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ON THE EQUIVALENCE OF DIFFERENT DEFINITIONS OF R-OPERATION*

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Three well-known definitions of R-operation in the BPHZ formalism are presented. The equivalence between the Zimmermann's forest formula and the factorized version of R-operation is proved.

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1. *Introduction*

The renormalization theory provides a general framework for carrying out perturbation calculations of QFT in a well-defined and unique way. The most general scheme for extracting finite parts from divergent Feynman amplitudes is the BPHZ formalism. This well-known procedure was given and elaborated by Bogoliubov, Parasiuk, Hepp, Zimmermann and Zavyalov [1-6]. In this approach, the renormalized amplitude is obtained by using some suitable set of subtractions, known as R-operation, which removes the ultraviolet infinities from the Feynman amplitude while preserving unitarity, Lorentz invariance and causality (locality). The essence of this method is that if we choose a renormalization point in the space of the external momenta and we expand the Feynman amplitude in a Taylor series about this point, the divergences appear only in the leading coefficients of the Taylor expansion. Furthermore, the R-operation is a solution to Bogoliubov's recursive relation (see [4]), what implies that the subtracted terms can be directly related to Lagrangian counter terms and, hence, in the case of renormalizable theories, we obtain the interpretation in terms of field, coupling constant and mass renormalization.

A convenient method for executing the R-operation, adopted throughout this paper, is to perform subtractions on the parametric function in the integrand of Feynman amplitude in the absence of regularization. While this procedure introduces considerable simplifications it also deals with integrands of non-existing integrals (the original theory deals with complete Feynman's integrals and the integrands in themselves have no phys-

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ical meaning). For this reason careful treatments of renormalization always use some sort of regularization. Nevertheless, the familiar renormalization prescriptions, which involve the analytical, dimensional or Pauli-Villars regularization, give the same results as the scheme based on the idea of performing suitable subtractions on the integrand of the Feynman amplitude (see [6, 7]). From this fact follows the equivalence of these quoted regularization methods, motivated by the concept that the final results should not depend on a choice of regularization. However, one could choose such a "pathological" regularization that the commutativity of the subtraction procedure with the integrations would be violated. Considering such regularization as improper [7], we assume that the R-operation can be applied either to the whole integral or to the integrand. Choosing the latter way, we can dispense with regularization.

In Section 2, we wish to present three known definitions of the R-operation, introduced by Appelquist [8], Zimmermann [3, 5] and Bergere and Zuber [9, 10]. The most popular and useful is the Zimmermann formula incorporating the sum over all forests consisting of divergent subdiagrams (renormalization parts). Sometimes it is desirable, from both a formal and calculational point of view, to use a factorized version of the R-operation being the single product of all individual subtraction operators. Such a subtraction formula was proposed by Appelquist [8]. However, adopting the factorized version of R-operation one should take care of the peculiarities connected with the overlapping of divergent subdiagrams (this problem is not solved in Appelquist's work). To keep the mathematical correctness one must generalize slightly the parametric κ-representation for subtractions [11]. Moreover, the equivalence between the factorized and the recursive renormalization formulae was proved in [8] only for the case of renormalizable theories. Bergere and Zuber [9] defined the R-operation as the product of subtraction operators over all possible, whether divergent or not, subdiagrams of the Feynman diagram. This formulation is almost independent of the topological structure of the diagram. Nevertheless, they proved it is equivalent to Zimmermann's R-operation (up to a finite renormalization for theories with spins).

In Section 3, we prove the equivalence between the Appelquist factorized definition of the R-operation and the Zimmermann forest formula. For the sake of simplicity, we limit ourselves to the spinless theory with no derivative couplings. Referring to a special class of Feynman diagrams, namely T-diagrams, this equivalence was established in [11]. In this paper, we prove it without limiting to any special topology of divergent diagrams.

The notation, terminology and mathematical concepts applied in this paper are explained in Appendix.

2. *R-operation*

To present different definitions of the R-operation, we begin with introducing some background to describe the subtractions procedure.

