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OVERVIEW REMARKS ON HOMOGENEOUS, $N = 1$, $d = 11$ SUPERGRAVITY COSMOLOGIES*

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The dynamics of the full class of homogeneous $N = 1$, $d = 11$ supergravity world models is investigated. By using the classification of Lie algebras of Lie groups which act simply transitively on 6- and 7-dimensional compact spaces some conclusions are drawn concerning the non-existence of the chaotic regime near the singularity. This is illustrated with some new solutions having a richer structure of the microspace. The significance of known solutions is briefly discussed.

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1. Introduction

The hunt for a consistent theory of quantum gravity and a theory unifying all physical interactions has in recent years evoked an interest in multidimensional cosmologies of superstring (Schwartz, 1985), supergravity and Kaluza-Klein types (Duff et al. 1986). All these theories, different as they are, refer to the very early Universe, and they presuppose, to a certain extent, similar cosmological setting.

Certain multidimensional homogeneous world models have been constructed and analysed in the context of the Kaluza-Klein theory and supergravity theory (Demiański

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et al. 1986a, Demaret et al. 1985a, 1985b, Lorenz-Petzold, 1984, 1985a, 1985b). A systematic approach to the problem has been proposed by Demiański et al. (1987). By generalizing the classical Bianchi classification of all spatially homogeneous world models to nine or ten dimensions, these authors have created a cosmological framework for considering the unifying theories near the Planck era. The aim of the present work is to discuss some aspects of the supergravity theory within this framework.

Demiański et al. assumed that the total space of the homogenous world model had the product structure $P = M_3 \times B_7$, where M_3 is 3-dimensional macroscopic space and B_7 is 7-dimensional microscopic space. The authors classified all 7-dimensional compact Lie algebras of the isometry group of the space B_7 and obtained the following three possibilities:

$$1.\ \bigoplus_{i=1}^{7} L_1^i, \quad 2.\ \bigoplus_{i=1}^{4} L_1^i \oplus B(\mathrm{IX}), \quad 3.\ L_1 \oplus B(\mathrm{IX}) \oplus B(\mathrm{IX}),$$

where L_1 is the 1-dimensional Abelian algebra and $B(\mathrm{IX})$ is the Lie algebra of the group SO(3). By combining these algebras with the ordinary Bianchi classification one obtains the full classification of 11-dimensional homogeneous world models. This classification

TABLE I

The algebraic classification of 11-dimensional homogeneous world models

B_7	M_3 (The Bianchi type)									
	I	II	VI_0	VII_0	VIII	IX	V	IV	VI_h	VII_h
A	0	3	5	5	6	6	3	5	6	6
B	6	9	11	11	12	12	9	11	12	12
C	12	15	17	17	18	18	15	17	18	18

$A = \bigoplus_{i=7}^{7} L_1^i \quad B = \bigoplus_{i=1}^{4} L_1^i \oplus B(\mathrm{IX}), \quad C = L_1 \oplus B(\mathrm{IX}) \oplus B(\mathrm{IX}).$

is shown in Table I. The structure constants determine the structure of P up to Ellis identification (Ellis, 1971). The numbers in the Table I are dimensions of equivalence classes of the structure constants in 10-dimensional homogeneous spaces. One can see that the following types have the highest dimension

$$1.\ \mathrm{VI}_h \oplus B(\mathrm{IX}) \oplus B(\mathrm{IX}) \oplus L_1, \quad 2.\ \mathrm{VII}_h \oplus B(\mathrm{IX}) \oplus B(\mathrm{IX}) \oplus L_1,$$

$$3.\ B(\mathrm{VIII}) \oplus B(\mathrm{IX}) \oplus B(\mathrm{IX}) \oplus L_1, \quad 4.\ B(\mathrm{IX}) \oplus B(\mathrm{IX}) \oplus B(\mathrm{IX}) \oplus L_1.$$

This means that every open set, in the space of all initial data, intersecting the subspace of the homogeneous initial data, must intersect the subspace of the initial data of one of these types. In this sense, only this types are "generic" in the space of all homogeneous initial data (for details see Collins, Hawking [1973]).

