[Review of] Bernd Rüdiger, Rainer Gebhardt & Menso Folkerts (eds), Adam Ries, Coß 1. Annaberg-Buchholz: Adam-Ries Bund, 2023

Høyrup, Jens

Published in:
MathSciNet

Publication date:
2023

Document Version
Publisher's PDF, also known as Version of record

Citation for published version (APA):
Høyrup, J. (2023). [Review of] Bernd Rüdiger, Rainer Gebhardt & Menso Folkerts (eds), Adam Ries, *Coß 1.* Annaberg-Buchholz: Adam-Ries Bund, 2023. *MathSciNet*.

Folkerts, Menso; Hellmann, Martin
Rüdiger, Bernd (ed.); Gebhardt, Rainer (ed.)
[Ries, Adam]

Adam Ries *Coß* 1. In 2 volumes. Volume 1. Text volume. Volume 2. Commentary volume. The commentary on *Coß* 1. Transcribed and edited by Bernd Rüdiger and Rainer Gebhardt. (Adam Ries *Coß* 1. Band 1. Textband. Band 2. Kommentarband. Der Kommentar zur *Coß* 1.) (German) Zbl 07697377

Quellen zum Leben und Wirken Adam Ries' und seiner Söhne 6. Annaberg-Buchholz: Adam-Ries-Bund (ISBN 978-3-944217-52-9/hbk). iv, 320 p./v.1; iv, 278 p./v.2 (2023)

It was known from Adam Ries's (1492–1559) "Third Rechenbuch", published in 1550, that Ries had written a *Coß*, an algebra in the German style of the century, and that he hoped to get it into print. He did not succeed before his death, and the manuscript was only located in 1855; a description with extracts of the text was published by Bruno Berlet in 1860. Berlet found out that it consisted in three main parts: A completed algebra, now known as *Coß* 1, finished in 1524; a re-elaboration that was never brought to an end – now *Coß* 2; and Ries's paraphrase of Jordanus de Nemore's *De numeris datis*, a re-interpetation of that work in algebraic key.

In 1992, Wolfgang Kaunzner and Hans Wußing published a facsimile of the manuscript, accompanied by a commentary and transcribed text excerpts. The script is careful and decorative, but the facsimile cannot be used by readers without palaeographic training and adequate familiarity with *Frühneuhochdeutsch*.

Now, as part of a larger project, Bernd Rüdiger, Menso Folkerts and Rainer Gebhardt have published an edition of *Coß* 1. The other parts of the project are a sorely needed restoration of the manuscript; and a digitization made publicly available at https://sachsen.digital/sammlungen/adam-ries-museum-annaberg-buchholz (accessed 9 June 2023).

Together with the digitized manuscript, this edition has everything that can be asked for. Volume 1 contains a careful transcription of the text prepared by Bernd Rüdiger and Rainer Gebhardt – faithful to the division of the text in pages and lines, which makes comparison with the manuscript easy.

Volume 2 is dedicated to two commentaries. The first, due to Menso Folkerts with the assistance of Martin Hellmann, goes through Ries's preface and the dedication as well the initial arithmetic and the algebraic generalia (the text itself as well as historical background). The well- known development of Latin Hindu-Arabic arithmetic from the 12th century onward is dealt with quite briefly by Folkerts and Hellmann. The early history of German algebra is less familiar and therefore gets more attention.

The second commentary, by Rainer Gebhardt, discusses the collection of problems (322 in total), offering for each single problem a close paraphrase in modern German but using Ries's algebraic notation, followed by a modern solution, mostly kept close to Ries's way.

The divisions of the commentaries corresponds to those of Ries's text. After a pseudo-historical preface and an informative dedicatory letter it contains an arithmetic (manuscript pages 5–89); an introduction to algebra (ms. pp. 109–122 – the pages from 90 to 108 are empty), and a collection of problems (ms. pp. 122–324).

