A general two phase depth integrated model considering pore water pressure and dewatering

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Abstract. Debris flows can be considered a type of landslide with large velocities and long run-out distances. There are many types of debris flows, depending on the properties of the solid and fluid components of the mixture. The triggering and propagation of debris flows can be studied using a single 3D mathematical model. The computational cost can be very high because of their length, and depth-integrated models provide a good combination of accuracy and cost. Both types of models can be combined in the analysis, using 3D models for initiation and at singular points where more accuracy is wanted. As in a chain where the strength is never higher than that of the weaker link, we have to ensure that all the models are accurate enough in a joint model. This paper deals with a new depth-integrated model which can take into account the changes caused by dewatering in a debris flow.

1 Introduction

Among all types of landslides, shallow landslides on saturated materials, especially those where excess pore water pressure has developed, present large runout distances and velocities. This is the case of flow slides and debris flows. Regarding the former, in some cases, the pore air pressure can raise and cause "dry liquefaction".

It is important, therefore, to describe the main phenomena on which pore pressure depends, and how to implement them in mathematical and numerical models. These phenomena are:

- Erosion of pressurized saturated basal materials
- Dewatering caused by pore fluid flowing out the body of the landslide through the basal surface or abandoning the landslide when the interaction forces are small (low equivalent permeability).
- Pore pressure build-up and dissipation along the vertical direction caused by changes in height, porosity, or consolidation

All these mechanisms can be characterized by suitable time scales which allow writing the governing equations in non-dimensional form. We can introduce:

i. The erosion time $T_e = h/e_r$ where e_r is the erosion rate.
ii. The dewatering time (in the case of materials with high permeability), given by $T_{dewat} = h/\bar{k}_w$ where $\bar{k}_w$ is the perm-eability (m/s).
iii. The consolidation time, given by $T_c = h^2/C_v$ where C_v is the coefficient of con-solidation ($\mathrm{m^2 s^{-1}}$).
iv. The propagation time T_{prop}.

All these mechanisms can be characterized by suitable time scales which allow writing the governing The relation between these characteristic times is fundamental when choosing which model has to be used. Here we will consider only dewatering and consolidation, as erosion has been described in Pastor et al. (2022). Our contribution will include some examples showing the importance of the characteristic times presented above.

2 Mathematical model

We will consider the general case of the soil column sketched in **Fig. 1**, where only the lower part of the landslide is saturated, the upper being dry. This situation happens when (i) the initial sliding mass has this structure, for instance, when water infiltrates from the basal surface into the soil, and (ii) when horizontal velocities for water are larger than those of the solid, and move ahead of the sliding mass (iii) when the porosity of the bed material is high and so is its permeability, as it happens when the material flows over basal grids, water flows out of the soil column, and (iv) when the debris flows arrives to a permeable wall or barrier. We have denoted by α_{sat} the fraction of the total height h which is saturated. This height will be denoted as $h_{sat} = \alpha_{sat} h$.

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The general model for the saturated part is based on the work of Biot (1955), the group of the late Prof. Zienkiewicz at Swansea University (Zienkiewicz and Shiomi (1984), Zienkiewicz et al. (1990,1999) and Anderson and Jackson (1967), and the application to depth integrated debris flows proposed by Pitman and Le (2005) and Pudasaini (2007). The model was recently extended to debris flows with excess pore pressure by Pastor et al. (2021).

The model is based on the assumption that the mixture is composed of a solid phase and a fluid phase. The equations are (i) Balance of mass and (ii) Balance of linear momentum for the constituents and the mixture, (iii) constitutive or rheological laws describing the material behaviour of all constituents, and (iv) kinematics relations linking velocities to rate of deformation tensors.

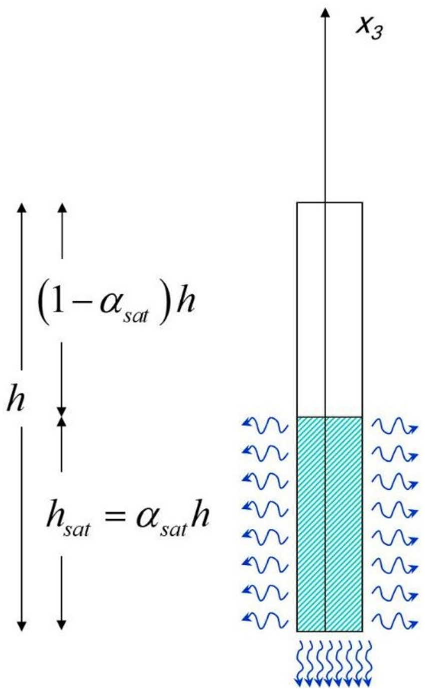

Fig. 1. Representative column of soil with two layers.