The classification presented in Table I can be easily adapted to theories other than the Kaluza-Klein theory. By omitting one Abelian algebra in cases A, B and C one obtains the classification adapted to the superstring theory and the 10-dimensional supergravity.

In the supergravity, world models with the microspace having the torus structure, have been investigated (Demaret et al. 1985a, b, Lorenz-Petzold, 1984, 1985a, b, Demiański et al. 1987). From the above consideration one can see that such solutions form a zero-measure subsets in the space of homogeneous initial data, and more general conclusions extrapolated from such models may easily be misteading. In the present work we shall show that the vanishing of chaotic oscillations is a typical behaviour in a neighbourhood of the velocity dominated singularity. We shall present some homogeneous solutions with a richer structure of the microspace for $N = 1$, $d = 11$ supergravity.

2. *Dynamical equations in $N = 1$, $d = 11$ supergravity. General properties*

In the bosonic sector of the $N = 1$, $d = 11$ supergravity the Einstein equations assume the form

$$R_{MN} = E_{MN}, \tag{1}$$

where

$$E_{MN} = -\tfrac{1}{6}\left(F_{MPQR}F_N^{PQR} - \tfrac{1}{12}\, g_{MN}F_{PQRS}F^{PQRS}\right)$$

and $F_{MNPQ} = \nabla_{[M}A_{NPQ]}$ satisfies the equation of motion:

$$\nabla_M F^{MNPQ} = -\frac{\sqrt{2}}{48}\,\varepsilon^{NPQM_1\cdots M_8}F_{M_1\cdots M_4}F_{M_5\cdots M_8},$$

$$\nabla_{[M}F_{NPQR]} = 0, \tag{2}$$

R_{MN} is the Ricci tensor.

If for the fields F_{MNPQ} the Freund-Rubin-Duff-Pope (FRDP) Ansatz is assumed, one has

$$F^{MNPQ} = \begin{cases} \dfrac{f(t)\varepsilon^{\mu\nu\varrho\delta}}{\sqrt{-g_4}} & M, N = \mu, \nu, \\[2ex] 0 & M, N = m, n, \end{cases} \tag{3}$$

where $\mu, \nu = 0, \ldots, 3$; $m, n = 4, \ldots, 10$; f is a function of time only, and the metric has a block form $g = (g_4, g_7)$.

From Eq. (3) one obtains $\nabla_\mu F^{\mu\nu\varrho\delta} = 0$ which implies:

$$f^2(t) = \frac{f_0^2}{-g_7}$$

f_0 being a constant.

Since we are interested in homogeneous world models with simply transitive groups of isometries, we may use the tetrad formalism (Landau, 1971) to compute the components of the Ricci tensor. The metric is assumed to be:

$$ds^2 = dt^2 - \eta_{ab}(t)e^a_i(x)e^b_j(x)dx^i dx^j, \tag{4}$$

where η_{ab} is a function of time only, and has a diagonal form $\eta_{ab} = \text{diag}(R_1^2, \ldots, R_{10}^2)$, $\omega^a = e^a_i dx^i$ are basis one-forms and $a, b, i, j = 1, \ldots, 10$. With this metric the components of the Riccis tensor are:

$$-R^0{}_0 = \tfrac{1}{2}\dot{\kappa}^a_a + \tfrac{1}{4}\kappa^a_b\kappa^b_a \tag{5a}$$

$$-R^0{}_a = \tfrac{1}{2}\dot{\kappa}^b_a(C^c{}_{ba} - \delta^c_a C^d{}_{db}) \tag{5b}$$

$$-R^a{}_b = P^a{}_b + \frac{1}{2\sqrt{-\eta}}\frac{d}{dt}(\sqrt{-\eta}\,\kappa^a_b) \tag{5c}$$

$$\kappa^a_b = \dot{\eta}_{bc}\eta^{ca}; \quad \eta = \det(\eta_{ab}); \text{ dot denotes } \frac{d}{dt}$$

$$-P_{ab} = \Gamma^c_{ad}\Gamma^d_{bc} + C^d{}_{dc}\Gamma^c_{ab} \tag{5d}$$

$$\Gamma^a_{bc} = \tfrac{1}{2}(C^a{}_{bc} + C^d{}_{be}g_{dc}g^{ae} + C^d{}_{ce}g_{db}g^{ae}), \tag{5e}$$

where P_{ab} is the Ricci curvature tensor of constant time sections.

