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A comparative analysis of the fuzzy and intuitionistic fuzzy environment for group and individual equipment replacement Models in order to achieve the optimized results

Vijaya Kumar Saranya and Shanmuga Sundari Murugan*

Department of Mathematics and Statistics, CSH, SRMIST, Kattankulathur, Chennai 603202, Tamil Nadu, India

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Abstract. The main goal of this research is to compare group and individual replacement models based on fuzzy replacement theory and intuitionistic fuzzy replacement theory. The capital costs are assumed to be triangular fuzzy numbers, triangular intuitionistic fuzzy numbers, and trapezoidal intuitionistic fuzzy numbers, respectively. As a result, interpreting the direct relationship between volatility and ambiguity is critical. It is difficult to predict when specific equipment will unexpectedly fail. This problem can be solved by calculating the probability of failure distribution. Furthermore, the failure is assumed to occur only at the end of period t. In this situation, two types of replacement policies are used. The first is the Individual Replacement Policy, which states that if an item fails, it will be replaced immediately. The Group Replacement Policy states that all items must be replaced after a certain time period, with the option of replacing any item before the optimal time. The dimensions of the prosecution are fuzzy, and they are then assessed using mathematical and logical procedures. The fuzzy assessment criteria of the replacement model are provided as a set of outcomes, whereas the intuitionistic fuzzy replacement model has many advantages. A methodological technique is used to determine quality measurements in which fuzzy costs or values are kept without being merged into crisp values, allowing us to draw mathematical inferences in an uncertain setting. A comparison conceptualise is created for each fuzzy number, and in an uncertain environment, a comparison study on group and individual replacement was also conducted.

Keywords: Theory of fuzzy sets / fuzzy logic / logic of vagueness / intuitionistic mathematics

1 Introduction

Electronic items such as bulbs, resistors, tube lights, and so on typically fail all at once rather than gradually. The sudden failure of the item causes the entire system to fail. The system could include a collection of such items or just one, such as a single tube light. As a result, we employ a replacement policy for such items in order to reduce the possibility of total breakdown. In this paper we compared individual and group replacement policy. Individual replacement policy requires that each item be replaced as soon as it fails. A decision is made regarding the replacement at what equal internals; all the items are to be replaced simultaneously with a provision to replace the items individually which fail during the fixed group replacement period under Group replacement policy. Xu et al. [3] developed a performance evaluation of an optimal cache replacement policy for wireless data dissemination that uses stretch as the primary performance metric because it accounts for data service time and is thus fair when items have different sizes. Kin-Yeung Wong [4] developed a Web cache replacement policy which is used to identify the appropriate policies for proxies with different characteristics, such as proxies with a small cache, limited bandwidth, and limited processing power. From a system perspective, Liu and Huang [5] established a policy for optimal replacement for a multistate system with imperfect maintenance. Some efforts are being made to determine policy from a systemic standpoint. Young Hyun Yoo [6] developed the maintenance technique used a group replacement policy based on failure frequency; they are all replaced after a specified number of failures occur. Barron [7] implemented the group replacement policies for a repairable cold reserve system with specified lead times. Park and Pham [8] developed cost models for warranty age replacement policies and block replacement plans. Zhao [9] models of a parallel system with a fixed and random

* e-mail: sv9558@srmist.edu.in/shanmugam@srmist.edu.in

number of units are considered. The models of expected cost rates and optimal replacement times to minimize and were discussed analytically and numerically. Chiu, Chang, and Yeh [10] discussed about Group replacement procedures for repairable N-component parallel systems. Diniz, Sessions [11] presents a model that uses detailed equipment maintenance schedules to aid in the decision-making process for equipment replacement in the Brazilian forestry sector. For three different scenarios, the strategies depend on the economic life (EL) method. Liu [12] proposed Internet of Things (IoT) conditioned-based group replacement decision-making system first creates a discounted cost framework for a production/service system with numerous detached working servers. To stimulate the proof procedure, the original discounted cost model is revised into an equivalent model using the different approach. Finkelstein et al. [13] discussed a new approach for the preventive maintenance of deteriorating items. It combines the traditional age-replacement strategy, in which a system is replaced either on failure or when it reaches a predetermined age, with replacement when it reaches a predetermined level of deterioration at a certain intermediate time. Garg et al. [14] introduce the new concept of an interval-valued picture/image uncertain linguistic set, which composes the grade of truth [15], abstention, and falsity as a subset of the unit interval. van Staden et al. [16] investigates the extent to which historical machine failures and maintenance records can be used to forecast future machine failures and, as a result, prescribe advances in scheduled preventive maintenance interventions. Forootani et al. [17] constructed a stochastic dynamic programming technique to solve the machine replacement problem. Makwana et al. [18] proposed a new hypothesis and solution for fuzzy equations. As a result, the illustrious scientist Zadeh [1] proposed Fuzzy Sets, an extension of classical set theory, in the 1960 s. The fuzzy set theory and the classical set theory approach ambiguity differently. In 1986, Atanassov [2] proposed and demonstrated intuitionistic Fuzzy Sets as a fuzzy set generalization. The main goal of this study is to find the best policy from individual and group replacement in a fuzzy and intuitionistic fuzzy environment, and we also demonstrated that intuitionistic fuzzy produced more generalised results than fuzzy. A numerical example is also provided to demonstrate its advantages.

2 Preliminaries

The goal of this division is to provide basic definitions, annotations, results that will be used in subsequent calculations.

