BOOST-INVARIANT MOTION OF RELATIVISTIC PERFECT FLUID*

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Equations of motion of relativistic perfect fluid subject to Bjorken's boost-invariant conditions are analysed. General relations between the gradient of temperature and the shape of stream lines are derived. The case of pure transverse motion (vanishing radial velocity) is studied in some detail. It is shown that the stable solution exists only for a very restricted class of the equations of state of the fluid.

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1. Introduction

In this paper we study the equations of motion of the relativistic perfect fluid satisfying the Bjorken condition [1] of boost-invariance. It was argued in [1] that these equations describe the evolution of quark-gluon plasma possibly created in the central region of the collision of heavy ions at very high energies.

Some particular cases of such boost-invariant motion of relativistic fluid were already considered by several authors. Bjorken [1] derived the solution depending only on z and t and independent on x, y variables. Baym et al. [2] considered radial motion of the fluid with rotational symmetry around the collision (z) axis and found several solutions corresponding to Riemann initial conditions. In Ref. [3] solutions for static initial conditions were presented. Finally, in Ref. [4] the purely transverse motion (vanishing radial velocity) again with rotational symmetry around the collision axis was considered. It was proven that if equation of state is that of an ideal gas, no solution exists.

In this paper we continue investigations along the lines suggested in Refs. [1–4]. We study the rotationally-symmetric motions but do not restrict the fluid velocity otherwise. An interesting relation is found between the gradient of temperature and the shape of flux lines. Namely, for the temperature decreasing with increasing distance, far from

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the centre, the fluid expands along the radial lines. We also discuss the purely transverse motion and find a class of equations of state of the fluid which admit a stable solution. Some comments are given on the problem of general equations of motion in case of weak dependence on azimuthal angle.

Evolution of the perfect fluid is governed by the conservation laws for energy and momentum

$$\partial_\mu T^{\mu\nu} = 0 \tag{1.1}$$

with the stress tensor given by

$$T^{\mu\nu} = (\varepsilon+p)u^\mu u^\nu + pg^{\mu\nu}. \tag{1.2}$$

Here ε is the energy density, p the pressure, $g^{\mu\nu}$ the metric tensor, u^μ is the four-velocity

$$u^\mu = \gamma(1, \vec{v}), \quad \gamma = (1-v^2)^{-1/2}. \tag{1.3}$$

Taking the four-divergence of Eq. (1.2) we obtain

$$u^\nu[u^\mu\partial_\mu(\varepsilon+p)+(\varepsilon+p)\partial_\mu u^\mu]+(\varepsilon+p)u^\mu\partial_\mu u^\nu-\partial^\nu p = 0. \tag{1.4}$$

Multiplication by u_ν gives

$$u^\mu\partial_\mu\varepsilon+(\varepsilon+p)\partial_\mu u^\mu = 0 \tag{1.5}$$

thus (1.4) can be written as

$$u^\nu u^\mu\partial_\mu p-\partial^\nu p+(\varepsilon+p)u^\mu\partial_\mu u^\nu = 0. \tag{1.6}$$

The last equations are relations between ε, p and $\vec{v}$. It is convenient for further discussion to consider thermodynamic variables: entropy and temperature rather than ε and p. This change of variables can be performed using the relations (they are valid provided chemical potential vanishes — this will be assumed throughout all the paper)

$$d\varepsilon = Tds, \quad dp = sdT, \quad \varepsilon+p = Ts, \tag{1.7}$$

where s is the entropy density.

One obtains, after some manipulations

$$\partial_\mu(su^\mu) = 0, \tag{1.8}$$

$$u^\mu\partial_\mu(Tu^\nu) = \partial^\nu T. \tag{1.9}$$

Only 3 components of the last equations are independent, since $u^\mu u_\mu = 1$. Therefore we have 4 equations for 5 unknowns: v_x, v_y, v_z, T, s. This system should be supplemented by equation of state

$$s = f(T). \tag{1.10}$$

In the next Section we rewrite Eqs. (1.8) and (1.9) using the Bjorken condition

$$v_z = z/t \tag{1.11}$$

and the requirement that all other quantities depend only on $\tau = \sqrt{t^2 - z^2}$ and transverse coordinates. In that Section we consider relations between the gradient of temperature and the shape of stream lines. The case of purely transverse motion (vortex) is discussed in Section 3. Our conclusions are listed in the last Section. Finally, in the appendices details of calculations are given.