By using the standard procedure, one can show that for macrospaces of the Bianchi class A, the metric can always be reduced to the diagonal form, whereas the macrospace of the Bianchi class B to the form with one non-diagonal component (with the exception when the macrospace is of B(V) type) (Bogoyavlensky, 1980). For the macrospace of class A, equations (5b) are trivially satisfied, whereas for the macrospace of class B the equation $R^0_1 = E^0_1$ is non-trivial. We shall also consider the case when the total space t = const has a SO(N) symmetry group. The structural constants of this group have been constructed by Ishihara (1985). The structure constant $C^a{}_{bc}$ is non-zero if $(a, b, c) \in Q$ where $Q = \{(i, k, m) : [T_i, T_k] = T_m\}$. This enables one to express the tensor in a simple manner

$$P^i_i = \tfrac{1}{2}\sum_{(ijk)\in Q}(R_i R_j R_k)^{-2}[R_i^4 - (R_j^2 - R_k^2)^2]$$

$$\text{(no summation with respect to } i). \tag{6}$$

Now we will investigate certain general properties of Equations (5a)–(5d) in the supergravity theory. In this theory the energy-momentum tensor for the fields F_{MNPQ} is of the form

$$T_{MN} = F_{MPQR}F_N^{PQR} - \tfrac{1}{8}F_{PQRS}F^{PQRS}g_{MN}.$$

We present T_{MN} in the "hydrodynamical" form (Gleiser, 1985)

$$T_{MN} = -pg_{MN} + (\varrho + p)U_M U_N + \varepsilon p' g_{MN}, \tag{7a}$$

$$U_0 = 1; \quad U_i = 0; \quad \varepsilon = \begin{cases} 0 & \text{for} \quad M, N = \mu, \nu \\ 1 & \text{for} \quad M, N = m, n \end{cases}$$

$$p = -\varrho \quad \text{and} \quad p' = \varrho.$$

The condition $T^M_{N;M} = 0$ yields

$$\varrho = \frac{g_0^2}{|g_7|}; \qquad g_0 = \text{const.} \tag{7b}$$

The existence of a metric-type singularity ($\det \eta_{ab} = 0$) turns out to be a general property of homogeneous supersymmetric models. Indeed, to see this it is enough to demonstrate the existence of the monotonic function

$$F = \frac{d}{dt} |\eta|^{1/2n}. \tag{8}$$

By using Eq. (5a), together with the algebraic inequality

$$(\kappa^a_a)^2 \leqslant -\frac{1}{n} |\eta|^{1/2n} \left(T^0{}_0 - \frac{1}{n-1} T \right), \tag{9}$$

where $T = \operatorname{tr}(T^a_a)$ n is the dimension of the total space $t = \text{const}$, and remembering that in our case $T^0_0 - \frac{1}{n-1} T \geqslant 0$ and this implies that $|\eta|$ is a convex function having only one local maximum and, in general having no minima. If $F > 0$ the total volume increases starting from the singularity.

The existence of the solutions with the static microspace is important from the point of view of the stability problem. It is easy to see that for the homogeneous supersymmetric cosmology such solutions do not exist. This is connected with the fact that the microspace admits at least one 1-dimensional Abelian Lie algebra.

An interesting general property of the homogeneous supersymmetric cosmology is the existence of solutions with the negligible curvature P^a_b and matter fields in a vicinity of the singularity, that is, the existence of asymptotics of the vacuum B(I) form. Solutions for the supersymmetric B(I) case have been found by Demaret (1985b) and Lorenz-Petzold (1984, 1985a, 1985b, 1985c)

Let us consider the model $B(\text{IX}) \times T^1$ or $B(\text{VIII}) \times T^1$. The fact that curvature is negligible in a vicinity of the singularity $P^a_a \to 0$ as $t \to 0$ together with $R_i \propto t^{p_i}$ ($i = 1, \dots 4$) leads to the following system of inequalities:

$$\begin{aligned} &p_4 + 2p_1 > 0; \quad &&p_4 + 2p_2 > 0; \quad &&p_4 + 2p_3 > 0, \\ &p_1 < 1 &&p_2 < 1 &&p_3 < 1. \end{aligned} \tag{10}$$