The 3D model is integrated along depth, including both saturated and unsaturated layers. We will introduce for convenience three auxiliary variables h_s, h_a and h_w which characterize the solid, air and fluid contents in a column of water of height h (**Fig. 2**).

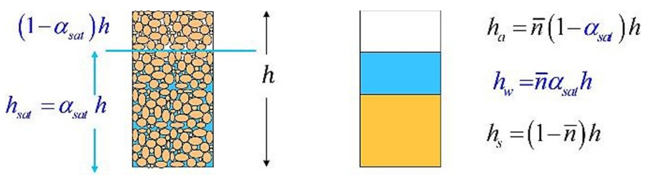

Fig. 2. Magnitudes characterizing the unsaturated debris flow components.

The relations between them are:

$$h = h_s + h_a + h_w \tag{1}$$

where

$$h = \frac{h_s + h_w}{1-\bar{n}(1-\alpha_{sat})} \tag{2}$$

$$\bar{n} = \frac{h_w}{h_w + \alpha_{sat} h_s} \tag{3}$$

$$\alpha_{sat} = \frac{1-\bar{n}}{\bar{n}} \frac{h_w}{h_s} \tag{4}$$

Using the proposed approach, once a threshold value of porosity $\bar{n}_{min}$ is reached, it will keep constant, with the saturated layer height decreasing,

$$\alpha_{sat} = \frac{1-\bar{n}_{min}}{\bar{n}_{min}} \frac{h_w}{h_s} \tag{5}$$

$$h_{sat} = \frac{1-\bar{n}_{min}}{\bar{n}_{min}} \frac{h_w}{h_s} h \tag{6}$$

In the limit, we see that $h_w \to 0$ and $h_{sat} \to 0$.

3 Modelling of excess pore water pressure

The finite difference equation describing the evolution of excess pore water pressure is

$$\left.\frac{\partial \Delta p_w}{\partial t}\right|_j^n = C_v \left.\frac{\partial^2 \Delta p_w}{\partial {x_3}^2}\right|_j^n - \frac{x_3}{h_{sat}} \frac{dh_{sat}}{dt} \left.\frac{\partial \Delta p_w}{\partial x_3}\right|_j^n - E_m \frac{1}{1-\bar{n}} \frac{d^{(s)}\bar{n}}{dt} - \rho' b_3 \frac{d^{(s)}h}{dt}\left(1-\frac{x_3}{h}\right) \tag{7}$$

where the partial derivative $\partial \Delta p_w/\partial t|_j^n$ where the partial derivative j. C_v is the coefficient of consolidation. E_m the oedometric modulus. $\bar{n}$ the depth averaged porosity.

The first term of the RHS characterizes consolidation, the second takes into account that the mesh is of arbitrary lagrangian eulerian type, the third takes into account the changes in porosity and the fourth the changes in total stress induced by changes of height.

The boundary conditions are: (i) at the saturated-dry interface where $\Delta p_w = 0$ and (ii) at the bottom, where the condition is $p_w = p_{atm} \to \Delta p_w = \rho_w b_3 \alpha_{sat} h$ in the case of dewatering.

This equation includes an advective term depending on T_{cons}/T_{dew}. This term can be approximated as:

$$\frac{T_{cons}}{T_{dew}} = \frac{h_0 \rho_w g}{E_m} \tag{8}$$

which does not depend on permeability and, in most cases, tends to zero. In those cases where propagation and dewatering times are similar, consolidation happens much faster, and we can approximate the excess pore pressure distribution along depth for that of steady state. For instance, in the case of a dewatering column. We can assume that the excess pore pressure varies between zero at the surface of the saturated part and $\rho_w b_3 h_{sat}$ at the bottom - where we assume pore pressure is atmospheric. If the time of consolidation is comparable to the time of propagation, the dewatering time will be much larger than both.

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