ELASTIC p—p CROSS-SECTION AND MULTIPLE SCATTERING OF QUARKS

By A. Białas*, K. Fiałkowski, W. Słomiński and M. Zieliński

Institute of Physics, Jagellonian University, Cracow**

(*Received May 23, 1977*)

The elastic p – p cross-section is calculated in the quark model using Glauber multiple scattering expansion. We find that: (a) recoil effects are crucial for momentum transfers higher than 2 $(\mathrm{GeV}/c)^2$; (b) multiple scattering terms are larger than required by the data. We interpret this last result as evidence for negative correlations at short distances (in the impact parameter plane) between quarks in the proton.

1. *Introduction*

Most of the existing evidence for composite structure of hadrons has been derived from the studies of lepton-hadron interactions. The emerging picture is that hadrons are composed of quarks carrying the quantum numbers and neutral glue which is responsible for binding forces. It is, however, much less clear how this composite structure is reflected in strong interactions. Many different possibilities are thus still open.

The ambiguity arises because the information obtained from lepton-hadron processes about distribution and properties of glue is rather indirect. Indeed, glue, being neutral, does not interact with leptons. On the other hand the glue interacts strongly with quarks (providing the binding force) and with itself[1]. Consequently, it must play a very essential role in the strong interaction of hadrons. Thus hadronic strong interactions seem to be the natural place for studying the properties of glue.

This point of view was emphasized recently in a series of papers by Van Hove and collaborators [1–4]. It was assumed that (*i*) glue is the only part of the hadron which undergoes strong interactions and (*ii*) that the distribution of glue is not related to the distributions of quarks (except for constraints implied by conservation laws). The model

* Also at Institute of Nuclear Physics, Cracow.

** Address: Instytut Fizyki UJ, Reymonta 4, 30-059 Kraków, Poland.

[1] At least in colour gauge theories.

turned out to be successful in semi-quantitative description of high-energy pp-interactions (diffractive and non-diffractive). Recently, it was shown that it can also describe very precisely [4] the complicated structure observed in pp elastic scattering at ISR energies [5].

In this paper we study another model of the glue interaction and apply it to the description of the elastic pp interactions at high energies.

Our starting point is the observation that the additivity rules are satisfied to a very good approximation not only for inelastic but also for diffractive (vacuum exchange) processes [6]. This suggests that either the quarks are responsible also for strong interaction of hadrons or, if glue plays an essential role, the amount of glue is proportional to the number of quarks in the hadron. The simplest way to understand this fact is to assume that hadrons are built from quarks "dressed" with glue, i.e. that most of the glue is concentrated around the point-like quarks[2]. Such picture is not new. It was implicitly used in all early discussions of the additive quark model [6].

We are thus led to the picture of hadrons in which quasi-free dressed valence quarks are confined into the region of the hadronic size. This picture resembles the one known in nuclear physics.

Having adopted the picture of the hadron as a bound state of the dressed quarks, it seems natural to describe the elastic hadron-hadron scattering by means of Glauber model [8, 9], which proved so succesful in studying the interactions with nuclei at high energies. In this paper we present such a Glauber model calculation for elastic pp scattering.

The Glauber model was already applied to elastic hadron-hadron scattering by several authors [10, 11]. Most of this work was devoted to study of meson-nucleon total cross--sections and elastic scattering at small momentum transfers [10]. In particular, the deviations from simple additivity rules were discussed in detail. Nucleon-nucleon elastic scattering was considered in Ref. [11]. Apart from several technical details (e.g. the explicit choice of the wave functions and quark-quark interactions) our calculations agree with general qualitative results of Ref. [11]. However, since we compare our results with the data at much higher energies than those available to the authors of Ref. [11], our conclusions differ substantially.

Our main conclusions are:

(a) The recoil corrections are essential for the description of the region beyond the first diffractive minimum. This indicates that only finite number of constituents takes part in the collision (the recoil corrections vanish for infinite number of constituents).

(b) In the simple calculation presented in this paper, the multiple scattering corrections are by far too strong. Several possibilities of improving this result are discussed.

In the next Section the model is formulated in more detail. In Section 3 we define our choice of quark densities and amplitudes. The general discussion of multiple scattering corrections and recoil effects is given in Sections 4 and 5. Our numerical results are presented in Section 6. The conclusions are listed in the last section. The summary of the formulae used in numerical calculations is given in the Appendix.

---

[2] An alternative explanation was given in Ref. [7].

2. Formulation of the model

In this section we summarize the basic assumptions of the model and the main formulas used.

We describe p−p interactions as a collision of two systems, each one composed of three dressed valence quarks[3]. The quark-quark interaction is assumed to be purely absorptive. According to the Glauber model [8] the elastic amplitude for a collision of particles A and B with the quarks frozen in transverse positions $\{\vec{s}_i\}$ has the form:

$$\Gamma(\vec{b}; \{\vec{s}_i\}) = 1 - \prod_i \prod_j (1 - \gamma_{ij}(\vec{b} + \vec{s}_i^A - \vec{s}_j^B)), \tag{2.1}$$