2. *General equations of motion of boost-invariant fluid*

Using the Bjorken ansatz (1.11) for v_z we are left with two-dimensional problem, in the plane perpendicular to the collision axis. If we represent the two-dimensional velocity vector $\vec{v}$ by its length $v \equiv |\vec{v}|$ and the angle α between $\vec{v}$ and the radial versor $\vec{e}_R$ (see Fig. 1) we have

$$v_R = v \cos \alpha, \quad v_T = v \sin \alpha, \tag{2.1}$$

where v_R and v_T are radial and transversal components of the velocity.

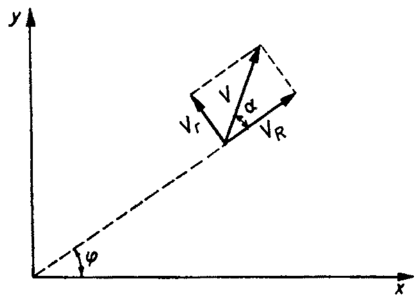

Fig. 1. Definition of kinematic variables

Without any loss of generality, we can restrict our considerations to $z = 0$, where $v_z = 0$ and $\tau = t$.

Using standard methods of differential geometry we then obtain from Eqs. (1.8) and (1.9) the following three conditions:

(a) Equation for entropy

$$\partial_t(rts\gamma) + \partial_r(rts\gamma v \cos \alpha) + \partial_\varphi(ts\gamma v \sin \alpha) = 0. \tag{2.2}$$

(b) Equations for temperature and velocity

$$\partial_t(rT\gamma v) + r \cos \alpha\, \partial_r(T\gamma) + \sin \alpha\, \partial_\varphi(T\gamma) = 0, \tag{2.3}$$

$$\sin \alpha\, \partial_r T - \frac{\cos \alpha}{r} \partial_\varphi T = T\gamma^2 v \left(\frac{v \sin \alpha}{r} + \frac{d\alpha}{dt} \right), \tag{2.4}$$

where the differential operator $\frac{d}{dt}$ is defined as

$$\frac{d}{dt} \equiv \partial_t + v \cos \alpha\, \partial_r + \frac{v \sin \alpha}{r} \partial_\varphi \tag{2.5}$$

and ∂_t, ∂_r, ∂_φ denote partial derivatives with respect to the corresponding variables. The operator (2.5) represents the change of any quantity in the co-moving frame, i.e. in the frame where the local velocity of the fluid vanishes.

In case of full rotational symmetry T, s, v and α do not depend on φ (small non-symmetric perturbations are discussed in Appendix 3). Consequently, Eqs. (2.2)–(2.4) reduce to

$$\partial_t(rts\gamma)+\partial_r(rts\gamma v\cos\alpha) = 0, \tag{2.6}$$

$$\partial_t(T\gamma v)+\cos\alpha\,\partial_r(T\gamma) = 0, \tag{2.7}$$

$$\sin\alpha\,\partial_r T = T\gamma^2 v\left(\frac{v\sin\alpha}{r}+\frac{d\alpha}{dt}\right). \tag{2.8}$$

For $\alpha = 0$ we recover the Eqs. of Ref. [2] and for $\alpha = \pi/2$ we recover the Eqs. of Ref. [4].

Eq. (2.8) has an interesting consequence. Considering the motion of a small fragment of the fluid we may treat φ and α as functions of t. Let us observe that the quantity

$$\frac{v\sin\alpha}{r} = \frac{v_T}{r} = \frac{d\varphi}{dt} \equiv \omega$$

is the angular velocity of the fragment.