Definition 2.1 A fuzzy number $\tilde{A}$ is 'defined on set of real numbers R is called a triangular fuzzy number (TFN) if the membership function (MF) $\mu_{\tilde{A}}: R \to [0,1]$ of $\tilde{A}=\{a_1,a_2,a_3\}$ has the' following conditions:

$$\mu\tilde{A}(x) = \begin{Bmatrix} \dfrac{x-a_1}{a_2-a_1}, & \text{for} & a_1 \le x \le a_2 \\ = 1, & \text{for} & x = a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & \text{for} & a_2 \le x \le a_3 \\ = 0, \text{ otherwise} & & \end{Bmatrix}.$$

Definition 2.2 The fuzzy arithmetic operations are extended to the set of (TIFN) triangular intuitionistic fuzzy numbers based on location indices and also fuzziness indices. Two arbitrary (TIFN) triangular intuitionistic fuzzy number

$$\tilde{A}^{\mathrm{IFN}} \approx (a_1, a_2, a_3; c_1, c_2, c_3),\ \tilde{A}^{\mathrm{IFN}} \approx (b_1, b_2, b_3; d_1, d_2, d_3)$$

Can be written as

$$\begin{aligned}\tilde{A}^{\mathrm{IFN}} &\approx (a_\mu, \alpha_{\mu_a}, \beta_{\mu_a}; a_\gamma, \alpha_{\gamma_a}, \beta_{\gamma_a}) \text{and} B^{\mathrm{IFN}} \\ &\approx (b_\mu, \alpha_{\mu_b}, \beta_{\mu_b}; b_\gamma, \alpha_{\gamma_b}, \beta_{\gamma_b})\end{aligned}$$

here $a\mu$, b_μ are the mid value of the fuzzy number, $\alpha_{\mu_a}, \alpha_{\mu_b}$ are right spread of MF, $\beta_{\mu_a}, \beta_{\mu_b}$ are left spread of MF, $\alpha_{\gamma_a}, \alpha_{\gamma_b}$ are right spread of NMF, $\beta_{\gamma_a}, \beta_{\gamma_b}$ are left spread of NMF and $* \in \{+, -, x, /\}$then the arithmetic operations on (TIFN) triangular intuitionistic fuzzy number are

$$\tilde{A}^{\mathrm{IFN}} * \tilde{B}^{\mathrm{IFN}} \approx \begin{pmatrix} a_\mu b_\mu, \max\{\alpha_{\mu_a}, \alpha_\mu\}, \max\{\beta_{\mu_a}, \beta\}; \\ a_\gamma * b_\gamma, \max\{\alpha_{\gamma_a}, \alpha_\gamma, \max\{\beta_{\gamma_a}\}, \beta_\gamma\} \end{pmatrix}.$$

2.1 A centroid point based ranking algorithm

In this section, we calculate the centroid point of the trapezoidal & triangular intuitionistic fuzzy numbers. The geometric centre of a trapezoidal intuitionistic fuzzy number is used in the method of ranking trapezoidal intuitionistic fuzzy numbers with centroid index. We can derive a ranking of triangular fuzzy numbers from this. The geometric centre corresponds to the horizontal axis's x value and the vertical axis's y value. Consider a triangular or trapezoidal intuitionistic fuzzy number $\tilde{A}$, its membership function (MF) & the non membership function (NMF) are defined by

$$\mu_A = \begin{Bmatrix} 0 & a_1 > x \\ f_A^L(x), & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ f_A^R(x), & a_3 \le x \le a_4 \\ 0, & a_4 \le x \end{Bmatrix} \quad \nu_A = \begin{Bmatrix} 0 & d_1 > x \\ g_A^L(x) & d_1 \le x \le d_2 \\ 0 & d_2 \le x \le d_3 \\ g_A^R(x) & d_3 \le x \le d_4 \\ 1 & d_4 < x \end{Bmatrix}$$

$$z_\mu = \frac{1}{3}\left[\frac{\begin{matrix}2\alpha_\mu^2 + 3a_2a_1 - 3a_1^2 + 3(a_3^2 - a_2^2) \\ -2\beta_\mu^2 - 3a_4a_3 + 3a_4^2\end{matrix}}{a_4 + a_3 - a_2 - a_1}\right]$$

The centroid point of the trapezoidal intuitionistic fuzzy number $\tilde{A}=(a_1,a_2,a_3,a_4;d_1,d_2,d_3,d_4)$ can be written as

$$\tilde{z}_\mu(\tilde{A}) = \frac{\int_{a_1}^{a_2} \frac{x^2-xa_1}{\alpha_\mu}dx + \int_{a_2}^{a_3} xdx - \int_{a_3}^{a_4} \frac{x^2-xa_1}{\beta_\mu}dx}{\int_{a_1}^{a_2} \frac{x^2-a_1}{\alpha_\mu}dx + \int_{a_2}^{a_3} dx - \int_{a_3}^{a_4} \frac{x^2-a_1}{\beta_\mu}dx}.$$

After the integration the centroid point $\tilde{z}\mu(\tilde{A})$ of IFN $\tilde{A}$ can be written as

$$\tilde{z}_\mu(\tilde{A}) = \frac{\frac{1}{\alpha_\mu}\left[\frac{2(\alpha_\mu^3)+6a_2a_1\alpha_\mu -3\alpha_\mu(a_2a_1+a_1^2)}{6}\right] + \frac{(a_3^2-a_2^2)}{2} - \frac{1}{\beta_\mu}\left[\frac{2(\beta_\mu^3)+6a_4a_3\beta_\mu -3\beta_\mu(a_3a_4+a_4^2)}{6}\right]}{\frac{1}{\alpha_\mu}\left[\frac{\alpha_\mu(a_2+a_1)}{2} - \alpha_\mu a_1\right] + (a_3-a_2) - \frac{1}{\beta_\mu}\left[\frac{\beta_\mu(a_4+a_3)}{2} - \beta_\mu a_4\right]}$$

$$\tilde{z}_\mu(\tilde{A}) = \frac{1}{3}\left[\frac{2\alpha_\mu^2+3a_2a_1-3a_1^2+3(a_3^2-a_2^2) -2\beta_\mu^2-3a_4a_3+3a_4^2}{a_4+a_3-a_2-a_1}\right]$$