We consider an arbitrary proper Feynman diagram appearing in the D-dimensional scalar field theory with no derivative couplings, that contains N vertices and L internal lines. The Feynman amplitude corresponding to this diagram can be presented in the standard parametric integral representation (omitting a numerical factor) (see [4–6])

$$I_\Gamma = \delta^{(D)}\left(\sum_{n=1}^{N} k_n\right)\int_0^\infty \cdots \int_0^\infty d\boldsymbol{\alpha} D^{-D/2}(\boldsymbol{\alpha}) \exp\left(i\frac{A(\boldsymbol{\alpha}, \boldsymbol{k})}{D(\boldsymbol{\alpha})}\right) \tag{1}$$

$$\times \exp\left(-i\sum_{l=1}^{L} \alpha_l(m_l^2 - i\varepsilon)\right),$$

where $\boldsymbol{k} = (k_1, \ldots, k_N)$ denotes the set of external momenta, $\alpha = (\alpha_1, \ldots, \alpha_L)$, $d\boldsymbol{\alpha} = d\alpha_1 \ldots d\alpha_L$. The functions appearing in (1) are defined as

$$D(\boldsymbol{\alpha}) = \sum_{T_1}\left(\prod_{l \notin T_1} \alpha_l\right), \tag{2}$$

where the sum runs over all trees in Γ,

$$A(\boldsymbol{\alpha}, \boldsymbol{k}) = \sum_{T_2}\left\{\left(\prod_{l \notin T_2} \alpha_l\right)\left(\sum_{i \in V_1} k_i\right)^2\right\}, \tag{3}$$

where the sum runs over all two-trees in Γ (each of the two-tree naturally separates the vertices into two disjoint non-empty sets V_1 and V_2).

Let us denote by ϕ this part of the integrand which reflects the behaviour in the ultraviolet region (related to the lower limit of the parametric integration), namely

$$\phi = D^{-D/2}(\boldsymbol{\alpha}) \exp\left(i\frac{A(\boldsymbol{\alpha}, \boldsymbol{k})}{D(\boldsymbol{\alpha})}\right). \tag{4}$$

A family of all divergent subdiagrams (renormalization parts) included in the diagram Γ we denote by $\mathscr{R}_\Gamma$

$$\mathscr{R}_\Gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_R\}. \tag{5}$$

To realize the subtraction in external momenta for any divergent subdiagram γ_r, we introduce the κ-parametrization (see [4, 6, 9–11]). The parametrized function $\phi(\boldsymbol{\kappa})$ is obtained from ϕ by dilatation of all $\alpha_l \in \gamma_r$ by κ_r (for any $1 \leqslant r \leqslant R$) and multiplication by the factor $\prod_{r=1}^{R} \kappa_r^{\frac{1}{2}Dp_r + \Omega_r}$

$$\phi(\boldsymbol{\kappa}) = \prod_{r=1}^{R} \kappa_r^{\frac{1}{2}Dp_r + \Omega_r} D_\kappa^{-D/2}(\boldsymbol{\alpha}) \exp\left(i\frac{A_\kappa(\boldsymbol{\alpha}, \boldsymbol{k})}{D_\kappa(\boldsymbol{\alpha})}\right), \tag{6}$$

where p_r denotes the number of loops in the subdiagram γ_r.

The subtraction operation O_{γ_r}, which truncates the Taylor expansion about the external momenta corresponding to the subdiagram γ_r, may be regarded as the truncation of the Taylor expansion with respect to the parameter κ_r

$$O_{\gamma_r} = 1_{\gamma_r} - M_{\gamma_r}, \tag{7}$$

$$1_{\gamma_r}\psi \equiv \psi(\kappa_r)|_{\kappa_r=1}, \tag{7a}$$

$$M_{\gamma_r}\psi \equiv \sum_{n=0}^{\frac{1}{2}\omega_r+\Omega_r} \frac{1}{n!}\frac{\partial^n}{\partial\kappa_r^n}\psi(\kappa_r)|_{\kappa_r=0}, \tag{7b}$$

where ψ denotes ϕ or any expression of the form $O_{\gamma_{i_1}}\dots O_{\gamma_{i_k}}\phi$. The auxiliary number Ω_r is large enough to make ψ and its derivatives non-singular at all points $\kappa_r \in [0, 1]$. Although the definition of the subtraction operation (7) involves pretty arbitrary parameter Ω_r, the result is always the same no matter which value is chosen for Ω_r. Nevertheless, Ω_r cannot be fixed because its smallest possible value is always determined by the actual structure of the argument ψ. The following lemma will be useful for us

LEMMA 1: If a family of divergent subdiagrams $\gamma_{i_1}, \dots, \gamma_{i_k}$ forms a tree family (see Appendix), then the product

$$M_{\gamma_{i_1}} M_{\gamma_{i_2}} \dots M_{\gamma_{i_k}}\phi$$

does not depend on the order of subtraction operators M_{γ_i} and all auxiliary parameters Ω_i may be fixed to be zero. The proof is given in Ref. [11].

