

Simple, stable and efficient nonlinear pulse compression through cascaded filamentation in air

Tao Pu¹, Kan Tian¹, Bo Hu¹, Zhongjun Wan¹, Linzhen He¹, Xuemei Yang¹, Han Wu¹, Yang
Li¹, Weizhe Wang^{1,*} and Houkun Liang^{1,*}

¹ College of Electronics and Information Engineering, Sichuan University, Chengdu 610064,
China

*Correspondence to: H. K. Liang, College of Electronics and Information Engineering, Sichuan
University, Chengdu 610064, China. Email: hkliang@scu.edu.cn, W. Z. Wang, College of
Electronics and Information Engineering, Sichuan University, Chengdu 610064, China. Email:
wangweizhe_1997@163.com.

Abstract Nonlinear compression has become an obligatory technique along with the
development of ultrafast lasers in generating ultrashort pulses with narrow pulse widths and
high peak power. Particularly, techniques of nonlinear compression have experienced a rapid
progress as ytterbium (Yb)-doped lasers with pulse widths in the range of hundreds
femtoseconds to a few picoseconds stepping into the mainstream laser tools for both
scientific and industrial applications. Here, we report a simple and stable nonlinear pulse
compression technique with high efficiency through cascaded filamentation in air followed
by dispersion compensation. Pulses at a center wavelength of 1040 nm with millijoule pulse
energy and 160 fs pulse width from a high-power Yb:CaAlGdO₄ regenerative amplifier are
compressed to 32 fs, with only 2.4% loss from the filamentation process. The compressed
pulse has a stable output power with a root-mean-square variation of 0.2% over one hour
time.

Key words: femtosecond pulse; nonlinear compression; filamentation

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1 *I. INTRODUCTION*

2 Ultrafast lasers with millijoule pulse energy, tens-watts average power, and a pulse width
3 of few to few-tens fs have found diversified applications such as generations of high-order
4 harmonics [1], intrapulse difference-frequency waves [2], terahertz pulses [3], and isolated
5 attosecond pulses [4]. In the last decade, ytterbium (Yb)-doped lasers have developed rapidly with
6 the superior power scaling capability thanks to the small quantum defects and the availability of
7 high-power laser diodes as the pump source [5], gradually replacing titanium-doped sapphire
8 (Ti:sapphire) lasers which have long been serving as workhorses of scientific ultrafast lasers.
9 Despite of many superior characters, the pulse width of Yb-doped lasers is usually greater than
10 100 fs [6], limited by the emission bandwidth, which severely hinders the popularization of Yb-
11 doped lasers for the above-mentioned applications.

12 To further compress the pulse width of Yb-doped lasers, a relatively straightforward
13 strategy is nonlinear pulse compression, with the principle based on the nonlinear spectral
14 broadening and dispersion compensation [7]. A number of techniques have been used for nonlinear
15 spectral broadening of millijoule-level Yb-doped lasers [8-10]. The hollow core fiber (HCF)
16 technique uses hollow waveguides filled with noble gases to extend the nonlinear interaction
17 length between pulses and nonlinear medium, which supports the generation of few-cycle pulses
18 with good beam quality [11]. However, the transmission efficiency of HCF is difficult to exceed
19 70% [12-14]. Careful alignment and good beam pointing of laser system are also required in HCF
20 compressor. Multiple thin plates using sequences of thin dielectric plates as nonlinear media serve
21 as an effective way for pulse compression with high efficiency [15-17]. However, damages to thin
22 plates and beam quality degradation could be an issue of the laser system stability and long-term
23 operation. Multi-pass cell (MPC) compressors have proven to be able to achieve the extreme

