A dilatant two-phase debris flow model with erosion validated by full-scale field data from the Illgraben test station

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Abstract. We propose a dilatant, two-layer debris flow model validated by full scale density/saturation measurements obtained from the Swiss Illgraben test site. Like many existing models we suppose the debris flow consists of a matrix of solid particles (rocks, boulders) that is surrounded by muddy fluid. However, we split the muddy fluid into two fractions. One part, the inter-granular fluid, is bonded to the solid matrix and fills the void space between the solid particles. The combination of solid material and inter-granular fluid forms the first layer of the debris flow. The second part of the muddy fluid is not bonded to the solid matrix and can move independently from the first layer. This free fluid forms the second layer of the debris flow. During flow the rocky particulate material is sheared which induces dilatant motions that change the solid/fluid concentration of the first layer and then his density. As suggested by real data of Illgraben, the rheology used is not constant and uniform but a function of the flow composition/ density. The model is then compared to real debris flow data of Illgraben and tested on a real event in Ritigraben for which erosion data are available.

1 Introduction

The assessment of debris flow hazard relies on both numerical simulation models and empirical methods. Most numerical approaches solve shallow-water type equations [1-5] and therefore can be effectively applied to predict flow heights and debris flow runout distances. Nonetheless, the application of numerical models in hazard engineering practice remains limited. This is due to two salient problems. Firstly, it is difficult to accurately quantify the initial starting and entrainment masses for a specific torrent. And secondly, historical case studies are still necessary to calibrate the rheological parameters that govern debris flow motion at a specific site, and therefore possible inundation area. Without this information, the motion of a debris flow is difficult to model because it depends strongly on the relative amounts of solid and fluid masses. In this contribution we present a two-phase debris flow model, that splits the fluid phase into bonded and non-bonded (free) parts. The dilatancy of the solid phase controls the distribution of bonded fluid in the debris flow. Field data from Illgraben and Ritigraben are used to calibrate and validate the model. These two torrents are located in Canton Wallis, Switzerland.

2 Model definitions and equations

We model debris flows with a two-layer, depth-averaged approach [6], see Fig.1. The debris flow contains two material components: a solid component (subscript *s*) consisting of coarse granular sediment, associated with a density ρ_s, and a fluid component (subscript *m*) consisting of fine sediment likely to behave as suspended sediment (e.g. sand, silt, clay), hereafter referred to as the muddy fluid content, whose density is denoted by ρ_m.

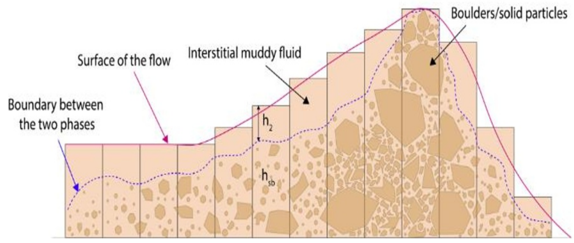

Fig. 1. Sketch of a debris flow. We divide the flowing material in two layers. The first one is a mixture consisting of all the solid material and a part of the fluid called interstitial fluid. The second one is composed by the rest of the fluid, called free fluid.

The solid and fluid components are divided into two layers. The first phase/layer (subscript *1*) contains the granular solid material and a part of the fluid (interstitial fluid). The fluid is contained in the interstitial space between particles and is assumed to be bonded to the solid particles (subscript *b*). The second phase/layer (subscript *2*), is formed by the fluid which can flow independently from the first layer. We term it the free fluid (subscript *f*). As we are dealing with a two-phase model with three

