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Robust extrapolation for systems with unknown input and interval uncertainty in system and observations

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Abstract. The problem of robust extrapolation for discrete linear system with unknown input and uncertain interval parameters in system and model of observations is considered. The probabilistic approach is used, which is based on replacing uncertain parameters of interval type by independent random variables with uniform distribution in recursive Kalman schemes. The LSM algorithms and nonparametric smoothing procedures are applied for estimating unknown input. The proposed algorithms can be used in control systems with incomplete information. Simulation results are presented and discussed.

Keywords: robust extrapolation; interval parameters; unknown input; nonparametric smoothing.

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Робастная экстраполяция для систем с неизвестным входом и интервальной неопределенностью в объекте и наблюдениях

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Аннотация. Рассмотрена задача робастной экстраполяции для дискретного объекта с неизвестным входом и интервальными параметрами в модели объекта и наблюдениях. Используется вероятностный подход, в основе которого лежат замена неопределенных параметров интервального типа независимыми случайными величинами с равномерным распределением, алгоритмы оценивания неизвестного входа, рекуррентные схемы экстраполяции на один такт (экстраполятор Калмана), метод наименьших квадратов и сглаживающие непараметрические процедуры. Представлены результаты моделирования.

Ключевые слова: робастная экстраполяция; интервальные параметры; неизвестный вход; непараметрическое сглаживание.

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Introduction

The problem of synthesis of filters, extrapolators and observers for dynamical systems with uncertain parameters, in particular, with interval parameters, was considered in [1–5]. These papers use methods of robust data processing. The robust Kalman filter is obtained for time-varying discrete-time linear systems by solving an optimization problem such that the upper bound on the variance of estimation error to be minimized [1]. In [2], the problem of state estimation and determining at any moment the smallest set containing all the possible values of the state vector, simultaneously compatible with the state equations and with a priori known bounds of the uncertain parameters, is considered. In [3], a problem was considered in the case when all the matrices of both the system and the observations model are subjected to norm-limited parametric uncertainties. The robust regularization is implemented using the penalty function method. Robust Kalman filter is proposed in [4], where the problem was solved by using a linear matrix inequality optimization problem. In [5], was proposed the robust Kalman filtering framework for systems with probabilistic uncertainty in system parameters. The uncertainty is propagated using conditional expectations and polynomial chaos expansion framework. Methods of data processing using estimates of unknown input are given in [6, 7]. In these papers Least Squares Method (LSM) was used to obtain estimates of unknown input. In [8–10] it was proposed to use a compensatory approach to calculate estimates of unknown input. In [11, 12] there were used additionally the algorithms of nonparametric smoothing to increase the accuracy of estimating unknown input.

This paper considers the problem of robust extrapolation in discrete systems with additive perturbations with unknown input and with interval parameters. It is based on a probabilistic approach to solving problems for model with interval parameters, which consists in the fact that the interval parameter is replaced by a uniformly distributed random variable [13].

The results of the work generalize results of the paper [14] to the case of the presence interval parameters not in the systems model only but in the observations model also.

1. The problem statement

Consider the linear discrete system with interval parameters, described by the difference equation

$$x(k+1) = \tilde{A}x(k) + f(k) + \tilde{B}q(k), \;\; x(0) = x_0, \tag{1}$$

where $x(k) \in \mathrm{R}^n$ is the state vector at time instant k, $f(k)$ is the unknown input vector; x_0 is the random vector with known mathematical expectation and covariance matrix $N_0 = \mathrm{E}[(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^{\mathrm{T}}]$; $\tilde{A}$ is the state transition matrix with interval uncertainty (with the given lower and upper bounds of $\underline{A}$ and $\bar{A}$, respectively), $q(k) \in \mathrm{R}^{n_1}$ is the random perturbations with the following characteristics: $\mathrm{E}[q(k)] = 0$, $\mathrm{E}[q(k)q^{\mathrm{T}}(j)] = I\delta_{kj}$, $\tilde{B}$ is the perturbations transition matrix with interval uncertainty (with the given lower and upper bounds of $\underline{B}$ and $\bar{B}$), I is the identity matrix. Here δ_{kj} is the Kronecker symbol.