1 compression factors with minimum losses, excellent pointing sensitivity, and good beam quality
2 [18,19]. However, traditional MPC modules are based on Herriot cavities which impose relatively
3 complicate beam routings, and require delicate dielectric coatings and gas chambers. Filamentation
4 in gases has been adapted over the past decade to achieve ultrashort pulses. The laser filamentation
5 is sustained when Kerr effect is balanced by the diffraction and plasma defocusing [20], which
6 enables much longer non-diffractive propagation compared to the Rayleigh range with a high
7 intensity. As a result, spectral broadening induced by self-phase modulation (SPM) becomes more
8 prominent. Equipped with a high-pressure gas cell, 0.68 mJ pulses from a Ti:sapphire laser system
9 with a pulse duration of 33 fs were compressed to 5.1 fs, nevertheless, the optical transmission in
10 two cascaded filaments is only 26% owing to the strong ionization loss [21]. In addition, 4 fs pulses
11 were obtained via a single filamentation in a semi-infinite argon gas cell, pumped by 35 fs input
12 pulses, with a low compression efficiency [22]. Besides the large loss, the strong ionization also
13 induces substantial spatial chirp, and confines the good temporal shape only in the beam center.
14 Without the requirement of vacuum apparatuses nor careful control of the gas pressure, pulse
15 compression through filamentation in air has also been demonstrated, generating 22 fs pulses out
16 of 100 fs input pulses. However, the energy coupled into the inner core of the filament which
17 generates the spectral broadening is only 20% of the total input pulse energy, again owing to the
18 strong ionization [23]. In addition, the output stability is another important concern of nonlinear
19 pulse compression. Strong ionization and spectral modulation could induce plasma instability
20 [24,25], which should be avoided in the design of Yb-laser compressors.

21 In this work, a simple, stable and efficient nonlinear pulse compression technique based on
22 cascaded laser filamentation in air is demonstrated. Pulses with an energy of 1 mJ from Yb-doped
23 laser are compressed from 160 fs to 32 fs with only 2.4% loss from the filamentation process. The

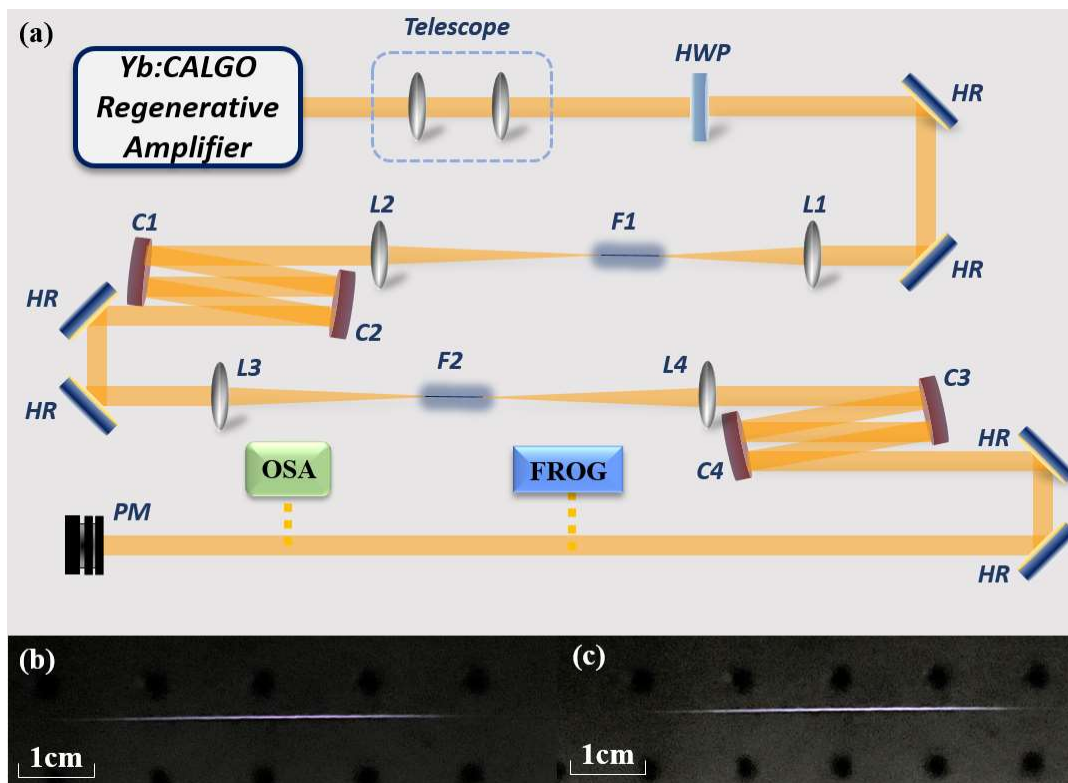
1 peak power of the Yb-doped laser system is enhanced from 6.25 GW to 24.7 GW by ~4 times,
2 with good beam quality and spectral homogeneity across the beam profile. The cascaded air
3 filamentation compressor has a superior long-term stability, with the measured power variation <
4 0.2% over one hour. The demonstrated technique is simple, robust and economical for the efficient
5 pulse compression of Yb-doped lasers, with moderate compression factors (5 times for 2 cascaded
6 compression stages), towards to the acquisition of high peak and average power output.

