COHERENT PRODUCTION ON NUCLEI AND MEASUREMENTS OF TOTAL CROSS SECTIONS FOR UNSTABLE PARTICLES

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(*Received March 3, 1980*)

The Kölbig–Margolis formula is fitted to some explicitly nonperturbative models of diffractive production. It is shown that, in spite of the fact that the standard procedure of fitting the integrated cross sections may give acceptable fits, thus obtained "cross sections of unstable particles", σ_2, grossly disagree with the "true" cross sections known exactly from the models.

1. *Introduction*

There have been many "measurements" of the total cross sections of unstable particles from their diffractive production on nuclear targets [1–4]. It has also been clearly realized that these "measurements" are model dependent because the numerical characteristics of attenuations of produced unstable particles have been determined from an optical model analysis of diffractive production [5].

This approach is reasonably reliable in the case of e.g. photoproduction of vector mesons on nuclei [6, 7] because of the perturbative nature of such processes. The situation however is not at all clear when the incident particles are hadrons, as it is the case in experiments of Refs [1–4]. There is even one well known example of hadronic diffraction on nuclei where the model of Ref. [5] gives wrong cross section for an unstable particle: This is $K_L \rightarrow K_S$ regeneration. The explanation is also well known: The relevant attenuations are not the attenuations of K_L and K_S but, rather, of $K^0 = \frac{1}{\sqrt{2}}(K_L+K_S)$ and

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$\overline{K}^0 = \frac{1}{\sqrt{2}}(K_L - K_S)$. This means that this process, in spite of being weak (in the sense that, on one nucleon, $\sigma_{K_L \to K_S} \ll \sigma_{K_L, \text{TOTAL}}$), cannot be treated perturbatively but, rather, in a *nonperturbative* way proposed long time ago in Ref. [8].

The nature of diffractive processes in the case of incident pions or protons [1–4] is indeed an open problem [12]. The existing analyses [13] treat these processes *perturbatively*, following [5], and obtain cross sections of unstable objects, σ_2, strongly vaiying from channel to channel. Thus a relevant question arises what happens when the standard formulae of [5] are forced to fit an explicitly nonperturbative mechanism of Ref. [8]. What shall we get then for cross sections of unstable particles? Shall we be able to get reasonably good fits to the optical model formulae of Ref. [5]? Will thus obtained cross sections of unstable particles have physical meaning they are supposed to have?

Such questions were already asked in Refs [9, 10]. This paper employs a well defined and soluble nonperturbative model of diffractive excitations to test the physical meaning of the standard parametrisations of the experimental data of diffractive dissociation on nuclear targets.

2. *The model*

The model we employ is a specific nonperturbative realisation of the scheme of Ref. [8]. We reject the assumption [1–5] that diffractive production occurs on one nucleon at a definite point inside of the target nucleus, and that the attenuations inside the nucleus are merely the attenuations of the incident particle and of the produced physical object detected outside of the nucleus. Rather, the propagation inside of the target nucleus consists of a multitude of diffractive transitions between various possible states and the relevant attenuations are not of the physical states but of the "eigenstates of diffraction", which states diagonalize the diffractive part of the S-matrix (the "bare particle" states of Ref. [8]). The unitary transformation between the physical states $|1\rangle$, $|2\rangle$ ... $|k\rangle$ ... and the "eigenstates of diffraction" $|\psi_1\rangle, |\psi_2\rangle \dots |\psi_l\rangle \dots$

$$|\psi_l\rangle = \sum_k |k\rangle \langle k|\psi_l\rangle = \sum_k U_{lk}|k\rangle, \qquad U_{lk} = \langle k|\psi_l\rangle, \tag{2.1}$$

diagonalizes the matrix of all possible amplitudes of diffractive transitions in hadron–nucleon collisions $\gamma = \{\gamma_{kk'}\}$

$$U^{-1}\gamma U = \begin{pmatrix} \lambda_1 & 0 & \\ & \lambda_2 & \\ 0 & & \ddots \end{pmatrix}. \tag{2.2}$$

The operator of diffractive transitions in hadron–nucleon collisions is

$$T(B) = \sum_{n=1}^{N} |\psi_n\rangle\lambda_n(B)\langle\psi_n|, \qquad T(B)\,|\psi_n\rangle = \lambda_n(B)\,|\psi_n\rangle, \tag{2.3}$$

where N is the number of the "eigenstates of diffraction" and $\boldsymbol{B}$ is the impact parameter. $\lambda_n(\boldsymbol{B})$ we call the profile of the n-th eigenstate of diffraction.

The operator of diffractive transitions in hadron–nucleus collisions we define as follows

$$T_A(\boldsymbol{B}) = \sum_{n=1}^{N} |\psi_n\rangle \Gamma_n(\boldsymbol{B}) \langle\psi_n|,$$

$$\Gamma_n(\boldsymbol{B}) = 1-[1-\int d^2 s \lambda_n(\boldsymbol{B}-\boldsymbol{s}) D(\boldsymbol{s})]^A, \tag{2.4}$$

where $D(\boldsymbol{s}) = \int_{-\infty}^{\infty} dz \varrho(\boldsymbol{s}, z)$, ϱ — single nucleon density in the target nucleus, A — atomic number. Thus the transition amplitude between two physical states is

$$\langle k|T_A(\boldsymbol{B})|p\rangle = \sum_n \langle k|\psi_n\rangle [1-(1-\lambda_n * D)^A] \langle\psi_n|p\rangle$$

$$= \sum_n U_{pn}^{-1}[1-(1-\lambda_n * D)^A] U_{nk}, \tag{2.5}$$

where $\lambda_n * D$ denotes the convolution (see Eq. (2.4)).