We have now set up enough mathematical concepts to describe the R-operation. Let us start with the standard Zimmermann's definition [3]

$$R_Z = 1 + \sum_{\mathscr{F}\subset\mathscr{R}_\Gamma} \{\prod_{r\in\mathscr{F}} (-M_{\gamma_r})\}, \tag{8}$$

where the sum runs over the set of all non-empty forests.

Since any forest is a tree family, from Lemma 1 we see that working with Zimmermann's formula for the R-operation (8) we are allowed to set all auxiliary parameters Ω_r in the κ-representation for subtractions equal to zero. Moreover, any product of subtraction operators M_γ appearing in the (8) does not depend on the order of its factors.

A precursor of Zimmermann's formula is the natural definition being the product of all subtraction operators

$$R_F \equiv O_{\gamma_1} O_{\gamma_2} \dots O_{\gamma_R} = \prod_{\gamma\in\mathscr{R}_\Gamma} O_\gamma. \tag{9}$$

This definition was first proposed by Appelquist [8]. He proved that this expression is well-defined, namely it does not depend on the order of application of the subtraction operators O_{γ_r} over ϕ, moreover, he demonstrated that this definition is equivalent to Bogoliubov's renormalization formula in the renormalizable scalar theory case. In Section 3, we will prove the equivalence of (8) and (9) for any would-be scalar theory (the result can be simply generalized to theories with spins and derivative couplings). Let us remark that the product (9) is independent of the order of its factors, but the subtraction operators do not commute in general. Moreover, the auxiliary parameters Ω_r are no longer fixed to be zeros and their choice depends on the order of applications of the subtraction operators.

Bergere and Zuber [9] put forward the third possible formula for the subtraction procedure

$$R_{\mathrm{BZ}} \equiv \prod_{\gamma \subset \Gamma} O_{\gamma}, \tag{10}$$

where the product is taken over all possible subdiagrams of Γ (a subdiagram is defined here as a set lines).

This definition is completely independent of the topology of the Feynman diagram except for the total number of internal lines. The independence of the order of subtraction operators and the equivalence with the Zimmermann forest formula are proved in Ref. [9]. The above equivalent form of the R-operation provides us with remarkable insight nto the mechanism of removing the ultraviolet divergences.

3. *Demonstration of the equivalence of Zimmermann's and factorized formulae*

In this section, we intend to give a general proof of the equivalence of the R-operation definitions (8) and (9).

Since the definition of the R-operation is independent of the order of the subtraction operators, we are allowed to order it in such a way that O_{γ_r} stands to the left of O_{γ_s} if $\gamma_r \subset \gamma_s$. We are intending to establish recursively the following identity

$$\prod_{k=1}^{R} O_{\gamma_k}\phi = \prod_{k=1}^{r} O_{\gamma_k}[1+ \sum_{\mathscr{F} \subset \mathscr{R}_{r+1}} \prod_{i \in \mathscr{F}} (-M_{\gamma_i})]\phi. \tag{11}$$

where the sum runs over the non-empty forests included in the set

$$\mathscr{R}_{r+1} \equiv \{\gamma_{r+1}, \gamma_{r+2}, \ldots, \gamma_R\}. \tag{12}$$

The first step is the trivial check that the identity (11) is satisfied for $r = R$-1. Next, assuming that the identity (11) is satisfied for order r we shall prove it for order (r-1). Let us consider the right hand side of (11)

$$(\prod_{k=1}^{r-1} O_{\gamma_k})O_{\gamma_r}[1+ \sum_{\mathscr{F} \subset \mathscr{R}_{r+1}} \prod_{i \in \mathscr{F}} (-M_{\gamma_i})]\phi. \tag{13}$$

