REGGE LIMIT IN QCD*

By M. Praszałowicz

Institut of Physics, Jagellonian University, Cracow**

(Received March 9, 1981)

It is shown that the perturbative calculations in Regge limit in nonabelian gauge theories are in agreement with the hypothesis of the reggeization of vector meson (gluon). The infrared properties of the integral equations for various quantum numbers exchange in the t-channel are studied in the spontaneously broken theory; the role of the Higgs particles is investigated. Some connections with deep inelastic scattering are also discussed. The integral equation describing the three gluon exchange in a colour singlet state is formulated and its infrared properties are studied. It is argued that it generates a fixed branch point in j-plane.

PACS numbers: 12.40.-y, 12.40.Mm

1. Introduction

The problem of high energy scattering in nonabelian gauge theories (NAGTs) has been widely discussed in the literature [1–8]. The calculations have been performed for various classes of processes in different approximation schemes. The picture which emerges seems to prove QCD to be the theory of strong interactions.

In this talk we would like to present the results which have been obtained by various authors [1–6, 8–21] in so called Regge limit of QCD, where s is large and t fixed (by s and t we denote the usual Mandelstam variables).

Perturbative calculations based on the usual Feynman techniques have been performed up to the 12$^{\text{th}}$ order in the leading $\ln s$ approximation [1–3]. To avoid infrared divergencies the gluon mass λ has been introduced by means of Higgs mechanism, in some cases the limit $\lambda \to 0$ can be taken and we again arrive at the massless theory. It has been shown that for vector meson quantum numbers exchange in t-channel perturbative results agree with the hypothesis of the vector-meson reggeization in NAGTs [9–11]. This will be demonstrated in Section 2 in the two lowest orders of the perturbation expansion.

Next we shall briefly sketch the alternative technique for the summation of the perturbation [4–6] series based on unitarity and analyticity. In this context we shall explain the role of the Higgs particles in the whole scheme.

* Presented at the XX Cracow School of Theoretical Physics, Zakopane, May 29 — June 11, 1980.

** Address: Instytut Fizyki, Uniwersytet Jagielloński, Reymonta 4, 30-059 Kraków, Poland.

The picture which emerges is the reggeization of vector meson in NAGTs. We shall next, in Section 3, use the Regge-like formulae for the scattering amplitudes to show how one can derive the integral equations for various quantum numbers exchange [4–6]. The results for SU(3) are as follows: antisymmetric octet (gluon) reggeizes, symmetric octet also has a Regge pole structure, singlet (the Pomeron) has a fixed branch point in complex angular momentum j-plane. The integral equations for the octets exchanges suffer from infrared divergencies (one has to keep $\lambda \neq 0$) but in the Pomeron case all infinite terms cancel out — this we shall explain in details — so one can take $\lambda = 0$ limit.

Some connections of the Pomeron equation with deep inelastic scattering (DIS) will be also discussed [7, 8, 15, 21].

The procedure presented in Sections 2 and 3 leads to the Gribov Reggeon calculus of reggeized gluons [22, 23] with calculable vertices [5, 6, 15–17]. In our case for leading $\ln s$ approximation only the $2 \to 2$ vertices are needed explicitly. Using the rules of the Reggeon calculus [22] we derive in Section 4 an integral equation for the three reggeized gluon system exchange in a colour singlet state [19]. This amplitude has C-parity $C = -1$ and therefore differs from the Pomeron. Phenomenologically it can be responsible for the behaviour of F_3 structure function in neutrino-hadron scattering for $x \to 0$. We shall investigate the infrared properties of this equation and argue that it generates a fixed branch point in j-plane [19]. Some authors [15–17] go beyond the leading logarithmic approximation in order to construct a complete Reggeon calculus with all types of vertices. In this paper however we will not consider this problem.

2. *Reggeization of high energy amplitudes*

At the begining of this section we shall recall what reggeization of a particle in a given theory means [10, 15]. We shall be working with the Sommerfeld-Watson representation of the scattering amplitude

$$A(s, t) = s \int \frac{d\omega}{2\pi i} s^{\omega} \frac{e^{-i\pi\omega} - \sigma}{\sin \pi\omega} F_{\omega}(t), \tag{2.1}$$

where $j = \omega + 1$ is complex angular momentum variable and σ denotes signature.

For the moment let us consider a fermion-fermion amplitude in any gauge theory. In the first order of the perturbation expansion (one spin $j = 1$ vector meson exchange) $F_{\omega}(t)$ is nonanalytic in ω:

$$F_{\omega}(t) \sim \delta_{\omega,0}.$$

If the higher orders remove this nonanalyticity and the amplitude $F_{\omega}(t)$ will have a t dependent pole in ω-plane (Regge pole) one would say that the vector meson reggeizes [10]

$$A(s, t) \sim s^{\alpha(t)+1} \frac{e^{-i\pi\alpha(t)} - \sigma}{\sin \pi\alpha(t)} \alpha(t), \tag{2.2}$$

where the trajectory $\alpha(t)$ is a calculable function of t, proportional to the coupling constant g^2, vanishing for $t = \lambda^2$ (λ — vector meson mass), and signature $\sigma = -1$. Some authors

[1–4] have successfully undertaken the efforts to calculate the scattering amplitudes up to the 12[th] order of the perturbation theory. The calculations were performed in a leading logarithmic approximation where terms $(g^2 \ln s)^n$ were picked up. These results were compared [10] with the expansion of Eq. (2.2) and it turned out that for vector meson quantum numbers exchange in the t-channel perturbative results were in agreement with the Regge

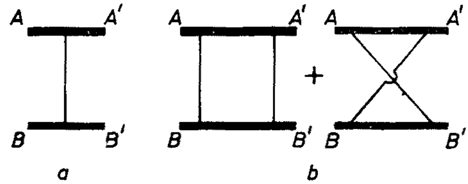

Fig. 1. Fermion-fermion scattering in the first and second order in α_s

pole structure of $F_\omega(t)$. This means that vector meson in NAGT reggeizes. In fact reggeization occurs in all gauge theories based on a semisimple Lie group [13], whereas in QED photon does not reggeize.

To illustrate this let us consider the second order fermion-fermion amplitude (Fig. 1b). In Feynman gauge for SU(N) theory one obtains in the first and second order

$$A_1(s, t) = \frac{g^2}{2} \frac{s}{m^2} \frac{\delta_{\lambda_A \lambda_A'} \delta_{\lambda_B \lambda_B'}}{t-\lambda^2} \vec{T}^{(A)} \vec{T}^{(B)},$$

$$A_2(s, t) = \frac{g^2}{2} \frac{s}{m^2} \frac{\delta_{\lambda_A \lambda_A'} \delta_{\lambda_B \lambda_B'}}{t-\lambda^2} \alpha(t) \left[i\pi \frac{N+1}{4N^2} \mathbf{1}^{(A)} \mathbf{1}^{(B)} \right.$$

$$\left. + \left(\ln \frac{s}{m^2} - \frac{i\pi}{2} \right) \vec{T}^{(A)} \vec{T}^{(B)} + i\pi \frac{N^2-4}{2N^2} \vec{T}^{(A)} \vec{T}^{(B)} \right], \tag{2.3}$$

$$\alpha(t) = \frac{Ng^2}{2(2\pi)^3} (t-\lambda^2) \int \frac{d^2 k_\perp}{(k_\perp^2 - \lambda^2)((k_\perp - (p_A - p_{A'}))^2 - \lambda^2)}, \tag{2.4}$$

where $\vec{T}$ are group couplings of vector meson to fermion, $\mathbf{1}$ unit matrix (for singlet exchange), m and λ denote fermion and vector meson mass respectively, $\delta_{\lambda_A \lambda_A'}$ stands for fermion helicity conservation.

