MODIFICATION OF THE PERTURBATIVE QCD TOWARDS CONFINEMENT*

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(*Received June 19, 1981*)

Modification of the low momentum behaviour of the perturbative SU(2) gauge theory is proposed. The modification is closely related (although not equivalent) to a nonstandard choice of boundary condition for the Euclidean 2-point gluonic Green function. In the resulting theory already single graphs lead to the confining potential between heavy, static quarks, $V(r) = ar^2$ for $r \to \infty$.

PACS numbers: 12.40.-y

1. Introduction

The aim of this paper is to present a modification of the low momentum QCD which might lead to the explicit confinement of quarks already in finite order perturbation theory. We present this modification for SU(2) gauge theory — the generalization to SU(3) is trivially performed by an embedding SU(2) into SU(3). The most interesting feature of the modified QCD is that already single graph contributions lead to the confining potential between heavy, static quarks. Precisely this result encourages us to present the modification of QCD in more detail.

The modified QCD (in short QCD′) differs from the standard one only in the infrared region. Therefore, it is reasonable to expect that it would lead to the small distance behaviour of the same type as it has been calculated within the standard QCD. In particular, we expect that QCD′ is asymptotically free and that QCD′ is compatible with high energy phenomenology to the same extent as QCD.

Unfortunately, the obtained potential is of ar^2 type, and therefore it is not in agreement with phenomenological fits to the spectra of charmonium and bottomium. However, this potential was obtained in a rather naive way, as a contribution of a single graph in weak

* This paper is supported in part by the Polish Ministry of Higher Education, Science and Technology, Project MR.I.7.

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coupling limit of ordinary perturbation theory, and without any deeper analysis of infrared problems which are present in QCD′. More complete study of QCD′ can lead to a better potential. Anyway, the very fact that one obtains the confining potential in four-dimensional field theory seems to be interesting by itself.

In this general presentation we would like also to remark that QCD′, formulated in this paper on the level of formal perturbation theory, described in a compact way by generating functionals for the Euclidean Green functions, is not equivalent to a standard, local field theory. There are two reasons for this. Firstly, finite order QCD′ can be described by a lagrangian which is nonlocal. This nonlocality is generated much in the same manner as the well-known nonlocality of the standard QCD, [1] is generated by the elimination of Faddeev-Popov ghots in, e.g., Lorentz gauge. The nonlocality of QCD′ is an attractive feature because it is consistent with the common expectation that the long distance structure of color interactions should effectively be described by a theory of interacting strings or bags, nonlocal from the outset.

Secondly, we do not expect that the Hilbert space of states in QCD′ is spanned by states obtained by acting on vacuum with fundamental (quark and gluon) field operators. We base this expectation on the fact that QCD′ seems to confine quarks even in finite order perturbation theory. Thus, the notion of the single quark is empty even in perturbation theory. The complicated problem of clarifying the precise field-theoretical structure of QCD′ we postpone to another investigation.

We construct the modified QCD in Euclidean space-time. What QCD′ after the analytic continuation to Minkowski space-time, $x_4 \to -ix_0$, looks like, remains to be investigated. However, the Euclidean formulation is sufficient in order to calculate the potential energy of heavy, static quark-antiquark pair from the well-known Wilson loop formula, [2].

Finally, we would like to present the origin of the idea of modifying QCD. While solving the classical Yang-Mills equations with a point-like external source one arrives at Poisson equation

$$\Delta A_0^a(\vec{x}) = gI^a\delta(\vec{x}), \tag{1}$$

where (I^a) is a constant color spin vector of the external source, [3]. This equation has solutions of the form

$$A_0^a(\vec{x}) = -\frac{1}{4\pi}\frac{gI^a}{|\vec{x}|} + F^a(\vec{x}), \tag{2}$$