Some subdiagrams belonging to the set $\mathscr{R}_{r+1}$ may overlap with the subdiagram γ_r. Let $\{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_m\}$ denote the subset of $\mathscr{R}_{r+1}$ composed of all elements overlapping with γ_r. Using the fact that the subtraction operators commute when the corresponding product is associated with non-overlapping subdiagrams (see Lemma 1), the expression (13) can be written in the form

$$(\prod_{k=1}^{r-1} O_{\gamma_k}) [1+ \sum_{\mathscr{F} \in \mathscr{R}_r} \prod_{i \in \mathscr{F}} (-M_{\gamma_i})+M_{\gamma_r} \sum_{k=1}^{m} A_k M_{\varepsilon_k}$$
$$-M_{\gamma_r} \sum_{k<l} A_{kl} M_{\varepsilon_k} M_{\varepsilon_l} + \ldots +(-1)^{m+1} M_{\gamma_r} A_{12\cdots m} M_{\varepsilon_1} M_{\varepsilon_2} \ldots M_{\varepsilon_m}]\phi. \tag{14}$$

The "coefficients" A are defined by

$$A_{k_1k_2\cdots k_n} = \sum_{\mathscr{F}\in\mathscr{A}_{k1k2\cdots kn}} \prod_{i\in\mathscr{F}} (-M_{\gamma_i}), \tag{15}$$

where $\mathscr{A}_{k_1k_2\dots k_n}$ is the set of forests $\mathscr{F}$ which obey the following conditions

$$\mathscr{F} \subset \mathscr{R}_{r+1}\backslash\{\varepsilon_1, \varepsilon_2, \dots \varepsilon_m\} \tag{15a}$$

$$\mathscr{F} \bigcup \{\varepsilon_{k_1}, \varepsilon_{k_2}, \dots, \varepsilon_{k_n}\} \text{ is a forest.} \tag{15b}$$

Let us consider some term in (14) corresponding to subdiagrams overlapping with γ_r

$$(\prod_{k=1}^{r-1} O_{\gamma_k})M_{\gamma_r}A_{k_1k_2\cdots k_n}M_{\varepsilon_{k_1}}M_{\varepsilon_{k_2}} \dots M_{\varepsilon_{k_n}}\phi. \tag{16}$$

To achieve our recursive proof one have to convince that each term like (16) vanishes. We pick among $\varepsilon_{k_1}, \varepsilon_{k_2}, \dots, \varepsilon_{k_n}$ the maximal subdiagrams. A subdiagram is said to be maximal if it is not contained in any other subdiagram ε_{k_j}. For pure convenience we number the maximal subdiagrams overlapping with γ_r as $\varepsilon_{k_1}, \varepsilon_{k_2}, \dots, \varepsilon_{k_f}$ $(f \leqslant n)$. Obviously, the expression $A_{k_1k_2\dots k_n}$ is identically equal to zero unless the corresponding maximal subdiagrams are mutually disjoint. For what follows, we introduce the following definitions

$$\alpha_j \equiv \gamma_r \bigcup \varepsilon_{k_1} \bigcup \varepsilon_{k_2} \bigcup \dots \bigcup \varepsilon_{k_j}, \quad j = 1, 2, \dots, f. \tag{17}$$

Keeping in mind that the subdiagrams ε_{k_i} are overlapping with γ_r and mutually disjoint, one observes that

$$\begin{gathered} \varepsilon_{k_1} \text{ overlaps with } \gamma_r, \\ \varepsilon_{k_{i+1}} \text{ overlaps with } \alpha_i, \quad i = 1, 2, \dots, f-1. \end{gathered} \tag{18}$$

Let us make some further remarks. First, we see that if some forest $\mathscr{F}$ satisfies the conditions (15ab) then the forest $\mathscr{F} \cup \{\alpha_f\}$ does it as well. As a result we are allowed to rewrite expression (15) as

$$A_{k_1k_2\cdots k_n} = [\sum_{\mathscr{F}\in\mathscr{A}^*_{k1k2\cdots kn}} \prod_{i\in\mathscr{F}} (-M_{\gamma i})]O_{\alpha_f}, \tag{19}$$

where the asterisk means that the sum is taken over the forests which do not contain the subdiagram α_f. Note that the above relation remains valid if the subdiagram α_f is not divergent, that is it has a negative index, because in this case the subtraction operator reduces to unity