By using the Kasner condition $\sum_{i=1}^{4} p_i = \sum_{i=1}^{4} p_i^2 = 1$ two of the Kasner parameters can be eliminated. The existence of the real p_2 in the space of the remaining parameters, determine the region

$$U = \{(p_3, p_4) : 1-3p_3^2-2p_3p_4+2p_3+2p_4-3p_4^2 \geqslant 0)\}.$$

This region is shown in Fig. 1 as the interior of the elipse.

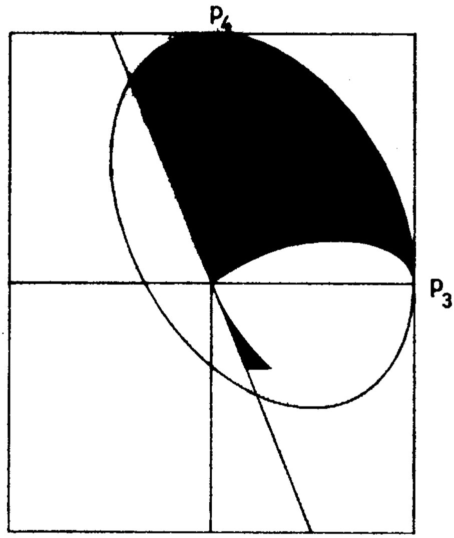

Fig. 1. The shaded region represents the set of the Kasner exponents leading to the Kasner asymptotic behaviour in a neighbourhood of the singularity in $B(\text{IX})\times T^1$ vacuum models

The dashed region represents sets of the Kasner parameters for which both the curvature and matter terms are negligible, and inequalities (10) are satisfied.

This property holds for the full class of homogeneous supersymmetric solutions.

3. *Solutions with the richer structure of the microspace*

Some homogeneous supersymmetric solutions have been obtained for the microspace having T^7 structure (Demaret, 1985a, 1985b, Lorenz-Petzold, 1984, 1985a, 1985b). As we have seen (see, Table I), such solutions are rather exceptional. Solutions presented by Lorenz-Petzold (with the exception of the generalized $B(\text{I})$ case) are special solutions obtained by assuming the proportionality of the curvature potential to the source potential. Such an Ansatz remains in agreement with the dynamics (Jantzen, 1986). It seems interesting to look for homogeneous supersymmetric solutions with the richer structure of the microspace. First, we consider the world models in which the total space have the following structures

1. $P = M_3 \times B_7 = B(\text{I})\times B(\text{IX})\times T^4$, 2. $P = M_3 \times B_7 = B(\text{I})\times B(\text{IX})\times B(\text{IX})\times T^1$.

A. $P = B(\mathrm{I}) \times B(\mathrm{IX}) \times T^4$

The Einstein equations have the form

$$\sum_{i=1}^{3} \frac{\ddot{R}_i}{R_i} + \sum_{m=1}^{3} \frac{\ddot{\varrho}_m}{\varrho_m} + \sum_{a=1}^{4} \frac{\ddot{r}_a}{r_a} = -\frac{\alpha^2}{u^2}, \tag{11a}$$

$$(\ln R_i)^{\cdot\cdot} + (\ln R_i)^{\cdot}(\ln V)^{\cdot} = -\frac{\alpha^2}{u^2}, \tag{11b}$$

$$(\ln \varrho_m)^{\cdot\cdot} + (\ln \varrho_m)^{\cdot}(\ln V)^{\cdot} + \frac{\varrho_m^4 - (\varrho_s^2 - \varrho_n^2)^2}{\varrho_m^2 \varrho_n^2 \varrho_s^2} = \frac{1}{2}\frac{\alpha^2}{u^2}, \quad m \neq n \neq s, \tag{11c}$$

$$(\ln r_a)^{\cdot\cdot} + (\ln r_a)^{\cdot}(\ln V)^{\cdot} = \frac{1}{2}\frac{\alpha^2}{u^2}, \tag{11d}$$