The observations model is determined by the formula

$$y(k) = \tilde{S}x(k) + \tilde{F}v(k), \tag{2}$$

where $y(k) \in \mathrm{R}^l$ is the observations vector, $\tilde{S}$ is the observations transition matrix with interval uncertainty (with the given lower and upper bounds of $\underline{S}$ and $\bar{S}$), $v(k) \in \mathrm{R}^{l_1}$ is the observations noise ($\mathrm{E}[v(k)] = 0$, $\mathrm{E}[v(k)v^{\mathrm{T}}(j)] = I\delta_{kj}$), $\tilde{F}$ is the observations transition matrix with interval uncertainty (with the given lower and upper bounds of $\underline{F}$ and $\bar{F}$).

It is assumed that the sequences $q(k)$, $v(k)$ and x_0 are mutually independent, system (1), (2) is observable under parametric perturbations of the dynamics matrix $\tilde{A}$ and observations matrix $\tilde{S}$. Using the information available to the time $k \in [0; T]$, it is required to construct the forecast $\hat{x}(k+1)$ by minimizing the following criterion

$$J(0;T) = \mathrm{E}[\sum_{k=0}^{T} (x(k) - \hat{x}(k))^{\mathrm{T}} (x(k) - \hat{x}(k))]. \quad (3)$$

2. Robust Extrapolator

To solve the problem, we will use the recurrent extrapolator Kalman (EK), while using the probabilistic approach to find its transition matrix. The essence of the method lies in the fact that the interval parameters are replaced by independent random variables that are uniformly distributed over the uncertainty intervals.

Then, using the probabilistic approach, we will replace the uncertain interval matrices $\tilde{A}, \tilde{B}, \tilde{S}, \tilde{F}$ by the matrices whose elements depend on random variables

$$A(\theta) = (A + \sum_{i=1}^{m} A_i \theta_i), \ B(\theta) = (B + \sum_{i=m+1}^{m+m_1} B_i \theta_i),$$
$$S(\theta) = (S + \sum_{i=m+m_1+1}^{m+m_1+m_2} S_i \theta_i), \ F(\theta) = (F + \sum_{i=m+m_1+m_2+1}^{m+m_1+m_2+m_3} F_i \theta_i), \quad (4)$$

where θ_i are independent uniformly distributed random variables according on the interval $[-1, +1]$ ($-1 \le \theta_i \le 1 \, (i = \overline{1, m+m_1+m_2+m_3})$). We will assume that the random variables θ_i are independent of x_0, $q(k)$ and $v(k)$. In (4), the matrices $A = \frac{1}{2}(\underline{A} + \overline{A})$, $B = \frac{1}{2}(\underline{B} + \overline{B})$, $S = \frac{1}{2}(\underline{S} + \overline{S})$ and $F = \frac{1}{2}(\underline{F} + \overline{F})$ are the medians of the interval matrices $\tilde{A}, \tilde{B}, \tilde{S}$ and $\tilde{F}$. The matrices A_i, B_i, S_i and F_i can be set so that one element corresponding to the uncertain element of the matrices $\tilde{A}, \tilde{B}, \tilde{S}$ and $\tilde{F}$ remains nonzero. Its value can be determined by the width of the interval uncertainty of the elements of the matrices $\tilde{A}, \tilde{B}, \tilde{S}$ and $\tilde{F}$.
In this case, the model of the system (1) and observations (2) takes the form

$$x(k+1) = A(\theta)x(k) + f(k) + B(\theta)q(k), \ x(0) = x_0, \quad (5)$$

$$y(k) = S(\theta)x(k) + F(\theta)v(k). \quad (6)$$

However, we restrict ourselves to characterizing the first two moments of $x(k)$ as defined below, since we apply the EK. To obtain the estimate, we use the recurrent algorithm

$$\hat{x}(k+1) = A\hat{x}(k) + f(k) + K(k)(y(k) - S\hat{x}(k)), \ \hat{x}(0) = \overline{x}_0, \quad (7)$$

where the matrix of the extrapolator transition coefficients $K(k)$ is determined by the optimization of criterion (3), taking into account the type of distribution of the parameter θ and assuming that the vector $f(k)$ is known.