where $\alpha_\mu = a_2 - a_1$(left spread) of IFN &$\beta_\mu = a_4 - a_3$(Right spread) of IFN for membership function

$$\tilde{z}(\tilde{A}) = \frac{\int_{d_1}^{d_2} \frac{x^2-xd_2}{-\alpha_\nu}dx + \int_{d_2}^{d_3} xdx + \int_{d_3}^{d_4} \frac{x^2-xd_3}{\beta_\nu}dx}{\int_{d_1}^{d_2} \frac{x^2-d_2}{-\alpha_\nu}dx + \int_{d_2}^{d_3} dx + \int_{d_3}^{d_4} \frac{x^2-d_3}{\beta_\nu}dx},$$

$$\tilde{z}_\nu = \frac{1}{3}\left[\frac{2\alpha_\mu^2+3a_2a_1-3a_1^2+3(a_3^2-a_2^2)-2\beta_\mu^2-3a_4a_3+3a_4^2}{a_4+a_3-a_2-a_1}\right],$$

$$\tilde{z}_\nu(\tilde{A}) = \frac{-\frac{1}{\alpha_\nu}\left[\frac{2(\alpha_\nu^3)+6d_2d_1\alpha_\nu-3\alpha_\nu(d_2d_1+d_1^2)}{6}\right] + \frac{(d_3^2-d_2^2)}{2} + \frac{1}{\beta_\nu}\left[\frac{2(\beta_\nu^3)+6d_4d_3\beta_\nu-3\beta_\nu(d_3d_4+d_4^2)}{6}\right]}{-\frac{1}{\alpha_\nu}\left[\frac{\alpha_\nu(d_2+d_1)}{2}-\alpha_\nu d_2\right]+(d_3-d_2) + \frac{1}{\beta_\nu}\left[\frac{\beta_\nu(d_4+d_3)}{2}-\beta_\nu d_3\right]},$$

$$\tilde{z}\left(\tilde{A}\right) = \frac{1}{3}\left[\frac{-2\alpha_\nu^2-3d_2d_1+3d_1^2+3(d_3^2-d_2^2) +2\beta_\nu^2+3d_4d_3-3d_3^2}{d_4+d_3-d_2-d_1}\right],$$

where α_ν = left spread of IFN and β_ν = Right spread of IFN for Non membership function

$$\tilde{w}_\mu\left(\tilde{A}\right) = \frac{\int_0^1 \left(\left(\alpha_\mu\right)y^2 + a_1y\right)dy - \int_0^1 ((-\beta_\mu)y^2 + a_4y)dy}{\int_0^1 \left(\left(\alpha_\mu\right)y + a_1\right)dy - \int_0^1 ((-\beta_\mu)y + a_4)dy}.$$

After the integration the centroid point $\tilde{w}\mu(\tilde{A})$ of IFN $\tilde{A}$
can be written as

$$\tilde{w}_\mu = \frac{\frac{2\alpha_\mu+3a_1-3a_4+2\beta_\mu}{6}}{\frac{\alpha_\mu}{2}+a_1+\frac{\beta_\mu}{2}-a_4},$$

$$\tilde{w}_\mu = \frac{1}{3}\left[\frac{2\alpha_\mu+3a_1-3a_4+2\beta_\mu}{\alpha_\mu+2a_1-2a_4+\beta_\mu}\right],$$

where α_μ = left spread of IFN and β_μ = right spread of IFN for membership function

$$\tilde{w}_\nu\left(\tilde{A}\right) = \frac{\int_0^1 \left(\left(-\alpha_\nu\right)y^2 + d_2y\right)dy - \int_0^1 ((\beta_\nu)y^2 + d_3y)dy}{\int_0^1 \left(\left(-\alpha_\nu\right)y + d_2\right)dy - \int_0^1 ((\beta_\nu)y + d_3)dy},$$

$$\tilde{w}_\nu = \frac{\frac{1}{6}[-2\alpha_\nu+3d_2-2\beta_\nu-3d_3]}{-\frac{\alpha_\nu}{2}+d_2-\frac{\beta_\nu}{2}-d_3},$$

$$\tilde{w}_\nu = \frac{1}{3}\left[\frac{2\alpha_\nu-3d_2+3d_3+2\beta_\nu}{\alpha_\nu-2d_2+2d_3+\beta_\nu}\right],$$

where $\alpha_\nu = d_2 - d_1$ (left spread) of IFN and $\beta_\nu = d_4 - d_3$ (Right spread) of IFN for Non membership function.

2.2 Ranking of triangular intuitionistic fuzzy number:

$$R\left(\tilde{A}^{\mathrm{IFN}}\right) = \sqrt{\frac{1}{2}\left(\left[\tilde{z}_\mu(\tilde{A}) - \tilde{w}_\mu(\tilde{A})\right]^2 + \left[\tilde{z}_\nu(\tilde{A}) - \tilde{w}_\nu(\tilde{A})\right]^2\right)}$$

where $\tilde{z}\mu$ $(\tilde{A})$,$\tilde{w}\mu$ $(\tilde{A})$,$\tilde{z}$ $(\tilde{A})$,$\tilde{w}\nu$ $(\tilde{A})$ are centroid point of the membership and non-membership functions and it can be define by

$$\tilde{z}_\mu = \left[\frac{(a_3+a_1+a_2)}{3}\right], z_\nu = \left[\frac{(d_1-(d_2-d_1)+2d_3)}{3}\right]$$

$$\tilde{w}_\mu = \frac{1}{3}\left[\frac{(a_1-a_3)}{(a_1-a_3)}\right] = \frac{1}{3}, w_\nu = \frac{1}{3}\left[\frac{2(d_3-d_1)}{(d_3-d_1)}\right] = \frac{2}{3}.$$

For the intuitionistic Fuzzy Number $\tilde{A}^{\mathrm{IFN}}$.

Table 1. Probability of the failure for each period.