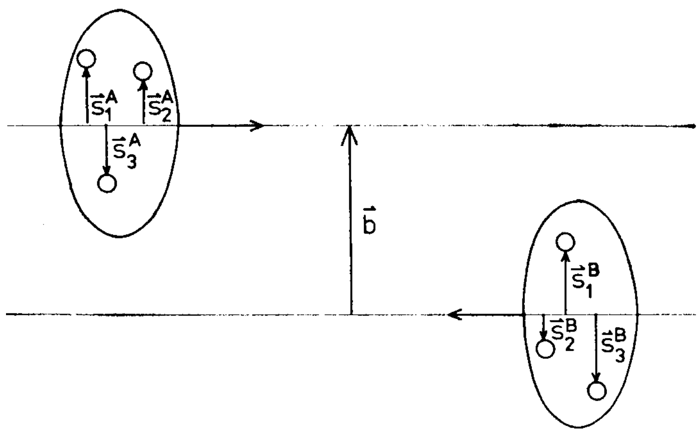

Fig. 1. Proton-proton scattering in the quark model

where γ_{ij} denotes the elastic amplitude of the i-th and j-th quarks interaction and $\vec{b}$ is the impact parameter (see Fig. 1). The elastic amplitude in momentum transfer representation is a Fourier transform of the amplitude in impact parameter space:

$$T(\vec{\Delta}) = \int d^2b \langle \Gamma(\vec{b}; \{\vec{s}_i\}) \rangle_{AB}\, e^{i\vec{\Delta}\cdot\vec{b}}, \tag{2.2}$$

where $\langle\ \rangle_{AB}$ denotes averaging over quark density distribution $D(\vec{s}_1, \vec{s}_2, \vec{s}_3)$ in CM systems of particles A and B, respectively:

$$\Gamma(\vec{b}) \equiv \langle \Gamma \rangle_{AB} = \int d^2s_1^A \ldots d^2s_3^B D_A(\{\vec{s}_i^A\}) D_B(\{\vec{s}_j^B\}) \Gamma(\vec{b}; \{\vec{s}_i\}). \tag{2.3}$$

The detailed description of this formula is given in the Appendix.

With this normalization the cross sections are given by the following formulae: total cross section:

$$\sigma_{tot} = 2\,\mathrm{Re}\, T(0), \tag{2.4}$$

[3] As explained in the introduction we assume that all glue and possible $q\bar{q}$ pairs are concentrated around valence quarks.

elastic differential cross section:

$$\frac{d\sigma}{dt} = \frac{1}{4\pi}|T(\vec{\Delta})|^2, \quad t \simeq -\Delta^2, \tag{2.5}$$

single diffraction (of B particle) cross section:

$$\sigma_{SD} = \int d^2b \langle\langle \Gamma(\vec{b}; \{\vec{s}_i\})\rangle_A^2\rangle_B - \sigma_{el}, \tag{2.6}$$

total diffraction cross section:

$$\sigma_{TD} = \int d^2b \langle \Gamma^2(\vec{b}; \{\vec{s}_i\})\rangle_{AB} - \sigma_{el}. \tag{2.7}$$

It is seen from these formulae that the cross-sections (2.4)-(2.7) are specified by the q$-$q elastic amplitudes γ_{ij} and quark densities $D(\{\vec{s}_i\})$. In the next Section we discuss our choice of these functions.

3. Quark densities and quark-quark amplitudes

We are now going to discuss some plausible choices of the unknown functions in our model. There are two such functions in the formulae of Section 2: $D(\vec{s}_1, \vec{s}_2, \vec{s}_3)$ and $\gamma(\vec{b})$. The function $D(\{\vec{s}_i\})$ describes the distribution of quarks in the impact parameter plane. $\gamma(\vec{b})$ is the quark-quark scattering amplitude.

To compute the scattering amplitude in different orders of expansion (2.3) we have to perform the integration over several two-dimensional variables. To control the dependence on parameters we considered distributions which can be at least partly integrated analytically. The obvious first choice for $D(\{\vec{s}_i\})$ and $\gamma(\vec{b})$ is therefore a Gaussian function

$$D_A(\{\vec{s}_i\}) = \frac{1}{(\pi A)^3} \exp\left(-\frac{s_1^2+s_2^2+s_3^2}{A}\right), \tag{3.1}$$

where A denotes the average radius squared for particle A (analogous formula holds obviously for particle B) and

$$\gamma(\vec{b}) = \gamma(0) \exp\left(-\frac{b^2}{r^2}\right). \tag{3.2}$$

Due to the normalization condition $\int \{d^2s_i\} D(\{s_i\}) = 1$ there is only one free parameter in the function $D_A(\{\vec{s}_i\})$. For $\gamma(\vec{b})$ we have in principle two parameters. However, it can be shown that the value of $\gamma(0)$ should be close to 1 if we want to produce sizeable inelastic diffractive cross-section by means of formula (2.7). In fact even for this maximal value allowed by unitarity for purely absorptive amplitude, inelastic diffraction in this case is slightly too small [1]. It can be corrected by flattening of $\gamma(\vec{b})$ at $b \simeq 0$ in the analogous way as in the glue model of Van Hove [4]. However in our case a small change of shape of $\gamma(\vec{b})$ is not very crucial for the elastic scattering, as we will discuss later. Anyway, in the following we will always use $\gamma(\vec{b})$ satisfying the condition

$$\gamma(0) = 1. \tag{3.3}$$

The choice of Gaussian function in all arguments for $D(\{\vec{s}_i\})$ is not quite unreasonable, since in this case one gets Gaussian p–p amplitude in the first order approximation (corresponding to a simple exponential in t). It has been conjectured by many authors that the dip-bump structure in differential cross-section is due to the interference between single and double scattering both being monotonic functions in t. Our choice can be regarded as the simplest ansatz satisfying this scheme.