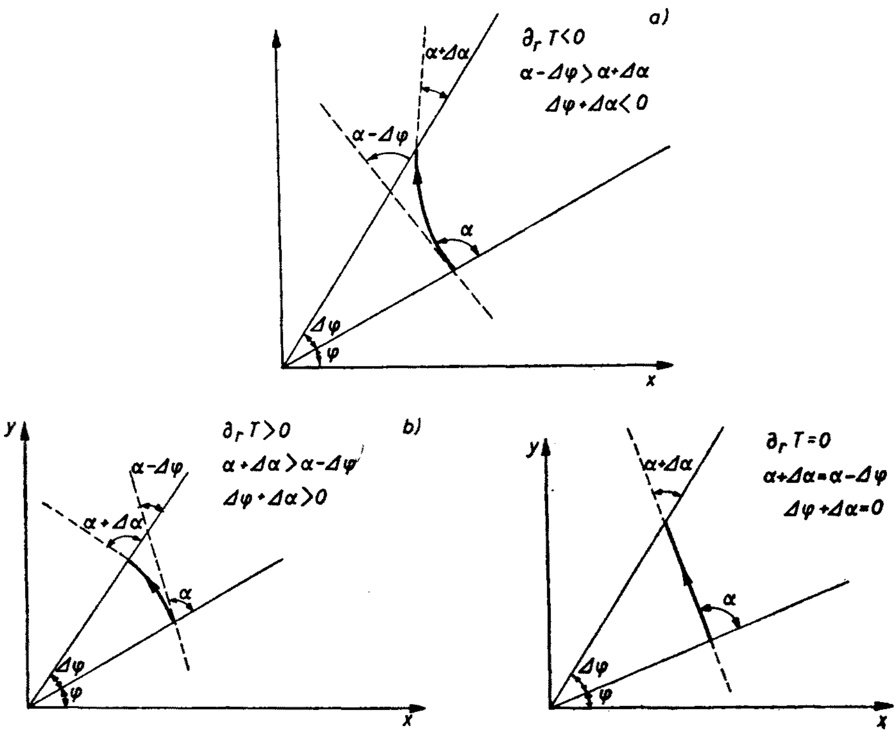

Fig. 2. The shape of the stream lines for (a) temperature decreasing with increasing distance from the centre; (b) temperature increasing with increasing distance from the centre; (c) constant temperature

Furthermore, one sees that the quantity

$$\frac{v \sin \alpha}{r} + \frac{d\alpha}{dt} = \frac{d}{dt}(\varphi + \alpha + \text{const})$$

is the rotational speed of the velocity vector, i.e. the rate of change of the angle between the velocity vector and a certain radial line. Consequently, the behavior of the angle $\alpha + \varphi + \text{const}$ is closely related to the sign of $\partial_r T$. To be specific, let us assume that $\omega > 0$, i.e. $\alpha \in (0, \pi)$. Then, in the co-moving frame:
for $\partial_r T < 0$ – this angle decreases with increasing t;
for $\partial_r T > 0$ – this angle increases with decreasing t;
for $\partial_r T = 0$ – this angle is constant and the stream line is a straight one.

These relations are illustrated in Fig. 2. One sees that for the most interesting case of temperature decreasing with increasing r ($\partial_r T < 0$), the rotational speed is negative. In the co-moving frame α must be a monotonic function of time, decreasing with increasing t, to balance the increase of φ ($\omega > 0$).

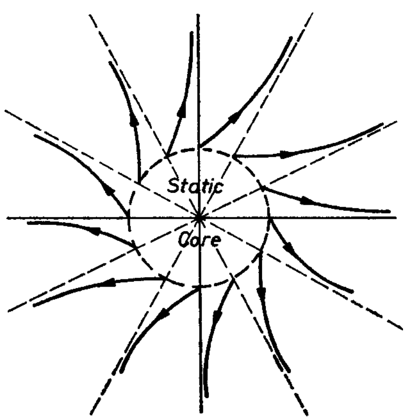

Fig. 3. General view of the stream lines and central static region

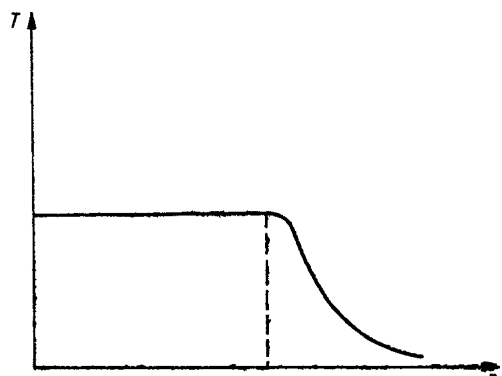

Fig. 4. Distribution of temperature corresponding to picture in Fig. 3

Since α is positive, it has a lower boundary and the increase of φ must be limited and far away from the centre it makes the radial motion predominating.

For such a shape of motion there is an area the stream lines cannot enter: the interior of a certain circle (see Fig. 3). Hence, we have a static hot core surrounded by the expanding fluid. However, since $v = 0$, Eqs. (2.7) and (2.8) imply $\partial_r T = 0$ inside the core. Thus, the temperature as a function of r must be given by a bell-shaped function (see Fig. 4), or temperature increases with increasing r somewhere.