where $\Delta F^a = 0$. In order to eliminate $F^a(\vec{x})$ one has to assume the boundary condition that $A_0^a(\vec{x})$ tends to zero when $|\vec{x}|$ increases to infinity. However, such a choice of the boundary condition is a physical assumption. This assumption is certainly correct for a point-like electric charge in vacuum. On the other hand, it is by no means obvious that the same boundary condition should be adopted for the color charge, because physical properties of chromodynamical systems are expected to be different from those of electrodynamical systems. On the simple level of the classical equation (1) and the solution (2) we find it difficult to construct a satisfactory theory in which $F^a(\vec{x}) \neq 0$. However, the same problem appears when one considers the equation for free Green functions present in perturbation theory. And there it is possible to construct the corresponding nonstandard theory, namely QCD'. This construction is presented below in Sections 2 and 3. In Section 4 we calculate the single graph contribution to the static quark-antiquark potential within the scheme of QCD'.

It is interesting that this construction cannot be carried out for gauge theories which do not contain SU(2) group as a subgroup of the gauge group. In particular, the construction does not work for the abelian U(1) gauge group.

2. The modified free gluon propagator

We consider Euclidean SU(2) gauge theory in the functional formulation. This formalism is here regarded only as a convenient tool for generating a perturbative expansion. We choose the Feynman gauge, $\alpha = 1$.

By the well-known manipulations, described, for example, in [1], we identify the free Euclidean gluon propagator $G^{ab}_{\mu\nu}$, $a, b = 1, 2, 3$, $\mu, \nu = 1, 2, 3, 4$, as the solution of the equation

$$\Delta_4 G^{ab}_{\mu\nu}(x_\varrho) = \delta_{ab}\delta_{\mu\nu}\delta^{(4)}(x), \tag{3}$$

where $\Delta_4 = \partial_\mu\partial_\mu$ is the four-dimensional laplacian. The propagator $G^{ab}_{\mu\nu}$ should be SO(4) invariant in order to ensure the Lorentz invariance of the theory after the inverse Wick rotation, $x_4 \to -ix_0$, to the Minkowski space-time.

The standard solution to (3) is

$$G^{ab}_{0\mu\nu}(x) = -\frac{1}{4\pi^2}\delta_{ab}\delta_{\mu\nu}\frac{1}{x^2}, \quad x^2 = x_\varrho x_\varrho, \tag{4}$$

which leads in momentum space to

$$G^{ab}_{0\mu\nu}(k) \equiv \int \exp(-ik_\varrho x_\varrho) G^{ab}_{0\mu\nu}(x) d^4x = -\delta_{ab}\delta_{\mu\nu}\frac{1}{k^2}. \tag{5}$$

Eq. (3) has also infinitely many other solutions, obtained by adding to (4) any solution of the four-dimensional Laplace equation,

$$\Delta_4 F(x) = 0,$$

regular at the origin. Any solution of (6) can be written in the form of the series, [4],

$$F(x_\varrho) = \sum_{l=0}^{\infty}\sum_{K,K'} a^l_{KK'} x^l \Xi^l_K(\theta_1, \theta_2, \theta_3), \tag{7}$$

where $x = \sqrt{x_\varrho x_\varrho}$, $a^l_{KK'}$ are arbitrary constants, K, K' are multindices, $K = (k_1 \pm k_2)$, $l \geq k_1 \geq k_2 \geq 0$, k_1, k_2 integers, and Ξ^l_K are 4-dimensional spherical harmonics, related to the Gegenbauer polynomials. $\theta_1, \theta_2, \theta_3$ are the spherical angles in 4-dimensional space-time. $x^l\Xi^l_K$ is a harmonic polynomial of order l in variables x_ϱ, [4].

The essence of QCD′ consists of keeping $F(x_\varrho) \neq 0$. However, this can not be achieved by the simple change of the free gluon propagator, $G_0 \to G_0+F$. This would lead to a violation of SO(4) symmetry, as it will be explained below, and to a violation of the positivity requirement, [5]. The requirement of SO(4) symmetry forces us to introduce the term $F(x)$ in a very particular manner. Then, SO(4) symmetry can be realised in the generalized sense, [6], that the effect of SO(4) rotation can be compensated by a sort of gauge transformation. We do not know whether QCD′ obeys an appropriate version of the positivity postulate.