Year	1	2	3	4	5	6
possibility of failure	0.10	0.17	0.25	0.35	0.11	0.02

2.3 Ranking of trapezoidal intuitionistic fuzzy number:

The ranking approach can be used to do the comparison of the two different trapezoidal intuitionistic fuzzy numbers.

$$R\left(\tilde{A}^{\mathrm{IFN}}\right)=\sqrt{\frac{1}{2}\left(\left[\tilde{z}_{\mu}(\tilde{A})-\tilde{w}_{\mu}(\tilde{A})\right]^{2}+\left[\tilde{z}(\tilde{A})-\tilde{w}_{\nu}(\tilde{A})\right]^{2}\right)},$$

where

$$\tilde{z}_{\mu}=\frac{1}{3}\left[\frac{2\alpha_{\mu}^{2}+3a_{2}a_{1}-3a_{1}^{2}+3(a_{3}^{2}-a_{2}^{2})-2\beta_{\mu}^{2}-3a_{4}a_{3}+3a_{4}^{2}}{a_{4}+a_{3}-a_{2}-a_{1}}\right],$$

$$\tilde{z}_{\nu}=\frac{1}{3}\left[\frac{-2\alpha_{\nu}^{2}-3d_{2}d_{1}+3d_{2}^{2}+3(d_{3}^{2}-d_{2}^{2})+2\beta_{\nu}^{2}+3d_{4}d_{3}-3d_{3}^{2}}{d_{4}+d_{3}-d_{2}-d_{1}}\right],$$

$$\tilde{\tilde{w}}_{\mu}=\frac{1}{3}\left[\frac{2\alpha_{\mu}+3a_{1}-3a_{4}+2\beta_{\mu}}{\alpha_{\mu}+2a_{1}-2a_{4}+\beta_{\mu}}\right],$$

$$\tilde{\tilde{w}}_{\nu}=\frac{1}{3}\left[\frac{2\alpha_{\nu}-3d_{2}+3d_{3}+2\beta_{\nu}}{\alpha_{\nu}-2d_{2}+2d_{3}+\beta_{\nu}}\right],$$

Which represents the centroid of the Trapezoidal IFN. The ranking can be define by
(i)$\tilde{A}^{\mathrm{IFN}}\succ\tilde{B}^{\mathrm{IFN}}\Leftrightarrow R(\tilde{A}^{\mathrm{IFN}})\succ R(\tilde{B}^{\mathrm{IFN}})(ii)\tilde{A}^{\mathrm{IFN}}\prec\tilde{B}^{\mathrm{IFN}}\Leftrightarrow R(\tilde{A}^{\mathrm{IFN}})\prec R(\tilde{B}^{\mathrm{IFN}})(iii)\tilde{A}^{\mathrm{IFN}}\approx\tilde{B}^{\mathrm{IFN}}\Leftrightarrow R(\tilde{A}^{\mathrm{IFN}})\approx R(\tilde{B}^{\mathrm{IFN}})$.

3 Replacement of low-cost items in bulk quantities (Group replacement)

The items which fail during a fixed period t_1 are replaced individually when they fail, and all the items (including those not failed/new) are replaced at some optimal interval of time. Therefore here we have to determine the value of n for which the average cost per period is minimum. Assume that the items which fail during as period t_1 are replaced at the end t_1.

Let

$\tilde{N}$ = Total number of equipments/items.

$_{N(x)}$ = Number of equipments/items which is failed in the xth period

$\tilde{C}g$ = Group/block replacement fuzzy cost per item.

$_{Ci}$ = Fuzzy cost of the Individual replacement.

$$\begin{aligned}\tilde{C}\left(n\right)&=NC_{g}+C_{i}[N\left(1\right)+N\left(2\right)+\cdots+N\left(n-1\right)]\\&=NC_{g}+C_{i}\sum_{x=1}^{(n-1)}N(x)\end{aligned}$$

Average/mean fuzzy cost

$$\tilde{A}(\tilde{n})=\frac{\tilde{C}(n)}{n}$$

Since in this case n is the discrete variable
$\tilde{A}$ (n)is min

$$\begin{aligned}&[\Delta\tilde{A}\ (n-1)/1<0<\Delta\tilde{A}(\tilde{n})/1]\Delta\tilde{A}(\tilde{n})\\&\quad=\tilde{A}(\tilde{n+1})/1-\tilde{A}(\tilde{n})/1\\&\quad=\tilde{C}(\tilde{n+1})/n+1-\tilde{C}(\tilde{n})/n\end{aligned}$$

From $\tilde{C}$ (n), we get "$\tilde{C}(\tilde{n+1})=\tilde{C}(\tilde{n})+\tilde{C}_{i}\tilde{N}(\tilde{n})$

$$\Delta\tilde{A}(\tilde{n})=\frac{\tilde{C}(n)/1+\tilde{C}_{i}\tilde{N}(n)/1}{(n+1)/1}\frac{-\tilde{C}(n)/1}{(n)/1}$$

$$\Delta\tilde{A}(n-1)=\frac{\tilde{C}_{i}\tilde{N}(n-1)/1-\tilde{C}(n-1)/(n-1)}{(n)/1}$$

Similarly we get

$$\begin{aligned}&\tilde{C}_{i}\tilde{N}(\tilde{n-1})/1-\tilde{C}(\tilde{n-1})/(n-1)<0\\&\quad<\tilde{C}_{i}\tilde{N}(n)/1-\tilde{C}(n)/(n)\end{aligned}$$

$$\tilde{C}_{i}\tilde{N}(n-1)/1<\tilde{C}(\tilde{n-1})/(n-1)$$

As a result of the above equations", we should find the optimal replacement time when the average annual fuzzy cost or intuitionistic fuzzy cost reaches its minimum.