3. Boost-invariant vortex

For purely transversal motion ($v_R = 0$, $\alpha = \pi/2$) Eqs. (2.6)–(2.8) simplify into

$$\partial_t(ts\gamma) = 0, \tag{3.1}$$

$$\partial_t(T\gamma v) = 0, \tag{3.2}$$

$$\partial_r T = \frac{T\gamma^2 v^2}{r}. \tag{3.3}$$

These Eqs were studied in Ref. [4] where it was shown that their solutions are inconsistent with the equation of state for perfect fluid, i.e.

$$s = \text{const} \cdot T^3. \tag{3.4}$$

In Appendix 1 we show that the only equations of state consistent with Eqs. (3.1)–(3.3) are of the form

$$s = \text{const} \cdot T \cdot \exp(cT^2), \tag{3.5}$$

and

$$s = \text{const} \cdot T \cdot \frac{\exp(cT^2)}{\sqrt{T^2+k^2}}, \tag{3.6}$$

where c and k are constants.

Let us first discuss the solution (3.5). As derived in Appendix 1, temperature and velocity of the fluid in this case are given by

$$T = k\sqrt{\frac{1}{r_0^2(t)} - \frac{1}{r^2}}, \tag{3.7}$$

and

$$v = \frac{r_0(t)}{r}. \tag{3.8}$$

$r_0(t)$ is an increasing positive function of time. It is plotted in Fig. 6 for several values of parameters c and k.

As seen from Eq. (3.7) for $r < r_0(t)$, T^2 becomes negative and thus the solution does not have a physical meaning. We also observe that at $r = r_0(t)$, $v = 1$. For $r > r_0(t)$ the solutions (3.7) and (3.8) are plotted in Fig. 5.

As seen from Fig. 5 the temperature is an increasing function of the distance from the centre, starting from $T = 0$ at $r = r_0(t)$. Thus this solution describes an expanding infinite

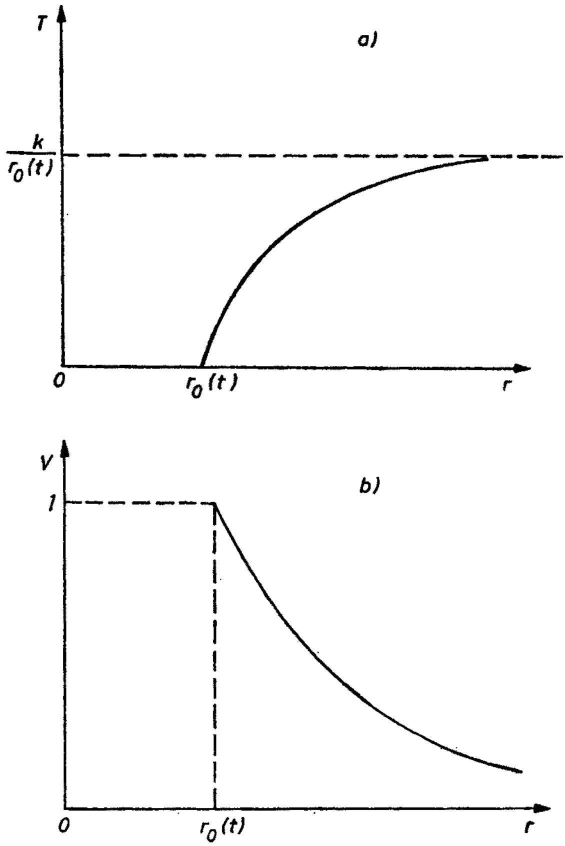

Fig. 5. The stable solution for the vortex: (a) temperature; (b) velocity

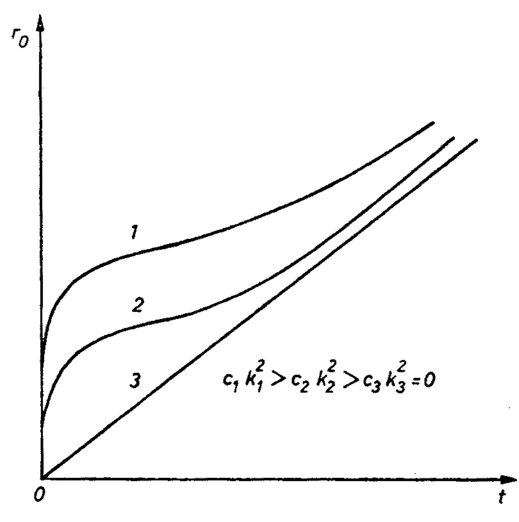

Fig. 6. The radius of a circle of indefinable temperature as a function of time

ring of plasma with temperature increasing from inner to outer region. The ring cools down with increasing time, the limiting temperature at $r \to \infty$ being $k/r_0(t)$. It is also worth to notice that this solution satisfies the standard condition for nonrelativistic vortices, namely that its circulation is independent of r. However, it does depend on time. This is easily seen from Eq. (3.8). It is proven in Appendix 2 that this solution is stable against small perturbations introducing radial component of velocity.