$$O_{\alpha_f} = 1 \quad \text{if} \quad \omega_{\alpha_f} < 0. \tag{20}$$

For current purposes, we evaluate the following identity

$$\begin{aligned} 1 &= (O_{\alpha_1}+M_{\alpha_1})(O_{\alpha_2}+M_{\alpha_2}) \dots (O_{\alpha_{f-1}}+M_{\alpha_{f-1}}) \\ &= O_{\alpha_1}+M_{\alpha_1}O_{\alpha_2}+M_{\alpha_1}M_{\alpha_2}O_{\alpha_3}+ \dots +M_{\alpha_1}M_{\alpha_2}M_{\alpha_3} \dots M_{\alpha_{f-1}}. \end{aligned} \tag{21}$$

Putting all this together, one can rewrite the term (16) as

$$(\prod_{k=1}^{r-1} O_{\gamma_k})M_{\gamma_r}[\sum_{\mathscr{F}\in\mathscr{A}^*_{k1k2\cdots kn}} \prod_{i\in\mathscr{F}}(-M_{\gamma_i})]M_{\varepsilon_{kn}} \dots M_{\varepsilon_{k1}}$$

$$\times[O_{\alpha_1}+M_{\alpha_1}O_{\alpha_2}+M_{\alpha_1}M_{\alpha_2}O_{\alpha_3}+ \dots +M_{\alpha_1}M_{\alpha_2}M_{\alpha_3}\dots M_{\alpha_{f-1}}]O_{\alpha_f}\phi. \tag{22}$$

To complete our proof we verify that each component of (22) vanishes, namely

$$(\prod_{k=1}^{r-1} O_{\gamma_k})M_{\gamma_r}[\sum_{\mathscr{F}\in\mathscr{A}^*_{k1k2\cdots kn}} \prod_{i\in\mathscr{F}}(-M_{\gamma_i})]M_{\varepsilon_{k_n}} \dots M_{\varepsilon_{k_1}}$$

$$\times M_{\alpha_1}M_{\alpha_2}\dots M_{\alpha_{g-1}}O_{\alpha_g}M_{\alpha_f}\phi = O \quad \text{for} \quad g = 1, 2, \dots, f. \tag{23}$$

We know that the subdiagrams ε_{k_i} $(i = 1, \dots, f)$ overlap with α_{g-1}, so we choose a family of mutually disjoint divergent subdiagrams $\gamma_{l_1}, \gamma_{l_2}, \dots, \gamma_{l_s}$, which are included in $\varepsilon_{k_g} \cap \alpha_{g-1}$ and their indices satisfy the following inequality

$$\omega_{l_1}+\omega_{l_2}+ \dots +\omega_{l_s} \geqslant \omega_{\cap}, \tag{24}$$

where $\omega_{\cap}$ denotes the index of the subdiagram $\varepsilon_{k_g} \cap \alpha_{g-1}$. Because the subdiagram α_g is the sum of ε_{k_g} and α_{g-1}, one can check the following identity (for details see references [11] or [9])

$$(\prod_{i=1}^{s} O_{\gamma_{l_i}})M_{\varepsilon_{k_g}}M_{\alpha_{g-1}}O_{\alpha_g}\phi = O. \tag{25}$$

The generalization of the above identity to the case (23) is trivial. Because we are not intending to repeat the highly technical proof of (25) given in Ref. [11], pages 17–21, we shall content ourselves with mentioning the most important steps and stress the differences connected with the generalization to the case (23). It suffices to show that after the $(\kappa_{l_1}, \dots, \kappa_{l_s})$-parametrization the expression of the form

$$M_{\varepsilon_{k_g}} \dots M_{\varepsilon_{k_2}}M_{\varepsilon_{k_1}}M_{\alpha_1}M_{\alpha_2}\dots M_{\alpha_{g-1}}O_{\alpha_g}M_{\alpha_f}\phi \tag{26}$$

is a polynomial with respect to the parameters $\kappa_{l_1}, \dots, \kappa_{l_s}$.