where R_i, ϱ_m, r_a are the scale factors of the $B(\mathrm{I})$, $B(\mathrm{IX})$ and T^4, respectively, $u = \prod_{m=1}^{3} \varrho_m \prod_{a=1}^{4} r_a$; $V = u \prod_{i=1}^{3} R_i$; $\alpha^2 = 2f_0^2/3$. Assuming that the source potential of the microspace is proportional to the curvature potential of the $B(\mathrm{IX})$ sector, we obtain the solutions:

$$R_i = R_{0i} \exp[A_i \tau] \operatorname{ch}^{-1/3}[D\tau], \tag{12a}$$

$$r_a = r_{0a} \exp[A_a \tau] \operatorname{ch}^{1/6}[D\tau], \tag{12b}$$

$$\varrho = \varrho_0 \exp[C\tau] \operatorname{ch}^{-1/3}[D\tau]; \quad \varrho = \varrho_m, \quad m = 1, 2, 3, \tag{12c}$$

where the τ parameter is defined by $dt = V d\tau$ and the integration constants satisfy the following conditions:

$$\prod_{i=1}^{3} R_{0i} = \prod_{a=1}^{4} r_{0a} = 1; \quad D^2 = 3\alpha^2; \quad A = -2C,$$

$$\sum_{i=1}^{3} A_i = \sum_{a=1}^{4} A_a = 0; \quad \sum_{i=1}^{3} A_i^2 + \sum_{a=1}^{4} A_a^2 + 4C^2 - D^2 \doteq 0; \quad \varrho_0 = (3\alpha^2)^{1/4}.$$

Our Ansatz leads to the identical, isotropic solutions in the $B(\mathrm{IX})$ sector. Note that isotropy of $B(\mathrm{IX})$ sectors does not imply that source potential is proportional to the curvature terms.

B. $P = B(\mathrm{I}) \times B(\mathrm{IX}) \times B(\mathrm{IX}) \times T^1$

The Einstein equations have the form

$$\sum_{i=1}^{3} \frac{\ddot{R}_i}{R_i} + \sum_{m=1}^{3} \frac{\ddot{\varrho}_m}{\varrho_m} + \sum_{m=1}^{3} \frac{\ddot{\varrho}'_m}{\varrho'_m} + \frac{\ddot{r}}{r} = -\frac{\alpha^2}{u^2}, \tag{13a}$$

$$(\ln R_i)^{\cdot\cdot} + (\ln R_i)^{\cdot}(\ln V)^{\cdot} = -\frac{\alpha^2}{u^2}, \tag{13b}$$

$$(\ln \varrho_m)^{\cdot\cdot} + (\ln \varrho_m)^{\cdot}(\ln V)^{\cdot} + \frac{\varrho_m^4 - (\varrho_s^2 - \varrho_n^2)^2}{\varrho_m^2 \varrho_n^2 \varrho_s^2} = \frac{1}{2}\frac{\alpha^2}{u^2}, \tag{13c}$$

$$(\ln \varrho'_m)^{\cdot\cdot} + (\ln \varrho'_m)^{\cdot}(\ln V)^{\cdot} + \frac{\varrho_m'^4 - (\varrho_s'^4 - \varrho_n'^2)^2}{\varrho_m'^2 \varrho_n'^2 \varrho_s'^2} = \frac{1}{2}\frac{\alpha^2}{u^2}, \quad m \neq n \neq s, \tag{13d}$$

$$(\ln r)^{\cdot\cdot} + (\ln r)^{\cdot}(\ln V)^{\cdot} = \frac{1}{2}\frac{a^2}{u^2}, \tag{13e}$$

where R_i, ϱ_m, ϱ'_m, r are the scale factors of the $B(\mathrm{I})$, $B(\mathrm{IX})$, $B(\mathrm{IX})$, T^1 respectively, $u = r \prod_{m=1}^{3} \times \varrho_m \prod_{m=1}^{3} \varrho'_m$; $V = u \prod_{i=1}^{3} R_i$.