In the second order only singlet and adjoint representations are exchanged, and in fact both of them appear in Eq. (2.3) with nonvanishing imaginary parts. In other words vector meson quantum numbers can be exchanged in the second and in the first order as well. This is not true for QED since photon has C-parity $C_\gamma = -1$ and two photon state can have only $C_{2\gamma} = +1$. There is no 3γ coupling and as a consequence photon does not reggeize.

In SU(2) theory the last term in (2.3) vanishes and the amplitude for vector meson quantum numbers exchange (term proportional to $\vec{T}^{(A)} \cdot \vec{T}^{(B)}$) agrees with the expansion of Eq. (2.2) with $\sigma = -1$. But for $N > 2$, in SU(3) theory for example, there is another term proportional to $\vec{T}^{(A)} \cdot \vec{T}^{(B)}$ which seems to spoil the reggeization pattern (2.2). This term, however, arises from the contraction of the symmetric structure constants d_{abc} and should be therefore identified with the symmetric octet exchange [11, 20], whereas the second term in (2.3) comes from the antisymmetric octet (gluon) exchange. Symmetric octet exchange, as seen from Eq. (2.3), is one power of logarithm down with respect to the antisymmetric octet.

In the Clebsch-Gordan series for the direct product of two octets (adjoint representations of SU(N) group) octet (adjoint representation) appears twice

$$8 \otimes 8 = 1 \otimes 8_a \otimes 8_s \otimes \ldots$$

These two octets differ by generalized C-parity [24]: $C_{8_a} = -1$, $C_{8_s} = +1$ and therefore they should not be mixed in the fermion-fermion scattering amplitudes. Having this in mind we see that up to the 2[nd] order amplitude with gluon quantum numbers exchange has Regge form of Eq. (2.2). In the symmetric octet case it can be shown that reggeization also occurs [11, 20] but the relevant trajectory differs in mass term from that of Eq. (2.4).

There is another trouble about Eq. (2.3); in any realistic theory we would like to have massless gluons, but the limit $\lambda \to 0$ for the amplitude (2.3) does not exist. We will show however, that in renormalizable SU(N) theory with massive "gluons" and Higgs particles the colour singlet exchange amplitude is infrared safe [6, 10] and λ can be put equal to 0. For other representations this limit does not exist, but their infrared behaviour is crucial for the finitness of the singlet exchange.

Now we shall come deeper into the question of the reggeization in SU(N) theory and the Pomeron. In what follows we shall use the unitarity and dispersion relations. This method was proposed in Refs. [4–6] and we shall only quote the results trying to describe physics rather than technical problems. In order to regularize the infrared divergencies in this approach one introduces Higgs particles [9, 25]. In SU(N) theory ($N > 2$)

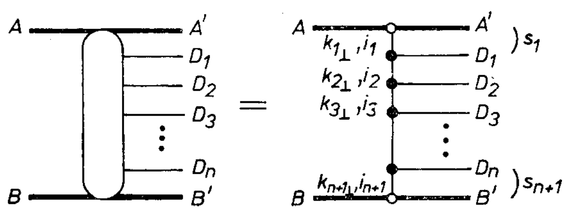

Fig. 2. Multigluon production in A+B scattering; an illustration of Eq. (2.5)

the Higgs sector is more complicated than in SU(2) model. For instance in SU(3) it consists of scalars in octet representation which couple to gluons with d_{abc} couplings and colour singlets as well [13, 18, 25]. It should be noted that the general approach to this problem in massless theory in Coulomb gauge has been worked out in Ref. [16].

Let us consider a multigluon production in $A+B \to A'+B'+D_1+D_2+ \ldots D_n$ collision where A(A'), B(B') stand for gluons, quarks or hadrons (see Fig. 2). We will show that in the high energy limit this amplitude can be written in a Regge form [5]

$$A_{2\to 2+n}(s,t) = s\Gamma^{i_1}_{AA'}\frac{\left(\frac{s_1}{m^2}\right)^{\alpha(k_{1\perp}^2)}}{k_{1\perp}^2-\lambda^2}\gamma^{D_1}_{i_1 i_2}(k_{1\perp},k_{2\perp})\dots\gamma^{D_n}_{i_n i_{n+1}}(k_{n\perp},k_{n+1\perp})$$

$$\times\frac{\left(\frac{s_{n+1}}{m^2}\right)^{\alpha(k_{n+1\perp}^2)}}{k_{n+1\perp}^2-\lambda^2}\Gamma^{i_{n+1}}_{BB'}, \tag{2.5}$$

where $\Gamma^i_{AA'}$ is a coupling to the external particles and $\gamma^D_{ii'}$ are Lipatov-Dickinson vertex functions describing the emission of gluons (or Higgs particles) [4–6].

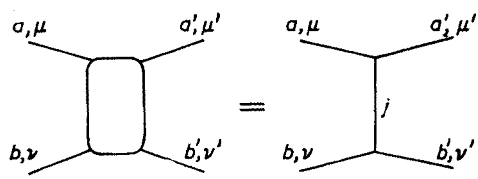

Fig. 3. The leading contribution for gluon-gluon scattering (each *ggg* vertex is a sum of three terms)

In the first order the dominant contribution comes from the gluon exchange (see Fig. 3). In what follows we assume that also external particles are gluons

$$A^{(1)}_{GG\to GG}(s,t) = g^2\frac{2s}{t-\lambda^2}f_{aja'}f_{b'jb}g_{\mu\mu'}g_{\nu\nu'}, \tag{2.6}$$

where we have suppressed Lorentz and colour indices of $A_{GG\to GG}$. Amplitude (2.6) should be contracted with the polarization 4-vectors of external gluons

$$\varepsilon(p,\vec{s}) = \left(\frac{\vec{s}\cdot\vec{p}}{\lambda},\ \vec{s}+\frac{\vec{p}}{\lambda}\frac{\vec{s}\cdot\vec{p}}{p_0+\lambda}\right),$$

where $\vec{s}$ is a spin vector of a particle at rest. Formula (2.6) gives the leading contribution only for transverse polarizations, whereas for longitudinally polarized gluons we have in fact to count all 9 terms which are represented by the graph in Fig. 3. This leads effectively to the replacement (for $t\sim\lambda^2$)

$$g_{\mu\mu'}\to\delta_{\lambda_A\lambda_A'}a_{\lambda_A}\equiv\begin{cases}-1 & \text{for transverse polarizations}\\ -\frac{1}{2} & \text{for longitudinal polarizations}\end{cases}$$

where by λ_A we have denoted polarization of a particle A (transverse $\lambda_A = 1, 2$ and longitudinal $\lambda_A = 3$). The quantity a_{λ_A} is usually included in the definition of the vertex function $\Gamma^i_{AA'}$ [4–6].