We will compare the results obtained using Eq. (3.1) and the following alternative forms of $D(\{\vec{s}_i\})$

$$D(\{\vec{s}_i\}) = \frac{3}{\pi^2 A^2} \exp\left(-\frac{s_1^2+s_2^2+s_3^2}{A}\right)\delta^{(2)}(\vec{s}_1+\vec{s}_2+\vec{s}_3), \tag{3.4}$$

$$D(\{\vec{s}_i\}) = \int_0^\infty dx\, xe^{-x}\frac{1}{(\pi xA)^3}\exp\left(-\frac{s_1^2+s_2^2+s_3^2}{xA}\right), \tag{3.5}$$

$$D(\{\vec{s}_i\}) = \int_0^\infty dx\, xe^{-x}\frac{3}{(\pi xA)^2}\exp\left(-\frac{s_1^2+s_2^2+s_3^2}{xA}\right)\delta^{(2)}(\vec{s}_1+\vec{s}_2+\vec{s}_3). \tag{3.6}$$

The functions (3.4) and (3.6) differ from (3.1) and (3.5) by additional δ-functions. Due to these δ-functions unphysical transverse momentum distribution of center of mass of the particles is avoided [9] or, in other words, the recoil effects are properly taken into account[4].

The function (3.6) may be regarded as a superposition of Gaussians with different widths. It has a virtue of producing dipole proton formfactor:

$$F(\vec{\Delta}) = \int d^2b \exp(i\vec{\Delta}\cdot\vec{b})\int d^2s_1 d^2s_2 d^2s_3 \delta^{(2)}(\vec{b}-\vec{s}_1)D(\{\vec{s}_i\}) = \left(1+\Delta^2\frac{A}{6}\right)^{-2}. \tag{3.7}$$

Note that analogous formula for pion (two quark system) produces simple pole formfactor. The function (3.6) is therefore a simple example of a choice producing correct quark counting rules.

The shape of the function $\gamma(\vec{b})$ seems to make not much difference in the elastic amplitude. As noted above, the magnitude of inelastic diffractive cross-section suggests that quarks are "black" i.e. $\gamma(0) = 1$. We have checked that two other choices which satisfy the condition (3.3)

$$\gamma(\vec{b}) = 1-(1-e^{-\frac{b^2}{R^2}})(1-\alpha e^{-\frac{b^2}{\beta R^2}}), \tag{3.8}$$

$$\gamma(\vec{b}) = [1-\exp(-\lambda e^{-\frac{b^2}{R^2}})]/(1-e^{-\lambda}), \tag{3.9}$$

give p–p amplitudes similar to that obtained from the Gaussian form (3.2).

---

[4] For the quark model it is not quite obvious that the choices (3.4) or (3.6) are superior; some part of the momentum can be carried by "bag walls", "additional glue" or other hypothetical objects (absent in nuclear physics) and some transverse oscillations of "quarks CM" are in principle possible. Comparison of the results obtained with both choices may help to understand better the quark dynamics.

Since the form (3.8) corresponds to a Gaussian with additional "tail" (for $\beta \gg 1$) and the form (3.9) interpolates for different values of λ between a Gaussian and "black disc"

$$\lim_{\substack{\lambda\to\infty \\ R^2\to 0 \\ R^2 \ln\lambda = C}} [1-\exp(-\lambda e^{-\frac{b^2}{R^2}})]/(1-e^{-\lambda}) = \theta(C-b^2),$$

we can regard our three cases as a fair representation of different possibilities. Hence we conclude that change in $\gamma(\vec{b})$ does not influence significantly the shape of the elastic cross--section.

Our choice of functions D may seem to be rather arbitrary. However, it seems to be difficult to find a function differing significantly from (3.6) or (3.5) and producing dipole formfactor, if we do not introduce correlations between quarks other than δ-factor. Therefore, similar results for significantly different choices (3.4) and (3.6) or (3.1) and (3.5) suggest the conclusion that these results are representative for quite a wide class of density functions D.

This class is defined mainly by three requirements:

(*i*) quarks are weakly correlated,

(*ii*) all the quarks interact with the same amplitude,

(*iii*) distribution of quarks is consistent with electromagnetic formfactor of the proton.

We shall see whether with these assumptions our model is able to describe the data.

4. *Multiple scattering corrections*

In order to estimate the magnitude of multiple scattering we present here a simplified argument which is independent of particular choice of functions $D(\{\vec{s}_i\})$ and $\gamma(\vec{b})$.

The geometrical picture suggests that n-th order contribution to the total cross-section behaves like $w_n \cdot (x/2)^n$, where

$$\frac{x}{2} = \frac{r^2}{2\langle\vec{s}^2\rangle} \tag{4.1}$$

is approximately the probability of two quarks (each from another proton) to interact, and $w_n = \binom{9}{n}$ is a number of different choices of pairs of interacting quarks. r^2 is squared radius of q$-$q interaction which measures the effective size of quark in transverse plane and $\langle\vec{s}^2\rangle$ is mean squared value of transverse quark position in the proton.

A lower bound for x can be obtained from experimental data on total p$-$p cross--section σ_{tot} and elastic slope in forward direction B. Because of shadowing effects the following inequality holds:

$$\sigma_{\text{tot}} < 9\sigma_{\text{tot}}^{\text{qq}}, \tag{4.2}$$

where $\sigma_{\text{tot}}^{\text{qq}}$ denotes the total quark-quark cross-section and equals to $2\pi r^2$ by definition of r. Similarly the slope B is increased by the multiple scattering and so it is greater than the slope for point-like quarks $B_{\text{p}} = \langle \vec{s}^2 \rangle$.