Using the property of the trace operation ($y^{\mathrm{T}} A y = \mathrm{tr} A y y^{\mathrm{T}}$) and the rules for differentiating the trace function from the matrix multiplication [15]:

$$\frac{\partial \, \mathrm{tr}\, AXB}{\partial X} = A^{\mathrm{T}} B^{\mathrm{T}}, \ \frac{\partial \, \mathrm{tr}\, A^{\mathrm{T}} X B^{\mathrm{T}}}{\partial X} = BA, \quad (8)$$

from the equation

$$\frac{\partial J(0;T)}{\partial K} = 0, \quad (9)$$

we obtain an analytical expression for the matrix $K(k)$

$$K(k) = AN(k)S^{\mathrm{T}}(FF^{\mathrm{T}} + SN(k)S^{\mathrm{T}} + V(k))^{-1}, \quad (10)$$

where the matrix $V(k)$ is determined by the formula

$$V(k) = \tfrac{1}{3}\sum_{i=m+m_1+1}^{m+m_1+m_2} S_i N(k) S_i^{\mathrm{T}} + \tfrac{1}{3}\sum_{i=m+m_1+1}^{m+m_1+m_2} S_i \hat{x}(k)\hat{x}(k)^{\mathrm{T}} S_i^{\mathrm{T}} + \tfrac{1}{3}\sum_{i=m+m_1+m_2+1}^{m+m_1+m_2+m_3} F_i F_i^{\mathrm{T}}, \quad (11)$$

and $N(k)$ satisfies the difference matrix equation

$$N(k+1) = (A - K(k)S)N(k)(A - K(k)S)^{\mathrm{T}} + \tfrac{1}{3}\sum_{i=1}^{m} A_i N(k) A_i^{\mathrm{T}} + \tfrac{1}{3}\sum_{i=1}^{m} A_i \hat{x}(k)\hat{x}(k)^{\mathrm{T}} A_i^{\mathrm{T}} +$$
$$+ BB^{\mathrm{T}} + \tfrac{1}{3}\sum_{i=m+1}^{m+m_2} B_i B_i^{\mathrm{T}} + K(k)(FF^{\mathrm{T}} + SN(k)S^{\mathrm{T}} + V(k))K(k)^{\mathrm{T}},\ N(0) = N_0. \quad (12)$$

In model (5) due to the fact that the median of the interval matrix $\tilde{A}$ is used as the dynamics matrix EK, the vector of the unknown input will change (this vector is denoted by $r(k)$)

$$r(k) = f(k) + \sum_{i=1}^{m} A_i \theta_i x(k),\ \ -1 \le \theta_i \le 1\ (i = \overline{1, m}), \quad (13)$$

where the second term is an additional unknown input arising from the uncertainty of the state transition matrix.

As an algorithm for estimating the unknown input $r(k)$, we will use the LSM method, in this case, the estimate can be constructed on the basis of minimizing the additional criterion [6, 7]

$$I_1 = \sum_{t=1}^{k} \left\{ \left\| y(t) - S(A\hat{x}(t-1) + r(t-1)) \right\|_C^2 + \left\| r(t-1) \right\|_D^2 \right\}, \quad (14)$$

In (14) $C,\ D$ are positive definite weight matrices. The LSM estimates of the unknown input, based on the minimization of criterion (14), will take the form

$$\hat{r}^{\langle \mathrm{LSM} \rangle}(k) = [S^{\mathrm{T}}CS + D]^{-1} S^{\mathrm{T}} C[y(k) - SA\hat{x}(k-1)]. \quad (15)$$

To increase the accuracy of estimating an unknown input, we will additionally use nonparametric algorithms [11, 12] for smoothing the innovation process $y(k) - SA\hat{x}(k-1)$

$$\hat{r}^{\langle \mathrm{NP} \rangle}(k) = [S^{\mathrm{T}}CS + D]^{-1} S^{\mathrm{T}} C\hat{\Omega}, \quad (16)$$

where the j component of the vector $\hat{\Omega}(k)$ can be calculated by

$$\hat{\Omega}_j(k) = \frac{\sum_{t=1}^{k} [y(t) - S(A\hat{x}(t-1)]_j\, G\left(\dfrac{k-t+1}{\mu_j}\right)}{\sum_{t=1}^{k} G\left(\dfrac{k-t+1}{\mu_j}\right)}. \quad (17)$$

In formulas (17) $G(\cdot)$ is a kernel function, μ_j is a bandwidth parameters.

