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FORWARD-BACKWARD CORRELATIONS IN THE MULTIPLE PRODUCTION

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We discuss the minimal model for the forward-backward correlations in the hadron- and lepton-induced multiple production. In this model correlations are generated by averaging over the components corresponding to the independent emission of clusters with different average multiplicities. We show that the model corrected approximately for phase-space effects describes reasonably well the high energy hadronic data. On the other hand, the test of its applicability to the lh or e^+e^- data requires higher energies than presently available.

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1. Introduction

By "forward-backward" (FB) correlations one means usually the correlations between the global numbers of particles observed in CM hemispheres in multiple production. With the advent of colliding beams experiments such correlations became particularly easy to measure, since they do not require particle identification and momentum measurements. Even for the fixed target experiments FB correlations are useful if the statistics is too low to investigate in detail the differential correlation function. On the other hand, measuring the FB correlations may allow already to discriminate between various models of multiple production and different hadronization mechanisms. Thus the experimental and theoretical investigations of FB correlations have been performed recently by many authors. In particular, the UA5 collaboration at SPS collider has presented data and discussed them in terms of a very simple model [1, 2].

In this model, called further the Minimal Model (MM) the clusters of hadrons are randomly emitted according to the longitudinal phase space. The multiplicity of cluster decay is assumed in agreement with earlier short-range correlations analysis and the experimental multiplicity distribution determines the cluster multiplicity distribution. It is possible to derive the approximate predictions of MM for FB correlations without performing the Monte Carlo generation of events and the results are good.

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In this paper we show the equivalence of MM of Ref. [1] with a similar model constructed earlier to describe the ISR data [3, 4, 5]. Then we propose a simple ansatz for phase-space corrections allowing for approximate description of FB correlations in the ISR-collider energy range. Further, we discuss the possible applicability of MM to the lh or e^+e^- data, extending the former analysis of Ref. [6]. We conclude with remarks on the relation of MM to some more elaborate existing models of multiple production and to the apparent universality observed recently in data by Wróblewski [7].

2. *Approximate predictions of MM*

Let us recall here first a simple derivation of a MM formula for the correlation parameter b presented in Ref. [1]. Here b is defined by the linear relation

$$\bar{n}_F(n_B) = \bar{n}_F + b(n_B - \bar{n}_B), \tag{1}$$

where $n_{F(B)}$ denotes the number of particles in forward (backward) hemisphere, $\bar{n}_F(n_B)$ denotes the average for fixed n_B and $\bar{n}_{F(B)}$ are averages over all events. This relation agrees quite well with high-energy data and is expected to work always when the multiplicity distribution in each hemisphere can be approximated reasonably well by a Gaussian curve. It is easy to check that

$$b = \frac{\overline{n_F n_B} - \bar{n}_F \bar{n}_B}{\overline{n_B^2} - \bar{n}_B^2} \equiv \frac{D_{FB}}{D_B^2}. \tag{2}$$

In the following we consider the symmetric case, for which

$$D^2 = D_F^2 + D_B^2 + 2D_{FB} = 2D_F^2(1+b). \tag{3}$$

Thus we can separate the effects of global multiplicity fluctuations and the effects of fluctuations in one hemisphere for fixed global multiplicity. Denoting by $\langle\rangle_n$ an average for fixed global multiplicity n, and by bar averaging over n we have

$$D_F^2 = \overline{\langle D_F^2\rangle_n} + \overline{\langle n_F\rangle^2} - \bar{n}_F^2 = \langle D_F^2\rangle_n + \tfrac{1}{4}\,\overline{n^2} - \tfrac{1}{4}\,\bar{n}^2 = \overline{\langle D_F^2\rangle_n} + \tfrac{1}{4}\,D^2. \tag{4}$$

Thus

$$b = \frac{D^2}{2D_F^2} - 1 = \frac{D^2 - 4\overline{\langle D_F^2\rangle_n}}{D^2 + 4\overline{\langle D_F^2\rangle_n}}. \tag{5}$$