The next step is to calculate the imaginary part of the second order amplitude [4, 5]. This can be done via unitarity since we know the leading form of the first order amplitude

$$\mathrm{Im}\, A^{(2)}_{AB\to A''B''}(s,t) = \tfrac{1}{2} \sum_{A',B'} \int (2\pi)^4 \delta(p_A + p_B - k_{A'} - k_{B'}) \frac{d^3\vec{k}_{A'}}{2E_{A'}(2\pi)^3} \frac{d^3\vec{k}_{B'}}{2E_{B'}(2\pi)^3}$$

$$\times s^{\alpha(k_{A'}^2)+\alpha(k_{B'}^2)+2} \frac{\Gamma'^{i}_{AA'}\Gamma^{i}_{BB'}\Gamma^{j*}_{A'A''}\Gamma^{j*}_{B'B''}}{(k^2_{A'\perp}-\lambda^2)(k^2_{B'\perp}-\lambda^2)}. \tag{2.7}$$

The sum goes over gluons and Higgses in the intermediate states (the summation over the indices i, j is understood).

In order to calculate the full 2[nd] order amplitude we shall use dispersion relations with one subtraction [4, 5]. The result for octet exchange is given by

$$A_{AB\to A'B'} = \Gamma^i_{AA'} \frac{s}{t-\lambda^2} \Gamma^i_{BB'} \left(1 + \left(\ln s - \frac{i\pi}{2}\right)\alpha(t)\right). \tag{2.8}$$

Formula (2.8), as should be expected, is in agreement with our reggeization conjecture (2.5). One thing however should be noted at this point. Suppose that only gluons occur as intermediate particles A′, B′ in Eq. (2.7), then $A_{AB\to A'B'}(s,t)$ would be proportional to

$$a^2_{\lambda_A} \cdot a^2_{\lambda_B}$$

whereas from (2.8) it is clear that

$$A_{AB\to A'B'} \propto a_{\lambda_A} \cdot a_{\lambda_B}$$

(remember that each vertex $\Gamma^i_{AA'}$ includes a_{λ_A}). This matters, of course, only for longitudinal polarizations (since $a_{\lambda_A=3} = -\frac{1}{2}$).

We can rewrite $a^2_{\lambda_A}$ in a form

$$a^2_{\lambda_A} = -a_{\lambda_A}(1 - \tfrac{1}{2}\delta_{\lambda_A,3}).$$

It turns out that $\frac{1}{2} a_{\lambda_A}\delta_{\lambda_A,3}$ term is cancelled by the diagrams with Higgses emission, so that in fact longitudinally polarized internal gluons do not contribute to the scattering amplitude. Not only Higgs particles regularize infrared divergencies, but also cancel the contribution from the unphysical polarizations of the intermediate gluons.

The next step is to calculate the $A_{2\to 2+1}$ amplitude. In the lowest g^3 order this amplitude can be represented as a sum of five terms (see Fig. 4) each being a priori a different function of s and t, and having a different group factor.

It can be seen, however, that the energy momentum dependence of graphs (b) and (c) or (d) and (e) differs only in sign. Consider graphs (b) and (c): for diagram (c) $p_C^2 = s_1 \simeq 2p_{A'}p_D$ whereas for (b) $p_C^2 = -2p_A p_D \simeq -s_1$ if $t \ll s$ (i.e. $p_A \simeq p_{A'}$), so that both graphs have the same energy-momentum dependence. The group theoretical factor for sum of these two graphs can be therefore calculated using Jacobi identity [18] (see Fig. 5). The resulting group factor is exactly the same as for graph (a) of Fig. 4. The same procedure can be repeated for the lower parts of diagrams (d) and (e). Hence the amplitude $A_{2\to 2+1}$

has the following form: a group factor as for "central" gluon production of Fig. 4a multiplied by a function of s_1, s'_1, $\vec{p}_{C\perp}$, $\vec{p}_{C'\perp}$ which is the sum of the contributions of all diagrams (a)–(e), that is

$$A_{2\to 2+1}(s,t) = s\Gamma^{C}_{AA'}\frac{1}{\vec{p}_C^{\,2}-\lambda^2}\gamma^{D}_{CC'}(\vec{p}_{C\perp},\vec{p}_{C'\perp})\frac{1}{\vec{p}_{C'}^{\,2}-\lambda^2}\Gamma^{C'}_{BB'}.$$

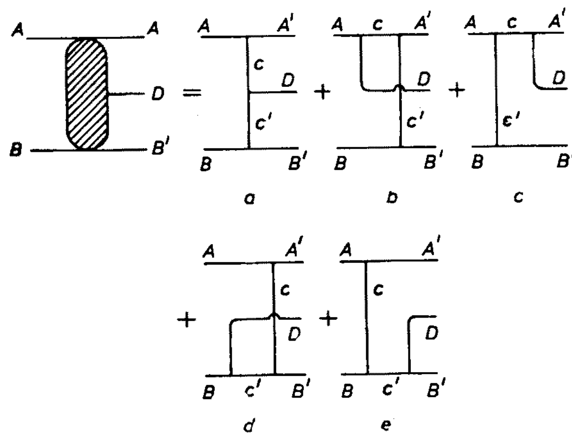

Fig. 4. One gluon production in gluon-gluon scattering

The explicit form of the functions $\gamma^{D}_{CC'}(\vec{p}_{C\perp}, \vec{p}_{C'\perp})$ can be found in Refs. [4–6]. The above reasoning can be extended [4, 5] for a multiparticle production amplitude $A_{2\to 2+n}$ yielding formula (2.8) in the lowest g^{2+n} order and can be represented by a graph on the r.h.s. of Fig. 2. The horizontal lines represent real QCD gluons (or Higgs particles), whereas the

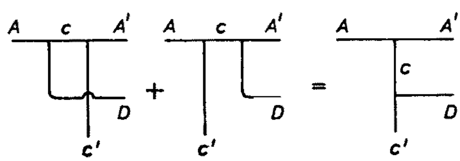

Fig. 5. Jacobi identity for graphs 4b and 4c

vertical lines stand for some "complex" objects obtained by a summation of a number of diagrams like those depicted in Fig. 4. These objects are just reggeized gluons and their properties will be sketched in Section 3.

Here we shall not come into the details of the higher order calculations [1–5]. The main features of these calculations are as in the lower orders, namely: longitudinal polarizations and Higgs particles emission cancel out, and the result agrees with our conjecture (2.5). Therefore in what follows we assume that formula (2.5) can be understood as an infinite sum of perturbation series in a leading ln s approximation. The procedure presented above leads to the full Reggeon calculus [22] of reggeized gluons with calculable vertices [15–17].

3. *Integral equations for Regge exchanges*

Now we come back to gluon-gluon (or in general hadron-hadron) elastic amplitude. So far we know that for gluon quantum numbers exchange this amplitude has a Regge pole given by Eq. (2.2). However the most interesting are, of course, colour singlet channels responsible for physical amplitudes in hadron scattering [6].