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components (solid, interstitial fluid and free fluid), mass conservation equations are of the number of three and can be written, per unit of area and density:

$$\frac{\partial h_1}{\partial t} + \vec{\nabla}(h_1 \boldsymbol{v_1}) = \frac{\rho_m}{\rho_s} Q + \frac{\rho_e}{\rho_s} E \qquad \text{(first phase)} \tag{1}$$

$$\frac{\partial h_2}{\partial t} + \vec{\nabla}(h_2 \boldsymbol{v_2}) = -Q \qquad \text{(second phase)} \tag{2}$$

$$\frac{\partial h_b}{\partial t} + \vec{\nabla}(h_b \boldsymbol{v_1}) = \frac{\rho_m}{\rho_s} Q + \frac{\rho_e - \rho_b}{\rho_b - \rho_s} E \,. \qquad \text{(bonded fluid)} \tag{3}$$

The symbol $\vec{\nabla}$ is the divergence operator in Cartesian coordinates. The right-hand side of the equations contains the term Q, [6], which is the mass exchange rate between the inter-granular and free fluid because of dilatant actions in the solid matrix (see following). It also contains the erosion rate, denoted by E and ρ_e is the density of the mass entrained. In order to compute the erosion rate E, we started with the model introduced in [7] and we adapted it for a two-phase model. In this model, the erosion rate E is not uniform, but considered to be a function of the flow composition. Note that E does not appear in the second layer equation, because we assume that the second layer cannot entrain mass.
Two momentum conservation equations are associated with each phase, which can be written, per unit of area and density:

$$\frac{\partial (h_1 \boldsymbol{v_1})}{\partial t} + \vec{\nabla} \cdot \left(h_1 \boldsymbol{v_1} \otimes \boldsymbol{v_1} + g \frac{h_1}{2} I \right) + g h_1 \vec{\nabla} \left(b + \frac{\rho_m}{\rho_s} h_2 \right) = -\frac{\boldsymbol{S_1}}{\rho_s} + \frac{\rho_m}{\rho_s} P \qquad \text{(first phase)} \tag{4}$$

$$\frac{\partial (h_2 \boldsymbol{v_2})}{\partial t} + \vec{\nabla} \cdot \left(h_2 \boldsymbol{v_2} \otimes \boldsymbol{v_2} + g \frac{h_2}{2} I \right) + g h_2 \vec{\nabla}(b + h_1) = -\frac{\boldsymbol{S_2}}{\rho_s} - P \,. \qquad \text{(second phase)} \tag{5}$$

The vectors $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ represent the velocities of the first and second layers, respectively; b denotes the bottom topography. Note that the term $g h_i \vec{\nabla} b$ represent the gravitation acting on the i-th layer. The symbol $\otimes$ denotes the tensor product and I is the two-dimensional unity matrix. The left side is the total variation of the momentum with respect to time, including the effect of gravitation and the influence of each phase on the other [8,9]. The right-hand side represents the change in momentum due to external forces (excluding gravitation). $\boldsymbol{S_i}$ is the shearing forces acting on the i-th layer and $\boldsymbol{P_i}$ are the momentum exchanges due to mass exchanges Q between the two phases. We use a Voellmy-Salm type friction for the debris flow,

$$\boldsymbol{S_i} = \left(\mu_i g_z h_i + \frac{\rho_i g}{\xi_i} v^2 \right) \boldsymbol{e_v} \,. \tag{6}$$

The symbol g_z denotes the slope perpendicular gravitational acceleration, $\boldsymbol{e_v}$ is a unit vector parallel to the flowing direction and μ_i and ξ_i the coulomb and turbulent coefficients friction.

The fluid mass exchange Q is a result of the dilatant actions of the solid particles in the first layer. When the flow is at rest, it is in the co-volume configuration, which corresponds to the one where the solid matrix is the most collapsed, right sketch on Fig. 2. However, under interactions with the rough bed of the channel, the solid matrix can expand its volume during flowing, leading to different flowing configurations, associated with different densities, left sketch on Fig. 2. These different configurations are obtained by exchanging fluid from one phase to the other, which is reflected in the eq. 1-3 by Q. Note that, even if the solid mass is conserved, the first layer density will vary because of the inter-granular fluid concentration, which is given by,