LEMMA 2: In the diagram Γ there exists a tree T_1 such that

$$w_{\varepsilon_{k_i}}(T_1) = p_{\varepsilon_{k_i}}+C(\varepsilon_{ki} \cap \alpha_{i-1})-1, \tag{27a}$$

$$w_{\alpha_i}(T_1) = p_{\alpha_i}, \quad i = 1, 2, \dots, g, f, \tag{27b}$$

$$w_{\gamma_{l_i}}(T_1) = p_{\gamma_{l_i}}, \quad i = 1, 2, \dots, s, \tag{27c}$$

where $C(\dots)$ denotes the number of connected components. The other notations are explained in Appendix.

PROOF: First we build trees in the subdiagrams $\varepsilon_{k_i} \cap \alpha_{i-1}$ (for $i = 2, 3, \dots, g$). We can do it because these subdiagrams are mutually disjoint. Then, this set of lines is extended in such a way that we create a tree in the subdiagram α_1. This possibility follows from the fact that each subdiagram $\varepsilon_{k_i} \cap \alpha_{i-1}$ is included in the α_1 (see relation (17)). The relation (17) implies also that

$$\alpha_1 \subset \alpha_2 \subset \ldots \subset \alpha_g \subset \alpha_f \subset \Gamma. \tag{28}$$

It means that it is possible to enlarge the tree in α_1 to be a tree in α_2, then to a tree in α_3 and so on. Finally, we obtain a tree T_1 in the whole diagram Γ and this tree, by construction, satisfies conditions (27abc).

Let us denote the collection of all trees (resp. two-trees) obeying (27abc) by $\mathcal{T}_1$ (resp. $\mathcal{T}_2$). The set $\mathcal{T}_2$ may be empty. Considering definitions (6) and (7), one can convince oneself that the expression (26) is of the form

$$\frac{B}{D^{*K}} \exp\left(i\frac{A^*}{D^*}\right), \tag{29}$$

where K denotes some positive number, B denotes some polynomial with respect to $\boldsymbol{\alpha}$ and

$$D^* = \sum_{T_1 \in \mathcal{T}_1} (\prod_{l \notin T_1} \alpha_l), \tag{29a}$$

$$A^* = \sum_{T_2 \in \mathcal{T}_2} \{(\prod_{l \notin T_2} \alpha_l)(\sum_{i \in V_1} k_i)^2\}. \tag{29b}$$

It is straightforward to deduce from the above and (27c) that the expression (26) is a polynomial with respect to the $(\kappa_{l_1}, \ldots, \kappa_{l_s})$, so due to applications of the subtraction operators $O\gamma_{l_i}$, which are included in the first term of (23), this polynomial is cancelled. This proves the identity (23).

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APPENDIX

In this Appendix we explain the mathematics and notation involved in Sections 2–3.

A subdiagram is said to be full if any two vertices in this subdiagram are joined by all the lines which already joined them in the original diagram.

A subdiagram is said to be one-particle irreducible (1PI) of it is connected and each its internal line belongs to at least one loop.

A subdiagram is said to be divergent if it is full, one-particle irreducible and its index is non-negative.

The index ω_r of the subdiagram γ_r is defined by

$$\omega_r = Dp_r - 2l_r,$$

where p_r and l_r are the numbers of loops and internal lines respectively. The number of loops can be calculated from

$$p_r = l_r - n_r + C(\gamma_r),$$

where n_r is the number of vertices and $C(\gamma_r)$ denotes the number of connected components.

We also use the notation

$$w_r(T_1)$$

to count the number of lines which belong to the subdiagram γ_r but not to the given tree T_1.

Two divergent subdiagrams γ_1 and γ_2 are overlapping if $\gamma_1 \cap \gamma_2 \neq 0$ and neither $\gamma_1 \subset \gamma_2$ nor $\gamma_2 \subset \gamma_1$.

A forest is a subfamily of non-overlapping divergent subdiagrams.

A family of divergent subdiagrams is called a tree family if there exist a tree T_1 of the diagram Γ such that for each of the divergent subdiagrams belonging to this family the intersection with T_1 is a tree of this subdiagram.

A Feynman diagram Γ is called a T-diagram if its family of divergent subdiagrams is a tree family.

A sum of divergent subdiagram γ_1 and γ_2 is said to be a full subdiagram composed of all vertices belonging to γ_1 and γ_2. This sum may contain such lines which belong neither to γ_1 nor to γ_2.

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