Once we have established the form of the n-particle production amplitude (2.5) we can derive the elastic amplitude via unitarity (see Fig. 6). In gluon-gluon scattering one can

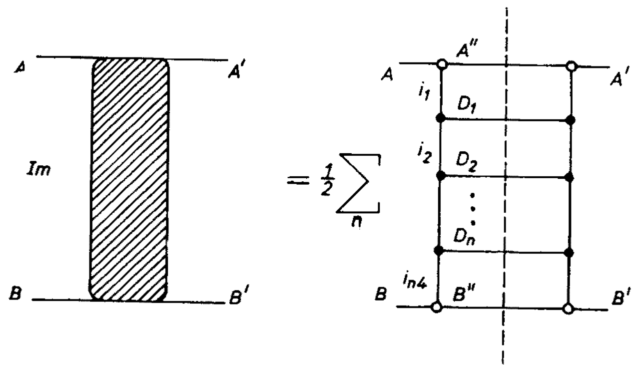

Fig. 6. Imaginary part of gluon-gluon elastic amplitude in terms of the multiperipheral-like multigluon amplitudes of Fig. 2

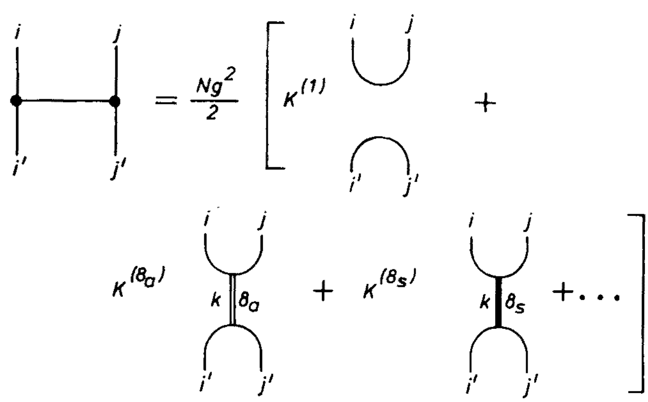

Fig. 7. Two vertex product projected onto the irreducible representations of SU(3) group

have singlet, symmetric (8_s) and antisymmetric (8_a) octets, and higher representations exchanged in t-channel. The behaviour of those amplitudes is governed by the product of two vertex functions (see Fig. 7) [5]

$$\sum_{D} \gamma^{D}_{ii'}(\vec{k}_{i\perp}, \vec{k}_{i'\perp})\gamma^{D*}_{jj'}(\vec{k}_{i\perp}-\vec{q}, \vec{k}_{i'\perp}-\vec{q}) = \frac{Ng^2}{2}\{K^{(1)}(\vec{k}_{i\perp}, \vec{k}_{i'\perp}; \vec{q})P_{iji'j'}(1)$$

$$+K^{(8a)}(\vec{k}_{i\perp}, k_{i'\perp}; \vec{q})P_{iji'j'}(8_a)+K^{(8s)}(\vec{k}_{i\perp}, \vec{k}_{i'\perp}; \vec{q})P_{iji'j'}(8_s)\},$$

where $P_{iji'j'}(\mu)$ are the group theoretical projection operators, μ denotes the representation, and the functions $K^{(\mu)}(k, k'; q)$ are given by [5, 20]:

$$K^{(1)}(k, k'; q) = K^{(1)}_{ns}(q^2) - 2K_s(k, k'; q),$$

$$K^{(\mu)}(k, k'; q) = K^{(\mu)}_{ns}(q^2) - K_s(k, k'; q), \tag{3.1}$$

where $\mu = 8_a$ or 8_s, transverse indices are suppressed. Here

$$K_s(k, k'; q) = \frac{(k^2-\lambda^2)((k'-q)^2-\lambda^2)+(k'^2-\lambda^2)((k-q)^2-\lambda^2)}{(k-k')^2-\lambda^2} \tag{3.2}$$

is singular for $k \to k'$, $\lambda = 0$. The nonsingular parts are given by [5, 20]:

$$K^{(1)}_{ns}(q^2) = 2q^2 - \tfrac{20}{9}\lambda^2,$$

$$K^{(8_a)}_{ns}(q^2) = q^2 - \lambda^2,$$

$$K^{(8_s)}_{ns}(q^2) = q^2 - \tfrac{13}{9}\lambda^2. \tag{3.3}$$

The difference in mass terms between $K^{(8_a)}$ and $K^{(8_s)}$ is caused by the emission of Higgs particles belonging to the symmetric octet [20]. It should be noted that the singular parts of $K^{(8_a)}$ and $K^{(8_s)}$ are the same and that the singular part of $K^{(1)}$ is two times larger than for octets.

In order to study the analytical structure of the elastic amplitude we shall make partial wave projection of this amplitude:

$$F^{(\mu)}_\omega(q^2) = \frac{1}{s}\int_1^\infty d\left(\frac{s}{\lambda^2}\right)\left(\frac{s}{\lambda^2}\right)^{-\omega-1} \operatorname{Im} A^{(\mu)}_{2\to 2}(s,t),$$

$$\omega = j-1. \tag{3.4}$$

The amplitude presented in Fig. 6 is given in terms of an infinite series which can be treated as the iteration of a certain integral equation. This equation is usually written for the function $f^{(\mu)}_\omega(k, k-q)$ defined by the equation [5, 6]:

$$F^{(\mu)}_\omega(q^2) = \frac{\pi}{2\omega}\frac{Ng^2}{2(2\pi)^3}\int \frac{d^2k' K^{(\mu)}_{ns}(q^2)}{(k'^2-\lambda^2)((k'-q)^2-\lambda^2)} f^{(\mu)}_\omega(k, k'-q)$$

and reads

$$[\omega - \alpha(k^2) - \alpha((k-q)^2)] f^{(\mu)}_\omega(k, k-q) = \frac{\omega}{K^{(\mu)}_{ns}(q^2)}$$

$$+\frac{Ng^2}{2(2\pi)^3}\int \frac{d^2k' K^{(\mu)}(k, k'; q)}{(k'^2-\lambda^2)((k'-q)^2-\lambda^2)} f^{(\mu)}_\omega(k', k'-q). \tag{3.5}$$

Let us check the infrared properties of Eq. (3.5). If we put $\lambda = 0$ for $\mu = 8_a$ or 8_s we obtain

$$\frac{K^{(\mu)}(k, k'; q)}{k'^2(k-q)^2} = -\frac{q^2}{k'^2(k'-q)^2} + \frac{1}{(k-k')^2}\left[\frac{k^2}{k'^2} + \frac{(k-q)^2}{(k'-q)^2}\right], \tag{3.6}$$

where by k we have denoted Euclidean transverse momenta. If $f_\omega^{(\mu)}$ is constant the second and the third term on the r.h.s. of Eq. (3.5) reproduce the trajectories (2.4) on the l.h.s. of Eq. (3.5). The remaining first term in Eq. (3.6) is not cancelled and therefore $\lambda \to 0$ limit does not exist for octet amplitudes. Keeping $\lambda \neq 0$ for $\mu = 8_a$ and 8_s we obtain

$$f_\omega^{(\mu)}(k, k-q) = \frac{\omega}{K_{ns}^{(\mu)}(q^2)} \frac{1}{\omega - \alpha^{(\mu)}(q^2)}, \tag{3.7}$$

where

$$\alpha^{(\mu)}(q^2) = \frac{Ng^2}{2(2\pi)^3} K_{ns}^{(\mu)}(q^2) \int \frac{d^2k}{(k^2-\lambda^2)((k-q)^2-\lambda^2)},$$

so that the amplitudes for octet exchanges have a moving Regge pole [5, 10, 11, 20].

For $\mu = 8_a$ this result was expected on the ground of the perturbation theory calculations presented in Section 2, hence for $\mu = 8_a$ (3.5) as a sort of self consistency check is often called a bootstrap equation. The new result is that the symmetric octet also has a pole [11, 20], but is one power of logarithm down with respect to the $\mu = 8_a$ exchange (see Eq. (2.3)). This is reflected in the signature factors of Eq. (2.2). Since antisymmetric and symmetric octets differ by their generalized C parities [24] their signature factors are also different. They are respectively $\exp\left(-i\frac{\pi}{2}\alpha^{(8_a)}(q^2)\right)\Big/\sin\left(\frac{1}{2}\pi\alpha^{(8_a)}(q^2)\right)$ and $i \cdot \exp\left(-i\frac{\pi}{2}\alpha^{(8_s)}(q^2)\right)\Big/\cos\left(\frac{1}{2}\pi\alpha^{(8_s)}(q^2)\right)$.