With the same Ansatz we obtain the following solutions

$$R_i = R_{0i} \exp[A_i\tau]\, \mathrm{ch}^{-1/3}[D\tau], \tag{14a}$$

$$r = r_0 \exp[A\tau]\, \mathrm{ch}^{1/6}[D\tau], \tag{14b}$$

$$\varrho = \varrho_0 \exp[C\tau]\, \mathrm{ch}^{-1/30}[D\tau]; \quad \varrho = \varrho'_m = \varrho_m, \quad m = \ ,2,3 \tag{14c}$$

with the conditions on the integration constants

$$\sum_{i=1}^{3} A_i = 0; \quad \sum_{i=1}^{3} A_i^2 + \tfrac{12}{5} A^2 - \tfrac{7}{10} D^2 = 0,$$

$$\prod_{i=1}^{3} R_{0i} = 1; \quad D^2 = 3\alpha^2; \quad A = -5C; \quad \varrho_0 = (\tfrac{5}{6}\alpha^2)^{1/10} B^{-1/5}.$$

In this case the $B(\mathrm{IX})$ sectors are also isotropic.

By assuming from beginning that the $B(\mathrm{IX})$ sectors of the microspace are isotropic, one can obtain the exact solutions of the field equation.

C. $P = B(\mathrm{I}) \times S^3 \times T^4$

$$R_i = R_{0i}\, \mathrm{tg}^{a_i}[\tfrac{1}{2}\eta]\, \mathrm{ch}^{-1/3}[\ln \mathrm{tg}^{3C}(\tfrac{1}{2}\eta)], \tag{15a}$$

$$r_\alpha = r_{0a}\, \mathrm{tg}^{b_a}[\tfrac{1}{2}\eta]\, \mathrm{ch}^{1/6}[\ln \mathrm{tg}^{3C}(\tfrac{1}{2}\eta)], \tag{15b}$$

$$\varrho = \varrho_0\, \mathrm{tg}^{b}[\tfrac{1}{2}\eta] \sin^{1/2}(\eta)\, \mathrm{ch}^{1/6}[\ln \mathrm{tg}^{3C}(\tfrac{1}{2}\eta)], \tag{15c}$$

where

$$\sum_{i=1}^{3} a_i = 0; \quad \sum_{i=1}^{3} b_a = -2b; \quad \prod_{i=1}^{3} R_{0i} = \frac{3C^2}{\alpha^2}; \quad \varrho_0^4 \prod_{a=1}^{4} r_{0a} = \frac{\alpha}{6C^2},$$

$$\sum_{i=1}^{3} a_i^2 + \sum_{a=1}^{4} b_a^2 + 2b^2 + \tfrac{9}{2} C^2 = \tfrac{3}{2}$$

and the η parameter is defined as following: $dt = \varrho d\eta$.

D. $P = B(\mathrm{I}) \times S^3 \times S^3 \times T^1$

$$R_i = R_{0i}\, \mathrm{tg}^{a_i}[\tfrac{1}{2}\eta]\, \mathrm{ch}^{-1/3}[\ln \mathrm{tg}^{3C}(\tfrac{1}{2}\eta)], \tag{16a}$$

$$r = r_0\, \mathrm{tg}^{b}[\tfrac{1}{2}\eta]\, \mathrm{ch}^{1/6}[\ln \mathrm{tg}^{3C}(\tfrac{1}{2}\eta)], \tag{16b}$$

$$\varrho = \varrho_0\, \mathrm{tg}^{a}\,[\tfrac{1}{2}\eta]\, \sin^{1/5}(\eta)\, \mathrm{ch}^{1/6}[\ln \mathrm{tg}^{3C}(\tfrac{1}{2}\eta)] \tag{16c}$$

and

$$\sum_{i=1}^{3} a_i = 0; \quad 5a+b = 0; \quad \prod_{i=1}^{3} R_{0i} = \frac{3C^2}{\alpha^2}; \quad r_0^2 \varrho_0^{10} = \frac{\alpha^2}{6C^2},$$