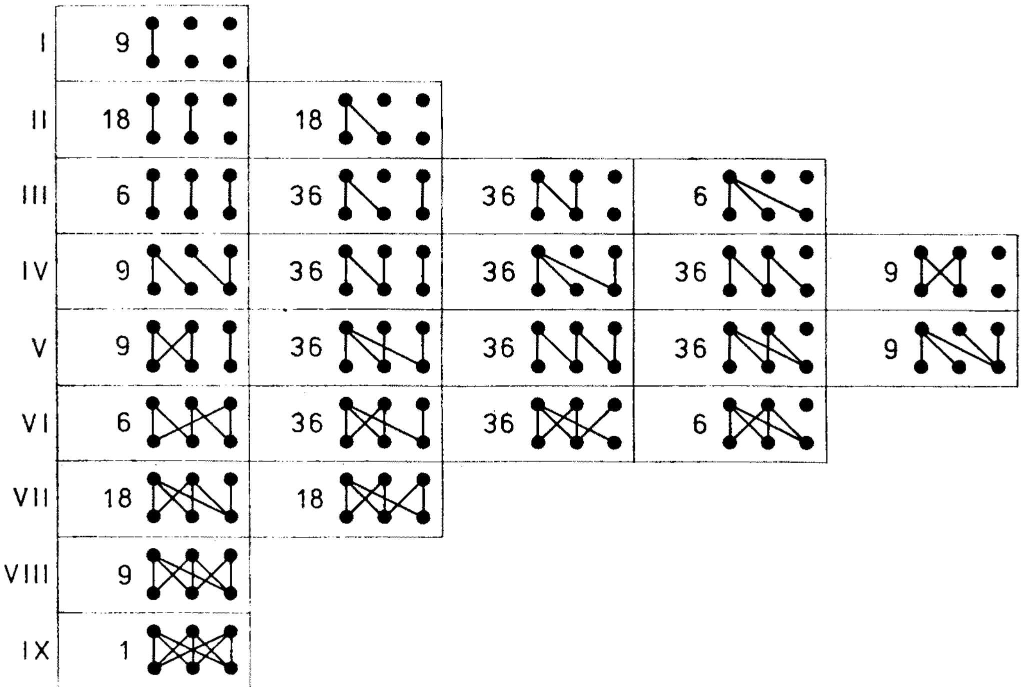

Fig. 2. Graphs representing different terms in the multiple scattering expansion of pp amplitude

Hence for x we have:

$$x = \frac{\sigma_{\text{tot}}}{2\pi\langle \vec{s}^2 \rangle} > \frac{\sigma_{\text{tot}}}{18\pi B} \simeq 0.2. \tag{4.3}$$

This estimate gives following ratios of subsequent orders of scattering:

$$9 : 3.6 : 0.8 : 0.1 : 0.0008 : 0.00004 : 0.000001 : 0.00000001$$

So one should expect several orders of scattering to be of similar magnitude. To illustrate this fact we show results for Gaussian $\gamma(\vec{b})$ (Eq. (3.2)) and $D(\{\vec{s}_i\})$ given by Eq. (3.4). Numerical values for r^2 and $\langle \vec{s}^2 \rangle$ are chosen to fit the total p$-$p cross-section and profile function at $b = 0 : \Gamma(0)$, for total CM energy $\sqrt{s} = 53$ GeV. The contributions of consecutive orders to these two quantities are the following

$$\sigma_{\text{tot}} = 42.5 \text{ mb} = (56.0-16.8+3.9-0.7+0.2-0.03+0.004-0.0004+0.00002) \text{ mb}, \tag{4.4}$$

$$\Gamma(0) = 0.75 = 1.37-0.95+0.47-0.18+0.06-0.02+0.003-0.0005+0.00003. \tag{4.5}$$

The contributions of multiple scatterings to $d\sigma/dt$ and to $\Gamma(\vec{b})$ are shown in Figs 3a and 3b (the numbers on the curves denote how many orders in multiple scattering are included).