In the MM of Ref. [1] one assumes random uniform distribution in rapidity for fixed n, i.e. the binomial distribution of n_F

$$P(n_F)|_n = \frac{1}{2^n}\binom{n}{n_F} \tag{6}$$

resulting in

$$\langle D_F^2\rangle_n = \tfrac{1}{4}\,n. \tag{7}$$

Thus

$$b = \frac{D^2 - \bar{n}}{D^2 + \bar{n}}. \tag{8}$$

In fact, the realistic model should take into account the existence of short-range correlations. It can be done by assuming random emission of clusters instead of single particles. If the cluster decay is independent of its formation, we obtain

$$\langle D_F^2 \rangle_n = \tfrac{1}{4} k_{\mathrm{eff}} \bar{n}, \tag{9}$$

where

$$k_{\mathrm{eff}} = \frac{\overline{k^2}}{\bar{k}} \tag{10}$$

parametrizes the cluster decay distribution with average decay multiplicity $\bar{k}$. Thus the final formula of MM as presented in Ref. [1] is

$$b = \frac{D^2 - k_{\mathrm{eff}} \bar{n}}{D^2 + k_{\mathrm{eff}} \bar{n}}. \tag{11}$$

In fact, a similar model has been considered earlier [3–5]. KNO scaling of multiplicity distributions have led many authors to describe the multiparticle production as the superposition of independent emission processes with various average multiplicities proportional to some physical parameter (related to inelasticity, impact parameter etc.). Then for the multiplicity distribution we obtain

$$P(n) = \int d\lambda \psi(\lambda) e^{-\lambda \bar{n}} \frac{(\lambda \bar{n})^n}{n!} \tag{12}$$

where

$$\int d\lambda \psi(\lambda) = \int \lambda d\lambda \psi(\lambda) = 1 \tag{13}$$

resulting in

$$D^2 = \bar{n} + \left[\int \lambda^2 d\lambda \psi(\lambda) - 1\right]^2 \bar{n}^2 \equiv \bar{n} + \delta^2. \tag{14}$$

Obviously, for fixed λ there are no FB correlations and

$$\langle D_F^2 \rangle_\lambda = \tfrac{1}{2} \langle D^2 \rangle_\lambda = \tfrac{1}{2} \lambda \bar{n}. \tag{15}$$

Thus

$$D_F^2 = \overline{\langle D_F^2 \rangle_\lambda} + \tfrac{1}{4} \bar{n}^2 \left[\int \lambda^2 d\lambda \psi(\lambda) - 1\right] = \tfrac{1}{2} \bar{n} + \tfrac{1}{4} \delta^2 \tag{16}$$

and

$$b = \frac{\delta^2}{\delta^2 + 2\bar{n}} = \frac{D^2 - \bar{n}}{D^2 + \bar{n}}. \tag{17}$$

Assuming again independent emission of clusters and not single particles we obtain

$$D^2 = k_{\text{eff}}\bar{n}+\delta^2, \tag{18}$$

$$D_F^2 = \tfrac{1}{2}\, k_{\text{eff}}\bar{n}+\tfrac{1}{4}\,\delta^2, \tag{19}$$

and

$$b = \frac{\delta^2}{\delta^2+2k_{\text{eff}}\bar{n}} = \frac{D^2-k_{\text{eff}}\bar{n}}{D^2+k_{\text{eff}}\bar{n}}\,. \tag{20}$$

Thus the two models are equivalent. However, now we see that the model can be justified by the idea of superposing components only for positive correlations, $b > 0$. Indeed, it is not reasonable to assume random emission of clusters for fixed global multiplicity, if the negative correlations (resulting e.g. from energy-momentum conservation) make the global multiplicity distribution of clusters narrower than Poissonian. In fact, we expect that the model will work well only if $D^2 \gg k_{\text{eff}}\bar{n}$, since one shall apply energy--momentum conservation for each λ, which may modify significantly formulae (18) and (19) even at quite high energies. This suspicion is supported by data. Whereas at collider energy formula (20) agrees with data for $k_{\text{eff}} = 2.5$–3 (as used to describe short-range correlations), we obtain too low values even in ISR range, as shown in Fig. 1. On the other hand, the Monte Carlo calculations of MM with phase-space effects built in have described ISR data very well [5]. Thus in the following we consider a simple ansatz for such effects modifying formula (20). Nevertheless, we restrict ourselves to the ISR–collider energy range, as for lower energy our approximations are unreliable.