The corresponding results for the solution (3.6) are

$$T = k\sqrt{2\ln\frac{r}{bt^m}}, \tag{3.9}$$

$$v = \frac{1}{\sqrt{1+2\ln\dfrac{r}{bt^m}}}, \tag{3.10}$$

where $b = \text{const}$ and $m = \dfrac{1}{2ck^2}$.

It is shown in Appendix 2 that this solution is unstable against small perturbations. We thus shall not discuss it any further.

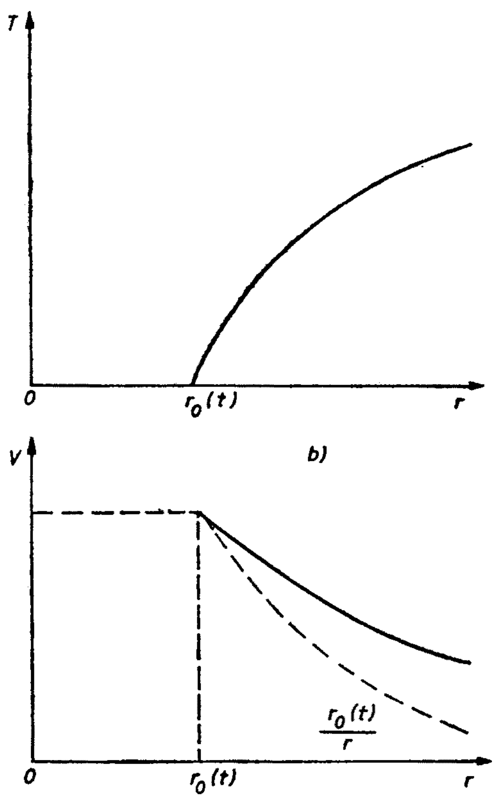

Fig. 7. The unstable solution for the vortex: (a) temperature; (b) velocity

4. Conclusions

We have investigated solutions of the hydrodynamical equations describing space-time evolution of the quark-gluon plasma with cylindrical symmetry along the collision axis.

The general relations between the gradient of temperature and the shape of stream lines show that temperature decreases with distance from the centre only when far from collison region the fluid expands outside along the radial lines.

Exact solutions of pure transversal motion (vortex) are given. It is interesting that in this case the form of the equation of state of the fluid is determined. This solution describes an expanding ring of rotating plasma. The temperature increases with increasing distance from the center and decreases with increasing time. It seems, however, unlikely that such a solution can be realizable in nature.

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APPENDIX 1

Symmetric vortex

In the case of no radial motion the equations reduce to those shown in Section 3. Equations (3.1) and (3.2) can be written

$$ts\gamma = A(r), \tag{A1.1}$$

$$T\gamma v = B(r), \tag{A1.2}$$

where $A(r)$ and $B(r)$ are certain functions of r.

We will show that the system of equations (A1.1), (A1.2), (3.3) and (1.10) is consistent only if special conditions for $A(r)$, $B(r)$, $f(T)$ are satisfied. Now we will find these relations.

Equations (A1.2) and (3.3) give

$$T^2 = X(t)+Y(r), \tag{A1.3}$$

where $Y(r) = 2\int B^2(r)\frac{dr}{r}$ and $X(t)$ is certain function of time. Let us assume that the functions $X(t)$ and $Y(r)$ can be inverted. Seeing that we can regard A and B as functions of Y and time as a function of X. We will use either X and Y or $X+Y$ and Y as independent variables in further considerations. Then, we transform Eq. (A1.1) and obtain

$$[X+Y+B^2(Y)]t^2(X)f^2(\sqrt{X+Y}) = (X+Y)A^2(Y). \tag{A1.4}$$

We differentiate the above equation with respect to Y while $X+Y$ is constant and then repeat twice differentiation with respect to X while Y is constant. After these manipulations the following equation is obtained

$$[X+Y+B^2(Y)]\phi'''(X)+2\phi''(X) = 0, \tag{A1.5}$$

where we introduced $\phi(X) = \ln t(X)$ and prim denotes the derivative with respect to X. The variable Y enters only in the expression

$$Y+B^2(Y),$$

thus $Y+B^2(Y)$ must be constant or $\phi(X)$ is a linear function. First we assume

$$Y+B^2(Y) = a = \text{const.} \tag{A1.6}$$

This condition leads to the form of $B(r)$ and $Y(r)$.

$$B(r) = \frac{k}{r}, \quad Y(r) = a - \frac{k^2}{r^2} \tag{A1.7}$$

(k is an arbitrary non-negative constant).