In the singlet case there is a complete cancellation of infrared divergencies [6]. Trajectory $\alpha(t)$ defined in Eq. (2.4) is logarithmically divergent, but since there are two sources of this divergence, namely $k \to 0$ and $k \to q$, trajectory $\alpha(q^2)$ is proportional to $2 \ln \frac{q^2}{\lambda^2}$. On the r.h.s. of Eq. (3.5) the infrared divergencies come from three regions of phase space:

1) $k' \to k$ giving $\left((2 \ln \frac{k^2}{\lambda^2} + 2 \ln \frac{k^2+q^2}{\lambda^2}\right) \cdot f_\omega^{(1)}(k, k-q)$. Factor 2 is just because $K^{(1)} = K_{ns}^{(1)} - 2K_s^{(1)}$. These logarithms and the trajectories on the l.h.s. of Eq. (3.5) cancel out.

2) $k' \to 0$ giving $2\left(\frac{-q^2}{k'^2q^2} + \frac{k^2}{k'^2k^2} + \text{finite terms}\right)$,

3) $k' \to q$ giving $2\left(\frac{-q^2}{q^2(k'-q)^2} + \text{finite terms} + \frac{(k-q)^2}{(k-q)^2(k'-q)^2}\right)$. In cases 2) and 3) the infrared divergencies also cancel because $K_{ns}^{(1)}$ and $K_s^{(1)}$ have opposite signs.

This proves that the integral equation (3.5) for the singlet exchange — the Pomeron — is infrared safe and we can put λ equal to 0.

Unfortunately, in the singlet case one can solve Eq. (3.5) only for $t = 0$. Let us rewrite Eq. (3.5) in the following form:

$$f_\omega(k_1^2, k_2^2) = \frac{\pi}{\omega} k_2^2 \delta(k_1^2 - k_2^2)$$

$$+ \frac{1}{\omega} \int \frac{d^2k'}{(k'^2)^2} \left[\frac{Ng^2}{(2\pi)^3} K^{(1)}(k_1, k'; 0) + 2\alpha(k_1^2)\delta^{(2)}(k_1 - k') \right] f_\omega(k'^2, k_2^2). \tag{3.8}$$

As one can easily check the effective kernel appearing in Eq. (3.8) is scale invariant and therefore the Pomeron equation can be diagonalized by a Mellin transform [6].

$$f_\omega(k_1^2, k_2^2) = \frac{(k_2)^2}{k_1^2} \int \frac{d\nu}{2\pi i} (k_1^2)^{-\nu} \varphi_\omega(\nu).$$

The solution for $\varphi_\omega(\nu)$ reads

$$\varphi_\omega(\nu) = \frac{(k_2^2)^\nu}{\omega - \dfrac{Ng^2}{(2\pi)^2} K(\nu)}, \tag{3.9}$$

$$K(\nu) = -2\gamma_E - \psi(1-\nu) - \psi(1+\nu) + \frac{1}{\nu}, \tag{3.9}$$

$\gamma_E = 0.57721...$ Euler constant, $\psi(z)$ digamma function [26].

The Mellin transform of the effective kernel of Eq. (3.8), $K(\nu)$ is a symmetric function of ν with respect to $\nu_0 = \frac{1}{2}$. Therefore if one performs the inverse Mellin transform integrating over $d\nu$ along a certain path there is a "pinch" of singularities and as a consequence $f_\omega(k_1^2, k_2^2)$ has a branch point in $\omega_0 = \dfrac{Ng^2}{(2\pi)^2} 2 \ln 2$ and a cut along the positive real axis in ω-plane [6]. The result $\omega_0 > 0$ (i.e. $j_0 > 1$) yields the Pomeron intercept above unity and therefore contradicts unitarity. This is of course due to our leading logarithmic approximation.

It was argued [6] that for the nonforward scattering the position of the branch point does not change. The discussion of this equation can be found in Ref. [6].

Since the Pomeron equation describes the propagation of gluons in a colour singlet state and does not depend on couplings to the external particles one can compare its solutions with the solutions of DDT [8] equation for small x [21]. It is well known that in a small x region the dominant contribution for DIS amplitudes comes only from gluon exchanges [7, 8]. So one can compare the solutions $f_\omega(k_1^2, k_2^2)$ for large k_2^2 with the gluon distribution functions [8] $D_{G\to G}(x)$ calculated for small $x (x \propto s^{-1})$.

In DIS one takes into account running coupling constant effects which are non-leading in the Regge limit, and therefore neglected [15]. If for the moment one also neglects running coupling constant in the DDT equation, one will find that the result in both limits for

$f_\omega(k_1^2, k_2^2)$ and $D_{G\to G}(x)$ is proportional to the modified Bessel function [21]

$$I_1\left(\frac{1}{2}\sqrt{\frac{Ng^2}{(2\pi)^2}\ln\frac{1}{x}\ln\frac{k_2^2}{\lambda^2}}\right).$$

This result was obtained by approximating the kernel $K(v)$ by its pole term (see Eq. (3.9)); $K(v) \approx \frac{1}{v}$.

One can attempt to include next-to-leading $\ln s$ term in the expansion for the amplitudes in the Regge limit [27]. This can be done by introducing the running coupling constant into the Pomeron equation [7, 21]:

$$\frac{Ng^2}{(2\pi)^3} \to \frac{16\pi^2}{b\ln\frac{k_2^2}{k_1^2}} \equiv \frac{\bar{g}^2}{4\pi\ln\frac{k_2^2}{k_1^2}}.$$

This replacement introduces a new scale into the effective kernel of Eq. (3.8) which is no longer scale invariant and therefore cannot be diagonalized by means of Mellin transform. However, if we perform Mellin transform with respect to the variable $\frac{k_2^2}{k_1^2}$, then the integral equation (3.8) can be converted into the differential one [21]:

$$\frac{d}{dv}\varphi_\omega(v) = \frac{\bar{g}^2}{\omega}\varphi_\omega(v)K(v) \tag{3.10}$$

and the solution is given by

$$\varphi_\omega(v) = \frac{1}{\omega}\exp\left(\frac{\bar{g}^2}{\omega}\int^v K(v')dv'\right). \tag{3.11}$$

It should be stressed that Eq. (3.10) is valid only for large k_2^2 since we have approximated running coupling by its asymptotic form [7, 8]. Therefore the solution (3.11) is also valid only for large k_2^2.

Function $\varphi_\omega(v)$ has a pole in $v = 0$ and it can be seen that leading $\ln k_2^2$ behaviour of $f\omega(k_1^2, k_2^2)$ is governed not approximately (like in case of Eq. (3.9)) but exactly by a pole term of the kernel $K(v)$. This leads to the essential singularity in $\omega_0 = 0$. In this approximation (large k_2^2) the Pomeron intercept is $j_0 = 1$.