We will consider the particular solution of (3),

$$G^{ab}_{\mu\nu}(x) = G^{ab}_{0\mu\nu}(x)+\lambda\tilde{G}^{ab}_{\mu\nu}(x), \tag{8}$$

where

$$\tilde{G}^{ab}_{\mu\nu}(x) = x^2 D^1_{ab}(x_\varrho/x)\delta_{\mu\nu}. \tag{9}$$

Here λ is a constant with dimension $[\lambda] = \text{cm}^{-4}$, and $D^1_{ab}\left(\frac{x_\varrho}{x}\right)$ are the matrix elements of spin 1 representation of SU(2) group. Such a choice for $F(x)$ is motivated by the simple transformation law for $G^{ab}_{\mu\nu}(x)$ under SO(4) rotation. As it was said earlier, $G^{ab}_{\mu\nu}$ cannot be straightforwardly adopted as the free gluon propagator. The problem of how to incorporate $\tilde{G}^{ab}_{\mu\nu}(x)$ in a more satisfactory manner we consider in the next Section. In this Section we will discuss transformation law for $\tilde{G}$ under SO(4) rotations.

The explicit formula for $D^1_{ab}(x_\varrho/x)$ is the following: We introduce matrix

$$\text{SU}(2) \ni u = \begin{pmatrix} a & b \\ -\bar{b} & \bar{a} \end{pmatrix}, \quad |a|^2+|b|^2 = 1, \tag{10}$$

where

$$a = \frac{x_4+ix_3}{x}, \quad b = \frac{x_2+ix_1}{x}. \tag{11}$$

Then,

$$D^1_{ab}(x_\varrho/x) \equiv D^1_{ab}(u), \tag{12}$$

where the form of $D^1_{ab}(u)$ is given, e.g., in [4]. Substituting there (11) we obtain

$$x^2D^1(x_\varrho/x) = \begin{bmatrix} x_4^2+x_2^2-x_3^2-x_1^2 & 2(x_3x_4-x_2x_1) & 2(x_4x_1+x_2x_3) \\ -2(x_3x_4+x_2x_1) & x_4^2+x_1^2-x_2^2-x_3^2 & 2(x_4x_2-x_1x_3) \\ 2(x_2x_3-x_4x_1) & -2(x_4x_2+x_3x_1) & x_4^2+x_3^2-x_1^2-x_2^2 \end{bmatrix}. \tag{13}$$

In fact, $D^1(u)$ is an orthogonal matrix. From (13) it is easy to see that $\tilde{G}^{ab}_{\mu\nu}(x)$ obeys equation (6). Observe also that $\tilde{G}^{ab}_{\mu\nu}(x)$ vanishes for small x. The Fourier transform of this function consists of second derivatives of $\delta^{(4)}(k)$. Therefore $G^{ab}_{\mu\nu}(k)$ differs from $G^{ab}_{0\mu\nu}(k)$ only at zero momentum, i.e., in the extreme infrared region.

Now, let us investigate the effect of an SO(4) rotation. It is a linear transformation, $(x_\varrho) \to (x'_\varrho)$ which does not change $x : x' = x$. On the other hand, consider matrices $w = xu = x_4\sigma^0 + i\vec{x}\vec{\sigma}$, where u is given by (10), (11), and the transformation

$$w' = u_1 w u_2^{-1}, \tag{14}$$

where $u_1, u_2 \in SU(2)$. Because $\det w = x_\varrho x_\varrho$, and because the correspondence between w and (x_ϱ) is linear, we conclude that any SO(4) rotation of (x_ϱ) is equivalent to the transformation

$$u' = u_1 u u_2^{-1} \tag{15}$$

of the unitary matrix u. The condition $u_1, u_2 \in SU(2)$ is necessary in order to obtain unitary u'. From (12) we see that

$$D^1_{ab}(x'_\varrho/x) = D^1_{ac}(u_1)D^1_{cd}(x_\varrho/x)D^1_{bd}(u_2). \tag{16}$$

If $u_1 = u_2$, which corresponds to "spatial" rotations ($x'_4 = x_4$), then from (16) it follows that the SO(4) rotation is equivalent to a constant gauge transformation of the color indices a, b in $\tilde{G}(x)$. However, the general transformation (16) does not have the form of a gauge transformation.