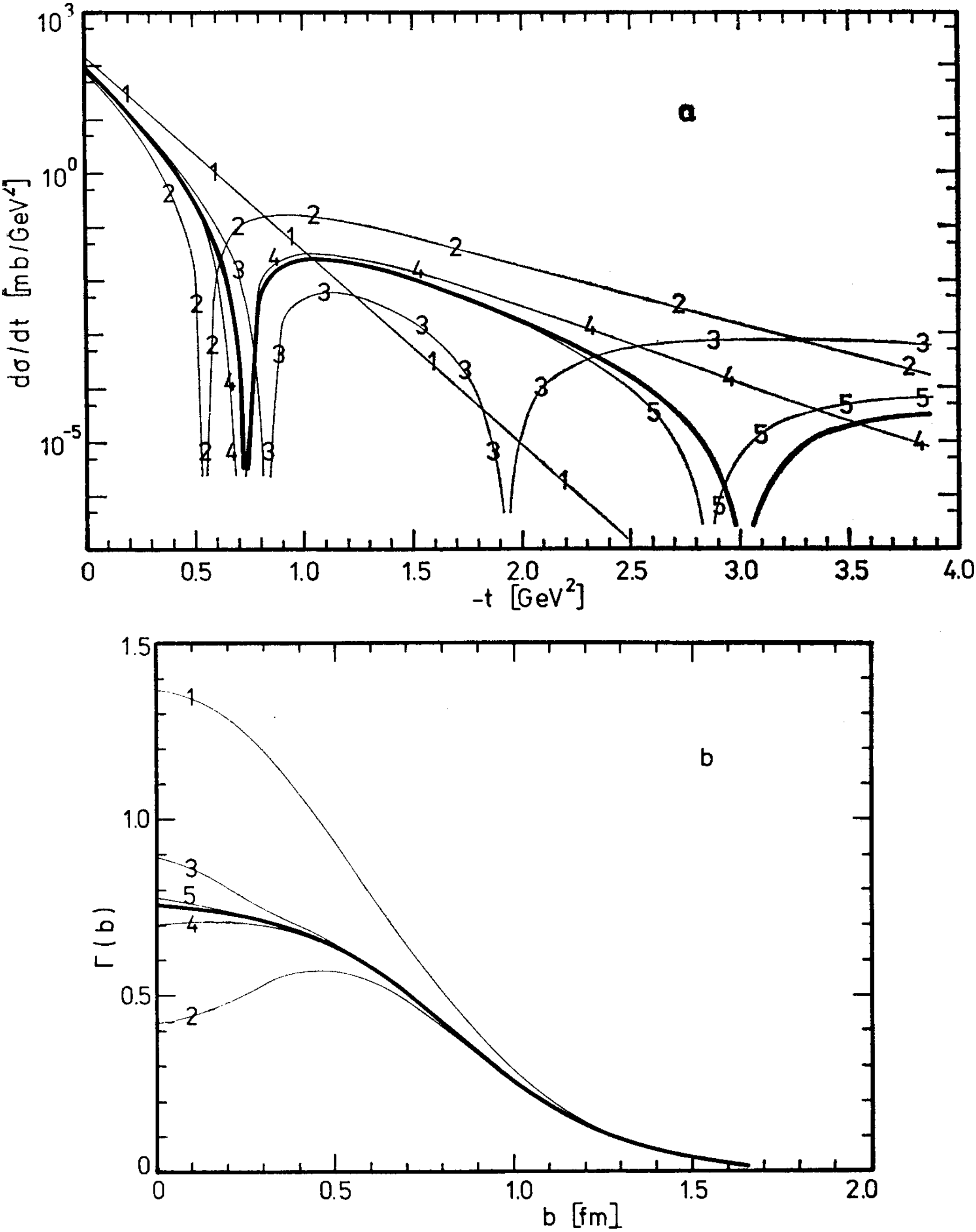

Fig. 3. a) Differential cross section $\frac{d\sigma}{dt}$ and b) impact parameter profile $\Gamma(\vec{b})$ calculated to the first (1), second (2), third (3), fourth (4), fifth (5) and ninth order of the multiple scattering expansion for the Gaussian density function (3.4) and Gaussian quark amplitudes (3.2) with the typical values of parameters

5. *Recoil effects for scattering at high momentum transfers*

It has been pointed out recently [13] that the ISR results for high momentum transfer elastic pp scattering [5] are rather embarrassing for "composite structure" models of pp collisions (as e.g. Chou-Yang model [14]). Indeed, although such models are able to predict a dip and secondary maximum corresponding to that observed in experiment for moderate momentum transfers, they fail to reproduce correctly differential cross section for higher t. In particular, the predicted decrease after the secondary maximum is too fast, and further dips (expected in the model) do not appear in data. Consequently, experimental cross section at momentum transfer, say, 5 $(\mathrm{GeV}/c)^2$ is by orders of magnitude higher than model predictions.

We would like to emphasize here that for the "composite structure" models with finite and small number of particle constituents (as in the quark model discussed in this paper) the result quoted above is valid only if one neglects recoil effects[5]. Introducing an additional $\delta^{(2)}(\sum_i \vec{s}_i)$ factor ($\vec{s}_i$ are positions of constituents in impact parameter plane) into the density functions of (3.1), (3.5) one can enhance the resulting cross section (at high momentum transfer) by orders of magnitude.

We shall demonstrate this statement for the simple case of Gaussian density functions and quark amplitudes. Denote any of the terms in the multiple scattering expansion of pp amplitude by $T_k(\vec{\Delta})$:

$$T_k(\vec{\Delta}) = \int d^2s_1^A \dots d^2s_3^B d^2b D_A(\{\vec{s}_i^A\}) D_B(\{\vec{s}_j^B\}) \exp\left(-\sum_{ij} m_{ij}(\vec{b}+\vec{s}_i^A-\vec{s}_j^B)^2\right) e^{i\vec{\Delta}\cdot\vec{b}}. \tag{5.1}$$

One can prove that the formulae obtained by integrating Eq. (5.1) with (T_k^{r}) and without recoil factor in $D\,(T_k^{\mathrm{nr}})$ are related by [9]

$$T_k^{\mathrm{nr}}(\vec{\Delta}) = e^{-\frac{\Delta^2 A}{6}} T_k^{\mathrm{r}}(\vec{\Delta}). \tag{5.2}$$

This does not mean yet that the result with recoil is much less steep, since A is a free parameter which should be fitted separately in both versions of the model. However, we can estimate its value by comparing the experimental forward slope with the slope of single scattering term (expected to dominate forward scattering). We have

$$B \equiv \frac{d}{dt}\left(\ln \frac{d\sigma}{dt}\right)\bigg|_{t=0} \simeq A_{\mathrm{nr}} + \frac{r^2}{4} = \tfrac{2}{3} A_{\mathrm{r}} + \frac{r^2}{4}.$$

Thus A_{nr} is about $10-12(\mathrm{c}^2/\mathrm{GeV}^2)$ and A_{r} about $15-18(\mathrm{c}^2/\mathrm{GeV}^2)$. We are now able to compute differential cross section in both models, but to get the idea of their relative magnitude it is enough to compare one of the least steep terms in amplitude, expected to be relevant in high momentum transfer region, $T_4(\vec{\Delta})$. It corresponds to three independent

---

[5] We would like to thank Professor Czyż for pointing out this to us.

collisions, in which all the quarks from both protons take part pairwise (first graph in III order in Fig. 2). This term is given in the discussed models by

$$T_4^{\mathrm{nr}}(\vec{\Delta}) = x_{\mathrm{nr}}^2 \pi r^2 \frac{\frac{2}{3}+\frac{1}{3}x_{\mathrm{nr}}}{(2+x_{\mathrm{nr}})^3} \exp\left(-\Delta^2 \frac{2A_{\mathrm{nr}}+r^2}{12}\right),$$

$$x_{\mathrm{nr}} = \frac{r^2}{A_{\mathrm{nr}}}, \tag{5.3}$$

and

$$T_4^{\mathrm{r}}(\vec{\Delta}) = x_{\mathrm{r}}^2 \pi r^2 \frac{\frac{2}{3}+\frac{1}{3}x_{\mathrm{r}}}{(2+x_{\mathrm{r}})^3} \exp\left(-\Delta^2 \frac{r^2}{12}\right). \tag{5.4}$$