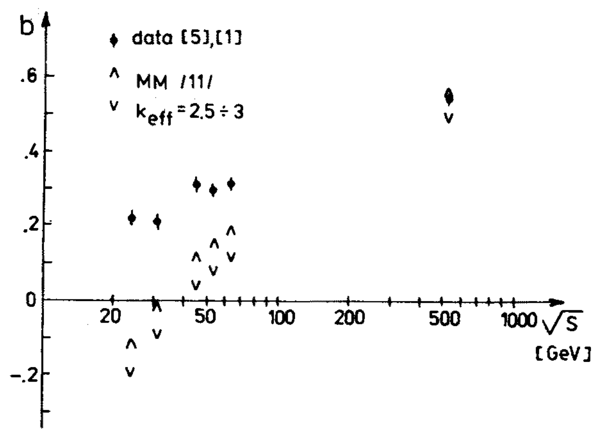

Fig. 1. Correlation parameter b in pp [5] and $\bar{p}$p [1] collisions at high energies (black points). Brackets show the limits of MM predictions (11) for $k_{\text{eff}} = 2.5$–3

3. Phase-space corrections to MM

The modifications of random emission resulting from the energy-momentum conservation are in general quite complicated [8] and it is customary to use Monte Carlo generation of events according to the cylindrical phase-space instead of analytical approximations.

However, for our purposes it is enough to note that for large $\bar{n}$ the dispersion of multiplicity distribution is reduced (relative to Poisson distribution) by an approximately constant term

$$D^2 = \bar{n} - c + O\left(\frac{1}{\bar{n}}\right). \tag{21}$$

Now, in MM we have assumed for fixed λ the Poisson shape for the cluster multiplicity distributions both globally and in a single hemisphere, which reflects the absence of FB correlations. It seems reasonable to assume that energy-momentum conservation does not introduce FB correlations, as the energy in each hemisphere may be distributed independently between different numbers of particles. Thus we can replace (15) by

$$\langle D_{\mathrm{F}}^2 \rangle_\lambda = \tfrac{1}{2} \langle D^2 \rangle_\lambda \simeq \tfrac{1}{2} \lambda \bar{n} - \frac{c_\lambda}{2} \tag{22}$$

and averaging over λ we obtain analogous result

$$\overline{\langle D_{\mathrm{F}}^2 \rangle_\lambda} = \tfrac{1}{2} \overline{\langle D^2 \rangle_\lambda} = \tfrac{1}{2} \bar{n} - \tfrac{1}{2} \bar{c}. \tag{23}$$

If this is assumed for cluster production, we get for hadrons

$$\overline{\langle D_{\mathrm{F}}^2 \rangle_\lambda} = \tfrac{1}{2} \overline{\langle D^2 \rangle_\lambda} = \tfrac{1}{2} k_{\mathrm{eff}} \bar{n} - \tfrac{1}{2} \bar{c} \bar{k}^2 \tag{24}$$

and the correlation parameter is given by

$$b = \frac{D^2 - k_{\mathrm{eff}} \bar{n} + c}{D^2 + k_{\mathrm{eff}} \bar{n} - c} \tag{25}$$

where

$$c = \bar{c} \bar{k}^2. \tag{26}$$

We treat c as a free parameter. Nevertheless, one may estimate that for standard phase-space parametrization $\bar{c}$ is of the order of few units, and since $\bar{k}$ is about 2, c is of the order of ten.