7 II. RESULT

8 The configuration of the experimental setup is shown in Fig. 1(a). A homemade
9 Yb:CaAlGdO₄ (Yb:CALGO) chirped-pulse regenerative amplifier, generating 160 fs pulses, at a
10 central wavelength of 1040 nm and a repetition rate of 20 kHz is used as the experimental platform.
11 In the experiment, 1 mJ pulses with a peak power of 6.25 GW is delivered. The cascaded
12 filamentation compressor consists of two identical modules. Nonlinear spectral broadening is
13 achieved mainly through SPM during the process of laser filamentation in air. Chirped mirrors are
14 placed behind each nonlinear spectral broadening stage to compensate the dispersion. The photos
15 of laser filamentations from the cascaded modules are shown in Fig. 1(b) and (c). The laser before
16 and after nonlinear compression is characterized by an optical spectral analyzer (Yokogawa
17 AQ6370D), a power meter (Ophir FL250A-BB-50), and a beam profiler (Dataray WinCamD). The
18 temporal profiles of the compressed pulses are measured by a commercial second-harmonic
19 generation frequency-resolved optical gating (SHG-FROG) setup (Mesa Photonics).

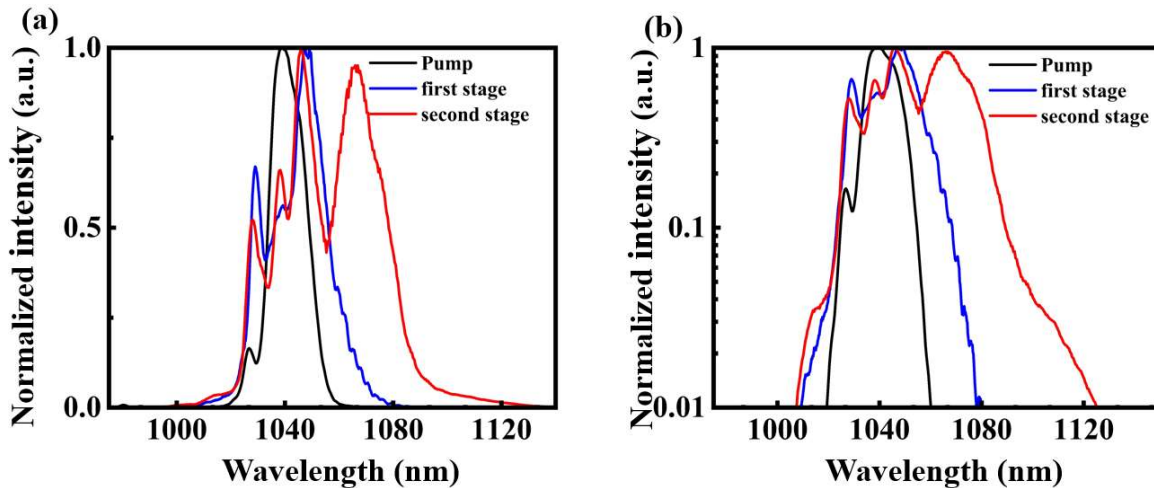
20 Prior to the nonlinear compressor, a telescope system is used to adjust the beam size which
21 is crucial for maximizing the spectral broadening, and at the same time minimizing the ionization
22 loss through the process of laser filamentation. After the telescope, p-polarization is adjusted with
23 a half-wave plate to reduce the polarization loss of the chirped mirrors. In the first compression

1 stage, pulses with an energy of 1 mJ and a peak power of 6.25 GW which is slightly greater than
 2 the critical power of self-focusing in air at a wavelength of 1040 nm are focused in air by using a
 3 lens with 500 mm focal length. Loose focusing is desired for a weak ionization and long interaction
 4 length. The beam diameter on the lens is ~ 3.5 mm generating a linear peak power density of 18.8
 5 TW/cm² at the focal spot. As shown in Fig. 1(b), laser filament with a length of ~ 50 mm is formed,
 6 through which SPM occurs. The divergent beam is collimated by a lens with a focal length of 750
 7 mm, and the diameter of the collimated beam is made ~ 5 mm.