From (2.3), (2.4) and (2.5) follow the well known formulae for the total and the inclusive diffraction cross sections

$$\sigma_{\mathrm{T}} = 2\int d^2 B \sum_{n=1}^{N} \langle 1|\psi_n\rangle \lambda_n(\boldsymbol{B}) \langle\psi_n|1\rangle = 2\int d^2 B \overline{\lambda(\boldsymbol{B})}$$

$$\sigma_{\mathrm{D}} = \int d^2 B \sum_{k\neq 1} |\sum_{n=1}^{N} \langle k|\psi_n\rangle \lambda_n(\boldsymbol{B}) \langle\psi_n|1\rangle|^2 = \int d^2 B(\overline{\lambda^2(\boldsymbol{B})} - \overline{\lambda(\boldsymbol{B})}^2) \tag{2.6}$$

for hadron–nucleon interactions, and

$$\sigma_{\mathrm{T}}^{(A)} = 2\int d^2 B \overline{\Gamma(\boldsymbol{B})},$$

$$\sigma_{\mathrm{D}}^{(A)} = \int d^2 B(\overline{\Gamma^2(\boldsymbol{B})} - \overline{\Gamma(\boldsymbol{B})}^2) \tag{2.7}$$

for hadron–nucleus interactions. $\overline{\lambda}_n$, $\overline{\Gamma}$ etc. are the averages taken over all eigenstates of diffraction with probabilities of their coupling to the incident particle state: $P_n = |\langle 1|\psi_n\rangle|^2$.

Two qualitative features of a nonperturbative description, with many eigenstates of diffraction at work, can at this point be emphasized. The first is that the orthogonality condition satisfied by

$$(U^{-1}U)_{kp} = \sum_n \langle k|\psi_n\rangle \langle\psi_n|p\rangle = \delta_{kp} \tag{2.8}$$

implies that, in general, $U_{ik} = \langle k|\psi_i\rangle$ fluctuate as one varies i with fixed k.

The second is that λ_i must vary as one varies i, to have a non zero *inclusive* diffractive production on one nucleon (compare Eq. (2.6)). In fact, in the numerical examples given below, one has to have as large as possible fluctuations of λ's in order to account for the observed diffractive production on one nucleon. These fluctuations of U_{ik} and λ_i result, in general, in a somewhat irregular dependence of the *exclusive* diffractive nuclear cross sections on both, the nuclear density (strictly speaking on the variable $\xi = AD$, see below) and on the number k which labels produced physical states, even if the underlying attenuations of the *physical* states do not depend on k at all (see Fig. 2). This should be contrasted with the standard description [1–5] of such processes which would give, for all *exclusive* diffractive production channels, the same A-dependences of the cross sections — if it reproduced correctly the true attenuations of produced states we assumed in our numerical examples given below.

Thus if nature realizes non perturbative mechanisms of diffractive production and we try to fit them by the standard [1–5] perturbative parametrisations, we should expect a mismatch. Our quantitative analysis shows below that this mismatch results in fluctuations of the "total cross sections of unstable particles", σ_2, which reflect this mismatch rather than very different attenuations of physical objects diffractively produced. In the next section we discuss this point in more details.

3. *Determination of the cross sections of unstable particles*

We shall limit our discussion to very high energies where the longitudinal momentum transfers and the invariant masses of the diffractively produced objects can be neglected. Although this approximation excludes any quantitative predictions and comparisons with available experimental results, it may be good enough for a qualitative analysis. In fact, this is our conjecture that one can extend our qualitative conclusions to energies as low as several GeV, where large body of the available data comes from.

We shall discuss the formulae for $A \gg 1$ (in the optical limit), but there is no problem in doing everything with A small. The standard [1–5] parametrisation of the cross sections in the impact parameter representation for the incident hadron, with the nucleon cross section σ_1, diffractively producing unstable particle (defined e.g. by its mass and quantum numbers) with the nucleon cross section σ_2, is

$$\sigma_A(\sigma_1\sigma_2; \boldsymbol{B}) = c\left|\frac{1}{\sigma_1-\sigma_2}\left(e^{-\frac{1}{2}\sigma_1 AD(\boldsymbol{B})}-e^{-\frac{1}{2}\sigma_2 AD(\boldsymbol{B})}\right)\right|^2 = c\left|\frac{1}{\sigma_1-\sigma_2}\left(e^{-\frac{1}{2}\sigma_1\xi}-e^{-\frac{1}{2}\sigma_2\xi}\right)\right|^2, \quad (3.1)$$

where c is an adjustable constant and $\xi = AD(\boldsymbol{B})$.