Substituting (A1.7) into Eq. (A1.4) we obtain (after adjustment of the constants of integration) the exact form of the relation between entropy and temperature, i.e. the equation of state

$$s = \text{const} \cdot T \exp(cT^2) \tag{A1.8}$$

(c is non-negative constant). We obtain also the implicit equation for $X(t)$

$$t = \text{const} \frac{\exp(-cX(t))}{\sqrt{X(t)+a}}. \tag{A1.9}$$

One observes that $X(t)+a$ is a monotonic function of time, exploding for time tending to zero and vanishing for large times like $1/t^2$.

Since the function $Y(r)$ becomes infinitely negative at $r \to 0$ the expression (A1.3) for T^2 is negative for r below some critical value $r_0(t)$. It means that the solution has no physical meaning for $r \leqslant r_0$. The radius $r_0(t)$ is given by

$$r_0(t) = \frac{k}{\sqrt{X(t)+a}}, \tag{A1.10}$$

$$\ln t = \ln r_0 - \frac{ck^2}{r_0^2} + \text{const.} \tag{A1.11}$$

The case of linear form of function $\phi(X)$ remains to be studied. One gets the solution by the method analogous to that used before. The corresponding equation of state is found to be

$$s = \text{const} \frac{T}{\sqrt{T^2+k^2}} \exp(cT^2), \tag{A1.12}$$

where c and k are positive constants.

It may be verified easily that function $B(r)$ is constant. The next appendix contains the argument which shows that $B(r)$ given by a constant leads to instability, so we will not discuss this case anymore. However, it is interesting to note that here also exists an expanding region of indefinable temperature ($T^2 < 0$).

APPENDIX 2

Stability of the solutions for a symmetric vortex

Let us assume that $\vec{v}$, at the plane $z = 0$, has transversal and radial components

$$\vec{v} = v_T\vec{e}_T + v_R\vec{e}_R, \tag{A2.1}$$

where

$$v_R \ll v_T. \tag{A2.2}$$

Because of cylindrical symmetry v_T, v_R are functions of r and t. Substituting (A2.1) into Eqs. (1.8)–(1.9) we obtain

$$\gamma\partial_t(T\gamma v_T) + \gamma v_R \partial_r(T\gamma v_T) + T\gamma^2 \frac{v_R v_T}{r} = 0, \tag{A2.3}$$

$$\gamma\partial_t(T\gamma v_R) + \gamma v_R \partial_r(T\gamma v_R) - T\gamma^2 \frac{v_T^2}{r} + \partial_r T = 0, \tag{A2.4}$$

$$\partial_t(s\gamma t) + \frac{1}{r}\partial_r(rs\gamma v_R) = 0. \tag{A2.5}$$

γ can be expanded in powers of v_R.

$$\gamma(v) = \gamma(v_T) + \gamma^3(v_T)\cdot\frac{v_R^2}{2} + \ldots, \tag{A2.6}$$

where

$$v = |\vec{v}| = \sqrt{v_T^2 + v_R^2}. \tag{A2.7}$$

Because $v_R \ll 1$ we neglect all the terms of higher than first order in v_R. This way we get equations of similar form to Eqs. (A2.3)–(A2.5), but $\gamma(v)$ in all expressions is replaced by $\gamma(v_T)$. In the next equations γ always means $\gamma(v_T)$.

$$\gamma\partial_t(T\gamma v_T) + \gamma v_R \partial_r(T\gamma v_T) + T\gamma^2 \frac{v_R v_T}{r} = 0, \tag{A2.8}$$

$$\gamma\partial_t(T\gamma v_R) - \frac{T\gamma^2 v_T^2}{r} + \partial_r T = 0, \tag{A2.9}$$

$$\partial_t(s\gamma t) + \frac{1}{r}\partial_r(rs\gamma v_R) = 0. \tag{A2.10}$$

Assuming that v_T is a solution of the above equations in the case $v_R = 0$, we obtain

$$\partial_r(T\gamma v_T) + \frac{T\gamma v_T}{r} = 0, \tag{A2.11}$$

$$\partial_t(T\gamma v_R) = 0, \tag{A2.12}$$

$$\partial_r(rs\gamma v_R) = 0. \tag{A2.13}$$

In Appendix 1 we have shown that in the case of a symmetric vortex two possibilities exist for function $B(r) = T\gamma v_T$:

(a) $T\gamma v_T = \frac{k}{r}$,

(b) $T\gamma v_T = \text{const.}$

Eq. (A2.11) shows that only the case (a) leads to stability.