4 Computational descriptions:

4.1 Comparison of individual and group replacement in fuzzy environment

There are street lights (9350, 10350, 11350) that must all be kept in functioning order. When a street lights fails in service, it costs Rs. (510, 610, 710) to replace it; however, if all of the lights are replaced at the same time, it costs Rs. (110, 210, 310) per light. Determine the optimum replacement policy if the fraction of lights failing in successive time intervals is known. The following types of street lights mortality rates have been observed (Tab. 1).

Let $\tilde{\eta}_n$ be the replacements at the end of nth year and $\tilde{\eta}_0$=(9350, 10350, 11350).

Determination for the number of replacements in 1st, 2nd, ..., 6th year$\tilde{\eta}_1$=(9350, 10350, 11350)×0.10=(1035,1000,1000)

Table 2. Determination of total cost of group fuzzy replacement and average Fuzzy cost per year.

End of year	Total cost of group fuzzy replacement	Average fuzzy cost / year
1	((1035, 1000, 1000) × 610) + ((9350, 10350, 11350) × 210)	(2803850, 2804850, 2805850)
2	((2898, 1000, 1000) × 610) + ((9350, 10350, 11350) × 210)	(1969640, 1970640, 1971640)
3	((5847.75, 1000, 1000) × 610) + ((9350, 10350, 11350) × 210)	(1912542.5, 1913542.5, 1914542.5) Replace (minimum)
4	((10340.68, 1000, 1000) × 610) + ((9350, 10350, 11350) × 210)	(2119329.46, 2120329.46, 2121329.46)
5	((13257.95, 1000, 1000) × 610) + ((9350, 10350, 11350) × 210)	(2051168.19, 2052168.19, 2053168.19)
6	((16023.78, 1000, 1000) × 610) + ((9350, 10350, 11350) × 210)	(1990336, 1991336, 1992336)

$\tilde{\eta}_2$=((10350,1000,1000)×0.17)+((1035,1000,1000)× 0.10)$\tilde{\eta}_2$=(1863,1000,1000)

We can calculate the number of replacements in the third, fourth, fifth, and sixth years in this manner.

$\tilde{\eta}_3$=(2949.75.15,1000,1000),$\tilde{\eta}_4$=(4493,1000,1000)

$\tilde{\eta}_5$=(2917.2,1000,1000),$\tilde{\eta}_6$=(2765.83,1000,1000)

Expected life time of street lights =

= 1(0.10) + 2(0.17) + 3(0.25) + 4(0.35) + 5(0.11) + 6 (0.02) = 3.26 years

The amount of failures each year on average

= $\tilde{\eta}$/average age=(9350, 10350, 11350)/3.26= (3175,1000,1000)

As a result, at Rs.(510,610,710) per light, the cost of individual replacement is (1936750, 1000, 1000).

Since replacing all (9350, 10350, 11350) lights at the same time costs Rs . (100, 110, 120) per light, the average cost for various group replacement strategies is shown in Table 2.

The average intuitionistic fuzzy cost is lowest in the third year, so group replacement after every third year is optimal.

4.2 Comparison of Individual and Group Replacement in intuitionistic fuzzy environment

There are street lights (9350, 10350, 11350; 8350, 10350, 12350) that must all be kept in functioning order. When a street lights fails in service, it costs Rs. (510, 610, 710; 410, 610, 810) to replace it; however, if all of the lights are replaced at the same time, it costs Rs. (110, 210, 310; 10, 210, 410) per light. Determine the optimum replacement policy if the fraction of lights failing in successive time intervals is known. The following types of street lights mortality rates have been observed in Table 3.

Let $\tilde{\eta}_n$ be the replacements at the end of nth year and $\tilde{\eta}_0$=(9350, 10350, 11350; 8350, 10350, 12350).

Determination for the number of replacements in 1^{st},2^{nd},....6^{th} year

$\tilde{\eta}_1$=(9350, 10350, 11350; 8350, 10350, 12350)X0.10= (1035,1000,1000;1035,2000,2000)

$\tilde{\eta}_2$=((10350,1000,1000; 10350,2000,2000)X0.17) +((1035,1000,1000; 1035,2000,2000)X0.10)$\tilde{\eta}_2$= (1863,1000,1000; 1863,2000,2000)

We can calculate the number of replacements in the third, fourth, fifth, and sixth years in this manner.

$\tilde{\eta}$3=(2949.75,1000,1000; 2949.75,2000,2000),$\tilde{\eta}$4= (4492.9,1000,1000;4492.9,2000,2000)

$\tilde{\eta}$5=(2917.25,1000,1000; 2917.25,2000,2000),$\tilde{\eta}$6= (2765.86,1000,1000; 2765.86,2000,2000)

Expected life time of street lights = $\sum_{i=1}^{6} x_i(P_{x_i})$

= 1(0.10)+2(0.17)+3(0.25)+4(0.35)+5(0.11)+6 (0.02) = 3.26 years

The amount of failures each year on average

= $\tilde{\eta}$/average age=(9350, 10350, 11350; 8350, 10350, 12350)/3.26=(3175,1000,1000;3175,2000,2000)

As a result, at Rs. (510, 610, 710; 410, 610, 810) per light, the cost of individual replacement is (1935750, 1936750, 1937750; 1934750, 1936750, 1938750).

Since replacing all (9450, 10450, 11450; 8450, 10450, 12450) lights at the same time costs Rs. (110, 210, 310; 10, 210, 410) per light, the average cost for various group replacement strategies is shown in Table 4.

The average intuitionistic fuzzy cost is lowest in the third year, so group replacement after every third year is optimal.

5 Results and discussions

The primary goal of this article is to provide industries with more precise results so that they can determine the best time to replace machines or equipment, as well as the best replacement policy.