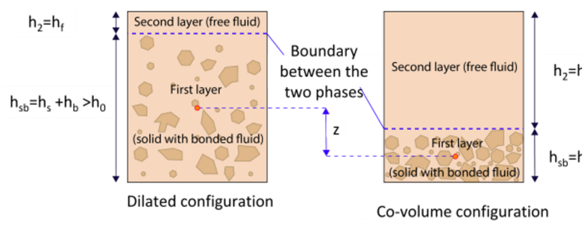

Fig. 2. Sketch of two different configurations. The left one is a dilated one, happening during flowing. The right one is the so-called co-volume configuration, which happens at rest. Even if the two configurations contain the same amount of fluid and solid, the first layer density varies from the left to the right one, due to the dilatant action of the solid matrix.

$$\rho_1 = \frac{\rho_s h_s + \rho_f h_f}{h_s + h_f} = \rho_s (1 - f) + \rho_f f \,. \tag{7}$$

The symbol f represents the volumetric fluid concentration in the first layer. Therefore, in our model, the dilatancy controls the entire space and time evolution of the debris flow density. For more details about the mathematical structure of the dilatancy, see [6].

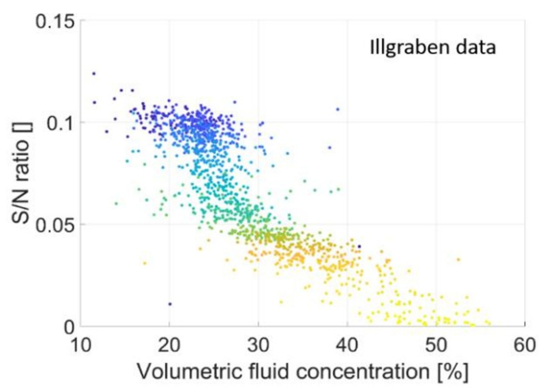

Fig.3. The ratio of the shear and the normal stress, plotted as a function of the volumetric fluid concentration. The curve indicates clearly a decrease of the shear stress intensity when increasing fluid content in the debris flow. Data are measured for real debris flow event in Illgraben, [10,11].

Real debris flow data of Ill graben, a debris flow test site located in Wallis (CH), [10,11], shows that the intensity of the shear forces, represented by the ratio of the shear and the normal stress, decreases with increasing the volumetric fluid fraction of a debris flow, Fig 3. Similar considerations have already been pointed by Iverson in his paper *the debris flow rheology myth* [12]. Therefore, in our model, the shear force acting on the first layer $\boldsymbol{S_1}$ is a function of the debris flow composition. Mathematically, it is given by the evolution of the shearing coefficient μ_1 and ξ_1:

$$\mu(f) = \mu_s(1-f) + \mu_f f = \mu_s(1-f) \tag{8}$$
$$\xi(f) = \xi(1-f) + \xi_f f \tag{9}$$

The friction parameters μ_s, ξ_s represent the coefficient of the solid material while μ_f, ξ_f are the one valid for the fluid. We generally assume $\mu_f = 0$. As the second layer is always composed only by fluid, its shearing coefficient are constant and uniform: $\mu_2 = \mu_f = 0$ and $\xi_2 = \xi_f$.

3 Results

Using the data (Shear and normal stress and flow height) measured at the Illgraben test site, [10,11], we can extract the debris flow density and therefore compare the measured density with the density computed by our model using the dilatancy theory [6]. The comparison is shown in Fig. 4 where the density of both experimental data (spared dots) and numerical output (circles) are plotted as a function of the norm al stress. This result indicates that the proposed model is able to simulate the longitudinal density profile of a real debris flow. The blue dots falling under the dataset represent the watery surges flowing before the front of the flow, which is not capture by the measurements.