$$\sum_{i=1}^{3} a_i^2 + 5a^2 + b^2 + \tfrac{9}{2}\, C^2 = \tfrac{7}{5}.$$

4. *Generalized mixmaster model*

As we have argued in the Introduction, the mixmaster model $B(\mathrm{IX}) \times B(\mathrm{IX}) \times B(\mathrm{IX}) \times T^1$ seems to be physically the most interesting solution. We are able to discuss the qualitative behaviour of this model near the singularity. The non-existence of chaotic oscillations for homogeneous multidimensional classical empty solutions of the Einstein field equation was motived by Furusawa and Hosoya (1985), and by Barrow and Stein-Schabes (for the $B(\mathrm{IX}) \times T^m$ model) (1985a). In both these cases the microspace has the torus structure. Tomimatsu and Ishihara (1986) discussed the breaking down effect of the chaotic behaviour in the model $B(\mathrm{IX}) \times B(\mathrm{IX})$. The non-chaotic regime for the generalized mixmaster model with the symmetry SO(N) was also noticed by Barrow and Stein-Schabes (1985b) and Ishihara (1985), Demaret et al. (1985c) shown that the generalized Kasner solution has a status of a "general solution" in a neighbourhood of the singularity for space times with the dimension $n \geqslant 11$. The problem of the chaotic behaviour near the singularity was also discussed for non-homogeneous Kaluza-Klein cosmologies (Hosoya, 1985, Demaret, 1986). How this problem looks in the case of homogenous supergravity world models? To asnwer this questions, let us assume that at a certain instant τ_0 the Kasner solution is valid, i.e. $R_i \propto t^{p_i}$ and $\sum_{i=1}^{10} p_i = \sum_{i=1}^{10} p_i^2 = 1$.

In order to present the vanishing of chaotic behaviour it is enough to show that chaos vanishes for certain choice of parameters p_i. Without any loss of generality we may assume that the set of Kasner parameters is such that p_{10} is the smallest one and negative. If $\tau \to -\infty$ ($dt = V d\tau$, $t \to 0$) the curvature terms become negligible. Our choice $p_{10} < 0$ is motivated by the fact that the equation for R_{10} contains no curvature term. In such a case the dynamical equations reduce to those of the n-dimensional $B(\mathrm{I})$ model in the supergravity theory. It is easy to show that if $\tau \to -\infty$ and $\tau \to +\infty$, the asymptotics are Kasner ones. Following Furusawa (1986) (see also Demaret, 1985c, Barrow, 1985b) we will speak that the system is non-chaotic if there is a finite region in the space of parameters p_i for which the transition to the next Kasner epoch is forbidden. This means that the dissipation of p_i has been completed and the system evolves in a non-chaotic manner. The geometric interpretation of this phenomena has been given by Hosoya (1986). By writing down the "bouncing rules" for the Kasner parameters, one easily demonstrates that such a non-zero measure region exists. This shows the non-existence of the chaotic oscillation phase for homogeneous $N = 1$ $d = 11$ supergravity cosmology.

So far we have assumed that the homogeneous spaces of constant time have the product structure $M_3 \times B_7$, and that B_7 is compact. One could ask to which extent the obtained results depend on this product structure. To answer this question we should have at our disposal the full classification of 10-dimensional homogeneous Lie algebras of the isometry groups. This is difficult to obtain. However, the program becomes solvable if we additionaly assume that the total space of constant time is compact. In this case, by a simple consequence of the classification theorem (Szydłowski, 1986) the following Lie algebras of the isometry group are possible:

$$1.\ \bigoplus_{i=1}^{10} L_1^{(i)} \qquad 2.\ \bigoplus_{i=1}^{7} L_1^{(i)} \oplus B(\mathrm{IX}),$$

$$3.\ \bigoplus_{i=1}^{4} L_1^{(i)} \oplus B(\mathrm{IX}) \oplus B(\mathrm{IX}), \qquad 4.\ B(\mathrm{IX}) \oplus B(\mathrm{IX}) \oplus B(\mathrm{IX}) \oplus L_1,$$

$$5.\ \mathrm{SU}(3) \oplus L_1 \oplus L_1, \qquad 6.\ \mathrm{SO}(5).$$

For cases (2)–(4) the problem of the chaotic behaviour has been solved. The case (5), for the time being, will be put aside, and now we focus on the space with the symmetry generated by the SO(5) group. When writing down the Einstein equations we make the use of formula (6). With the new parameter τ, introduced by the transformation $dt = V d\tau$, the field equations assume the form:

