ON THE DISTRIBUTION OF THE FLUCTUATING REACTION CROSS SECTION

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Exact formula for the statistical distribution function of the fluctuating cross section is presented. Comparison with the results obtained from the approximate formula shows that such a formula can be used for calculation of the cumulative distribution function in the range of its values from 0.01 to 0.99.

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Recently a great increase of interest in molecular resonances in heavy ion collision has been observed [1–3]. Usually such resonances are expected in the region of nuclear excitation where a strong fluctuating statistical component of the reaction cross section is also present. The aim of the analysis of the experimental data in a search for resonances consists in the first line in checking to what extent the observed structures in the excitation curves are explanable in terms of the statistical model. In the next step of analysis the possible resonances should be fished out from the fluctuating background. These tasks have given rise to a revival of interest in the methods of statistical analysis and their cricital examination [4, 5].

Rudimental for the construction of many test functions used in a such analysis (e.g. deviation function, cross correlation function) and for establishing their distributions is the knowledge of the distribution function of the cross section which follows from the statistical model of the reaction. An approximate formula is usually applied for this purpose [6]. A problem arises to what extent such an approximate formula is consistent with the exact one. The aim of the present work was to derive the accurate statistical distribution function

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of the cross section and to compare its predictions with those calculated with the approximate one.

The differential cross section for reaction to a specific final state is expressed as a sum of the partial cross sections corresponding to the different substates $\beta = (\mu, \mu', M, M')$ of nuclei participating in the reaction

$$\sigma = \sum_{\beta=1}^{N\beta} \sigma_\beta, \tag{1}$$

where N_β, the maximum number of independent partial cross sections is related to the number of magnetic substates in the entrance and exit channels of the reaction. In the presence of the direct reaction contribution the averaged cross section could be split into two incoherent parts corresponding to the two different reaction mechanisms:

$$\langle\sigma\rangle = \sum_\beta \langle\sigma_\beta\rangle = \sum_\beta (\sigma_\beta^{\mathrm{d}} + \langle\sigma_\beta\rangle^{\mathrm{CN}}) = \sigma^{\mathrm{d}} + \langle\sigma\rangle^{\mathrm{CN}}. \tag{2}$$

According to the statistical reaction model [7] the probability distribution function for the partial cross section normalized by its average value

$$y_\beta = \sigma_\beta / \langle\sigma_\beta\rangle$$

is given by the formula:

$$f_{y_\beta{}^{\mathrm{d}}}(y_\beta) = \exp\left[-(y_\beta + y_\beta^{\mathrm{d}})/(1-y_\beta^{\mathrm{d}})\right] J_0\left[2i\sqrt{y_\beta y_\beta^{\mathrm{d}}}/(1-y_\beta^{\mathrm{d}})\right]/(1-y_\beta^{\mathrm{d}}), \tag{3}$$

where $y_\beta^{\mathrm{d}} = \sigma_\beta^{\mathrm{d}}/\langle\sigma_\beta\rangle$ denotes the relative direct reaction contribution to the $\langle\sigma_\beta\rangle$ and $J_0(iz)$ is the cylindrical Bessel function of order zero of the imaginary argument iz.

Introducing a notation

$$p_\beta = \langle\sigma_\beta\rangle^{\mathrm{CN}} / \langle\sigma_1\rangle^{\mathrm{CN}},$$

where $\langle\sigma_1\rangle^{\mathrm{CN}}$ is a selected value of the average partial cross section we define

$$y \overset{\mathrm{df}}{=} \sigma/\langle\sigma\rangle = \sum_\beta \frac{p_\beta/(1-y_\beta^{\mathrm{d}})}{\sum_i p_i/(1-y_i^{\mathrm{d}})} \times \frac{\sigma_\beta}{\langle\sigma_\beta\rangle},$$

$$y_{\mathrm{d}} \overset{\mathrm{df}}{=} \sigma^{\mathrm{d}}/\langle\sigma\rangle = \frac{\sum_\beta p_\beta y_\beta^{\mathrm{d}}/(1-y_\beta^{\mathrm{d}})}{\sum_\beta p_\beta/(1-y_\beta^{\mathrm{d}})},$$

and

$$N_{\mathrm{eff}} \overset{\mathrm{df}}{=} \frac{(\sum_\beta \langle\sigma_\beta\rangle^{\mathrm{CN}})^2}{\sum_\beta (\langle\sigma_\beta\rangle^{\mathrm{CN}})^2} = \frac{(\sum_\beta p_\beta)^2}{\sum_\beta p_\beta^2}.$$