Taking into consideration (A2.11) we obtain from (A2.12)

$$v_R = \alpha(r) v_T, \tag{A2.14}$$

and from (A2.13)

$$\partial_r \left[\frac{s}{T} \alpha(r) \right] = 0, \tag{A2.15}$$

$\alpha(r)$ is arbitrary function of r.

Identity (a) implies an equation of state in the form

$$s = \text{const} \cdot T \exp(cT^2), \tag{A2.16}$$

where $T^2 = X(t) + a - \frac{k^2}{r^2}$. The solution of Eq. (A2.15) is $\alpha(r) = \alpha_0 \exp\left(\frac{ck^2}{r^2}\right)$. $\alpha(r)$ should be infinitesimal, therefore the solution is stable for all $r > 0$ only when $c = 0$ and $\alpha_0 \ll 1$.

APPENDIX 3

Non-azimuthally-symmetric perturbation

The general equations of hydrodynamics – Eqs. (1.8) and (1.9) (with the condition of Bjorken) can be written in a form alternative to that of Eqs. (2.2)–(2.4), shown in Section 2. Taking $\vec{v}$ as a pair of its components v_R and v_T one obtains:

(a) Equation for entropy

$$(1-v^2)\left(\frac{d\sigma}{dt} + \frac{v_R}{r} + \partial_r v_R + \frac{\partial_\varphi v_T}{r} + \frac{1}{t}\right) + v\frac{dv}{dt} = 0, \tag{A3.1}$$

where $\sigma = \ln s$, $v = |\vec{v}|$.

(b) Equations for temperature and velocity

$$(1-v^2)(v_R \partial_t \theta + \partial_r \theta) + \frac{dv_R}{dt} - \frac{v_T^2}{r} = 0, \tag{A3.2}$$

$$(1-v^2)\left(v_T \partial_t \theta + \frac{1}{r}\partial_\varphi \theta\right) + \frac{dv_T}{dt} + \frac{v_R v_T}{r} = 0, \tag{A3.3}$$

where $\theta = \ln T$. The operator $\dfrac{d}{dt}$ is defined by

$$\frac{d}{dt} \equiv \partial_t + v_R \partial_r + \frac{v_T}{r} \partial_\varphi. \tag{A3.4}$$

We assume the equation of state in the form

$$\sigma = \Psi(\theta). \tag{A3.5}$$

It is much more easy to obtain a solution when the unknowns do not depend on the azimuthal angle φ. However, if one knows such a solution, one can find an approximate result for the case with the functions slightly deformed. (The exact solution must be stable.) We will put all the dependence on the angle φ into these small perturbations. So, we assume that the unknowns depend on φ but it is a weak dependence. The quantities should be periodical functions of the angle (period $= 2\pi$) and we can represent them with their Fourier series:

$$\theta(r, \varphi, t) = \sum_n \theta^{(n)}(r, t) e^{in\varphi}, \quad \sigma(r, \varphi, t) = \sum_n \sigma^{(n)}(r, t) e^{in\varphi},$$
$$v(r, \varphi, t) = \sum_n v^{(n)}(r, t) e^{in\varphi}, \quad v_a(r, \varphi, t) = \sum_n v_a^{(n)}(r, t) e^{in\varphi}, \tag{A3.6}$$

where $a = \mathrm{R}, \mathrm{T}$ and $n = 0, \pm 1, \pm 2, \ldots$

We assume that all the coefficients with their indices different from zero are infinitesimal and we may neglect their squares and products. Some coefficients are related. One observes that

$$v_T^{(0)} v_T^{(k)} + v_R^{(0)} v_R^{(k)} = v^{(0)} v^{(k)}, \quad k = 0, \pm 1, \pm 2, \ldots \tag{A3.7}$$

since the Fourier expansion is unique and $(v_R)^2 + (v_T)^2 = (v)^2$. We can also write the expansion of $\sigma = \Psi(\theta)$

$$\sum_n \sigma^{(n)} e^{in\varphi} = \sigma = \Psi(\theta) = \Psi(\theta^{(0)}) + \Psi'(\theta^{(0)}) \sum_{n \neq 0} \theta^{(n)} e^{in\varphi}. \tag{A3.8}$$

Consequently, $\sigma^{(n)}$ and $\theta^{(n)}$ must satisfy the following conditions

$$\sigma^{(0)} = \Psi(\theta^{(0)})$$
$$\sigma^{(n)} = \Psi'(\theta^{(0)}) \theta^{(n)}, \quad n \neq 0. \tag{A3.9}$$