Obviously, in analogy to formula (20) we can expect that formula (25) will work only for high energies when b is definitely positive. We do not perform a detailed fit to the ISR and collider data, as the two experiments differ significantly in event selection and contain certainly different admixture of diffractive events (obviously not described by MM). However, we show in Fig. 2 the belt of predictions resulting from formula (25) with $k_{\mathrm{eff}} = 2.5$–3 and $c = 10$. Curves for e.g. $k_{\mathrm{eff}} = 2.5$, $c = 6$–10 and $k_{\mathrm{eff}} = 3$, $c = 10$–14 lie also inside this belt. We use here the phenomenological relation of Wróblewski [9]

$$D \simeq \frac{1}{\sqrt{3}} (\bar{n} - 1). \tag{27}$$

As we see, for this range of parameters MM describes the FB correlations quite well. As an example of a particular choice of parameters we show also points obtained for

$k_{eff} = 3$, $c = 13$ using measured values of D^2 and $\bar{n}$ (and not Wróblewski's relation), which agree with experimental values of b within errors. Let us note here that we are using the values of D^2 and $\bar{n}$ from Ref. [5] instead of more recent ones [10] to ensure the same choice of events used for determination of b and multiplicity distributions. We do not show the predictions of formula (25) for $\bar{n} < 8$, where the results become very sensitive to the choice of c and k_{eff} and where our simple ansatz for phase-space corrections in MM is no longer reliable.

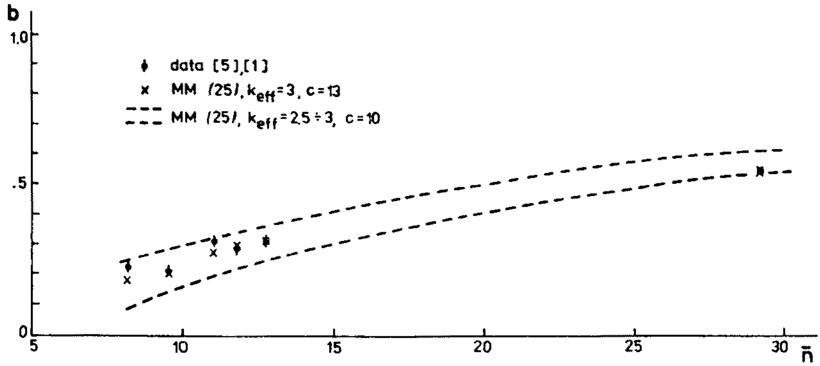

Fig. 2. Correlation parameter b in pp [5] and p$\bar{p}$ [1] collisions (black points) as a function of average multiplicity $\bar{n}$. Broken lines show the limits of MM predictions corrected for phase space (25) with $c = 10$, $k_{eff} = 2.5$–3 and crossed points correspond to the values of $c = 13$, $k_{eff} = 3$

4. *MM and lepton-induced multiple production*

Kühn and Schneider [6] have pointed out that FB correlations may help to answer the basic question concerning multiple production in the lh or e^+e^- collisions: are the two jets related to the q–$\bar{q}$ or q–qq separation fragmenting independently, or is there a colour tube or string connecting the original partons and fragmenting (uniformly?) into final hadrons?

The hadronization models currently used answer this question differently. In the Feynman-Field approach [11] independent fragmentation is assumed explicitly, and this idea persists in more modern models based on perturbative QCD [12]. The non-perturbative models [13] assume uniform particle creation within colour tube. The Lund model of hadronization [14], where string picture is assumed, seems closer to this approach. In general, however, the model predictions depend on many details and it is difficult to check by comparison with data which assumption works better. FB correlations provide us in principle with a unique opportunity to test this directly. Indeed, if the two jets fragment independently, the correlation parameter b should vanish at large $\bar{n}$. On the other hand, if we assume the "colour tube" picture and take into account the experimentally observed approximate KNO scaling, we expect that in this limit b approaches the value of 1 and that MM should work.