Of course, $G^{ab}_{0\mu\nu}(x)$ is SO(4) invariant.

3. *The modified QCD*

The use of $G^{ab}_{\mu\nu}(x)$, given by (8), as the free, Euclidean propagator for gluons leads to a theory which is not SO(4) invariant because $\tilde{G}^{ab}_{\mu\nu}(x)$ has different transformation properties than $G^{ab}_{0\mu\nu}(x)$. This difficulty can be circumvented at the price of introducing $\tilde{G}^{ab}_{\mu\nu}(x)$ in more refined manner than just a part of the gluon propagator.

Let us recall that the standard perturbative expansion in powers of coupling constant g in the pure Yang-Mills theory can be generated from the following expression for the generating functional for Green functions, [1],

$$Z(J) = V\left(\frac{\delta}{\delta J}, \frac{\delta}{\delta \eta}, \frac{\delta}{\delta \eta^*}\right) E(J, \eta, \eta^*)|_{\eta=\eta^*=0}, \tag{17}$$

where

$$V\left(\frac{\delta}{\delta J}, \frac{\delta}{\delta \eta}, \frac{\delta}{\delta \eta^*}\right) = \exp\left\{gV_3\left(\frac{\delta^3}{\delta J^3}\right) + g^2 V_4\left(\frac{\delta^4}{\delta J^4}\right) + gV_{\text{F-P}}\left(\frac{\delta}{\delta J}, \frac{\delta}{\delta \eta}, \frac{\delta}{\delta \eta^*}\right)\right\}, \tag{18}$$

and

$$E(J, \eta, \eta^*) = \exp \int d^4x d^4y [J^a_\mu(x) G^{ab}_{0\mu\nu}(x-y) J^b_\nu(y) + \eta^*_a(x) G^{ab}_{0\text{F-P}}(x-y)\eta_b(y)]. \tag{19}$$

In these formulae J^a_μ, η_a, η^*_a are external sources coupled to the gluon field $A^a_\mu(x)$ and Faddeev-Popov ghosts, correspondingly. V_3 denotes the three-gluon vertex, V_4—the four-gluon vertex. V_{FP} — vertex in which A^a_μ couples to the F-P ghosts.

The proposed modification of QCD consists of replacing $Z(J)$ by $Z'(J)$ defined as follows:

$$Z'(J) = V\left(\frac{\delta}{\delta J}, \frac{\delta}{\delta \eta}, \frac{\delta}{\delta \eta^*}\right) V\left(\frac{\delta}{\delta K}, \frac{\delta}{\delta \chi}, \frac{\delta}{\delta \chi^*}\right) E(J, \eta, \eta^*)$$

$$\times \exp\left[\lambda \int d^4x d^4y J^a_\mu(x) \tilde{G}^{ab}_{\mu\nu}(x-y) K^b_\nu(y)\right] E(K, \chi, \chi^*)|_{\substack{K=\eta=\eta^*=0,\\ \chi=\chi^*=0}} \qquad (20)$$