8
 9 **Fig. 1.** (a) Schematic diagram of the cascaded nonlinear compressor through filamentation in air.
 10 HWP, half-wave plate; L, lens; C, chirped mirror; F, filamentation; HR, high reflection mirror;
 11 PM, power meter. The photos of the generated filamentation in air in the first (b) and second stage
 12 (c).
 13 The reason for enlarging the beam size is to avoid damages on subsequent chirped mirrors. After
 14 the collimating lens, the beam is reflected 18 times on the chirped mirror pairs, which provide a

1 total negative dispersion of -2700 fs^2 over the spectral range of 850 -1200 nm. In the second stage,
 2 a lens with a focal length of 750 mm is used to focus the beam and a filament in air with a length
 3 of 55 mm is generated, as shown in Fig. 1(c). The peak power density is 36.2 TW/cm^2 . The
 4 diverged beam is then collimated by a 750 mm lens and reflected on the chirped mirror pairs for 6
 5 bounces, which supplies a total negative dispersion of -900 fs^2 .



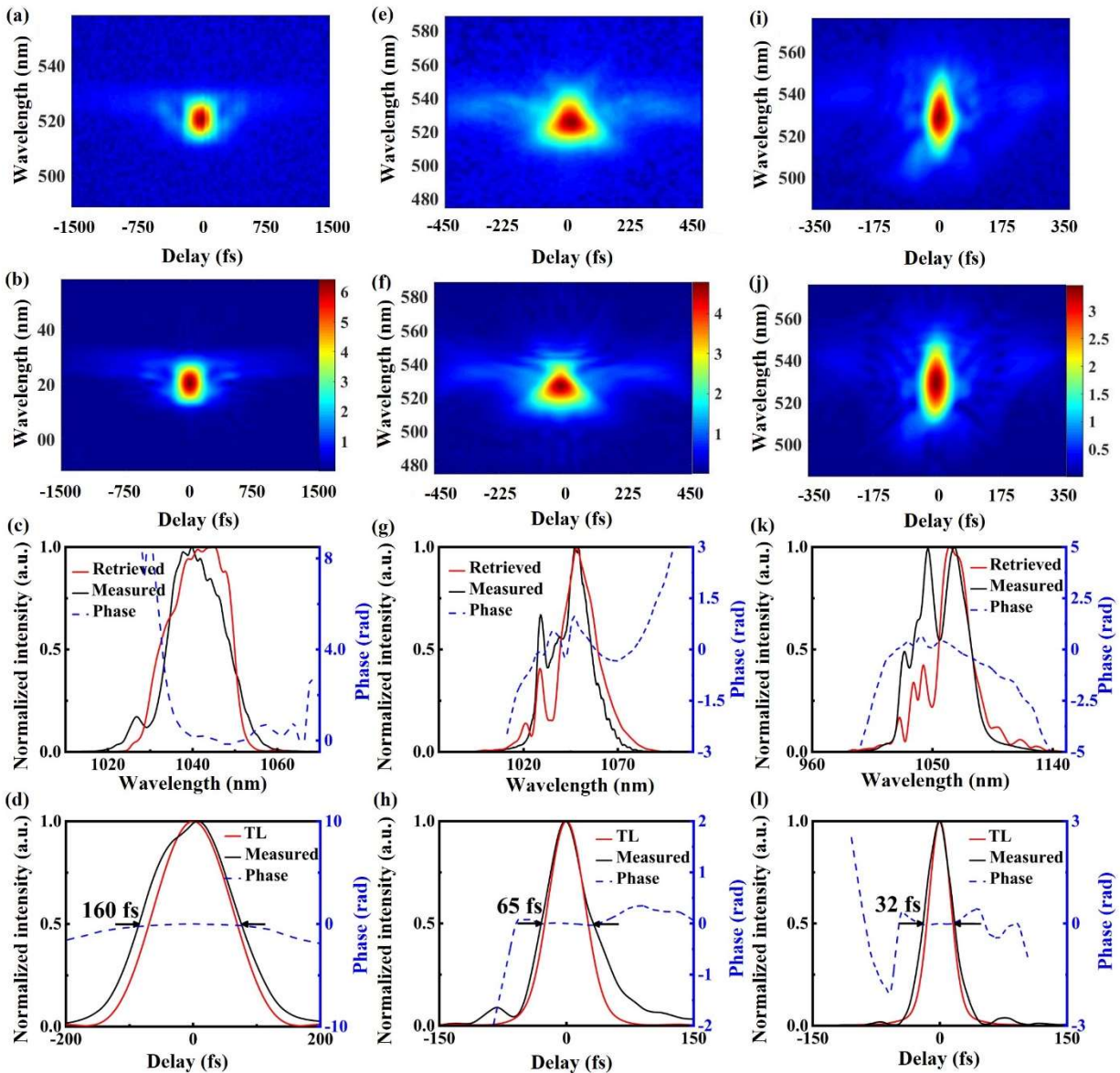
6
 7 **Fig. 2.** Spectra of the input and the output pulses of each nonlinear compression stage in the
 8 linear (a) and logarithm (b) scales, respectively. Black, blue and red curves are the pump
 9 spectra, the spectra after the first and second nonlinear compression stages, respectively.