We see that the value of A_{r} does not enter at all in the exponent. This reduces the estimated error of the ratio of (5.3) and (5.4) (x_{r} and x_{nr} are much smaller than 1 and not very different in all our fits). We have

$$\frac{T_4^{\mathrm{nr}}(\vec{\Delta})}{T_4^{\mathrm{r}}(\vec{\Delta})} \simeq e^{t\frac{A_{\mathrm{nr}}}{6}} \simeq e^{2t},$$

which corresponds to 8 orders of magnitude in the differential cross section at $t = -5\,(\mathrm{GeV}/c)^2$. Thus the disqualifying results of Ref. [13] do not apply to the quark models with small number of quarks and recoil effects taken into account. It is therefore interesting to try to fit these models to the data. Note that for the infinite number of constituents (optical limit, as in the Chou-Yang model [14]) the recoil effects are negligible [9]. The results of Ref. [13] seem to rule out all such models as possible pictures of high t scattering.

6. The results of numerical calculations

In this section we summarize the results of our calculations and compare them with experimental data. We have computed the elastic amplitude of p$-$p scattering in impact parameter representation and in momentum transfer representation as well, including all orders of scattering (Eq. (2.1)). The comparison of the results for Gaussian $\gamma(\vec{b})$ (Eq. (3.2)) and for three different $D(\{\vec{s}_i\})$ given by Eqs (3.1), (3.4), (3.6) was made. Such D functions lead to Gaussian (G) or dipole (d) electromagnetic proton formfactors — corrected for recoil effects (r) or not (nr). Thus we denote our three cases by: (G, nr), (G, r), (d, r) respectively. It is convenient to parametrize the results by q$-$q interaction radius squared r^2 and dimensionless parameter $x = r^2/\langle\vec{s}^2\rangle$ (Eq. (4.1)). For all amplitudes and cross-sections considered here these parameters are fixed by following requirements:

$$\sigma_{\mathrm{tot}} = 42.5 \text{ mb}, \tag{6.1}$$

$$\Gamma(b = 0) = 0.75, \tag{6.2}$$

according to experimental data at total CM energy $\sqrt{s} = 53$ GeV.

In Figure 4 we show the dependence of $\Gamma(b = 0)$ on parameter x. From this graph and Eq. (6.2) we obtain $x \cong 0.36$ for all three cases. Then the requirement (6.1) implies the following values of q−q interaction range r, transverse proton size[6] $R_p = \sqrt{\langle \vec{s}^2 \rangle}$ and logarithmic slope B of $d\sigma/dt$ (for small momentum transfer).

D function	r [fm]	R_p [fm]	B [GeV^{-2}]
(G, nr)	0.33	0.55	10.6
(G, r)	0.31	0.52	9.7
(d, r)	0.33	0.55	10.6

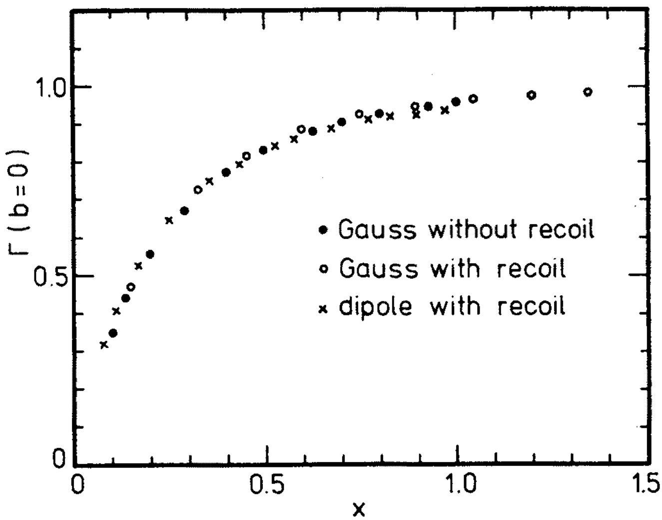

Fig. 4. The values of opacity for central collision $\Gamma(0)$ calculated for Gaussian density function with (3.4) and without recoil (3.1) and for dipole density function with recoil (3.6) as a function of parameter $x = \frac{r^2}{\langle \vec{s}^2 \rangle}$

[6] The quark density distribution (3.6) leads to electromagnetic proton formfactor of correct dipole form, so it may be interesting to compare the transverse proton size $R_p^2 = 0.31$ fm^2 resulting from our fit and $R_{em}^2 = 0.44$ fm^2 given by experimental proton formfactor

$$F_{exp}(t) = (1+t/0.71)^{-2}.$$

The difference between values of R_p^2 and R_{em}^2 has simple physical interpretation. The neutral glue does not take part in electromagnetic interactions and can be regarded as fourth component of proton, carrying a part of total proton momentum. Therefore there are no momentum conservation constraints on quark wave functions (the recoil effects are not important in this case). In the strong interaction processes the recoil corrections are necessary and the following relation should hold:

$$R_p^2 = \tfrac{2}{3} R_{em}^2.$$

This prediction is in good agreement with the values quoted above.