Kühn and Schneider [6] have elaborated such predictions basing on the non-perturbative models [12] and using MM analogous to the one described earlier [3, 4]. They do not present explicitly the obtained dependence of b vs $\bar{n}$, but using their approximate formula for b with $k_{eff} = 2.5$–3 and $\bar{n}/D = 2.6$–3.2 (quoted value is 2.6, which seems rather low) we find

$$b \simeq \frac{\bar{n}}{\bar{n}+c}, \tag{28}$$

where $c = 33$–60. Since at PETRA energies $\bar{n}$ (as measured recently by JADE collaboration [15]) varies from 8.4 to 13.6, we expect at highest energy $b = 0.18$–0.29, and in principle experiment should be able to distinguish such value easily from zero. Unfortunately, the multiplicity distribution used in [6] does not really correspond to the data; the value of parameter $\langle\lambda^2\rangle - 1$ which they use corresponds to the quoted $\bar{n}/D$ ratio only asymptotically, when

$$\left(\frac{D}{\bar{n}}\right)^2 = \frac{\overline{\langle D^2\rangle_\lambda}+\delta^2}{\bar{n}^2} \simeq \frac{\delta^2}{\bar{n}^2} = \langle\lambda^2\rangle - 1 \tag{29}$$

and at available energies this approximation is unreliable. In fact, using $k_{eff} = 2.5$ we find within PETRA energy range [15]

$$\overline{\langle D^2\rangle_\lambda} = k_{eff}\bar{n} > D^2. \tag{30}$$

Extrapolating the existing data one can expect the reversion of inequality (30) at $\bar{n}$ around 25. This means that formula (18) cannot describe data until much higher energies are achieved. Cluster multiplicity distribution is presently narrower than Poissonian and cannot be described as a superposition of Poisson distributions. This inapplicability of MM may be again attributed to the phase-space effects, which make the assumption of random emission at fixed λ unreliable, as in lower energy hadron data. Note that for the e^+e^- case we may expect from phase-space even stronger negative correlations than in hadron-hadron collisions at the same $\bar{n}$, since there are no leading particles, and consequently no distribution of inelasticity weakening the effect of energy conservation on the multiplicity distribution. In any case, if we use the original formula (20) instead of approximation (28) we see that MM cannot be used to describe PETRA data.

Let us check if using the ansatz of Section 3 to describe the phase-space effects we will be able to extend the applicability range of MM down to available energies. Using $k_{eff} = 2.5$, $c = 10$ and $\bar{n}/D = 3.2$ (as seen at highest PETRA energies) we find

$$b \simeq \frac{\bar{n}^2 - 25\bar{n} + 100}{\bar{n}^2 + 25\bar{n} - 100}. \tag{31}$$

Unfortunately, this expression is again negative, until $\bar{n}$ reaches the values about 20, expected at energies of the order of 100 GeV. Varying parameters within the limits compatible with other data we find the lower limit of $\bar{n}$ for which b is positive changing between 12 and 24. In any case, however, within the present energy range our ansatz is unreliable and we can neither support, nor disprove MM. Consequently, it seems that we have to wait till LEP energies to distinguish definitely between the two pictures of multiple production using FB correlation data (obviously, for lh processes the energies will be also too low till HERA).

To test MM one can also go beyond the crude approximation of our ansatz (21) and to perform Monte Carlo analysis of phase-space effects. However, such an analysis at PETRA energies is seriously influenced also by effects of various flavours of quarks, three-jet events, experimental biases and inefficiences. The only measurement of FB correlations in e^+e^- collisions performed at 34 GeV by TASSO [16] yielded the results incompatible with linear relation (1) and suggesting for "nearly linear" range of n_B the value of b around 0.2. However, these data are compatible with $b = 0$ when corrected foi acceptance, trigger conditions, and referred to definite flavour two-jet events. This shows that details of analysis produce effects larger than deviations from $b = 0$ predicted by MM, and the two pictures are unfortunately indistinguishable at available energies within the experimental and theoretical uncertainties. At higher energies this distinction should be more clear.

5. *Discussion and summary*

We have shown that MM allows for correct description of the FB correlations in the high energy hadron-hadron collisions even with very simple parametrization of phase--space effects. This suggests that MM can be used as the reliable model to be contrasted with the assumption of independent fragmentation for e^+e^- and lh collisions. Unfortunately, the energies available are too low to test MM there.

Nevertheless, we may wonder why such a primitive model works so well.