The idea underlying this modification is the following. Consider the term $JGJ = JG_0J + \lambda J\tilde{G}J$ which would be present in (19) if G_0 was replaced by $G_0 + \lambda\tilde{G}$. The term JG_0J is the standard one, so we would like to keep it unchanged. The term $J\tilde{G}J$ is not SO(4) invariant because under SO(4) rotation the indices a, b of $\tilde{G}$ transform in different manner, as it follows from (16). Now we shall consider Feynman graphs. If we replace G_0 by $G_0 + \lambda\tilde{G}$ on lines of a graph, we would obtain 2^I contributions, where I is the number of lines in the graph. In each such contribution a line corresponds to either G_0 or $\lambda\tilde{G}$. The (single) contribution such that all its lines correspond to G_0 is just the graph calculated within the standard QCD. It is SO(4) invariant, of course. The other contributions contain at least one $\tilde{G}$. In general they are not SO(4) invariant. However, some of them are SO(4) invariant—namely those in which all $\tilde{G}^{ab}_{\mu\nu}(x-y)$ lines, which we regard as directed from (x, a) to (y, b), end only in vertices which are common end points for $\tilde{G}$-lines (that is: no G_0-line is attached to such a vertex and no $\tilde{G}$-line starts at it). These SO(4) invariant contributions can be singled out formally by introducing an auxiliary current K^b_ν coupled to the end point of the $\tilde{G}$-line, i.e., by replacing

$$\lambda \int d^4x d^4y J^a_\mu(x) \tilde{G}^{ab}_{\mu\nu}(x-y) J^b_\nu(y)$$

by

$$\lambda \int d^4x d^4y J^a_\mu(x) \tilde{G}^{ab}_{\mu\nu}(x-y) K^b_\nu(y),$$

and by introducing the second vertex part

$$V_0\left(\frac{\delta}{\delta K}\right) = \exp\left[gV_3\left(\frac{\delta^3}{\delta K^3}\right) + g^2 V_4\left(\frac{\delta^4}{\delta K^4}\right)\right].$$

It is easy to see that all vertices generated by $V_0\left(\frac{\delta}{\delta K}\right)$ from $\exp(\lambda \int J\tilde{G}K)$ are SO(4) invariant. Namely, the transformation of the a index of $\tilde{G}$ is compensated by the constant gauge transformation of J^a_μ, corresponding to u_1 in (15), and the transformation of the b index, corresponding to u_2 in (15), is compensated by an independent constant gauge transformation of the current K^b_ν.

The other contributions are not SO(4) invariant. They are the contributions which contain at least one $\tilde{G}$-line ending at a vertex to which some G_0-lines are also attached or/and at which a $\tilde{G}$-line starts, or/and there are attached F-P ghosts lines. Some of these contributions can be made SO(4) invariant by reinterpreting some of G_0-lines and some of F-P ghost lines as being of the K-type, i.e., as transforming accordingly to u_2 under SO(4) rotations. This step can be formally described by introducing another set of F-P ghosts, described by the sources χ, χ^* and coupled to $\frac{\delta}{\delta K^a_\mu}$, and by introducing the standard G_0 propagator of the K-type. The resulting theory is described by $Z'(J)$ given by (20).

The modified QCD, defined by (20), is much richer than the standard one. The standard QCD is obtained as the zeroth order in λ, because then the factor

$$V\left(\frac{\delta}{\delta K}, \frac{\delta}{\delta\chi}, \frac{\delta}{\delta\chi^*}\right) E(K, \chi, \chi^*)|_{K=\chi=\chi^*=0}$$

drops out as a constant normalisation. However, QCD′ contains less graphs than the standard QCD with the straightforward replacement $G_0 \to G_0 + \lambda\tilde{G}$. For example, graphs containing closed loops formed by the $\tilde{G}$-lines are excluded from QCD′.

The current K^a_ν plays the auxiliary role, much like the external sources η, η^* introduced for F-P ghosts in standard QCD. There is no physical field coupled to K^a_ν. In principle, this current can be eliminated from the formula (20) for $Z'(J)$. This would lead to a nonlocal effective lagrangian, similarly as elimination of F-P ghosts leads to a nonlocal lagrangian containing the well-known Trlog, [1].