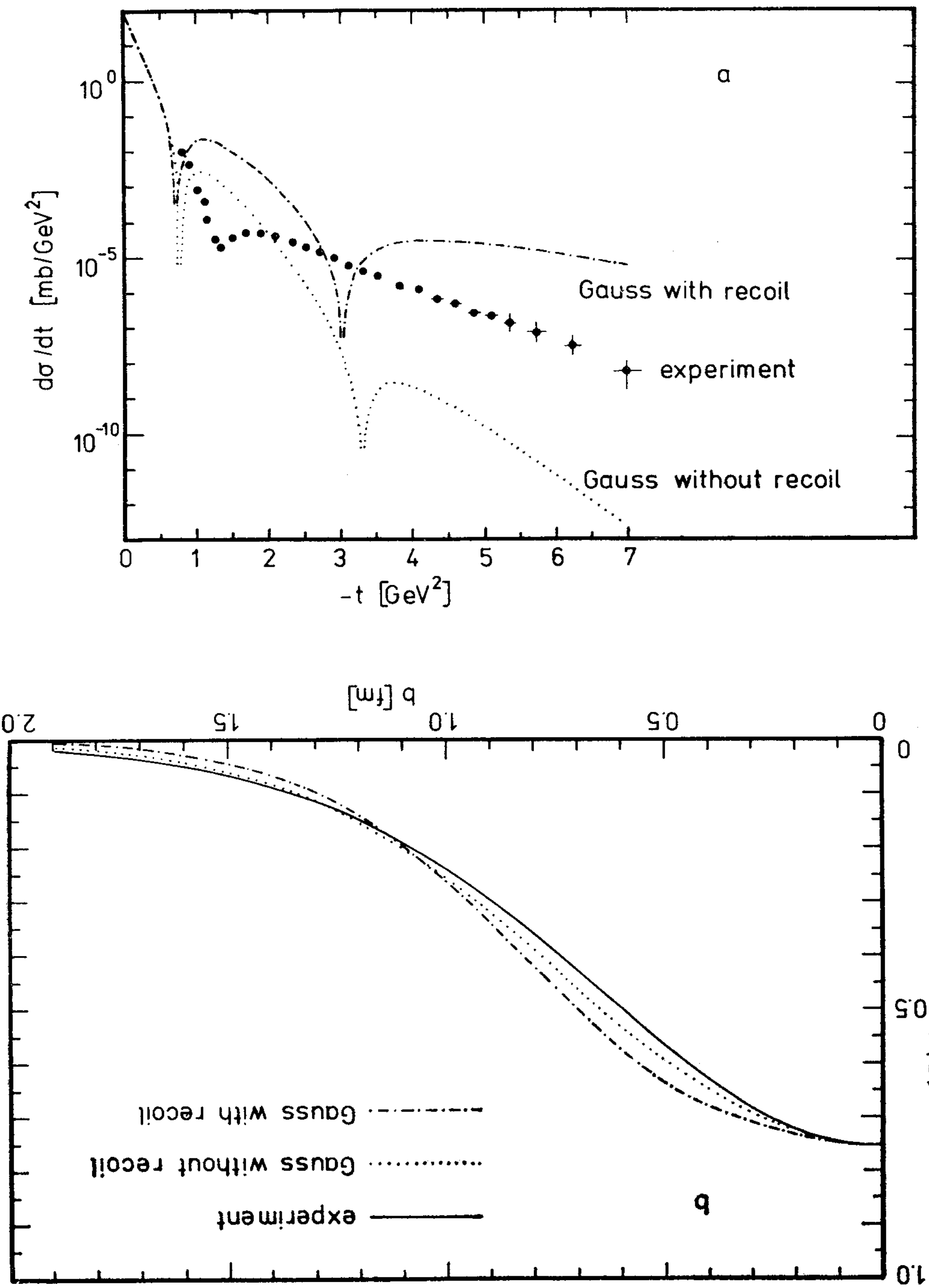

Fig. 5. Comparison of the experimental differential cross section a) and impact parameter profile b) with the model calculations for Gaussian density function with and without recoil for typical values of parameters