Robust extrapolation estimates in discrete systems with interval parameters were determined from the recurrent equation

$$\hat{x}(k+1) = A\hat{x}(k) + \hat{r}(k) + K(k)(y(k) - S\hat{x}(k)),\ \ \hat{x}(0) = \overline{x}_0, \quad (18)$$

where the matrix transition coefficients $K(k)$ was calculated by formulas (10)–(12), and the estimates of $\hat{r}(k)$ was determined by formulas (16), (17). Note that the medians of the matrices $\tilde{A},\ \tilde{S}$ are used to calculate the estimates of $\hat{r}(k)$.

3. Simulation Results

The simulation was performed for the following data $(m = 2,\ m_1 = 1,\ m_2 = 2,\ \ m_3 = 2)$:

$$A = \begin{pmatrix} 0 & 1 \\ 0{,}02 & 0{,}73 \end{pmatrix},\ S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ A_1 = \begin{pmatrix} 0 & 0 \\ 0 & 0{,}4 \end{pmatrix},\ A_2 = \begin{pmatrix} 0 & 0 \\ 0{,}04 & 0 \end{pmatrix},\ S_3 = \begin{pmatrix} 0{,}05 & 0 \\ 0 & 0 \end{pmatrix},$$

$$B = \begin{pmatrix} 0{,}1 & 0 \\ 0 & 0.15 \end{pmatrix},\ B_4 = \begin{pmatrix} 0{,}05 & 0 \\ 0 & 0 \end{pmatrix},\ B_5 = \begin{pmatrix} 0 & 0 \\ 0 & 0{,}1 \end{pmatrix},$$

$$F = \begin{pmatrix} 0{,}5 & 0 \\ 0 & 0{,}6 \end{pmatrix},\ F_6 = \begin{pmatrix} 0{,}4 & 0 \\ 0 & 0 \end{pmatrix},\ F_7 = \begin{pmatrix} 0 & 0 \\ 0 & 0{,}4 \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; D = \begin{pmatrix} 0{,}1 & 0 \\ 0 & 0{,}1 \end{pmatrix},$$

$$f(k) = \begin{pmatrix} 0{,}1 + 0{,}25\sin(0{,}1k) \\ 0{,}1 + 0{,}2\sin(0{,}15k) \end{pmatrix}.$$

The initial conditions are:

$$x(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \; \hat{x}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \; N(0) = \begin{pmatrix} 0{,}1 & 0 \\ 0 & 0{,}1 \end{pmatrix}.$$

In (17), a Gaussian kernel function was used.

Fig. 1 shows the results of comparing the standard errors of the deviations of the estimates of the state vector

$$\sigma_i = \sqrt{\frac{\sum_{k=1}^{N} (x_i(k) - \hat{x}_i(k))^2}{N-1}}, \; (i = \overline{1,2})$$

for five algorithms (with different implementations of the components of a random vector θ):

- optimal EK for systems with median matrices *A*, *B*, *S* and *F*, when *f*(*k*) is known (OEK);
- optimal EK for systems with median matrices *A*, *B*, *S* and *F*, when *f*(*k*) is unknown, estimate *f*(*k*) is not used (OEKUN);
- optimal EK for systems with median matrices *A*, *B*, *S* and *F*, LSM method (15) was used to calculate estimates of an unknown input, nonparametric smoothing was not applied for estimate unknown input (EK-LSM);
- optimal EK for systems with median matrices *A*, *B*, *S* and *F*, LSM method and nonparametric smoothing (formulas (16), (17)) was applied (EK-LSM-NP);
- proposed robust EK (10)–(12) for systems with interval parameters, LSM method and nonparametric smoothing were used (REK).

The table presents 10 realizations of the values of the components of the random vector θ distributed with uniform density. Simulation results are obtained for extrapolation algorithms (*N* = 200) and by averaging 100 realizations.