10

11 In Fig. 2, the spectra of the input and the output pulses of the two nonlinear compression
 12 stages are compared in both linear and logarithm scales. The black and blue curves present the
 13 spectra before and after the first filamentation stage. Spectral broadening spanning from 1010 to
 14 1080 nm at -20 dB is manifested through the first stage filamentation. The spectrum measured
 15 after the second stage filamentation is shown as red curves in Fig. 2, which is significantly
 16 broadened compared to blue ones in both linear and logarithm scales, covering a spectral range
 17 from 1010 nm to 1130 nm at -20 dB, which supports a transform-limited (TL) pulse width of 30
 18 fs. Notably, in both filamentation stages, red shift is the main force of spectral broadening, which

1 indicates that in our experiment the spectral broadening is mainly aroused by the Kerr and Raman
2 effects [26,27] in air, while the plasma generation associated with ionization loss is weak. It is
3 worth mentioning that when the pulse energy varying in the range of 0.8 to 1.5 mJ, similar spectral
4 broadening could be achieved by adjusting the beam size on the focusing lens via the telescope.

5 The temporal profiles of the pump pulse, the pulse after the first and second nonlinear
6 compression stages are characterized by SHG-FROG as shown in Fig. 3. The measured and
7 retrieved FROG traces of the pump pulse are shown in Fig. 3(a) and (b), respectively. And the
8 pump pulse has a bandwidth of 13 nm, as presented in Fig. 3(c), which supports a TL pulse of 138
9 fs. The retrieved temporal profile of the pump pulse indicates a pulse width of 160 fs as in Fig.
10 3(d). The spectral-temporal characteristics of the compressed pulses after the first stage are shown
11 in Fig. 3(e)-(h). The pulse width is compressed from 160 fs to 65 fs, with a TL pulse width of 58
12 fs, as manifested in Fig. 3(h). The pulse energy after the first nonlinear compression stage is 0.87
13 mJ, with 85.5% energy contained in the main pulse, generating a peak power of 11.5 GW. In this
14 stage, the losses caused by filamentation and reflections on chirped mirrors are 0.8% and 11.6%,
15 respectively. Fig. 3(i)-(l) characterize the second-stage nonlinear compression. The measured and
16 retrieved spectra have a relatively good agreement as presented in Fig. 3(k). In the second stage,
17 the pulse width is compressed from 65 fs to 32 fs with a pulse energy of 0.83 mJ. 95% of the total
18 energy is contained in the main pulse as presented in Fig. 3(l), which produces a peak power of
19 24.7 GW. In the second compression stage, losses from laser filamentation and reflections on
20 chirped mirrors are measured as 1.6% and 3%, respectively. After both compression stages the
21 measured and retrieved spectra cannot reproduce the short wavelength sides very well which is
22 mainly due to the slight misalignment during the FROG traces measurement.



1

2 **Fig. 3.** SHG-FROG measurements of the 160, 65, and 32 fs pulses from the Yb:CALGO
 3 regenerative amplifier, after the first and second compression stages, respectively. Input pulse:
 4 the measured (a) and retrieved (b) FROG traces. The FROG error is measured as 0.93%. (c)
 5 The retrieved spectral intensity and phase, compared to the spectrum independently measured
 6 using a spectral analyzer. (d) The retrieved and TL temporal profiles indicating a measured
 7 pulse width of 160 fs. The pulse from the first compression stage: (e) and (f) are the measured
 8 and retrieved FROG traces, respectively. The FROG error is measured as 0.94%. (g) The
 9 retrieved spectral intensity and phase, compared to the spectrum independently measured using

1 a spectral analyzer. **(h)** The retrieved and TL temporal profiles indicating a measured pulse
2 width of 65 fs. The pulse from the second compression stage: **(i)** and **(j)** are the measured and
3 retrieved FROG traces, respectively. The FROG error is measured as 1.09%. **(k)** The retrieved
4 FROG spectral intensity and phase, compared to the spectrum independently measured using
5 a spectral analyzer. **(l)** The retrieved and TL temporal profiles indicating a measured pulse
6 width of 32 fs.