- (From Fig. 1) In fuzzy environment individual replacement fuzzy cost is calculated from the expected life time of street lights and the total number of street lights. The individual replacement fuzzy cost of street lights is 1936750 where the left & right fuzziness of the (MF) membership function are 1935750 and 1937750, respectively.
- In fuzzy environment group replacement fuzzy cost is calculated from the total fuzzy cost of street lights. (From Tab. 1) The group replacement fuzzy cost of the street lights for first year is 2804850, where the left & right fuzziness of the (MF) membership function are 2803850 and 2805850, respectively. In the second year group replacement fuzzy cost of the street lights is1970640, where the left & right fuzziness of the (MF) membership function are 1969640 and 1971640, respectively.
- In the third year group replacement fuzzy cost of the street lights is 1913542.5, where the left & right fuzziness of the (MF) membership function are 1912542.5 and 1914542.5, respectively. In the fourth year group replacement fuzzy cost of the street lights is

Table 3. Probability of the failure for each period.

Year	1	2	3	4	5	6
possibility of failure	0.10	0.17	0.25	0.35	0.11	0.02

Table 4. Determination of total cost of group intuitionistic fuzzy replacement and average intuitionistic Fuzzy cost per year.

End of year	Total cost of group intuitionistic fuzzy replacement	Average intuitionistic fuzzy cost / year
1	((1035,1000,1000;1035,2000,2000)X610)+ ((9350, 10350, 11350; 8350, 10350, 12350)X210)	(2803850, 2804850, 2805850; 2802850, 2804850, 2806850)
2	((2898,1000,1000;2898,2000,2000)X610)+ ((9350, 10350, 11350; 8350, 10350, 12350)X210)	(1969640,1970640,1971640; 1968640,1970640, 1972640)
3	((5847.75,1000,1000;5847.75,2000,2000)X610)+ ((9350, 10350, 11350; 8350, 10350, 12350)X210)	(1912542.5, 1913542.5, 1914542.5; 1911542.51913542.5,1915542.5) Replace (min)
4	((10340.68,1000,1000;10340.68,2000,2000)X610)+ ((9350, 10350, 11350; 8350, 10350, 12350)X210)	(2119329.46,2120329.46,2121329.46; 2118329.46, 2120329.46,2122329.46)
5	((13257.95,1000,1000;13257.95,2000,2000)X610)+ ((9350, 10350, 11350; 8350, 10350, 12350)X210)	(2051168.19,2052168.19,2053168.19; 2050168.19,2052168.19,2054168.19)
6	((16023.75,1000,1000;16023.75,2000,2000)X610)+ ((9350, 10350, 11350; 8350, 10350, 12350)X210)	(1990336,1991336,1992336; 1989336,1991336,1993336)

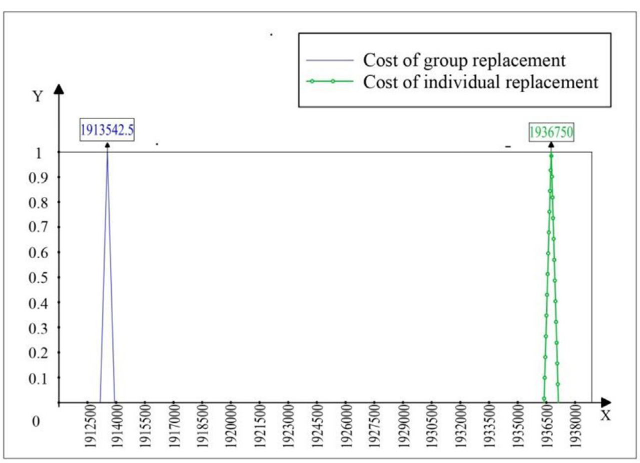

Fig. 1. The graph depicts comparison of Group and Individual replacement cost in fuzzy environment.

2120329.46, where the left & right fuzziness of the (MF) membership function are 2119329.46 and 2121329.46, respectively.

– In the fifth year group replacement fuzzy cost of the street lights is 2052168.19, where the left & right fuzziness of the (MF) membership function are 2051168.19 and 2053168.19, respectively. In the sixth year group replacement fuzzy cost of the street lights is 1991336, where the left & right fuzziness of the (MF) membership function are 1990336 and 1992336, respectively.

– Based on the discussion above, the individual replacement fuzzy cost of an street lights is 1936750 and the group replacement fuzzy cost of an street lights is 1913542.5. As a result, the fuzzy cost of group replacement is less than that of individual replacement. Therefore, the group replacement policy is the best policy for the aforementioned situation. Figure 1 shows the comparison of Group and Individual replacement cost in fuzzy environment.

– (From Fig. 2) In intuitionistic fuzzy environment individual replacement cost is calculated from the expected life time of street lights and the total number of street lights. The individual replacement fuzzy cost of an street lights is 1936749.5, which is approximately 1936750 where the left & right fuzziness of the (MF) membership function are 1935750 and 1937750, respectively, and where the left & right fuzziness of the (NMF) non membership function are 1934750 and 1938750, respectively.

– In intuitionistic fuzzy environment group replacement fuzzy cost is calculated from the total fuzzy cost of street lights. (From Tab. 2) The group replacement fuzzy cost of the street lights for first year is 2804849.5, which is approximately 2804850, where the left & right fuzziness of the (MF) membership function are 2803850 and 2805850, respectively, and where the left & right fuzziness of the (NMF) non membership function are 2802850 and 2806850, respectively. In the second year group replacement fuzzy cost of the street lights is 1970639.5, which is approximately 1970640, where the left & right fuzziness of the (MF) membership function are 1969640 and 1971640, respectively, and where the left & right fuzziness of the (NMF) non membership function are 1968640 and 1972640, respectively.

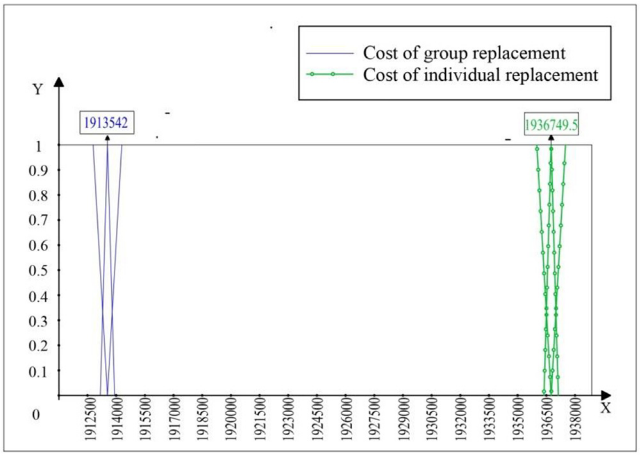

Fig. 2. The graph depicts comparison of Group and Individual replacement cost in intuitionistic fuzzy environment.