On the other hand in our nonperturbative calculations the cross section for producing the k-th state from the incident hadron (labelled $1 \neq k$) is (from (2.5))

$$\sigma_A(\sigma_1, k; \boldsymbol{B}) = |\sum_n \langle k|\psi_n\rangle e^{-\lambda'_n AD(\boldsymbol{B})}\langle\psi_n|1\rangle|^2 = |\sum_n \langle k|\psi_n\rangle e^{-\lambda'_n\xi}\langle\psi_n|1\rangle|^2. \quad (3.2)$$

In (3.1) and (3.2), for the sake of simplicity, we neglected the spatial extensions of all the objects propagating through the nucleus but this can be corrected, if necessary, by using convolutions instead of products ($\sigma D \to \sigma * D$, $\lambda'_n D \to \lambda_n * D$, compare (2.4) and the discussion of the finite size effects of $\lambda_n(\boldsymbol{B})$ at the end of this Section).

In order to see how the standard procedure [1–5] works, we calculated explicitly (3.2) for the two specific models described in the Appendix (Case I, Case II, see also Ref.

[1]) and then fitted (3.1) to these curves in a similar manner as one fits the experimental data: $\int d^2B\sigma_A(\sigma_1\sigma_2; \boldsymbol{B})$ is fitted to $\int d^2B\sigma_A(\sigma_1, \check{k}; \boldsymbol{B})$ by adjusting σ_2 and c. Since we know the cross sections for all states of our model, we can see whether the standard procedure gives σ_2's which are realistically close to their true values.

Figs 2,3 and Table I present some examples of the fits of (3.1) to the Case I, where a', b', c', d' are the transition amplitudes for hadron–nucleon diffractive processes: a' — elastic, b' — excitation of the "first" excited state, c' — of the "second", d' — of the "third" (the prime means integration over $\boldsymbol{B}$, e.g. $a' = \int d^2Ba(\boldsymbol{B})$). Thus on one nucleon only three states can be excited, all the others can be reached only through some intermediate states e.g. the "sixth" state can be reached by first exciting the "third" from which the "sixth" can be reached. The complete transition matrix, γ, between the physical states is given in the Appendix (A5). This transition matrix is diagonalised by

$$\langle k|\psi_l\rangle = U_{lk} = \sqrt{\frac{2}{N+1}}\sin\frac{\pi kl}{N+1}, \tag{3.3}$$

where N is the number of diffractive states (the rank of the transition matrix). The eigenvalues of γ come out to be

$$\lambda_n = a-c+2(b-2d)\cos\frac{\pi n}{N+1}+4c\cos^2\frac{\pi n}{N+1}+8d\cos^3\frac{\pi n}{N+1}. \tag{3.4}$$

One may easily construct many similar and more complex models but for qualitative discussion Case I is quite adequate as an illustration of a few relevant points.

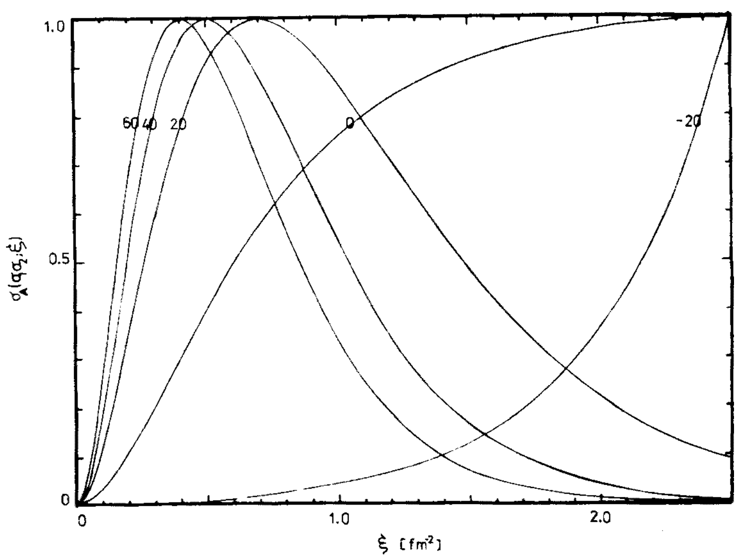

Fig. 1. The differential Kölbig-Margolis (KM) cross-sections $\sigma_A(\sigma_1\sigma_2, \xi)$ for σ_1 = 40 mb and different values of the parameter σ_2 (60, 40, 20, 0, -20 mb) which labels the curves. Each curve is normalized to 1 at its maximum

In Fig. 1 we show a typical sequence of $\sigma_A(\sigma_1\sigma_2; \xi)$ for fixed σ_1, and varying σ_2. We can see that the curves vary systematically with σ_2 : for $\sigma_2 > 0$ all of them have one maximum which shifts with decreasing σ_2 towards larger ξ's. We also continue to $\sigma_2 < 0$ curves which are monotonically increasing everywhere.