As the variable y is a linear combination of the statistically independent random variables y_β its characteristic function is equal

$$\Phi_y(t) = \prod_\beta \exp\left(-\frac{y_\beta^d}{1-y_\beta^d}\right) \exp\left[\frac{y_\beta^d}{(1-y_\beta^d)(1-ip_\beta t/A)}\right] 1/(1-ip_\beta t/A), \tag{4}$$

where

$$A = \sum_i p_i/(1-y_i^d).$$

The exact distribution function of y could be obtained as usual as the Fourier transform of the characteristic function:

$$f(y) = (2\pi)^{-1} \int_{-\infty}^{\infty} \exp(-ity)\Phi_y(t)dt. \tag{5}$$

In two particular cases the evaluation of the integral in formula (5) leads to the closed form analytical expressions:

1. When all the partial cross sections $\langle\sigma_\beta\rangle^{CN}$ have the equal average values and thus

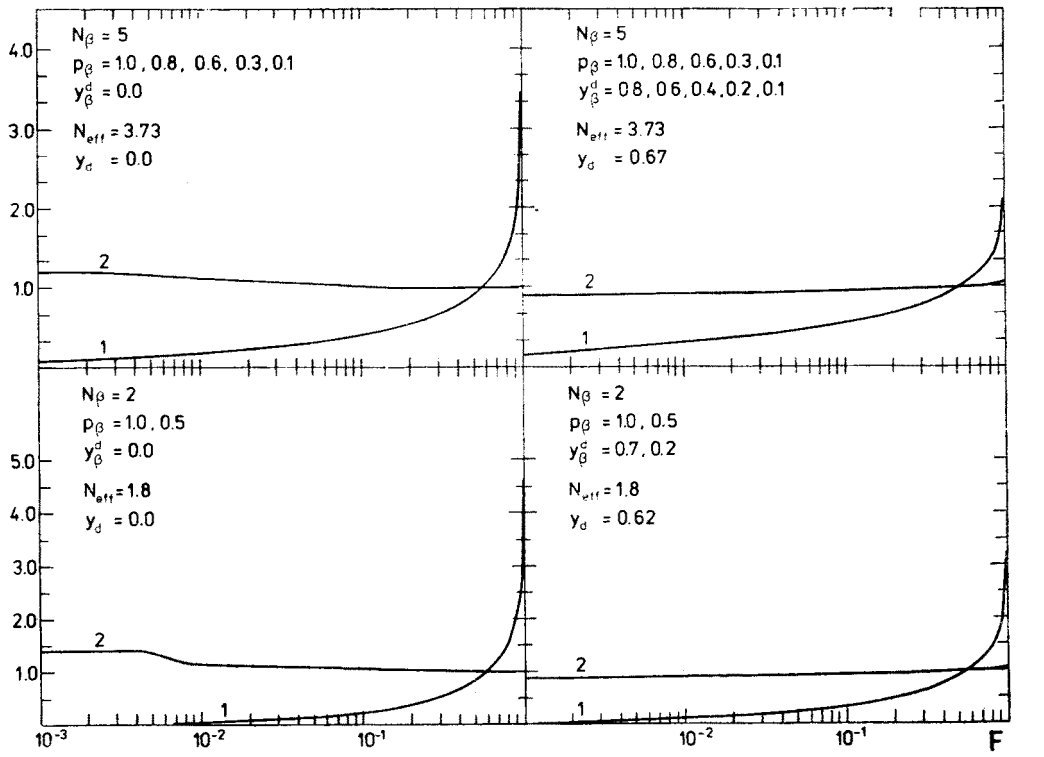

Fig. 1. Comparison of the exact and approximate distributions of the fluctuating reaction cross section. The abscisa are the values of the cumulative distribution function while the ordinates give the fractiles of the approximate distribution (curve *1*) or the ratio of fractiles for the exact and approximate distributions (curve *2*)

$p_\beta = 1$ for all β; then $N_{\text{eff}} = N_\beta$ and

$$f(y) = A^{N_{\text{eff}}} \exp\left[-(y+y_d)A\right] y^{N_{\text{eff}}-1} J_{N_{\text{eff}}-1}(2iA\sqrt{yy_d}) / \left[(iA\sqrt{yy_d})^{N_{\text{eff}}-1}\right]. \quad (6)$$