The answer is very simple: as far as we are investigating only FB correlation, MM is equivalent to many more elaborate models of multiple production. Indeed, the only relevant assumption is here the possibility to decompose multiple production into components, for which there are no long-range correlations (i.e. the multiplicity distribution of clusters is close to Poissonian and FB correlations are asymptotically negligible). In fact, at high energies when $D^2 \gg k_{\text{eff}}\bar{n}$, even quite significant deviations from Poisson distribution are irrelevant as long as their contributions to dispersion do not grow with energy. Such moderate assumption is present in the broad class of models, as the Lund model [14] or the dual parton model [17, 18], although the predictions of these models for more sophisticated experiments may differ significantly [18]. In any case, we do not regard as justified claims that all the FB correlations results follow just from the KNO scaling of multiplicity distributions [1]. One can construct as well independent fragmentation models with $b = 0$ and KNO scaling. The success of MM for hadronic collisions shows that the fluctuations of average density of particles in rapidity between events are here much stronger than the relative fluctuations in various intervals of rapidity (e.g. hemispheres). In the e^+e^- and lh collisions this effect may appear or not, and a test of this hypothesis is very interesting.

Finally, let us comment on the recently observed universality of the FB correlation data. Wróblewski [7] has compiled existing measurements of correlation parameter b for hadron-hadron collisions supplementing them with estimate for e^+e^- collisions based on formula (3). The results, when plotted versus $(\bar{n})^{-1/2}$ lie not far from a single straight line, suggesting the universal behaviour

$$b = 1 - \frac{c}{\sqrt{\bar{n}}}, \qquad c \simeq 2.4. \tag{32}$$

As we can see from formulae (20) or (25), MM predicts different asymptotic behaviour for hadron-hadron collisions (only integer powers of $\bar{n}$ occur). Nevertheless, for the ISR and collider data, where our MM calculations are reliable, the results of formulae (25) and (32) are practically indistinguishable, as shown in Fig. 3. The discrepancies at higher

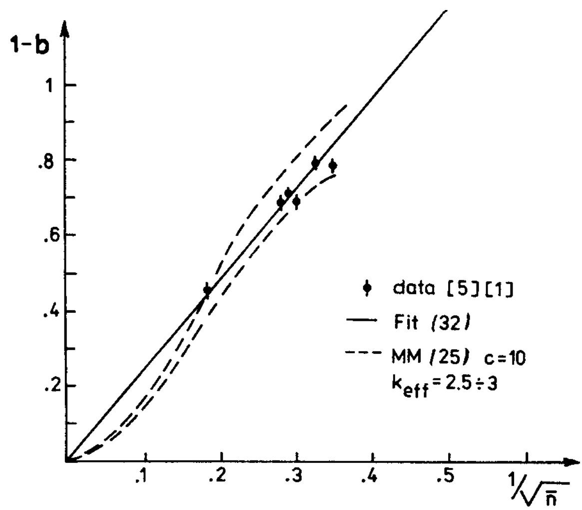

Fig. 3. Correlation parameter $1-b$ in pp [5] and $p\bar{p}$ [1] collisions (black points) as a function of $1/\bar{n}$. Solid straight line represents the universal fit [7] and broken lines show the limits of MM predictions corrected for phase space (25) with $c = 10$, $k_{\rm eff} = 2.5$–3

energies will be also hard to find. It would be perhaps interesting to test if Monte Carlo estimates of phase-space effects at lower energy can be also approximated by formula (32). The low energy νN data are subject to biases and certainly not accurate enough to draw any serious conclusions from their rough agreement with this formula. One can hope that new data (e.g. from EMC) will show if the universality of formula (32) is the real fact, or just a numerical accident for the limited set of inaccurate data. From Section 4 we could see that MM, if applicable at all for lh data, would predict the b versus $\bar{n}$ dependence different from that in hadronic collisions, as the D vs $\bar{n}$ dependence is not the same. For independent fragmentation models obviously b is asymptotically zero. Thus the universality would contradict practically all the existing models for multiple production in e^+e^- and lh collisions and it is an extremely interesting subject for further investigations.

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