Comparing the results for the solutions of the DDT and running coupling constant "improved" Pomeron equations we find that they are proportional to [21]:

$$I_1\left(\frac{1}{2}\sqrt{\bar{g}^2\ln\frac{1}{x}\ln\ln\frac{k_2^2}{\lambda^2}}\right).$$

It is worthwhile to note that we have introduced the Pomeron equation assuming the reggeization of the vector meson in NAGT. Although the amplitudes with colour quantum numbers exchange are unphysical, and therefore infrared divergent, their behaviour is essential for the infrared finiteness of the colour singlet exchanges. We have seen that divergent parts of the kernel of the Pomeron equation and the trajectories, which are formally infrared divergent cancelled out.

The multiperipheral-like equation (3.8) because of the scale invariance of its kernel generates a fixed branch point in j-plane at $j_0 > 1$. The asymptotic freedom corrections, valid only for large k^2 remove this scale invariance and push the leading singularity in -plane to 1.

4. *Three gluon integral equation*

In this section we shall present the detailed derivation of the integral equation for the exchange of the system of three reggeized gluons being in the colour singlet state [19], and therefore having the generalized C-parity [24] $C_{3G} = -1$ (in the Pomeron case $C_P = 1$). This amplitude can contribute to the "nonsinglet" structure function F_3 in deep inelastic neutrino scattering for $x \to 0$ [19]. This contribution is nonleading in the $\ln Q^2$ expansion but may be enhanced at $x \to 0$ since it corresponds to the Regge singularity with intercept above 1. But as we shall see the three gluon equation is interesting by itself because of mathematical complexity; noncompactness and nonseparability of its kernel.

In order to derive this equation we shall use the Reggeon calculus in ω-plane [22]. The main ingredients of this calculus have been already presented in the previous sections. We shall briefly remind the basic rules of the Reggeon calculus.

One Reggeon exchange amplitude, as seen from Eqs. (3.7) and (2.2) is given by

$$F_\omega^{(8a)}(t) = \frac{1}{t-\lambda^2} \cdot \frac{\frac{\pi}{2}\alpha(t)}{\omega-\alpha(t)}.$$

So with each Reggeon line carrying transverse momentum k we shall associate the propagator

$$G(k, \alpha(k^2)) = \frac{\frac{\pi}{2}\alpha(k^2)}{k^2-\lambda^2}. \tag{4.1}$$

Factor $(\omega-\alpha(k^2))^{-1}$ according to the Gribov rules appears for each one Reggeon intermediate state, whereas for each two or three Reggeons carrying momenta k_1, k_2 or k_1, k_2, k_3 one has the following factors:

$$(\omega-\alpha(k_1^2)-\alpha(k_2^2))^{-1} \quad \text{and} \quad (\omega-\alpha(k_1^2)-\alpha(k_2^2)-\alpha(k_3^2))^{-1}.$$

The interaction of two Reggeons beeing in the μ-th representation of SU(N) group is given by the coupling

$$\frac{Ng^2}{2} K^{(\mu)}(k_1, k_1'; q)\gamma_{1'2'},$$

$$k_2 = k_1 - q \quad \text{and} \quad k_2' = k_2 - q,$$

where $\gamma_{1'2'}$ is the signature factor associated with the "emission" of two Reggeons 1' and 2'. As it was previously mentioned, one Reggeon exchange amplitude has signature $\sigma = -1$, hence the resulting signature factor is

$$\eta_\sigma(\alpha) \equiv \frac{e^{-i\pi\alpha} - \sigma}{\sin \pi\alpha} \underset{\sigma=-1}{=} \frac{e^{-i\frac{\pi}{2}\alpha}}{\sin \frac{\pi}{2}\alpha}.$$

With n-Reggeon emission one associates the signature factor [22]:

$$\gamma_{12\ldots n} \equiv \mathrm{Im}\,(\eta_\sigma(\alpha_1) i\eta_\sigma(\alpha_2) \ldots i\eta_\sigma(\alpha_n)),$$

so that

$$\gamma_{12} = \frac{1}{\sin \frac{\pi}{2}\alpha_1 \cdot \sin \frac{\pi}{2}\alpha_2}, \tag{4.2a}$$

$$\gamma_{123} = \frac{-(\alpha_1+\alpha_2+\alpha_3)}{\sin \frac{\pi}{2}\alpha_1 \cdot \sin \frac{\pi}{2}\alpha_2 \cdot \sin \frac{\pi}{2}\alpha_3}. \tag{4.2b}$$

In the leading ln s approximation we keep only terms $(g^2 \ln s)^n$ and therefore expanding $\sin \frac{\pi}{2}\alpha$ in the denominators of Eqs. (4.2a, b) we notice that the resulting terms $\frac{\pi}{2}\alpha$ and the appropriate $\frac{\pi}{2}\alpha$ from the Reggeon propagator (4.1) cancel out. So in the weak coupling limit we obtain [10, 19]:

$$\gamma_{12} \to 1, \tag{4.3a}$$

$$\gamma_{123} \to -(\alpha_1+\alpha_2+\alpha_3) = -\omega, \tag{4.3b}$$

and

$$G(k^2, \alpha(k^2)) \to G(k^2) = \frac{1}{k^2-\lambda^2}. \tag{4.3c}$$

In this approximation we do not consider nonleading interactions as $3 \to 3$, or more Reggeon lines than the minimal number needed to exchange given quantum numbers in t-channel (one for $\mu = 8$ exchange, two for the Pomeron, three for the $C = -1$ singlet). Using the above rules one can easily obtain Eq. (3.8) for the Pomeron.

Now we shall come to the question of the three gluon exchange amplitude, whose partial wave projection can be written as below (see Fig. 8):

$$F_\omega(t) = \int \prod_{i=1}^{3} d^2k_i \delta^{(2)}\left(\sum_{i=1}^{3} k_i - \Delta\right) \tilde{T}(\omega; k_1, k_2, k_3) \frac{\omega}{\omega - \sum_{i=1}^{3} \alpha(k_i^2)} F_0(k_1, k_2, k_3).$$

Here $\tilde{T}$ is the 3 reggeized gluons-two hadrons amplitude, F_0 is the vertex function which can be expressed in terms of the infinite momentum wave function [28], ω is the signature factor in the weak coupling limit (4.3b), and $G(k^2)$ is the Reggeon propagator of Eq. (4.3c).

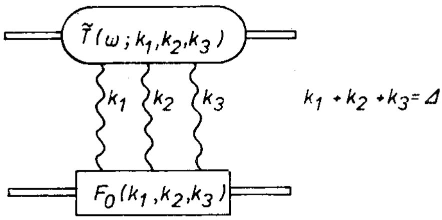

Fig. 8. Three reggeized gluon exchange amplitude

The leading ln s expansion of the amplitude $\tilde{T}$ in terms of Reggeon diagrams is depicted in Fig. 9. Using the Reggeon calculus, with $\gamma_{12} = 1$ one can write down, as in Pomeron case, the integral equation for the amplitude T defined below

$$T(\omega; k_1, k_2, k_3) = (\omega - \sum_{i=1}^{3} \alpha(k_i^2))\tilde{T}(\omega; k_1, k_2, k_3).$$

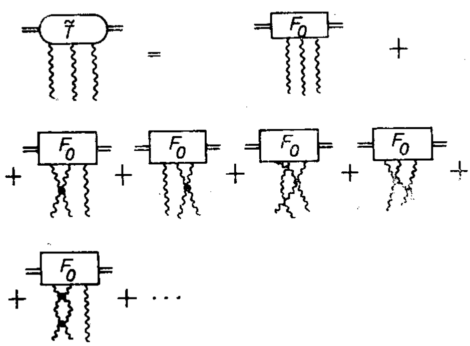

Fig. 9. Perturbative expansion of $\tilde{T}(\omega)$ defined in text in terms of the Reggeon diagrams

It should be stressed that each pair of two interacting Reggeons is in the symmetric octet state, so that all three reggeized gluons are in the $C = -1$ singlet state. Therefore the corresponding interaction vertex is just $K^{(8_s)}$). But, as we shall see, the amplitude T has no infrared divergencies and therefore we can put $\lambda = 0$. In this limit, however, there is no difference between $K^{(8_a)}$) and $K^{(8_s)}$).