2. Without contribution from the direct reaction processes $y_d = 0$ and

$$f(y) = \frac{\sum_\beta p_\beta}{\prod_\beta p_\beta} \sum_k \frac{\exp\left(-y\sum_i p_i/p_k\right)}{\prod_{l\neq k}(1/p_l - 1/p_k)}. \quad (7)$$

The cumulative distribution function has than the form:

$$F(y) = \int_0^y f(x)dx = \left(\prod_i p_i\right)^{-1} \sum_k \frac{p_k\left[1-\exp\left(-y\sum_i p_i/p_k\right)\right]}{\prod_{l\neq k}(1/p_l - 1/p_k)}. \quad (8)$$

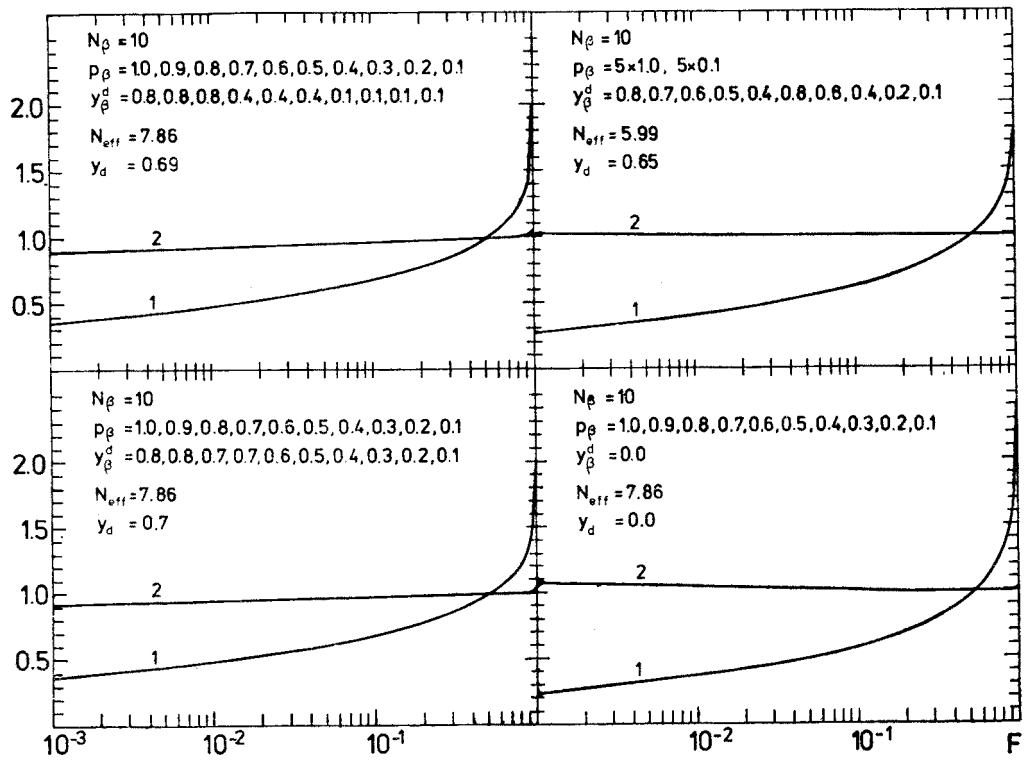

Fig. 2. The same as in Fig. 1

In general case however it is necessary to determine the density and cumulative distribution functions by means of the numerical integration of (5).

It should be pointed out however that the exact expressions for the distribution function developed here are not suitable for the practical calculations not only due to the rather complicated form but also due to the lack of the a priori information concerning

the values of parameters p_β and y_β^d appearing in the formula. Thus for the practical application an approximate formula was developed [6] and is in common use:

$$f_{y_d}^{N_{eff}}(y) = \left(\frac{N_{eff}}{1-y_d}\right)^{N_{eff}} y^{N_{eff}-1} \exp\left(-N_{eff}\frac{y+y_d}{1-y_d}\right) \times \frac{J_{N_{eff}-1}[2iN_{eff}\sqrt{yy_d}/(1-y_d)]}{[iN_{eff}\sqrt{yy_d}/(1-y_d)]^{N_{eff}-1}}. \tag{9}$$

