa - radial normal stress in the PMMA at a fixed radius r = b (taken from experiment);

$$U(r, 0) = 0 \ \Rightarrow \ C(0) = 0,$$

$$U'(r, 0) = 0 \ \Rightarrow \ C'(0) = 0.$$

By integrating the equation of motion (9) using the boundary conditions r = b, $\bar{r} = \bar{r}_*$, we obtain the expression for σ_r:

$$\sigma_r + P_2 = \mu\left[\ln\left(\frac{\bar{r}}{\bar{r}_*}\right)C''(\bar{t}) - 2\left(\frac{1}{\bar{r}^2} - \frac{1}{\bar{r}_*^2}\right)C(\bar{t})\right] \tag{13}$$

Using the second boundary condition at $\bar{r} = 1$, we obtain the equation for determining the function $C(\bar{t})$:

$$P_1 - P_2 = \mu\left[\ln \bar{r}_*^2 C''(\bar{t}) + 2\left(1 - \frac{1}{\bar{r}_*^2}\right)C(\bar{t})\right] \tag{14}$$

Solution of this equation under initial conditions C(0)=0, $C'(0) = 0$ has the following form:

$$C(\bar{t}) = \frac{P_1 - P_2}{2\mu}\frac{\bar{r}_*^2}{\bar{r}_*^2 - 1}(1 - \cos\omega\,\bar{t}) \tag{15}$$

Here

$$\omega^2 = \frac{2(\bar{r}_*^2 - 1)}{\bar{r}_*^2 \ln \bar{r}_*}(15)$$

The first and second time derivatives of the function $C(\bar{t})$ will look like:

$$C'(\bar{t}) = \frac{P_1 - P_2}{2\mu}\frac{\bar{r}_*^2}{\bar{r}_*^2 - 1}\omega \sin \omega\bar{t}, \tag{16}$$

$$C''(\bar{t}) = \frac{P_1 - P_2}{2\mu}\frac{\bar{r}_*^2}{\bar{r}_*^2 - 1}\omega^2 \cos\omega\bar{t} \tag{17}$$

Let us return to the question of choosing the value of the function C(t) in the solution of the incompressibility equation (10). Since we are interested only in the dependence of the radial stress amplitude on the radius during the compression wave propagation, the right part of equation (13) should be negative and maximal in modulo. It can be shown that the right side of this equation for the chosen parameters is reduced to the expression

$$\mu[-A - B\cos\omega\bar{t}],$$

here A and B — are positive numbers. This expression is negative and maximal in modulo when cos $\omega\bar{t} = 1$, that is when $\omega\bar{t} = 0$ ($\bar{t} = 0$), which satisfies the initial conditions. In view of the above, it can be noted that $C(0) = 0$ and

$$C''(0) = \frac{P_1 - P_2}{2\mu} \frac{\bar{r}_*^2}{\bar{r}_*^2 - 1} \omega^2,$$

then expression (13) can be written as:

$$\sigma_r + P_2 = \mu \left[\ln\left(\frac{\bar{r}}{\bar{r}_*^2}\right) \frac{P_1 - P_2}{2\mu} \frac{\bar{r}_*^2}{\bar{r}_*^2 - 1} \omega^2 \right] \quad (18)$$

Using this expression, the dependence of radial stress on the radius of the cylinder was plotted $\sigma_r(r)$ (solid line) shown in Fig. 4 in comparison with experimental data (dots). It should be noted that the simple mathematical model of dynamic deformation of a thick-walled cylinder constructed quite closely reflects the experimental results obtained.

Using this model, we calculate the stresses in a composite thick-walled cylinder consisting of a piece of aluminum tube into which a PMMA cylinder is pressed. The calculation is performed for two elements of the compound cylinder: in the PMMA cylinder with a diameter of 6 mm and in the aluminum shell with an outer diameter of 7 mm.

First, we determine the distribution of radial stress amplitude in the PMMA cylinder of 6 mm diameter. The condition at the boundary of the explosion channel of the conductor and PMMA has already been set. Contact pressure in the passed shock wave P_{pas} was 1067 MPa. From the graph of radial pressure vs. radius in PMMA (Fig. 3) at r = 3 mm (i.e., at the PMMA-aluminum shell boundary) we find the value of contact pressure: P_{inc} = 970 MPa. Fig. 5 shows a graph of radial stress vs. radius calculated from the model (18).

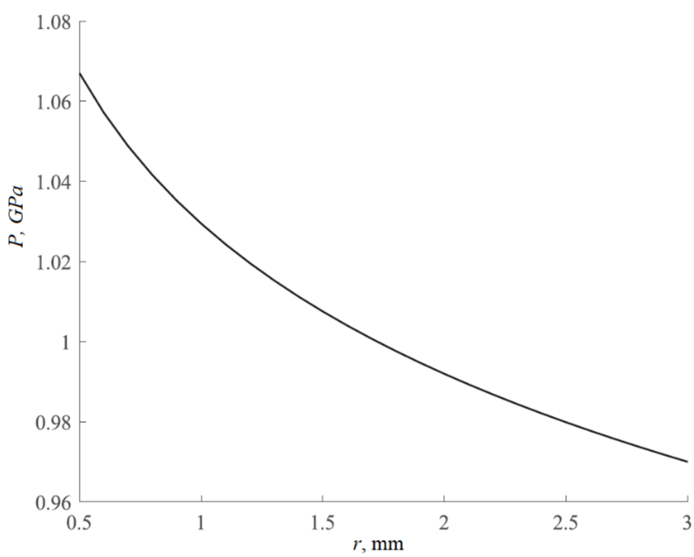

Fig. 5. Dependence of the radial stress on the radius in the PMMA sample.

Now let us determine the amplitude of the radial stress in the aluminum shell. To do this, we find the contact pressure in the shock wave that passed through the PMMA-aluminum shell boundary using the relation (3):

$$P_{pas} = \frac{2\rho_3 c_3}{\rho_3 c_3 + \rho_2 c_2} P_{inc}\,.$$

where ρ_3, c_3 determine density and wave velocity in aluminium (ρ_3 = 2.7*10^3 kg/m^3, c_3 = 6.3*10^3 m/s), ρ_2, c_2 — density and wave velocity in PMMA (ρ_2 = 1.18*10^3 kg/m^3, c_2 = 2.67 10^3 m/s), P_{inc} = 970 MPa. The result is P_{pas} = 1630 MPa.

It follows from [6] that in thick-walled cylinders there is little difference between the radial and circumferential stresses: $\sigma_r = P_{\text{pas}} = 1.1\sigma_\theta$. Hence, we have σ_θ = 1480 MPA. At this stress the aluminium sheath ruptured. Therefore, the result obtained can be regarded as a sheath rupture stress. The result is in agreement with [7,8].

4 Conclusions

As a result of the study the following results were obtained.
1. Based on the constructed equation of state for electrical explosion of conductor, the pressure in the explosion channel and the transmitted contact pressure in the shock wave through the boundary in PMMA have been estimated.
2. A mathematical model for constructing the radial stress fields in a thick-walled PMMA cylinder with regard to initial and boundary conditions has been developed and the stresses have been calculated. A comparison with experimental data is made.
3. The estimation of circumferential destructive stress in aluminum shell of composite thick-walled cylinder is performed. The obtained data do not contradict the known literature sources.

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