The integral equation for the amplitude T reads [19]

$$[\omega - \sum_{i=1}^{3} \alpha(k_i^2)]T(\omega; k_1, k_2, k_3) = F_0(k_1, k_2, k_3) + \int \prod_{i=1}^{3} d^2k_i'\delta^{(2)}(\sum_{i=1}^{3} k_i - \Delta) \sum_{l=1}^{3}$$
$$\times [G(k_{l-1}'^2)G(k_{l+1}'^2)K_l(k_l; k_{l+1}, k_{l-1}, k_{l+1}', k_{l-1}')\delta^{(2)}(k_l - k_l')]T(\omega; k_1', k_2', k_3'), \quad (4.4)$$

where $k_4 = k_1$, $k_5 = k_2$ etc.

The kernels $K_l(k_l; k_{l+1}, k_{l-1}, k'_{l+1}, k'_{l-1})$ (here by l we denote the number of the gluon line, $l = 1, 2, 3$) for massless theory are given by

$$K_l(k_l; k_{l+1}, k_{l-1}, k'_{l+1}, k'_{l-1}) = \frac{Ng^2}{2(2\pi)^3}\left[-k_l^2 + \frac{k_{l-1}^2 k'^2_{l+1} + k_{l+1}^2 k'^2_{l-1}}{(k_{l+1} - k'_{l+1})^2}\right].$$

To shorten our notation we define new kernels V_l

$$V_l \equiv G(k'^2_{l-1})G(k'^2_{l+1})K_l.$$

Now Eq. (4.4) can be written symbolically

$$[\omega - \alpha(1) - \alpha(2) - \alpha(3)]T(\omega; 1, 2, 3) = F_0(1, 2, 3) + (V_1 + V_2 + V_3) \otimes T(\omega; 1', 2', 3').$$

In what follows we shall put $\Delta = 0$. This implies that $k_1 + k_2 + k_3 = 0$ and $k'_1 + k'_2 + k'_3 = 0$. Again the infrared properties of Eq. (4.4) can be studied as in the Pomeron case.

The infrared divergencies introduced by the propagators $G(k^2)$ coming from the region $k'_{l+1} \to 0$ or $k'_{l+1} \to k_l$ (i.e. $k'_{l-1} \to k_l$ and $k'_{l-1} \to 0$) are cancelled by appropriate zeros of the kernel ($K_{ns}^{(8_s)}$ and $K_s^{(8_s)}$ have opposite signs) exactly in the same way as in the Pomeron case (see Section 2).

Another source of divergencies in the potentials V_l is the region where $k'_{l+1} \to k_{l+1}$ (i.e. $k'_{l-1} \to k_{l-1}, k'_l = k_l$). Consider $k'_1 \to k_1$ ($k'_2 \to k_2$), then there are two divergent terms in the whole kernel $V_1 + V_2 + V_3$ which reproduce trajectory $\alpha(k_1^2)$ standing on the l.h.s. of Eq. (4.4)

1) in $$V_3 : \frac{k_1^2}{(k_1 - k'_1)^2 k'^2_1} T(\omega; k_1, k_2, k_3),$$

2) in $$V_2 : \frac{k_1^2}{(k_2 - k'_2)^2 k'^2_1} T(\omega; k_1, k_2, k_3).$$

So again trajectory and the singular part of the kernel $\left(\text{both proportional to } 2\ln\frac{k_1^2}{\lambda^2}\right)$ cancel out. We should remind that in the Pomeron equation factor 2 which guaranties this cancellation is just the strenght of the kernel itself (see Eq. (3.1)), whereas here there are *two* divergent terms in $V_1 + V_2 + V_3$ which add up to produce the needed factor 2. The same can be repeated for k'_2 and k'_3. This proves that Eq. (4.4) is manifestly free from infrared divergencies. So in what follows we shall keep $\lambda = 0$.

It is easy to check that Eq. (4.4) has a trivial solution if F_0 = const. Then the trajectories and the appropriate parts of the kernel cancel out and the solution reads

$$T(\omega) = \frac{F_0}{\omega}.$$

This trivial solution generates a fixed pole in j-plane at $j_0 = 1$. In what follows we shall try to find out the position of the most right singularity in j-plane.

The main difficulty with Eq. (4.4) is, that because of $\delta^{(2)}(k_l - k'_l)$ function its kernel has an infinite Schmidt norm, and therefore the Eq. (4.4) is not of Fredholm type. The equations with such kernels were investigated by Fadeev [29] by means of so called Fadeev decomposition. We shall explain how it works in our case.

Consider an equation

$$\omega T(\omega; 1, 2, 3) = F_0(1, 2, 3)+(\tilde{V}_1+\tilde{V}_2+\tilde{V}_3)\otimes T(\omega; 1', 2', 3'), \tag{4.5}$$

where $\tilde{V}_i$ contains V_i and some combination of trajectories. One defines quasi two-body amplitude $t_1(1, 2, 3)$ as below:

$$t_1(1, 2, 3) = F_0(1, 2, 3)+V_1(2', 3') \otimes t_1(1, 2', 3') \tag{4.6}$$

and similarly for t_2 and t_3. As we see from Eq. (4.6) t_1 is defined in such a way that interaction takes place only between the second and the third line.

Now we decompose $\omega T-F_0$ into three parts with inhomogeneous terms given by t_i [19, 29]:

$$\omega T-F_0 = \omega \sum_{i=1}^{3} T^{(i)}$$

$$T^{(i)} = t_i+\tilde{V}_i\otimes(T^{(1)}+T^{(2)}+T^{(3)}). \tag{4.7}$$

Functions $T^{(i)}$ have the same analytical form

$$T^{(i)} = t_i(\omega; k_i; k_{i-1}, k_{i+1}),$$

they differ only by the order of arguments.

So far we have not specified the form of the kernels $\tilde{V}_i$. The only constraint is the infrared finitness of Eq. (4.7), and we present two possibilities below:

$$\begin{aligned}\omega T = F_0&+[V_1-\alpha(1)+\alpha(2)+\alpha(3)]\otimes T\\&+[V_2-\alpha(2)+\alpha(3)+\alpha(1)]\otimes T\\&+[V_3-\alpha(3)+\alpha(1)+\alpha(2)]\otimes T,\end{aligned} \tag{4.8a}$$

$$\begin{aligned}\omega T = F_0&+[V_1+\tfrac{1}{2}\,\alpha(2)+\tfrac{1}{2}\,\alpha(3)]\otimes T\\&+[V_2+\tfrac{1}{2}\,\alpha(3)+\tfrac{1}{2}\,\alpha(1)]\otimes T\\&+[V_3+\tfrac{1}{2}\,\alpha(1)+\tfrac{1}{2}\,\alpha(2)]\otimes T,\end{aligned} \tag{4.8b}$$