4. *An example — the perturbative confining potential*

In order to calculate the potential $E(r)$ between static quark and antiquark we use the well-known formula, [2, 7], relating $E(r)$ to the vacuum expectation value of the Wilson loop operator,

$$E(r) = -\lim_{T\to\infty} \frac{1}{T} \ln \langle W(C) \rangle_0, \tag{21}$$

where

$$W(C) = \mathrm{Tr}\,[P \exp (ig \oint_C \hat{A}_\mu dx^\mu)], \tag{22}$$

and $\hat{A} = A^a_\mu T^a$, $T^a = \sigma^a/2$ are generators of the fundamental representation of SU(2). The contour C is taken to be a rectangle laying in a plane parallel to the x_4 axis, e.g., in (x_3, x_4) plane, with corners at points $(x_3, x_4) = (0, -T/2), (r, -T/2), (0, T/2), (r, T/2)$. The orientation of C is chosen to be from $(0, -T/2)$ to $(0, T/2)$. P denotes the path ordering along C.

The usual formal manipulations on the Euclidean path integral formula in the standard Yang-Mills theory yield

$$\langle W(C) \rangle_0 = \mathrm{Tr}\left[P \exp\left(ig \oint_C T^a \frac{\delta}{\delta J^a_\mu(x)} dx^\mu\right)\right] V\left(\frac{\delta}{\delta J}, \frac{\delta}{\delta\eta}, \frac{\delta}{\delta\eta^*}\right) E(J, \eta, \eta^*)|_{J=\eta=\eta^*=0}.$$

In order to pass to QCD′, according to Section 3 we add $V\left(\frac{\delta}{\delta K}, \frac{\delta}{\delta\chi}, \frac{\delta}{\delta\chi^*}\right)$, $E(K, \chi, \chi^*)$ and $\exp(\lambda \int J\tilde{G}K)$. Thus, in QCD′

$$\langle W(C)\rangle_0 = \mathrm{Tr}\left[P\exp\left(ig\oint_C T^a \frac{\delta}{\delta J^a_\mu(x)} dx^\mu\right)\right] V\left(\frac{\delta}{\delta J}, \frac{\delta}{\delta\eta}, \frac{\delta}{\delta\eta^*}\right)$$

$$\times V\left(\frac{\delta}{\delta K}, \frac{\delta}{\delta\chi}, \frac{\delta}{\delta\chi^*}\right) E(J, \eta, \eta^*) E(K, \chi, \chi^*)$$

$$\times \exp\left[\lambda \int d^4x d^4y J^a_\mu(x)\tilde{G}^{ab}_{\mu\nu}(x-y)K^b_\nu(y)\right]\Big|_{\substack{J=K=\eta=\eta^*=0\\ \chi=\chi^*=0}}. \tag{23}$$

This formula is the starting point for the perturbative calculation of $\langle W(C)\rangle_0$ in QCD′ — from it one can write $\langle W(C)\rangle_0$ as a double power series in g and λ.

We shall systematize the terms in this expansion by writing it in the form

$$\langle W(C)\rangle_0 = \sum_{k=0}^{\infty} \lambda^k W_k(g, r)$$

where $W_k(g, r)$ are given as a power series in g. The zeroth order term, $W_0(g, r)$, give the contribution to the potential $E(r)$ coming from the standard QCD. The term

$$V\left(\frac{\delta}{\delta K}, \frac{\delta}{\delta\chi}, \frac{\delta}{\delta\chi^*}\right) E(K, \chi, \chi^*)|_{K=\chi=\chi^*=0}$$

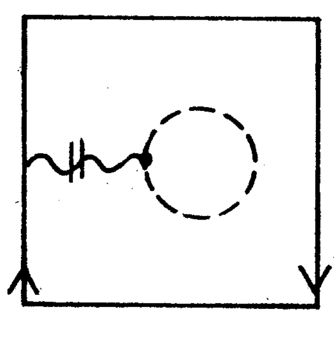

Fig. 1. Example of the tadpole. Double slashed wavy line denotes $\tilde{G}$-line, wavy line denotes G_0-line, dashed line denotes F-P ghost line

enters here only as a constant normalisation factor independent of r. When expanded in powers of g up to a finite order, $W_0(g, r)$ leads to the Coulomb type potential corrected by some logarithms of r. The renormalization group improvements also do not provide any confining potential, [8].