In Fig. 2 we show a sequence of $\sigma_A(\sigma_1, k; \xi)$ for fixed σ_1 and various excitations k (solid lines). All these curves correspond to similar attenuations of the produced physical states ($\sigma_k = 48$ mb for $k = 2-9$). The amplitudes a, b, c, d were chosen to give a reason-

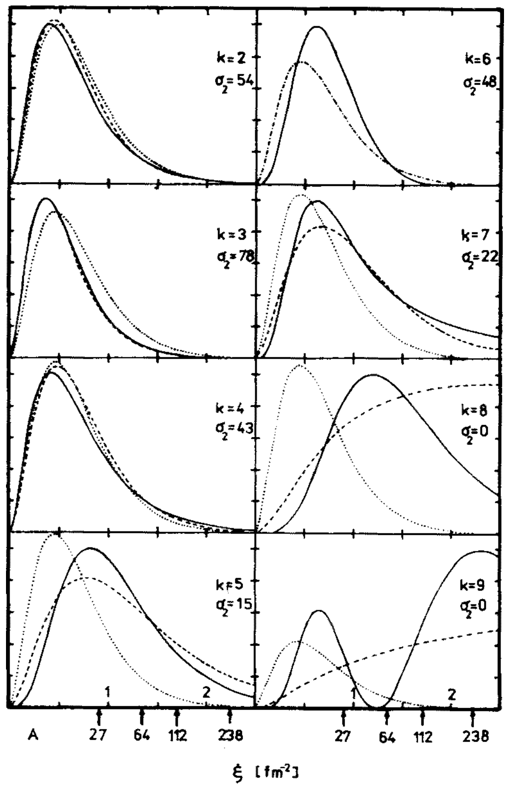

Fig. 2. An example of the model results for differential diffractive production cross-sections, $\sigma_A(\sigma_1, k; \xi)$, for the states $k = 2$—9 (solid lines). The curves result from Case I with the amplitudes $a' = 2$ fm^2, $b' = c' = d' = 0.4$ fm^2 and $N = 10$ (see the text for more details). Each model curve is compared with the corresponding KM cross-section $\sigma_A(\sigma_1\sigma_2, \xi)$ (dashed lines) with σ_2 fitted to the integrated cross-sections, see Fig. 3 (the values of σ_2 are given in the figure). Also shown are the KM curves corresponding to the true total nucleon cross-section of each state ($\sigma_2 = 48$ mb, dotted lines). The normalization is fixed by the model "data" for the integrated cross-sections at $A = 112$, similarly as in Fig. 3. Below the figure the values of ξ corresponding to central densities of some nuclei are marked

ably large diffraction on one nucleon and the same σ_1 as in Fig. 1. These curves are compared with $\sigma_A(\sigma_1\sigma_2; \xi)$ both for σ_2 obtained from the standard fit (dashed lines; the values of σ_2 are given in the figure), and for the true value of attenuation (dotted lines) which is $\sigma_2 = 48$ mb in all cases.

A comparison of the curves of Fig. 2 clearly shows these important characteristics of our nonperturbative production mechanism: The character of the wave function of the produced object, $\langle k|\psi_n\rangle$, is very essential for the shape of $\sigma_A(\sigma_1, k; \xi)$ .The oscillations of $\langle k|\psi_n\rangle$ and of λ_n with n (compare (3.3) and (3.4)) can cause some dramatic changes in shapes of the curves with neighbouring k's. As we can see more than one maximum is possible. Thus the differential characteristics of $\sigma_A(\sigma_1\sigma_2; \xi)$ and $\sigma_A(\sigma_1, k; \xi)$ can be very different.

However, the standard fitting procedure is done [1–4] with the *integrated* cross sections through their dependence on A. Fig. 3 shows that such fits can look quite respectable! Yet they give wrong σ_2's! Not only that σ_2 may come out very different from the "true"

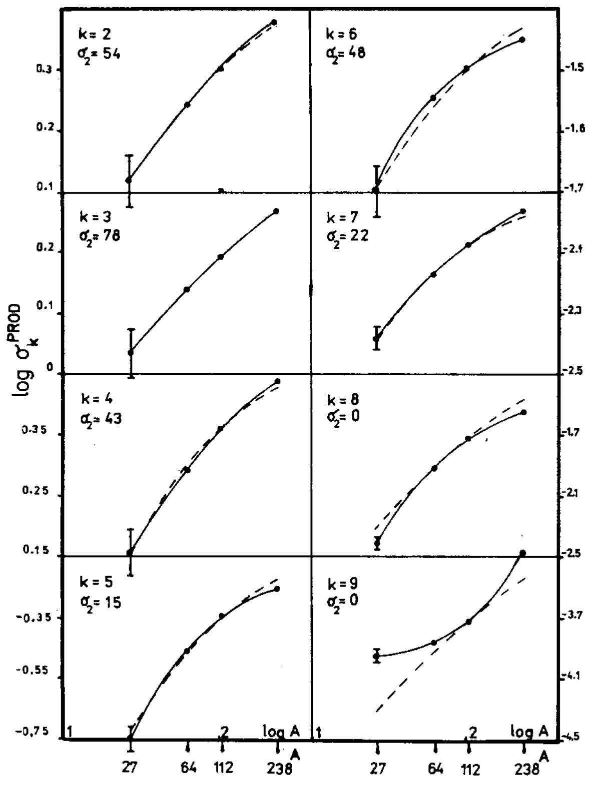

Fig. 3. A-dependences of the total production cross-sections for different states, $k = 2 \div 9$, resulting from the model (solid lines; the parameters are the same as in Fig. 2), and the corresponding Kölbig–Margolis fits (dashed lines) with normalization fixed by the model "data" at $A = 112$. To set a scale, 10% "error" bars of the model "data" are also shown

cross sections, they may also become negative! This collapse of the Kölbig–Margolis (K–M) description has its origin in the oscillations of the wave functions $\langle k|\psi_n\rangle$. They often lead to misfits in the differential behaviour of $\sigma_A(\sigma_1\sigma_2; \xi)$ and $\sigma_A(\sigma_1, k; \xi)$. But not always; e.g. an excellent fit of the differential shapes in the case $k = 3$ of Fig. 2 gives nevertheless wrong σ_2 (= 78 mb) because of these oscillations. On the other hand in the case of $k = 8$ we have a clear case of a gross misfit of the differential shapes and again wrong σ_2 ($= 0$).