(compare with Eq. (4.5)). Sign $\otimes$ is understood as the convolution in case of V_i and as a multiplication in case of $\alpha(i)$. To be more specific we shall rewrite Eq. (4.7) using the decomposition of Eq. (4.8a) [19]:

$$\omega t(\omega; k; k_1, -k_1-k) = t_0(k, k_1)+\frac{Ng^2}{2(2\pi)^3}\int\frac{d^2k_1'}{k_1'^2(k_1'+k)^2}$$

$$\times\left[-k^2+\frac{k_1'^2(k_1+k)^2+(k_1'+k)^2k_1^2}{(k_1'-k_1)^2}\right][t(\omega; k, k_1', -k-k_1')-t(\omega; k, k_1, -k-k_1)$$

$$+t(\omega; k_1', k, -k-k_1')-t(\omega; k_1, k, -k-k_1)+t(\omega; -k-k_1', k_1', k)$$

$$-t(\omega; -k-k_1, k_1, k)].$$

The analytic properties of $t(\omega)$ can be investigated by means of Mellin transform

$$t(\omega; k; k_1, -k-k_1) = \int \frac{d\lambda}{2\pi i} (k^2)^\lambda t(\omega; \lambda; x_1, n),$$

$$x_i = \frac{k_i}{|k_i|}, \qquad n = \frac{k}{|k|}$$

giving

$$\omega t(x, n) = t_0(x, n) + \frac{Ng^2}{2(2\pi)^3} \int \frac{d^2x'}{x'^2(x'+n)^2} \left[-1 + \frac{x'^2(x+n)^2 + (x'+n)^2 x^2}{(x'-x)^2} \right]$$

$$\times \left[t(x', n) - t(x, n) + t\left(-\frac{x'+n}{|x'|}, \frac{x'}{|x'|} \right)(x'^2)^\lambda - t\left(-\frac{x+n}{|x|}, \frac{x}{|x|} \right)(x^2)^\lambda \right.$$

$$\left. + t\left(\frac{n}{|x'+n|}, -\frac{x'+n}{|x'+n|} \right)(x'+n)^{2\lambda} - t\left(\frac{n}{|x+n|}, -\frac{x+n}{|x+n|} \right)(x+n)^{2\lambda} \right], \qquad (4.9)$$

where by x, n, k we have denoted 2 dim vectors and indices ω and λ have been skipped.

Unfortunately, due to the lack of separability of the kernel this equation cannot be solved in a close form. One can however argue that Eq. (4.9) generates a fixed branch point in j-plane [19] since, as in the Pomeron case, there is a pinch of the integration contour in complex λ-plane. However, the most interesting thing, the position of the singularity has not been found yet.

The integral equation formulated above, after the Pomeron equation is another example of the physical amplitude which can be calculated by means of the QCD-based Reggeon calculus. The structure of this equation is much more complicated than in the Pomeron case, but the qualitative result is the same, namely the fixed branch point in j-plane.

5. *Summary*

We have shown that the perturbative calculations in NAGTs [1–6] support the hypothesis of the vector meson reggeization [9–15]. The calculations were performed in the framework of the spontaneously broken theory [25] and we have shown that the Higgs particles produced, say, in gluon-gluon scattering cancel the contributions from unphysical longitudinal polarizations of gluons. The leading contribution comes always from spin one gluon exchanges in t-channel, so that Higgs particles can be produced but never exchanged.

The integral equations describing the nonsinglet exchanges are infrared divergent [4, 5], so we have to keep $\lambda \neq 0$, contrary to the singlet case [6, 19] where we can put $\lambda = 0$. The symmetric octet amplitude has a Regge pole structure [11, 20] as well as the antisymmetric one [2–5, 10], whereas the Pomeron is described by a fixed branch point in j-plane.

Using the results for 8_a amplitudes we have established the form of the interaction vertices for the QCD based Gribov Reggeon calculus [22] of reggeized gluons [14–17]. Then we have used this calculus to formulate the integral equation for three gluons beeing in a colour singlet state [19]. We have argued that this equation generates a fixed branch point in j-plane at $j_0 \gtrsim 1$. It should be noted that the gluonic degrees of freedom have the high intercept even for nonvacuum quantum numbers.

The author is indebted to Doctor J. Kwieciński for numerous discussions and helpful remarks and to Doctor T. Jaroszewicz for reading the manuscript.

REFERENCES

[1] B. Mc Coy, T. T. Wu, *Phys. Rev.* **D12**, 1076, 3257 (1976).
[2] L. Tyburski, *Phys. Rev.* **D13**, 1107, (1976).
[3] H. Cheng, C. Y. Lo, *Phys. Rev.* **D13** 1131 (1976); **D15**, 2959 (1977).
[4] L. N. Lipatov, *Yad. Fiz.* **23**, 642 (1976).
[5] E. A. Kuraev, L. N. Lipatov, V. S. Fadin, *Zh. Eksp. Theor. Fiz.* **71**, 840 (1976).
[6] Ya. Ya. Balitzkij, L. N. Lipatov, *Yad. Fiz.* **28**, 1597 (1978).
[7] Yu. L. Dokshitzer, *Zh. Eksp. Theor. Fiz.* **73**, 1216 (1977).
[8] Yu. L. Dokshitzer, D. I. D'yakonov, S. I. Troyan, *Phys. Rep.* **58**, 1 (1980).
[9] M. T. Grisaru, H. J. Schnitzer, H. S. Tsao, *Phys. Rev.* **D8**, 4498 (1973); **D9**, 2864 (1974).
[10] J. B. Bronzan, R. L. Sugar, *Phys. Rev.* **D17**, 585 (1978).
[11] J. B. Bronzan, R. L. Sugar, *Phys. Rev.* **D17**, 2813 (1978).
[12] A. R. White, *Nucl. Phys.* **B159**, 77 (1979).
[13] L. Łukaszuk, L. Szymanowski, *Nucl. Phys.* **B159**, 316 (1979).
[14] J. Bartels, *Nucl. Phys.* **B151**, 293 (1979).
[15] J. Bartels, *Acta. Phys. Pol.* **B11**, 281 (1980).
[16] T. Jaroszewicz, *Acta. Phys. Pol.* **B11**, 965 (1980), Triest preprint IC/80/175.
[17] J. Bartels, T. Jaroszewicz to be published.
[18] G. M. Cicuta, *Phys. Rev. Lett.* **43**, 826 (1979).
[19] J. Kwieciński, M. Praszałowicz, *Phys. Lett.* **94B**, 413 (1980).
[20] J. Kwieciński, M. Praszałowicz, *Acta. Phys. Pol.* **B12**, 103 (1981).
[21] M. Praszałowicz, *Acta Phys. Pol.* **B12**, 765 (1981).
[22] M. Baker, K. A. Ter-Martyrosjan, *Phys. Rep.* **28C**, 2 (1976).
[23] J. Bartels, *Phys. Rev.* **D11**, 2977, 2989 (1975).
[24] M. Gell-Mann, *Phys. Rev. Lett.* **12**, 155 (1964).
[25] K. Bardakci, M. Halpern, *Phys. Rev.* **D6**, 696 (1972).
[26] M. Abramowitz, I. A. Stegun, *Handbook of Mathematical Functions*, Dover. Pub., Inc., New York 1970.
[27] D. Heckatorn, *Phys. Rev.* **D18**, 1286 (1978).
[28] M. Fukugita, J. Kwieciński, *Phys. Lett.* **83B**, 119 (1979).
[29] L. D. Fadeev, *Zh. Eksp. Theor. Fiz.* **39**, 1459 (1960).