Realizations of the values of the components of the random vector θ distributed with uniform density

n/n	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
1	–0,97	0,99	–0,39	–0,73	–0,63	–0,24	0,35
2	–0,83	0,84	0,07	0,39	0,15	–0,18	–0,02
3	–0,82	0,30	0,57	–0,06	0,25	–0,48	0,18
4	–0,18	0,88	–0,56	–0,24	0,52	0,08	0,72
5	0,12	0,07	0,01	–0,73	–0,71	–0,04	0,38
6	–0,87	–0,06	0,25	0,19	–0,03	–0,73	0,92
7	–0,48	0,57	–0,15	–0,87	–0,98	–0,84	0,64
8	–0,80	0,38	–0,69	–0,94	0,08	–0,08	–0,94
9	0,56	0,72	0,08	–0,80	0,59	–0,13	0,92
10	0,13	–0,20	0,45	–0,85	–0,07	–0,77	–0,25

Fig. 1 shows that the procedures with robust extrapolation (REK) have the advantages in the accuracy compared to the known algorithms using the LSM estimates and LSM estimates with nonparametric smoothing. The advantage (REK) in accuracy compared to the (EK-LSM-NP) algorithm is from 3% to 15%.

The worst results were obtained using estimates of an unknown input using the LSM method without smoothing (EK-LSM). The reason for this is the high level of intensity of measurements errors, which was used in the example, which led to a low quality of the estimation of *f*(*k*) by the LSM method and, as a consequence, to a low accuracy of the estimation of the state vector. Smoothing the innovation process and proposed robust algorithm improve accuracy estimates of the state vector.

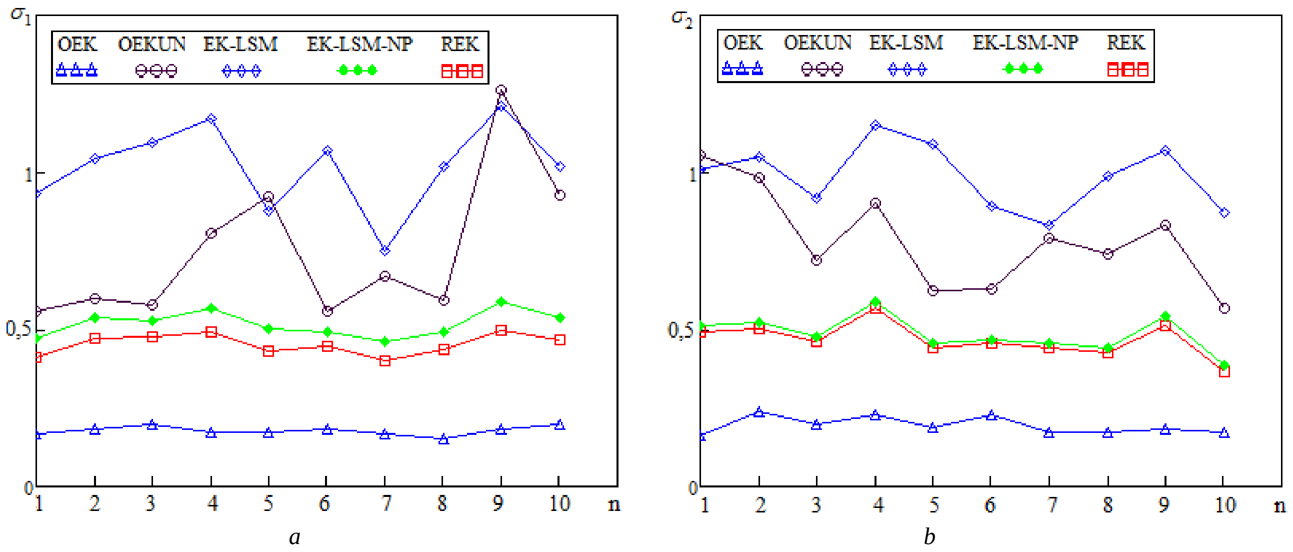

Fig. 1. Standard errors for extrapolation algorithms for ten simulation results
a) first component, *b*) second component

The algorithm (OEK) determines the potential capabilities of the EK algorithm, but algorithm (OEK) requires the exact values of the input vector $f(k)$. In our problem, the input vector $f(k)$ is not available for observation.

Conclusions

Using the probabilistic approach, algorithm for the synthesis of the robust extrapolator for discrete systems with unknown input and with interval parameters in the model and observations is proposed.

The problem is solved using recurrent algorithms, the LSM method and nonparametric smoothing procedures. The proposed method implements a decrease in the influence of uncertainties in the model and observations using replacing an interval uncertainty by the probabilistic uncertainty and taking into account estimates of the unknown input with additional smoothing.

The numerical example shows that the joint use of smoothing algorithms and robust approach can improve the estimation accuracy of the state vector.

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