The remnants of this overall mismatch of perturbative and nonperturbative descriptions which one still sees in the integral characteristics of diffractive production are strong fluctuations of the fitted σ_2.

Now we shall briefly discuss a limiting case where the K–M description has a chance of being acceptable. First we have to comment on the role of the finite spatial extensions of the diffractive eigenstates which is reflected in the impact parameter dependences of the eigenvalues $\lambda_n(\boldsymbol{B})$. So far we have used the approximation that $\lambda_n(\boldsymbol{B})$ are much narrower than $D(\boldsymbol{B})$, thus the convolution of Eq. (2.4) can be approximated as follows

$$\int d^2s\lambda_n(\boldsymbol{B}-\boldsymbol{s})D(\boldsymbol{s}) \approx \lambda_n' D(\boldsymbol{B}), \quad \lambda_n(\boldsymbol{B}) = \lambda_n' f_n(\boldsymbol{B}), \quad \int d^2Bf_n(\boldsymbol{B}) = 1. \tag{3.5}$$

At first one may think that introducing strong "spatial fluctuations", i.e. strong variations of $f_n(\boldsymbol{B})$ with n, one can achieve a nonperturbative realizations of the K–M parametrisation. Indeed, one may accept all λ_n' approximately equal and generate large enough σ_D for hadron–nucleon diffractive excitations by drastically changing $f_n(\boldsymbol{B})$ with n:

$$\sigma_D = \bar{\lambda}' \int d^2B(\overline{f^2(\boldsymbol{B})} - \overline{f(\boldsymbol{B})}^2). \tag{3.6}$$

Using the approximation (3.5) and the approximate equality of λ_n''s one obtains (3.1) from (3.2), hence one recovers the K–M formula, for the excited states which have sufficiently large probabilities of being excited in one step. Such are the cases $k = 2, 3, 4$ of the third, fifth and seventh rows of Table I. Note that in all other cases, even when fluctuations of λ_n' are small, one does not obtain correct K–M fits.

One should remember, however, that such mechanism of elementary diffraction leads to forward dips in *all* differential exclusive cross sections which fact disagrees with experiment. Indeed, from (2.3) we get the following amplitude for the exclusive excitation $1 \to k$ in the high energy limit

$$\begin{aligned} F_{k1}(\boldsymbol{q}) &= \frac{ip}{2\pi} \int d^2Be^{i\boldsymbol{q}\cdot\boldsymbol{B}} \sum_n \langle k|\psi_n\rangle \lambda_n(\boldsymbol{B}) \langle\psi_n|1\rangle \\ &= \frac{ip}{2\pi} \int d^2Be^{i\boldsymbol{q}\cdot\boldsymbol{B}} \sum_n \lambda_n' \langle k|\psi_n\rangle \langle\psi_n|1\rangle f_n(\boldsymbol{B}), \end{aligned} \tag{3.7}$$

where $\boldsymbol{q}$ is the momentum transfer and p is the incident momentum. As $\lambda_n' \approx \lambda_{n'}'$, we get from (3.5) and the completeness relation $\sum_n |\psi_n\rangle\langle\psi_n| = 1$, the following forward amplitudes

$$F_{k1}(0) = \frac{ip}{2\pi} \sum_n \lambda_n' \langle k|\psi_n\rangle \langle\psi_n|1\rangle \approx \frac{ip}{2\pi} \bar{\lambda}' \langle k|1\rangle = 0, \quad k \neq 1. \tag{3.8}$$

Thus for all $k \neq 1$ we have dips in the forward direction which are not observed. One should also point out that this special case leads to disappearance of the inelastic shadowing contribution to the total cross section which again contradicts the experiments which

TABLE 1

The ratios σ_2/σ_k^{tot} of cross-sections σ_2, given by the standard KM fitting procedure, to the "true" total nucleon cross-sections of k states. The following nuclear targets were used: Al, Cu, Cd, Pb, U. In all examples, the nucleon cross-section of incident particle equals 40 mb ($a' = 2$ fm^2) and $N = 10$. Normalisation of the KM formula was taken as a free parameter, fixed by the integrated cross-sections of our model at $A = 112$ (Cd). In Case I, nuclear cross-sections are entirely determined by a', b', c', d' which are given in the table. For description of Case II see Appendix. Percentage-wise, the contributions of inelastic screening to the total nuclear cross-sections in Al, Cu, Cd, Pb, U differ little for a given set of λ_n'. Therefore, in the last column we give their averages