7
8 The parameters of the cascaded air filamentation compressor such as the pulse energy, peak
9 power and efficiency are summarized in Table 1. After the cascaded air filamentation compressor,
10 32 fs pulses with an average power of 16.7 W are obtained and the peak power is increased from
11 6.25 GW to 24.7 GW. The compression factors of the first and second stage are 2.45 and 2,
12 respectively. Small nonlinear accumulation and weak ionization process in each compression stage
13 avoids the spatio-temporal inhomogeneous of the laser beam. The total efficiency of the nonlinear
14 compressor is 83% while the loss aroused by laser filamentation is only 2.4%, which indicates that
15 the loss is mainly due to the reflection of chirped mirrors. It is worth mentioning that limited by
16 the laboratory space and the availability of chirped mirrors, only two stages of filamentation air
17 compressor are performed in this experiment, but it is foreseeable that by using more stages of air
18 filamentation compressors it's possible for sub-10 fs pulse generation.

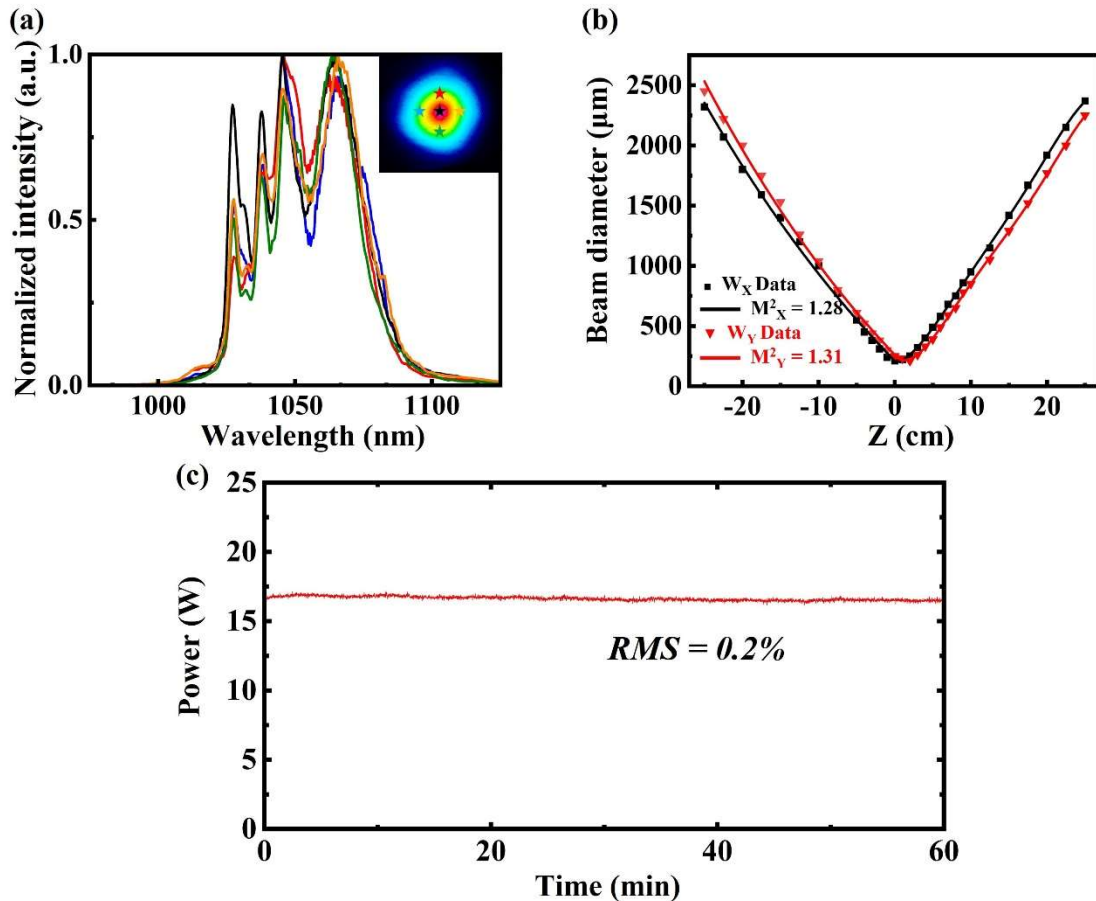
19 The spatial chirp of the output beam after the cascaded air filamentation nonlinear
20 compressor is also characterized as shown in Fig. 4(a). The spectral shapes remain nearly identical,
21 when scanning the beam across both axes, which indicates there is almost no spatial