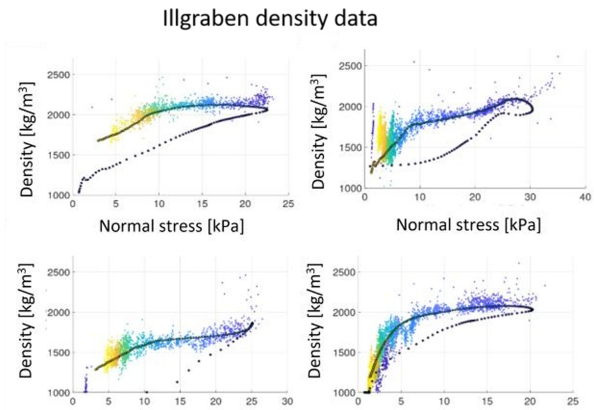

Fig. 4. Denisty profile of four real event in Illgraben, [10,11]. The spared dotes are the measured value of the density and the circles represent the numerical outputs.

Fig. 5 shows a simulation of a real event in the torrent of Ritigraben located in Wallis (CH). The density evolves from high value in the front (top left corner) to the tail (bottom right corner). This result agrees with the fact that the front of the debris flow is composed by large boulders and becomes more and more wet as we go toward the tail. Therefore, the model is not only able to predict the correct density profile but also the right density structure of a debris flow.

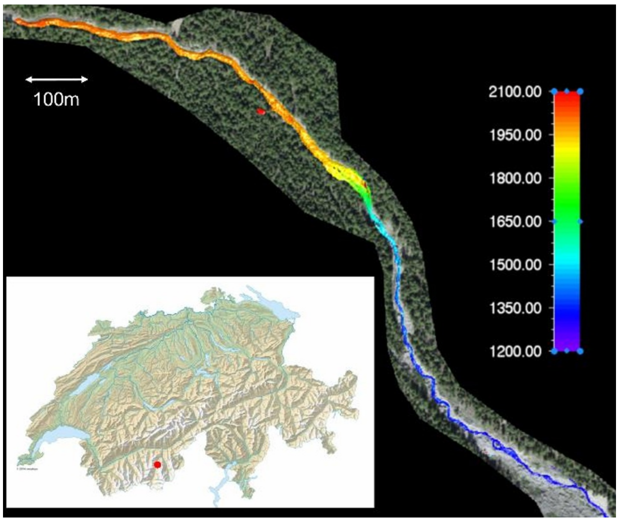

Fig. 5. Spatial density evolution from the front (top left corner) to the tail (bottom right corner) of a debris flow. As expected, the density varies from a high value in the front of the debris flow, composed by large boulders to a lower value in the muddy tail. The red circle in the lower left coner map indicates the Ritigraben torrent location.

According to Iverson [12], and to Fig. 3, the rheology of a debris flow is not constant and uniform but evolves with the flow composition, eq.8 and eq. 9. The spatial evolution of the Coulomb friction μ is shown on Fig. 6. The front, associated with high value of the density, is associated with a value of the coulomb friction $\mu \simeq 0.15$ while it goes to zero as we go to the tail. Fig. 5 and 6 come from the same simulation.
The event of Ritigraben was special from a debris flow mitigation point view because two drones flights, one before and one after the event, have been performed. Therefore, it gives accurate and rare data about erosion. The comparison with the model is shown on Fig. 7. The red curve is obtained with drone flight while the blue one is the model outputs. One can see that the general erosion pattern, as well as the entire eroded volume, can be captured by the model.