Case I			σ_2/σ_k^{tot}										
b' [fm^2]	c' [fm^2]	d' [fm^2]	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$	$k = 9$	$k = 10$	σ_{SD} [mb]	$\Delta\sigma_A/\sigma_A^{tot}$ [%]
0.4	0.4	0.4	1.13	1.62	0.9	0.31	1.02	0.41	0	−0.02	−0.38	~0.6	~2.1
0.1	0.1	0.1	1.11	1.17	1.14	0.28	0.38	0.40	−0.12	0	0.12	~0.04	~0.15
0.04	0.04	0.04	1.05	1.07	1.05	0.24	0.27	0.29	−0.17	−0.12	−0.10	~0.006	~0.03
0.4	−0.4	−0.4	1.12	0.72	0.81	−0.5	−0.06	−0.09	−0.5	−0.53	−0.4	~0.8	~3.4
0.1	−0.1	−0.1	1.05	0.98	1.02	−0.29	0.18	0.18	−0.29	−0.26	−0.22	~0.05	~0.2
0.6	0.6	−0.6	0.5	0.6	0.42	−0.17	0.08	0	−0.06	−0.21	−0.43	~1.5	~7.7
0.1	0.1	−0.1	0.95	0.97	0.93	−0.17	0.21	0.21	−0.17	−0.19	−0.23	~0.04	~0.2
0.2	−0.2	0.2	0.48	0.39	0.39	−0.17	−0.28	−0.35	−0.67	−0.75	−0.81	~0.3	~1.4
Case II													
			0.36	−0.15	−0.29	0.67	−0.17	−0.15	−0.21	1.6	0.68	~3.1	~16.0
			0.53	−0.12	−0.3	−0.34	−0.5	−0.1	−0.34	−0.5	−0.28	~1.2	~7.2
			0.65	0.42	0.28	0.17	0.06	−0.06	−0.17	−0.3	−0.35	~0.12	~1.2

clearly established its existence [14, 15]. So, the case of diffractive production generated *exclusively* from "spatial fluctuations" seems to be rather unrealistic.

This does not mean that the "spatial fluctuations" can be neglected altogether. One can even argue that they do contribute appreciably to diffractive production [16]. But even in this case it turns out that they cannot save the K–M parametrisation [9]. Although the role of "spatial fluctuations" as one of the mechanisms contributing to diffractive production deserves further analysis we shall not discuss it here. Let us only state that our numerical results do contain some contributions from "spatial fluctuations" because they are present in our $\lambda_n(\boldsymbol{B})$ (see Appendix).

Table I gives a sample of fits of (3.1) to Case I (see Appendix) with various elementary diffractive amplitudes a, b, c, d and to Case II (see Appendix). We shall not discuss Case II in any details because, though it gives larger diffractive production on one nucleon than Case I, it leads to very similar conclusions. If anything, the overall mismatch of Case II with perturbative description is still worse than of Case I.

Although in Table I we do have $\boldsymbol{B}$-dependence of $\lambda_n(\boldsymbol{B})$ which comes from the construction of the amplitudes a, b, c, d (see Appendix) and which is reflected in the single diffraction on one nucleon (σ_{SD}), but in calculating the *nuclear* cross sections we accepted the approximation (3.5). From Table I we can see that small σ_{SD} does not lead, in general, to a K–M description: *Even in the cases of negligible diffractive production on one nucleon we get for high enough excitations negative* σ_2. The nonperturbative character of our model leads to an overall mismatch which does not disappear in the limit $\sigma_{\mathrm{SD}} \to 0$.

We already stressed the point that one should rather study the differential diffractive cross sections than the integral. This differential behaviour can be — to some extent — extracted from the following procedure. Underneath Fig. 2 we marked regions of ξ occupied by various nuclei. We can see that the result of the fit which gives σ_2 depends on the choice of nuclei: For two different sets of nuclei the fitting procedure may pair differently $\sigma_A(\sigma_1\sigma_2; \xi)$ and $\sigma_A(\sigma_1, k; \xi)$. Thus the procedure of using two different sets of A is more sensitive to the differential characteristics of the cross sections than the standard one. We tried it on our model. In the case $k = 9$ we do get spectacular fluctuations of σ_2 around the fit which uses all A's: For the following three groups of nuclei (27, 56, 64, 112), (64 112, 184, 207, 238), (27, 64, 112, 207, 238) we get for σ_2, respectively, 58 mb, -6 mb and 0 mb. For the other cases the fluctuations are much smaller.

The moral of this exercise is that if doing similar things with experimental data we get large fluctuations of σ_2, it will be a signal of nonperturbative mechanism at work. Unfortunately, lack of large fluctuations cannot be interpreted as a proof of a perturbative nature of the process investigated.

4. *Summary and conclusions*

By solving completely some *nonperturbative* models of diffractive production and comparing the results with the standard parametrisation of diffractive production on nuclei [1–5] we have shown that the so called Kölbig–Margolis formula (K–M formula) gives wrong cross sections for the unstable diffractively produced objects.

This might be expected because the K–M formula is *perturbative* in its nature, except that even in the case of weak diffraction on one nucleon, in which case the K–M formula has a chance to work, the standard fitting procedure with K–M parametrization still gives wrong σ_2's for many channels of production.

It was also shown that when one applies this standard procedure to fit a nonperturbative diffraction, the fits of the integrated cross sections look as respectable as the fits in Refs [1–4]. There remains however a clear signal of an overall mismatch of perturbative vs nonperturbative descriptions even in the integrated cross sections. The K–M formula produces strong fluctuations, from channel to channel, of the cross sections of unstable particles: a frequent occurrence of "anomalously" small or large σ_2's. This feature is very much similar to what one observes in experiment.