- In the third year group replacement fuzzy cost of the street lights is 1913542, where the left & right fuzziness of the (MF) membership function are 1912542.5 and 1914542.5, respectively, and where the left & right fuzziness of the (NMF) non membership function are 1911542.5 and 1915542.5, respectively. In the fourth year group replacement fuzzy cost of the street lights is 2120329.46, where the left & right fuzziness of the (MF) membership function are 2119329.46 and 2121329.46, respectively, and where the left and right fuzziness of the (NMF) non membership function are 2118329.46 and 2122329.46, respectively.
- In the fifth year group replacement fuzzy cost of the street lights is 2052168.19, where the left & right fuzziness of the (MF) membership function are 2051168.19and 2053168.19, respectively, and where the left & right fuzziness of the (NMF) non membership function are 2050168.19 and 2054168.19, respectively. In the sixth year group replacement fuzzy cost of the street lights is 1991336, where the left & right fuzziness of the (MF) membership function are 1990336 and 1992336, respectively, and where the left & right fuzziness of the (NMF) non membership function are 1989336 and 1993336, respectively.
- Based on the discussion above, the individual replacement intuitionistic fuzzy cost of street lights is 1936749.5, and the group replacement intuitionistic fuzzy cost of street lights is 1913542 As a result, the intuitionistic fuzzy cost of group replacement is less than that of individual replacement. Therefore, the group replacement policy is the best policy for the aforementioned situation. Figure 2 shows the comparison of Group and Individual replacement cost in intuitionistic fuzzy environment. As a result, the intuitionistic fuzzy environment obtained a more generalized result than the fuzzy environment.

Example 4.3: The failure rates of certain items are observed as follows:

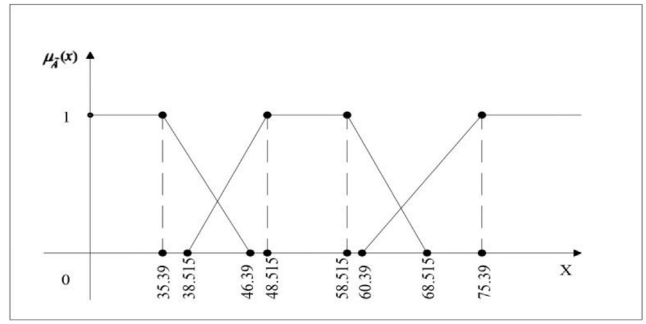

Fig. 3. Graphical representation of trapezoidal intuitionistic fuzzy number.

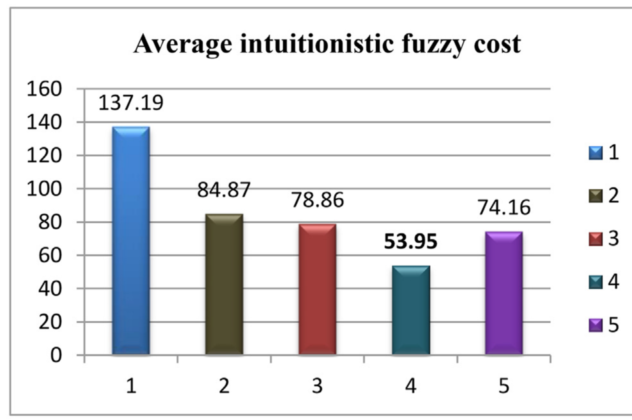

Fig. 4. Because the mean/average cost is lowest in the fourth month, here we must perform group replacement at the end of every fourth month. Figure 4 shows that the mean/average cost is $ 53.95 (the mean cost in the case of group replacement), the group replacement policy is preferable because which is less than the cost of individual replacement

The cost per item of individual replacement is $\tilde{p}_1=(1.35,1.40,1.45,1.50;1.25,1.40,1.55,1.70)$. The cost of group replacement is $\tilde{p}_2=(0.60,0.65,0.75,0.80;0.55,0.65,0.80,0.90)$ per item, Determine the optimal of replacement time as whole group/block. Also find whether individual replacement is preferable or group replacement.

Solution:
$\rho_1 = 0.30$
$\rho_2 = 0.40 - 0.30 = 0.10$
$\rho_3 = 0.70 - 0.40 = 0.30$
$\rho_4 = 0.85 - 0.75 = 0.15$
$\rho_5 = 1.0 - 0.85 = 0.15$

Let $\tilde{N}_i$ items be replaced at ith month end.

Ñ− Number of items at the beginning.

$\tilde{N}=\tilde{N}_0=(105,115,125,135;100,115,125,140)\tilde{N}_1=\tilde{N}_0\rho 1=(105,115,125,135;100,115,125,140)$

The parametric form of $\tilde{N}_0$

$\tilde{N}=\tilde{N}_0=(120,5,10,10;120,5,15,15)$

$\tilde{N}_1=\tilde{N}_0\rho_1=(36,5,10,10;36,5,15,15),\tilde{N}_2=\tilde{N}_0\rho_2+\tilde{N}_1\rho_1=(22.8,5,10,10;22.8,5,15,15)$

Table 5. Probability of the failure for each period.

End of period	1	2	3	4	5
probability of failure	0.3	0.4	0.7	0.85	1.0

Table 6. To determine the intuitionistic fuzzy cost of group replacement.