The arithmetic contains algorisms for integers and fractions, generally quite traditional but provided with extensive well-explained examples and systematic proofs by reverse calculation and by casting out sevens and nines. Outside the habitual falls an algorithm for "parts of parts" – $\frac{2}{3}$ of $\frac{1}{4}$ and similarly. Initially it teaches the reduction of these composites expressions to simple fractions; calculations in the ensuing algorism build on these. The proofs reveal the real purpose of this section: they refer to the subunits of the monetary system and fractions of these. In the very end comes an algorism "for signs, degrees, minutes, seconds, thirds, etc.", restricted to addition and subtraction but explaining that the principle applies not only to calculation of celestial distances but also to calculation in weight metrology.

The algebraic generalities start (ms. p. 109) by presenting the abbreviations used for the powers of the unknown: ϕ, "*dragma or numerus*", the "unit" for pure numbers; a contracted $re[s]$ explained to stand for "*radix or coß*"; a stylized z, "*zensus or quadratus*", the second power of the unknown; etc., until the "*cubus* *de cubo*", the ninth power. Next it points to the importance of observing their mutual distance when two or three powers occur in the same equation, which leads to a listing of eight equation types – in modern translation (Greek letters stand for implicit coefficients), (1) $\alpha x^n = \beta x^{n+1}$, (2) $\alpha x^n = \beta x^{n+2}$, (3) $\alpha x^n = \beta x^{n+3}$, (4) $\alpha x^n = \beta x^{n+4}$, (5) $\alpha x^n = \beta x^{n+1} + x^{n+2}$, (6) $\alpha x^n + \beta x^{n+2} = x^{n+1}$, (7) $\alpha x^n + \beta x^{n+1} = x^{n+2}$, (8) $\alpha x^{2n} + \beta x^{n+2} =$. For all cases, rules for solving them are given – for all except (6), correct. For (6), the case with a double solution, the rule is botched up – for the simple case $\beta = 1, n = 0$, the rule given is $x =$

$$\sqrt{\left(\frac{\gamma}{2}\right)^2 - \alpha} \pm \frac{\gamma}{2} \text{ instead of } x = \frac{\gamma}{2} \pm \sqrt{\left(\frac{\gamma}{2}\right)^2 - \alpha}.$$

Another list of 24 equation types follows on ms. p. 115, in Ries's opinion developed from the eight that were just presented (the actual historical order is the opposite). It comprises the 22 equation types of up to the fourth degree that can be reduced to the second degree or solved by means of a root extraction; these had already been known as a group in Italian abbacus algebra. Beyond these, the list includes equations that can be translated $\alpha x^2 = \sqrt{(\beta x)}$ and $\alpha x^2 = \sqrt{(\beta x^2)}$. Then follow the 322 problems. All but three are immediately of the first degree; #104, #135 and #322 seem to be of the second degree, but once the equation of formulated the zensus disappears. So, in spite of the initial presentation of many equation types of higher degree, Ries restricts himself to the domain where algebra can really serve to solve normal business problems – which does not mean, however, that all of Ries's *are* business problems or could be useful in normal business.

Many are pure-number problems. Others are genuine business problems, many more deal with situations involving trade or money but in a way that would never present itself in commercial real life, illustrating that commercial questions that can produce complicated mathematics tend to become recreational riddles; other recreational types are also present, often with unusual variations.

First-degree equations with a single unknown are never intricate. The complexity of Ries's problems, and what he trains extensively, is to be located in the formulation of problems and in their transformation into an equation, and that often asks for a clear mind.

As also illustrated by the immense success of his *Rechenbücher*, Ries was a born pedagogue. Other algebraic writings since the beginning of abbacus algebra (indeed since al-Khwārizmī) had excelled in the treatment of higher-degree problems, their main purpose being to display virtuosity. Ries, as he explains in the dedication, aims at selecting from what he has learned from predecessors that which can be useful for the common man, and to present that. As a great pedagogue he also combines thorough training with so much variation that boredom can be avoided. For this purpose, a collection of 322 first-degree problems is justified.

Reviewer: Jens Høyrup (Roskilde)

MSC:

01A40 History of mathematics in the 15th and 16th centuries, Renaissance

Keywords:

Ries; Coß; Algebra, før-moderne

Biographic references:

Ries, Adam