The differential characteristics of the production cross sections may perhaps be seen from experimental results when one fits σ_2 with at least two different groups of nuclei. The accuracy of the existing data is probably still too poor to do that.

It is suggested that one should make an effort to study the measured differential cross sections avoiding the σ_2's determined from the integrated cross sections.

The authors thank Professor A. Białas for discussion and encouragement.

APPENDIX

We adopt two forms of γ, called Case I and Case II, with which we study diffraction on nuclei.

Case I

$$\gamma = \gamma^{(1)}\gamma^{(2)} \dots \gamma^{(m)}, \qquad \gamma^{(i)} = \begin{bmatrix} \alpha_i & \beta_i & 0 & \dots & & \\ \beta_i & \alpha_i & \beta_i & 0 & \dots & \\ 0 & \beta_i & \alpha_i & \beta_i & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}. \tag{A1}$$

The unitary matrix

$$\langle k|\psi_l\rangle = U_{lk} = \sqrt{\frac{2}{N+1}} \sin \frac{\pi k l}{N+1} \tag{A2}$$

diagonalizes each $\gamma^{(i)}$:

$$U^{-1}\gamma^{(i)}U = \begin{pmatrix} \lambda_1^{(i)} & & 0 \\ & \lambda_2^{(i)} & \\ 0 & & \ddots \end{pmatrix}, \qquad \lambda_j^{(i)} = \alpha_i + 2\beta_i \cos \frac{\pi j}{N+1}. \tag{A3}$$

Thus U also diagonalizes γ

$$U^{-1}\gamma U = U^{-1}\gamma^{(1)}UU^{-1}\gamma^{(2)}U \dots = \begin{pmatrix} \lambda_1 & 0 & \\ & \lambda_2 & \\ 0 & & \ddots \end{pmatrix}, \qquad \lambda_j = \prod_{i=1}^{m} \lambda_j^{(i)}. \tag{A4}$$

γ of Eq. (A1) depends on $m+1$ independent parameters which are algebraic functions of $2m$ original parameters α_i, β_i; $i = 1, \ldots, m$. This way one can construct γ matrices which have an ample degree of complexity built in and are diagonalized by U. We study diffraction on nuclei using γ with at most four non zero functions of the impact parameter $\boldsymbol{B}$: $a(\boldsymbol{B})$, $b(\boldsymbol{B})$, $c(\boldsymbol{B})$, $d(\boldsymbol{B})$, but one could easily construct more complicated cases. From the product of three $\gamma^{(i)}$ we get

$$\gamma = \begin{bmatrix} a, & b, & c, & d, & 0, & \ldots & & & & \\ b, & a+c, & b+d, & c, & d, & 0, & \ldots & & & \\ c, & b+d, & a+c, & b+d, & c, & d, & 0, & \ldots & & \\ d, & c, & b+d, & a+c, & b+d, & c, & d, & 0, & \ldots & \\ 0, & d, & c, & b+d, & a+c, & b+d, & c, & d, & 0, & \ldots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \end{bmatrix}. \tag{A5}$$

The transformation U brings it to a diagonal form with the following eigenvalues

$$\lambda_j = a-c+2(b-2d)\cos\frac{\pi j}{N+1}+4c\cos^2\frac{\pi j}{N+1}+8d\cos^3\frac{\pi j}{N+1}. \tag{A6}$$

The matrix (A5) is a model of diffractive processes in hadron–nucleon collisions with four amplitudes. Having chosen a, b, c, d, and the number of diffractive channels N, one can study various relations between diffraction on nucleons and nuclei. Note that the transformation (A2) does not depend on the numerical values of the matrix γ which it diagonalizes. In the expansion (2.1) no one state dominates, very much like in $K_L \to K_S$ regeneration. In fact, our model is a direct generalization of $K_L \to K_S$ regeneration to many degrees of freedom. When we restrict (2.1) to just two pairs of states $|\psi_1\rangle$, $|\psi_2\rangle$ and $|1\rangle$, $|2\rangle$ and employ (A2) we get $|\psi_1\rangle = \frac{1}{\sqrt{2}}(|1\rangle+|2\rangle)$ and $|\psi_2\rangle = \frac{1}{\sqrt{2}}(|1\rangle-|2\rangle)$, which relations are the same as between $|K^0\rangle$, $|\bar{K}^0\rangle$ and $|K_L\rangle$, $|K_S\rangle$.

The four functions $a(\boldsymbol{B})$, $b(\boldsymbol{B})$, $c(\boldsymbol{B})$, $d(\boldsymbol{B})$ are not entirely arbitrary because they must satisfy the conditions which guarantee that they represent physically acceptable description of diffractive processes in hadron–nucleon interactions. These conditions are:

(*i*) The total cross section should be determined by $a(\boldsymbol{B})$

$$\sigma_T = 2\int d^2Ba(\boldsymbol{B}), \tag{A7}$$

which formula follows from (2.6), (A2) and (A6).