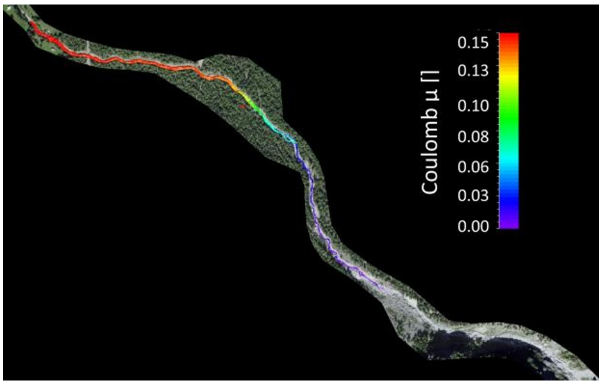

Fig 6. As suggested by real debris flow data, the rheology is a function of the flow composition. This fact is shown on this figure by the decrease of the coulomb friction coefficient μ, given by eq. 8. This figure comes from the same simulation as the one of Fig. 5.

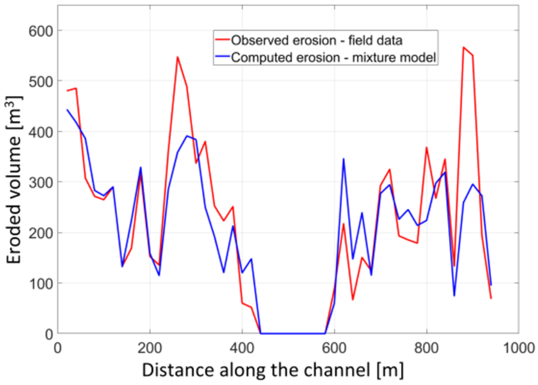

Fig. 7. Erosion pattern towards the channel. The red curve is obtained by drone flights while the blue curve is the numerical model outputs.

4 Conclusion

We applied a dilatant flow model to reproduce not only the correct shape of the flow but also the right structure, i.e. a flow with a dense front which become more and more wet as we move towards the tail. The model contains two types of fluid: bond and non-bonded (free). The fluid is not constrained to the solid matrix, and, therefore can flow well beyond the solid matrix. Thus, the model allows for de-watering simulations. We adapted the well-known Voellmy-Salm rheology to the debris flow problem, using real debris flow data as a guide. Finally, the erosion can be accurately simulated using a simple erosion model adapted for two-phase flow.
We conclude by noting that it is unlikely that the debris flow problem will be solved by purely theoretical considerations. More likely is the continual monitoring of debris flow sites coupled with the documentation of debris flow events and their persistent back-calculation and modelling. Central in this undertaking will be the application of a numerical model that effectively simulates the motion of debris flows of different muddy/granular (i.e. mass) compositions. Work on the experimental front is on-going.

References

1. J. Kowalski, Two-phase modeling of debris flows (ETH Zurich, 2008)
2. S.P. Pudasaini, Journal of Geophysical Research: Earth Surface 117(F3) (2012)
3. J. Kowalski, J.N. McElwaine, Journal of Fluid Mechanics 714, 434 (2013)
4. R.M. Iverson, D.L. George, Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 470(2170), 20130819 (2014)
5. T. Takahashi, Annual review of fluid mechanics 13(1), 57 (1981)
6. Meyrat G, McArdell B, Ivanova K, Müller C, Bartelt P. A dilatant, two-layer debris flow model validated by flow density measurements at the Swiss illgraben test site. Landslides. (2022)
7. F. Frank, B.W. McArdell, C. Huggel, A. Vieli, Natural Hazards and Earth System Sciences 15(11), 2569 (2015)
8. E. Savary, Y. Zech, Journal of Hydraulic Research 45(3), 316 (2007)
9. K.T. Mandli, Finite volume methods for the multilayer shallow water equations with applications to storm surges. Ph.D. thesis (2011)
10. F. Schlunegger, A. Badoux, B.W. McArdell, C. Wendeler, D. Schnydrig, D. Rieke-Zapp, P. Molnar, Quaternary Science Reviews 28(11-12), 1097 (2009)
11. F. Frank, B.W. McArdell, C. Huggel, A. Vieli, Natural Hazards and Earth System Sciences 15(11), 2569 (2015)
12. R.M. Iverson, Debris-flow hazards mitigation: mechanics, prediction, and assessment 1, 303 (2003)