REMARK ON THE COPENHAGEN VACUUM

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(*Received September 26, 1980*)

It is pointed out that the Yang Mills vacuum structure proposed by the Copenhagen group can be obtained very simply and within their assumptions, when their gauge condition is used instead of the linearised equations of motion. Other choices of gauge are shown to lead to higher energy densities than that chosen. The full equations of motion together with the gauge condition admit only a trivial solution with zero colour-transverse gluons.

PACS numbers: 11.10.Np, 12.20.—m

A tentative description of the ground state in pure Yang Mills theories has been developed by a group in Copenhagen [1–8]. Their proposal was triggered by the observation made by Savvidy [9] that, because of quantum fluctuations, the state which corresponds classically to a non-zero homogeneous colour magnetic field, may have lower energy than the state corresponding to zero colour fields. Nielsen and Olesen noticed that energy can be further lowered by adding gluons with magnetic moments parallel to the field [1, 2]. The search for a lowest energy state had been refined and finally a theory formally similar to the theory of type II superconductors in magnetic fields emerged [8]. It has been stressed that the ordered structure typical of superconductors is strongly modified by quantum fluctuations so that space filled with a "spaghetti of color magnetic tubes" [6, 7] is probably a better picture than a crystal. The fluctuation effects, however, have only been qualitatively discussed and we do not consider them here.

The vacuum structure worked out in Refs. [1–8] was obtained by solving linearized equations of motion corresponding to the standard Lagrangian of the Yang Mills theory, and choosing the solution for gluon fields which corresponds to the lowest classical energy.

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The following assumptions and/or results from Refs. [1–8] are used in our further analysis. The third component in colour space of the gluon field has been fixed by the assumption

$$A_\mu^3 \equiv A_\mu = Hx_1\delta_{\mu,2}, \tag{1}$$

where δ is a Kronecker delta. Following Refs [1–8], we use as colour-transverse fields

$$W_\mu = \frac{1}{\sqrt{2}}(A_\mu^1+iA_\mu^2) \tag{2}$$

and its complex conjugate. In this notation it is assumed (cf., e.g., [7]) that

$$W \equiv W_1 = -iW_2, \qquad W_3 = W_4 = 0. \tag{3}$$

Assumptions (1) and (3) have a simple classical interpretation: A_μ^3 provides the homogeneous magnetic field, which according to Savvidy [9] has lower energy than the perturbative vacuum, when quantum fluctuations are included, and $W_1 \neq 0$ with condition (3) corresponds to the presence of gluons all with magnetic moments parallel to the magnetic field. Finally, the energy density, which using standard formulae and assumptions (1) and (3) reads

$$\varepsilon = \tfrac{1}{2}(H-2g|W|^2)^2+2\left|\frac{\partial W}{\partial x_3}\right|^2+2\left|\frac{\partial W}{\partial x_4}\right|^2+\left|\left(\frac{\partial}{\partial x_1}+i\frac{\partial}{\partial x_2}+gHx_1\right)W\right|^2, \tag{4}$$

should be minimized with respect to W at fixed H. Stability with respect to changes in H is ascribed to quantum fluctuations, which however are supposed not to interfere with the determination of W from (4). In order to support this assumption one would have to calculate an effective Yang Mills lagrangian at least at the one loop level. This is a difficult problem (cf., e.g., [10]) not solved generally even in QED.

The gauge condition for field W is chosen as [2]

$$(\partial_\mu-igA_\mu)W^\mu = 0. \tag{5}$$

Substituting formulae (1) and (3) into condition (5) we find

$$\left(\frac{\partial}{\partial x_1}+i\frac{\partial}{\partial x_2}+gHx_1\right)W = 0. \tag{6}$$

Usually the gauge condition only eliminates unphysical degrees of freedom. Here, however, with only one unknown function W, it can replace the equations of motion. As shown below, the structure obtained in Refs. [1–8] follows easily from condition (6).

We observe that the general solution of equation (6) is

$$W(x_1, x_2, x_3, x_4) = \exp\left(-\frac{gHx_1^2}{2}\right)F(z, x_3, x_4), \tag{7}$$

where $z = x_1+ix_2$ and F is an arbitrary function differentiable with respect to z. From the minimum condition for the energy density (4), we immediately conclude that F should be independent of x_3 and x_4 and we have

$$W = \exp\left(-\frac{gHx_1^2}{2}\right)F(z), \tag{8}$$

which is the crucial result from Refs. [1–8]. It has been made plausible that the modulus $|W|$ should be periodic in both x and y. Many other choices lead to reductions in energy density, which on the average tend to zero, when the volume increases to infinity [5]. The assumption that W should be doubly periodic [5] may be equally plausible a priori, but it has been checked numerically that it leads to less reduction in the energy density [7]. Since $|W|$ is doubly periodic, the function $F(z)$ must be expressible in terms of elliptic functions [11]. The actual calculation involves no physics and has been described in detail in Ref. [7]. The remaining free parameters should be adjusted so as to minimize the energy density (4). This reproduces completely the results obtained in Refs [1–8]. Given the assumptions (1), (3), (4) and (5), the solution is exact. In particular, no linearisation is involved.

We conclude with a few remarks.

— The choice of gauge (5) is crucial for our argument. Another gauge condition would in general lead to a different solution for W. For example, the condition $\partial^\mu W_\mu = 0$ is inconsistent with (7). In Refs. [1–8] it was an open problem, why gauges other than (5) were excluded. The analysis from Refs. [1–8] shows that this problem is closely related to the problem of justifying the use of linearized equations of motion.

— It is not clear, whether the linearized equations of motion have much merit. The terms quadratic and higher order in W must be important, since they reduce the energy density by a factor of seven [7] from its "unperturbed" value $\frac{1}{2}H^2$. Besides the solution used in Refs [1–8], the linearized equations of motion have many other solutions, but they are all excluded by the gauge condition (5). Finally, the linearized equations suggest a harmonic time dependence of W (cf., e.g., [5], Eq. (10)), which has then to be dropped.

— Substituting solution (7) into the full (non linearized) equations of motion one finds $W = 0$ and there is no unstable mode. We conclude that the vacuum structure from Refs. [1–8] is non classical in the sense that it does not satisfy the classical equations of motion.

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