$$\alpha_a'' = V^2(P_a^a + E_a^a), \tag{17}$$

where comma denotes the differentiation with respect to τ, V is the total space volume and $\alpha_a = \ln R_a$. We assume as before, that at a certain instant $\tau = \tau_0$ the Kasner solution holds. Let us assume that the Kasner parameter p_1 is the smallest one and negative. In such a case a dominant contribution to the curvature, in a neighbourhood of singularity, comes from the term $R_1^2/(R_2^2 R_3^2)$, whereas the source term is negligible. With this approximation the Einstein equations are

$$\alpha_1'' = -\tfrac{1}{2} V^2 \frac{R_1^2}{R_2^2 R_3^2}, \tag{18a}$$

$$\alpha_2'' = \alpha_3'' = \tfrac{1}{2} V^2 \frac{R_1^2}{R_2^2 R_3^2}, \tag{18b}$$

$$\alpha'' = 0 \qquad \text{for} \qquad i = 4, \ldots, 10. \tag{18c}$$

The solutions of the Eqs. (18), having the Kasner asymptotics $R_i \propto t^{p_i}$ for $\tau \to -\infty$, posses such asymptotics $R_i \propto t^{p_i'}$ for $\tau \to +\infty$. In a similar manner to the previous case, we can determine the "bouncing rules" of Kasner parameters:

$$p_1' = -\frac{p_1+p}{1+2p_1+p}; \quad p_2' = \frac{p_2+2p_1+p}{1+2p_1+p},$$

$$p_3' = \frac{p_3+2p_1+p}{1+2p_1+p}; \quad p_m' = \frac{p_m}{1+2p_1+p}; \quad p = \sum_{m=4}^{10} p_m.$$

Now we can find the non-zero measure region:

$$U = \{(p_1, \ldots, p_{10}): p_1, p_2, p_3 < 0, p_1', p_2', p_3' < 0, p_j, p_j' > 0 \text{ for } j = 4, \ldots, 10)\}$$

in the space of the Kasner initial parameters p_i from which the transition to the new Kasner epoch is forbidden. In this way, the problem of the non-existence of the chaotic behaviour is reduced to that in the empty homogeneous Kaluza-Klein cosmology which has been solved by us (Szydłowski et al. 1986). It turns out, therefore, that the conclusion on the non-existence of the chaotic oscillation phase is, in a sense, independent of the assumption of the product structure $M_3 \times B_7$.

5. *Conclusions*

In the present work we have investigated the significance of the homogeneous $N = 1$ $d = 11$ supergravity world models (with the FRDP Ansatz) within the context of the full classification of homogeneous models having the product structure $M_3 \times B_7$, where M_3 and B_7 are homogeneous spaces with a simply transitive isometry group and B_7 has been additionally assumed to be compact. Many solutions of this type are known in the literature (mainly with the microspace of the torus structure). The point is which of the solutions should be regarded as "generic". We have proposed, as the "genericity" criterion, the maximal dimension of the equivalence class of the structure constants. In this sense the model $B(\text{IX}) \times B(\text{IX}) \times B(\text{IX}) \times T^1$ is privileged. We have shown that within this model there is no chaotic behaviour near the singularity (in contrast to the classical $B(\text{IX})$ model). We have also presented the full classification of 7-dimensional compact homogeneous spaces (according to their isometry group) and we have shown that conclusions concerning the chaotic behaviour do not depend, in a sense, of the product structure assumption.

It turns out that within the considered class there are no solutions with a static microspace. They also do not exist in any 11-dimensional homogeneous Universe with matter sources (e.g. in 11-dimensional Einstein-Maxwell theory, in the theory with the hydrodynamic energy-momentum tensor with the exception of the Zeldovich matter $p = p' = \varrho$). The non-existence of such solutions is a serious difficulty for the 11-dimensional homogeneous supergravity. However, such solutions do exist in the 10-dimensional supergravity.

It seems therefore that, both within the cosmological and field theoretical context, 10-dimensional version of the theory is more interesting. We will turn to this in a forthcoming work.

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