In cases when the contributions from the direct reaction processes as well as the average cross sections for the compound nucleus reactions are equal for all magnetic substates (i.e. $y_\beta^d = y_{\beta'}^d$ and $p_\beta = 1$) the approximate formula leads to the results identical with those obtained from the exact expression with $N_{eff} = N_\beta$. This could be considered as some justification of the approximate procedure. In that case for the analysis of experimental

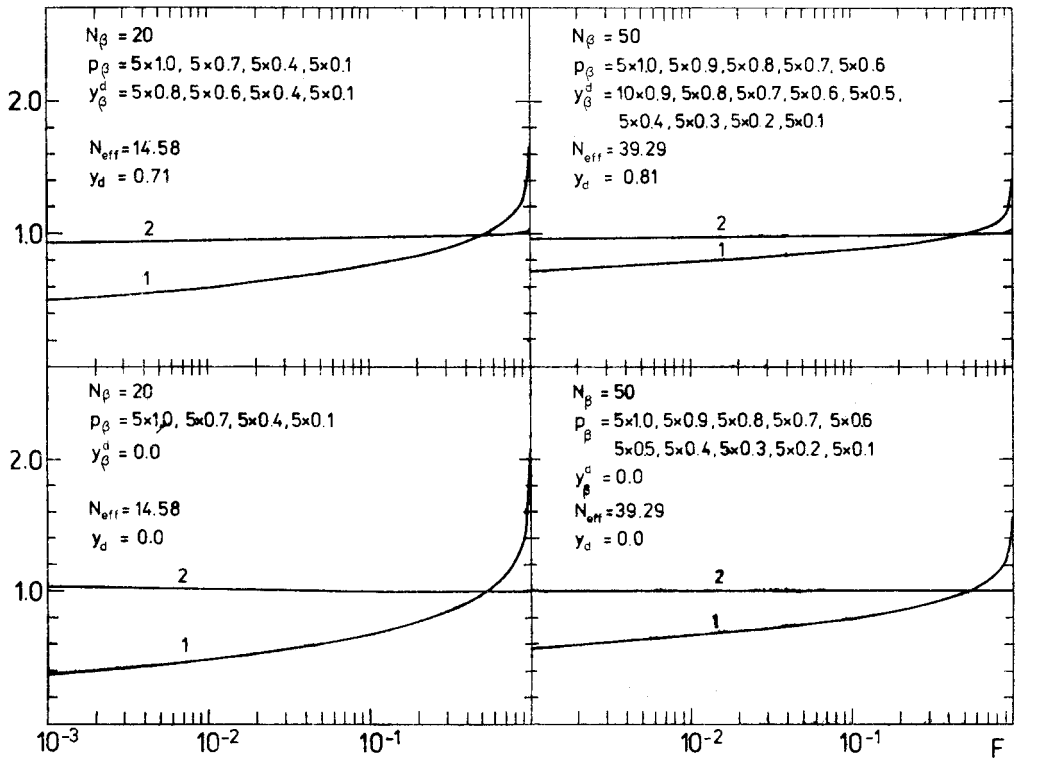

Fig. 3. The same as in Fig. 1

data only the knowledge of the compound nucleus contribution ($\langle\sigma\rangle^{CN}$, N_{eff}) and the average cross section, which could be obtained by appropriate averaging of experimental data, is demanded. However in a general case the agreement between the exact and approximate calculations should be tested by a direct comparison of the distribution calculated with both procedures.

In the present paper such calculations are performed for a considerable range of the values of N_{eff} and y_{d}. The results are presented in Figs. 1, 2, 3. In the diagrams the abscisa are the values of the cumulative distribution function while the ordinates give the fractiles of the approximate distribution (curve *1*), or the ratio of fractiles for the exact and approximate distributions (curve *2*).

The largest discrepancies in the values of fractiles appear for small values of cumulative distribution and for small values of N_{eff} and y_{d}. E.g. the disagreement between the fractiles for the exact and approximate formulas reaches ca 15% for $F \approx 0.01$, $N_{\text{eff}} \approx 2$ and $y_{\text{d}} \approx 0$. For $F \approx 0.99$ the discrepancies are not larger than ca 5% with the fractiles for the exact distribution always larger than those for the approximate one. With increasing N_{eff} and y_{d} these discrepancies tend rather fast to zero.

Thus the application of the approximate formula is justified for calculation of the fractiles in the range of values of the cumulative distribution function from 0.01 to 0.99.

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