In the first order in λ only tadpole graphs are possible, Fig 1. Such graphs are usually set to be zero. Anyway, they are the selfenergy graphs for the quark and antiquark, and therefore they do not give contribution depending on r.

In the second order in λ there are contributions to $E(r)$ coming from the graphs of the type presented in Fig. 2. In this figure we do not display the quark and antiquark selfenergy graphs, nor the graphs which give contributions trivially vanishing in the limit $T \to \infty$. In the momentum space the general structure of such graphs is

$$\oint_C \oint_C dx_\mu^1 dx_\nu^2 \int d^4p \exp[i(x^1 - x^2)p]\tilde{G}^{ab}_{\mu\sigma}(p) G^{(2)bc}_{\sigma\varrho}(p)\tilde{G}^{ac}_{\varrho\nu}(p),$$

where $G^{(2)}(p)$ is the two-point Green function of the gluon field A_μ^a. The propagator $\tilde{G}$ is given by the second derivatives of $\delta^{(4)}(p)$. It is well-known, [1], that in the perturbation theory $G^{(2)}(p)$ contains logarithms of p which diverge for $p \to 0$. Thus, we see that in

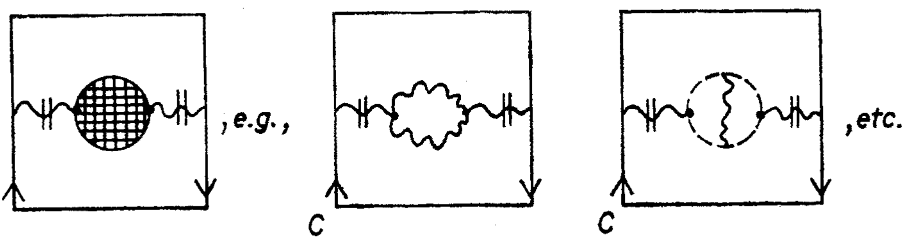

Fig. 2. The second order (in λ) contributions

such simple graphs there are divergences due to the singular infrared behaviour of perturbative logarithms in addition to the expected small p divergences induced by squaring of the derivatives of δ distribution. We cannot find out how to regularize and renormalize such graphs without a deeper analysis of physical features of QCD' — therefore we postpone calculation of these graphs to another investigation.

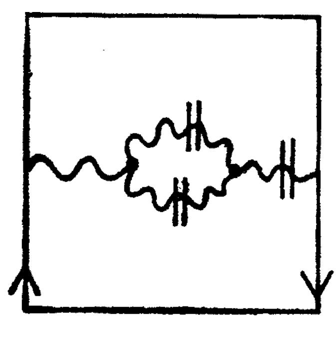

Fig. 3. The calculated λ^3 contribution

For this reason, we prefer to present as the example the contribution of a graph which does not involve any logarithms. There are no such graphs in the second order in λ apart from graphs containing tadpoles. We find such a graph in the order λ^3, Fig. 3.

Calculation of the contribution to $E(r)$ coming from this graph is somewhat tedious. For example, it involves contraction of three D^1 matrices with two ε_{abc}'s. Below we describe only the most important steps of the calculation.

We use the standard diagrammatic rules in the Euclidean momentum space. Expressions for vertices are of the same form as in the standard QCD. The slashed lines correspond to the Fourier transforms of $\tilde{G}^{ab}_{\mu\nu}(x)$, and therefore they are given by the second derivatives of $\delta(p)$. In order to avoid singularities due to multiplication of δ's we regularize the slashed lines by the replacement

$$\delta^{(4)}(p) \to (2\pi\varepsilon\mu^2)^{-2} \exp(-p^2/\varepsilon\mu^2). \tag{24}$$