(*ii*) The total diffractive dissociation cross section of the incident particles which we also get from (2.6), (A2) and (A6)

$$\sigma_{SD} = \int d^2B(|b(\boldsymbol{B})|^2+|c(\boldsymbol{B})|^2+|d(\boldsymbol{B})|^2), \tag{A8}$$

should be close to the known experimental estimates. Also the unitarity bound

$$\sigma_{SD}(\boldsymbol{B}) \leqslant \tfrac{1}{2}\sigma_T(\boldsymbol{B})-\sigma_{EL}(\boldsymbol{B}) \to [|b(\boldsymbol{B})|^2+|c(\boldsymbol{B})|^2+|d(\boldsymbol{B})|^2] \leqslant a(\boldsymbol{B})-|a(\boldsymbol{B})|^2 \tag{A9}$$

should not be violated. This is guaranteed by (*iii*). Note that in the *coherent* nuclear processes we are considering in this paper we are dealing only with single diffraction and we neglect double diffraction altogether.

(*iii*) The profiles of the eigenstates of diffraction should be within the limits

$$0 \leqslant \lambda_j(\boldsymbol{B}) \leqslant 1 \tag{A10}$$

of complete transparency and complete absorption. We use purely absorptive amplitudes, hence λ_j's are real.

The following example clarifies our construction of the amplitudes $a(\boldsymbol{B})$, $b(\boldsymbol{B})$, $c(\boldsymbol{B})$, $d(\boldsymbol{B})$. Only one of these amplitudes is determined directly from experimental data. For instance, the forward elastic proton–proton amplitude at high energies can be simply parametrized

$$a(\boldsymbol{B}) \cong 0.75 \exp(-B^2/r^2) \tag{A11}$$

with r^2 determined from $\sigma_T = 40$ mb using (A7). The other amplitudes are rather weakly restricted by the size of σ_{SD} and (*ii*), (*iii*). To simplify matters we consider the case when these amplitudes contribute to σ_{SD} with comparable strengths. So, let us take

$$|b(\boldsymbol{B})| = |c(\boldsymbol{B})| = |d(\boldsymbol{B})|. \tag{A12}$$

Then the problem is reduced to determining one amplitude, say $b(\boldsymbol{B})$, with four possible sign choices for $c(\boldsymbol{B})$ and $d(\boldsymbol{B})$ ($b(\boldsymbol{B})$ may always be taken positive by convention). In fact, as we argued above, the $\boldsymbol{B}$-dependence of the amplitudes is not very critical for our calculations, hence only the integral $b' = \int d^2Bb(\boldsymbol{B})$ should be evaluated with (*i*)–(*iii*) satisfied. It is easy to determine the largest value of b' allowed by unitarity which corresponds to the largest possible elementary diffraction. To do this one has to find, at each $\boldsymbol{B}$, the largest value of b not violating (*iii*), with (A11) and (A12) put into (A6). For instance, for $b(\boldsymbol{B}) = c(\boldsymbol{B}) = d(\boldsymbol{B})$ one obtains

$$b_{\max}(\boldsymbol{B}) \simeq \begin{cases} (1-a(\boldsymbol{B}))/9, & \text{for} \quad B \lesssim 0.96\ \text{fm} \\ a(\boldsymbol{B})/3, & \text{for} \quad B \gtrsim 0.96\ \text{fm} \end{cases}$$

which gives $b' = 0.4\ \text{fm}^2$, and $\sigma_{SD}^{\max} = 3\int d^2B|b_{\max}(\boldsymbol{B})|^2 \simeq 0.6$ mb. For all cases of our model $\sigma_{SD}^{\max}$ are significantly smaller than the experimental estimates $\sigma_{SD}^{\exp} = 2.5-3.5$ mb. The same procedure can be also carried out for any given ratio of $b(\boldsymbol{B})$, $c(\boldsymbol{B})$ and $d(\boldsymbol{B})$.

Case II

In the Case I it is difficult to have large σ_{SD}. The following construction provides us with a possibility of obtaining larger elementary diffraction (σ_{SD}). We *assume* $\lambda_n(\boldsymbol{B})$ with large enough fluctuations to produce σ_T and the desired σ_{SD} and then use the same U_{lk} as in Case I. The transition matrix is in this case

$$\gamma_{hm} = U_{kl}^{-1}\delta_{ln}\lambda_n U_{nm}. \tag{A13}$$

The largest diffraction saturating (A9) is obtained when $\lambda_n(B) = 1$ for some n, and is zero for the others. To control the size of diffraction we may take, for instance,

$$\lambda_n(B) = \begin{cases} h, & 1 \leqslant n \leqslant n_1(B) \\ h\,\dfrac{n-n_2}{n_1-n_2}, & n_1(B) < n < n_2(B) \\ 0, & N \geqslant n \geqslant n_2(B) \end{cases} \tag{A14}$$

where, at each B, the parameters h, n_1, and n_2 are fitted to reproduce the desired $\sigma_T(B)$ and $\sigma_{SD}(B)$. N is the number of diffractive eigenstates.

One may construct, of course, many similar models. We doubt whether they would bring anything qualitatively very different from what one can see from Cases I, II.

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Note added in proof. We thank dr N. N. Nikolaev for calling our attention to the paper by A. B. Zamolodchikov et al., *Zh. Eksp. Teor. Fiz.* **77**, 451 (1979), where the effect of "antishadowing" in diffractive processes on nuclei and its role in the measurements of cross sections of unstable particles are discussed.