For $n = 0$ we obtain equations equivalent to those obtained in case of rotational symmetry. For each $n \neq 0$ we have three linear equations for $\theta^{(n)}, \sigma^{(n)}, v_R^{(n)}, v_T^{(n)}$ with two independent variables: r and t. They are

$$[1-(v^{(0)})^2] v_R^{(0)} \partial_t \theta^{(n)} + [1-(v^{(0)})^2] \partial_r \theta^{(n)} + \partial_t v_R^{(n)} + v_R^{(0)} \partial_r v_R^{(n)}$$
$$+ \left\{ [1+(v_T^{(0)})^2 - 3(v_R^{(0)})^2] \partial_t \theta^{(0)} - 2 v_R^{(0)} \partial_r \theta^{(0)} + \partial_r v_R^{(0)} + \frac{in v_T^{(0)}}{r} \right\}$$
$$\times v_R^{(n)} - 2 v_T^{(0)} \left(v_R^{(0)} \partial_t \theta^{(0)} + \partial_r \theta^{(0)} + \frac{1}{r} \right) v_T^{(n)} = 0, \tag{A3.10}$$

$$[1-(v^{(0)})^2]v_{\rm T}^{(0)}\partial_t\theta^{(n)}+\partial_t v_{\rm T}^{(n)}+v_{\rm R}^{(0)}\partial_r v_{\rm T}^{(n)}$$

$$+\left\{[1-(v_{\rm R}^{(0)})^2-3(v_{\rm T}^{(0)})^2]\partial_t\theta^{(0)}+\frac{v_{\rm R}^{(0)}}{r}+\frac{inv_{\rm T}^{(0)}}{r}\right\}v_{\rm T}^{(n)}+[1-(v^{(0)})^2]\frac{in\theta^{(n)}}{r}$$

$$+\left[v_{\rm T}^{(0)}\left(\frac{1}{r}-2v_{\rm R}^{(0)}\partial_t\theta^{(0)}\right)+\partial_r v_{\rm T}^{(0)}\right]v_{\rm R}^{(n)}=0, \qquad \text{(A3.11)}$$

$$[1-(v^{(0)})^2]\left(\partial_t\sigma^{(n)}+v_{\rm R}^{(0)}\partial_r\sigma^{(n)}+\frac{inv_{\rm T}^{(0)}}{r}\sigma^{(n)}\right)+v_{\rm R}^{(0)}\partial_t v_{\rm R}^{(n)}$$

$$+[1-(v_{\rm T}^{(0)})^2]\partial_r v_{\rm R}^{(n)}+v_{\rm T}^{(0)}(\partial_t v_{\rm T}^{(n)}+v_{\rm R}^{(0)}\partial_r v_{\rm T}^{(n)})$$

$$+\left\{[1-(v^{(0)})^2]\left(\partial_r\sigma^{(0)}+\frac{1}{r}\right)-2v_{\rm R}^{(0)}\mathfrak{M}_0+\partial_t v_{\rm R}^{(0)}\right.$$

$$\left.+\frac{inv_{\rm R}^{(0)}v_{\rm T}^{(0)}}{r}+v_{\rm R}^{(0)}\partial_r v_{\rm R}^{(0)}+v^{(0)}\partial_r v^{(0)}\right\}v_{\rm R}^{(n)}$$

$$+\left\{[1-(v_{\rm R}^{(0)})^2]\frac{in}{r}-2v_{\rm T}^{(0)}\mathfrak{M}_0+\partial_t v_{\rm T}^{(0)}+v_{\rm R}^{(0)}\partial_r v_{\rm T}^{(0)}\right\}v_{\rm T}^{(n)}=0, \qquad \text{(A3.12)}$$

where

$$\mathfrak{M}_0=\partial_t\sigma^{(0)}+v_{\rm R}^{(0)}\partial_r\sigma^{(0)}+\frac{v_{\rm R}^{(0)}}{r}+\partial_r v_{\rm R}^{(0)}+\frac{1}{t}.$$

These equations are complicated, but they may simplify in some specific cases. E. g., for a vortex with small radial and transversal perturbations we may neglect all the terms multiplied by $v_{\rm R}^{(0)}$ in the above system and substitute temperature, velocity and entropy of the symmetric pure vortex (see Appendix 1), for $\theta^{(0)}$, $v_{\rm T}^{(0)}$, $\sigma^{(0)}$. Hence, we have three linear equations with known coefficients and some relations (A3.9) between the unknowns.

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