22 **Table 1. Experimental parameters in the two-stage cascaded air filamentation pulse**
23 **compressor**

	<i>Input energy</i>	<i>Output energy</i>	<i>Peak power</i>	<i>Loss by filamentation</i>	<i>Total efficiency</i>
<i>First stage</i>	<i>1 mJ</i>	<i>0.87 mJ</i>	<i>11.5 GW</i>	<i>0.8%</i>	<i>87.4%</i>
<i>Second stage</i>	<i>0.87 mJ</i>	<i>0.83 mJ</i>	<i>24.7 GW</i>	<i>1.6%</i>	<i>95.4%</i>

1
2 chirp resulted from the cascaded filamentation compressor. The M-square (M^2) factor of the output
3 beam after two stages of nonlinear compression is characterized in Fig. 4(b), revealing values of
4 M^2 of 1.28 and 1.31 along the x and y axis, respectively. Notably, that the M^2 factor of the pump
5 beam is 1.09 and 1.14 along the x and y axis, respectively, which indicates that after nonlinear
6 pulse compression the beam quality is only slightly deteriorated. It is therefore evident that the
7 laser output after the cascaded filamentation in air with weak ionization has a good Gaussian beam
8 profile, negligible spatial chirp, and small temporal pedestals, which indicates a uniform spatial
9 phase resulted from the nonlinear compression process.

10 To test the stability of the nonlinear compressor the laser system is operated for an hour
11 with an average power of 16.7 W and a pulse energy of 0.83 mJ after nonlinear compression. Fig.
12 4(c) shows the power stability of the laser output after the cascaded air filamentation compressor.
13 For the measurement time of one hour, the root-mean-square (RMS) power variation of $\sim 36,000$
14 consecutive points is $\sim 0.2\%$. The slight decreasing trend in average power is caused by the thermal
15 drift of the Yb:CALGO regenerative amplifier. It is worth noting that the nonlinear compression
16 method proposed here which relies on the cascaded weak ionization could minimize the plasma
17 instabilities in the conventional nonlinear compression techniques, and provide a route towards to
18 a super-stable and super-efficient nonlinear compression process, especially at high repetition rates.



1

2 **Fig. 4. (a)** The measured spectra at different spatial positions which are indicated by the five
 3 stars across the beam as shown in the inset. **(b)** Beam quality measurement after nonlinear
 4 compression. **(c)** The measured output power over 1 hour of continuous operation.

5

III. CONCLUSION

6

7 In conclusion, we demonstrate a simple, stable and efficient nonlinear pulse compression
 8 technique by using cascaded laser filamentation in air. The pulse width from a millijoule
 9 Yb:CALGO regenerative amplifier is compressed from 160 to 32 fs, with a total transmission
 10 efficiency of 83%, in which the loss aroused by filamentation is only 2.4%, and the major loss is
 11 from the reflection of chirped mirrors. Thus a peak power of 24.7 GW is achieved. Moreover, the
 demonstrated technique has a superior stability with a measured output power variation of 0.2%

1 over 1 hour. It is worth noting that with more stages of air filamentation compressor, it is possible
2 to obtain sub-10 fs pulses with high efficiency by using customized chirped mirrors. Compared to
3 the MPC, HCF and multi-plate compressors, fine alignment of the pump beam and precise pressure
4 control of gas chambers are not required. In addition, there is no risk of damaging the optical
5 components such as the HCF and thin plates in the nonlinear compressor. Therefore, we believe
6 the developed cascaded air filamentation compressor can provide a new route for generating high
7 energy pulses with a pulse width of 10-30 fs, which can extend the applications of Yb-doped lasers
8 such as high harmonics generation and terahertz rectification.

9

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