End of the month ‘t’	Individual Replacement $\sum_{i=1}^{t}\tilde{N}_i$	Group Replacement intuitionistic fuzzy cost $P_1(\sum_{i=1}^{t}\tilde{N}_i)+\tilde{P}_2\tilde{N}$
1	(36,5,10,10;36,5,15,15)	(135.3,5,10,10;140.1,5,15,15)
2	(58.8,5,10,10;58.8,5,15,15)	(167.79,5,10,10;173.73,5,15,15)
3	(105.24,5,10,10;105.24,5,15,15)	(233.967,5,10,10;242.229,5,15,15)
4	(150.22,5,10,10;150.22,5,15,15)	(214.06,5,10,10;221.57,5,15,15)
5	(198.5,5,10,10;198.5,5,15,15)	(366.86,5,10,10;379.78,5,15,15)

Table 7. To determine the Intuitionistic Average fuzzy cost of Group Replacement

End of the month ‘t’	Intuitionistic Average fuzzy cost = $\widetilde{p}_1(\sum_{i=1}^{t}\tilde{N}_i)+\widetilde{p}_2\,\tilde{N}/_t$
1	(135.3,5,10,10;140.1,5,15,15)
2	(83.89,5,10,10;86.86,5,15,15)
3	(77.98,5,10,10;80.74,5,15,15)
4	(**53.515**,5,10,10;53.39,5,15,15)(Replace)
5	(73.372,5,10,10;75.956,5,15,15)

$\tilde{N}_3=\tilde{N}_0\rho_3+\tilde{N}_1\rho_2+\tilde{N}2\rho_1=$(46.44,5,10,10;46.44,5,15,15)

$\tilde{N}_4=\tilde{N}_0\rho 4+\tilde{N}_1\rho 3+\tilde{N}_2\rho_2+\tilde{N}_3\rho_1=$ (44.98,5,10,10;44.98,5,15,15)

$\tilde{N}_5=\tilde{N}_0\rho_5+\tilde{N}_1\rho_4+\tilde{N}_2\rho_3+\tilde{N}_3\rho_2+\tilde{N}_4\rho_1=$ (48.28,5,10,10;48.28,5,15,15)

Table 5 depicts the failure probability, which was acquired in order to determine item's expected life time.

The expected life of any item $\sum_{i=1}^{5} i\rho_i = 2.75$

The average number of failures per month = (43.7, 5, 10, 10; 43.7, 5, 15, 15).

(1.425,0.025,0.05,0.05;1.475, 0.075,0.15,0.15) = (62.27,5,10,10;64.45,5,15,15)

Our accuracy function = 62.27

Average cost of group replacement policy.

Table 6 shows the determination of group replacement intuitionistic fuzzy cost, which is obtained from cumulative individual replacement intuitionistic fuzzy cost. From table 7 shows that the above replacement policy Intuitionistic Average fuzzy cost is continuously decreasing after some time it star increasing; hence the group/block replacement should be done at the end of the 4th year. In this paper a different solution approach has been followed that is used to find the optimal group replacement time without converting/changing the fuzzy parameters into the crisp one. Figure 3 shows the graphical representation of the result.

6 Conclusions

The components of electronics such as tube lights, bulbs, and resistors typically fail abruptly rather than gradually deteriorate. Because of the unexpected failure, the system has completely failed. As a result, we must implement a suitable replacement policy for such items in order to reduce the possibility of a complete breakdown. The replacement in this case can be any of the following: Individual replacement is the first. The second technique is group replacement. Individual replacement policies require that the item be replaced as soon as possible if it fails. We replace all items at the same time in group replacement. When dealing with prospective implementation in replacement models, this study shows that IFS is a more powerful tool than fuzzy set theory. We determined the minimum average cost without changing the capital cost of fuzzy to crisp. Another reason to use the proposed approach index is that it produces more minimum values in the replacement model with different types of membership functions (MF) while maintaining accuracy within the closed crisp interval. An intuitive fuzzy set (IFS) is defined by a membership function (MF) and a non-membership function (NMF), the sum of which is 1.IFN appears to reflect the uncertainty and lack of accuracy of data in a specific way. As a result, IFS has been used to assist humans in making better decisions and performing other tasks that require cognitive experience and expertise but are inherently imprecise or worthy. The individual replacement fuzzy cost of street lights, according to the preceding discussion, is 1936750, and the group replacement fuzzy cost is 1913542.5. As a result, group replacement is less expensive than individual replacement. Hence the group replacement policy is the best policy in the aforementioned situation. The individual replacement intuitionistic fuzzy cost of street light is 1936749.5, and the group replacement intuitionistic fuzzy cost is 1913542 based on the preceding discussion. As a result, the intuitionistic fuzzy cost of group replacement is lower than that of individual replacement. As a result, the group replacement policy is the best policy in the aforementioned situation. This paper considers individual and group replacement policies in fuzzy and intuitionistic fuzzy environments. The method is illustrated with numerical examples. Furthermore, this paper compares individual and group replacement models in fuzzy and intuitionistic fuzzy environments, and we obtained the same replacement time and cost in both cases. The proposed replacement model's accuracy is determined using TFN and TIFN arithmetic illustrations and also with trapezoidal intuionistic fuzzy number. IFS replacement models are far more generalized and easy to evaluate than fuzzy ones. According to the findings of this study, intuitionistic fuzzy replacement is one of the most advantageous techniques for computing evaluation standards because the evidence obtained appears to be more easily interpretable. The approach being suggested has significantly more advantages. Because the traditional model has some boundaries in explaining real-life situations, the fuzzy concept can take multiple values due to their uncertainty. The future work will involve interacting with a neutrosophic fuzzy environment.

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Conflicts of interest

The authors have no conflicts of interest.

Author's Contribution

Conceptualization, Methodology, Validation, Investigation, Recourses, Data curation, Visualization: V.K. Saranya and S.S. Murugan. Software, Writing: V.K. Saranya; Formal analysis, Supervision: S.S. Murugan.

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