We introduce here the dimensional constant μ in order to keep ε dimensionless. Then, we obtain the contribution to $\langle W(C)\rangle_0$ of the form

$$\Delta\langle W(C)\rangle_0 = Tg^4\lambda^3 c_0(\varepsilon\mu^2)^{-7} \int d^3\vec{p} \exp(i\vec{r}\vec{p} - 2\varepsilon^{-1}\mu^{-2}\vec{p}^2), \tag{25}$$

where c_0 is a positive constant, $\vec{x}^1$, $\vec{x}^2$ are spatial locations of the static quark and anti-quark, $\vec{r} = \vec{x}^1 - \vec{x}^2$. The integral over p_4 was performed due to $\delta(p_4)$ coming from

$$\int_{-T}^{T} dx_4^1 \int_{-T}^{T} dx_4^2 \exp[ip_\mu(x_\mu^1 - x_\mu^2)] \tag{26}$$

in the limit $T \to \infty$. From (26) comes also the factor T present in (25). From (25) we obtain

$$\Delta\langle W(C)\rangle_0 = c_0 T\lambda^3 g^4 u^{-11} \varepsilon^{-\frac{11}{2}} \exp[-\varepsilon\mu^2(x^1 - \vec{x}^2)^2]. \tag{27}$$

This expression becomes singular when we try to remove the cutoff, $\varepsilon \to 0$.

As usual in such circumstances, one can try to introduce a renormalization procedure. From (21) it follows that

$$E(\vec{x}^1 - \vec{x}^2) = -\lim_{T\to\infty} \frac{1}{T} \Delta\langle W(C)\rangle_0 \tag{28}$$

if we neglect all other contributions to $E(r)$. Therefore, the term in $\Delta\langle W(C)\rangle_0$, which is independent of $\vec{r}$, will lead to an unessential constant in the energy E. Subtracting from $\Delta\langle W(\vec{r})\rangle_0$ the term $\Delta\langle W(\vec{r} = 0)\rangle_0$ we obtain

$$E(\vec{r}, \varepsilon) = -c_0 g^4 \lambda^3 \mu^{-11} \varepsilon^{-11/2} (\exp[-\varepsilon\mu^2(\vec{x}^1 - \vec{x}^2)^2] - 1). \tag{29}$$

Next, we introduce the renormalized λ:

$$\lambda = \varepsilon^{3/2} \lambda_{\text{ren}}. \tag{30}$$

Observe that if λ_{ren} is kept finite, then the bare λ vanishes in the limit $\varepsilon \to 0$. Now, the limit $\varepsilon \to 0$ in (29) can be taken, yielding

$$E(\vec{r}) = c_0 g^4 \left(\frac{\lambda_{\text{ren}}}{\mu^3}\right)^3 \vec{r}^2. \tag{31}$$

The above renormalization procedure should be regarded merely as a proposal of how to extract finite $E(r)$ from divergent graphs. The reader should be aware of the fact that we have renormalized infrared divergencies. The usual approach is to leave them unrenormalized but, instead, to calculate only the quantities which are infrared finite due to cancellations of these infinities. It would be interesting to find out what are infrared finite quantities in QCD′. The proposed renormalization approach is to some extent justified, because the renormalized infinities appear in the perturbative expansion in powers of λ much like the well-known ultraviolet infinities appear in the perturbative expansion in powers of g.

5. *Ending remarks*

We have proposed the modification of perturbative QCD which essentially consists of introducing a new type of internal line in Feynman graphs. The corresponding propagator is proportional to the second derivatives of $\delta^{(4)}(p)$. In comparison to other modifications of QCD, e.g. [9, 10], our modification is very straightforward — it merely takes the advantage of the mathematical freedom of choosing boundary conditions for the free gluon propagator. However, it is just this choice which largely determines the long-distance physics.

We have left a number of unsolved problems, like the status of QCD′ from the point of view of general axiomatics, or problem of the infrared divergences. Nevertheless, it is interesting that starting from SO(4)-noninvariant propagator we can obtain SO(4) invariant theory which likely confines quarks. This is possible only due to the color degrees of freedom, described by the indices a, b in $G^{ab}_{\mu\nu}(x)$. In particular, there is no analogously modified and SO(4) symmetric QED. This we regard as one of the most attractive features of QCD′.

Finally, let us remark that our construction can be easily generalized to SU(n), $n > 2$, groups merely by embedding SU(2) in SU(n).

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