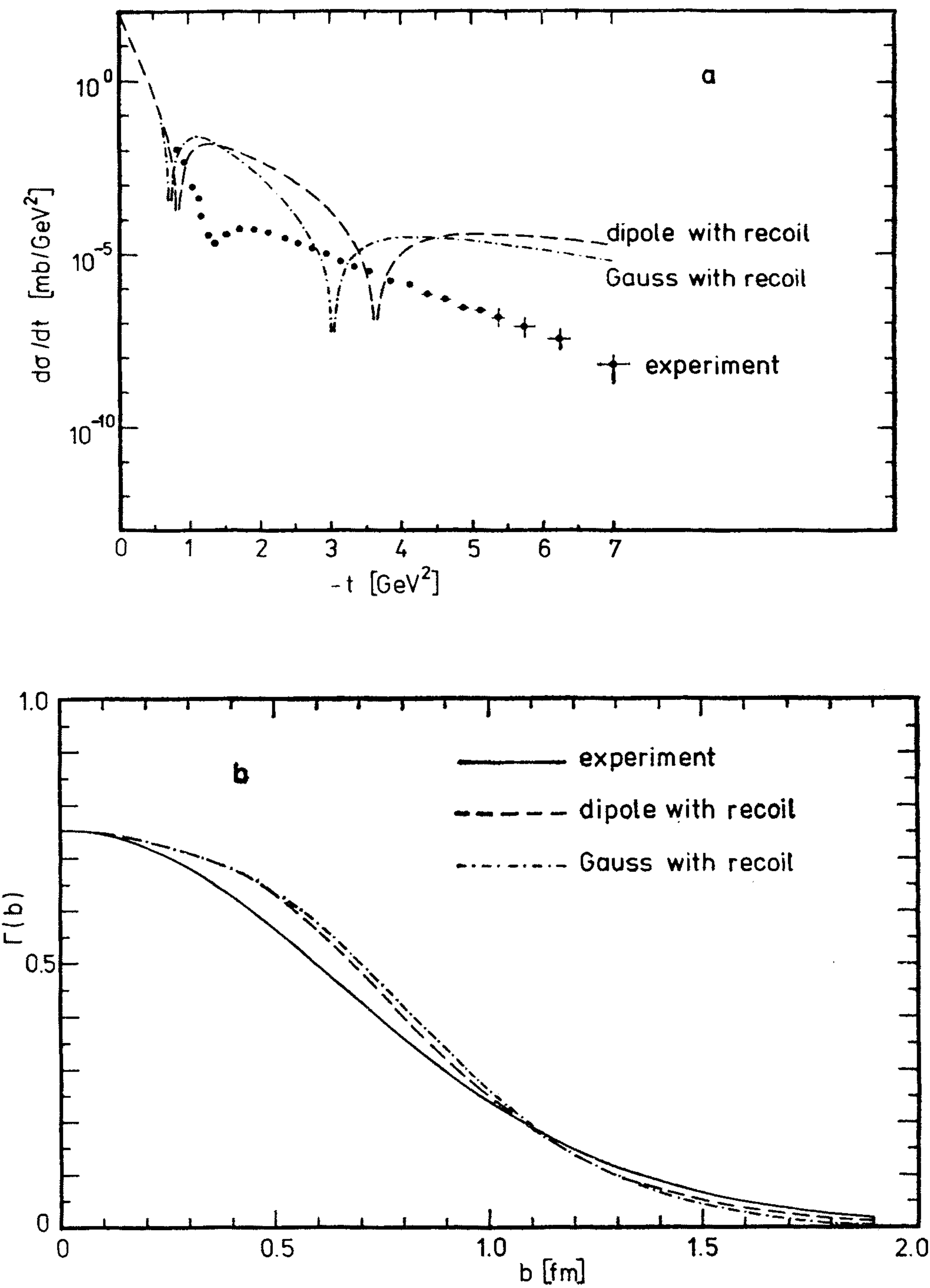

Fig. 6. Comparison of the experimental differential cross section a) and impact parameter profile b) with the model calculations for Gausssian and dipole density function with recoil for typical values of parameters

The results for differential cross-section $d\sigma/dt$ for cases (G, nr) and (G, r) are compared with experimental data in Fig. 5a. This picture shows that recoil effects are responsible for slow decrease and large values of $d\sigma/dt$ at large momentum transfer. (This fact was already discussed in Section 5 . However, none of the model curves agrees quantitatively with data.

For the set of D functions (Eqs (3.5), (3.6)) giving dipole formfactor we present only (d, r) curve (Fig. 6a) and compare it with (G, r) curve to exhibit the dependence of $d\sigma/dt$ on the shape of quark density distribution.

The proton profile in impact parameter space $\Gamma(\vec{b})$ obtained in all three cases is more "black" for small values and more "transparent" for large values of impact parameter

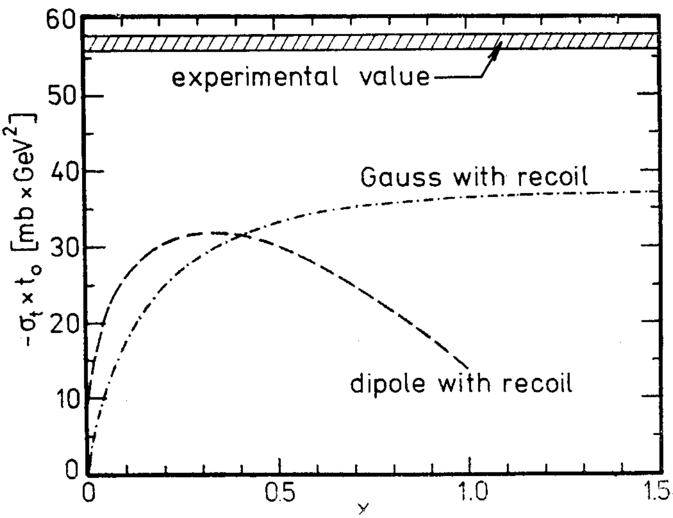

Fig. 7. The values of the product of total cross section and dip position $-\sigma_t t_0$ calculated for Gaussian and dipole density function with recoil as a function of parameter $x = \frac{r^2}{\langle \vec{s}^2 \rangle}$. Experimental value is shown as horizontal line

when compared with experimental distribution. The comparison of $\Gamma(\vec{b})$ with data (from Ref. [15]) is presented in Fig. 5b for (G, r) and (G, nr) cases and in Fig. 6b for (d, r) case.

In all our model curves the position of first dip in $d\sigma/dt$ is distincly shifted to smaller values of momentum transfer ($-t_0 = 0.7-0.8$ GeV^2 for all the cases) than the experimental value $-t_0 = 1.34$ and a second dip in $d\sigma/dt$ is present in contradiction with experimental data. For our simple choices for functions D and γ the position of the first dip cannot be fitted correctly without significant decrease of total cross-section. This is seen in Fig. 7 where the product $-t_0\sigma_{tot}$ (which is the function of x only) is plotted vs x for two cases: (G, r) and (d, r). Both curves lie below the experimental value.

7. Conclusions and outlook

We have presented the calculation of the elastic pp scattering in a specific quark model. The main assumption is that the proton consists of three valence quarks, each dressed with the corresponding piece of glue (and possibly $q\bar{q}$ pairs). All these dressed quarks contribute to the elastic scattering amplitude according to the multiple scattering expansion. We have assumed furthermore that quarks are uncorrelated except for momentum conservation constraints. To satisfy the experimental estimates for inelastic diffraction, quarks are assumed to be small and nearly "black", i.e. in central qq collision the unitarity is saturated. As long as this condition is fulfilled, the detailed shape of qq amplitude does not influence our results[7].

We find these assumptions strongly restrictive and determining the resulting elastic amplitude almost unambiguously. The numerical study of this amplitude led us to the following conclusions.

1. Recoil factors in the quark density functions (implied by the momentum conservation for quarks) play essential role for the momentum transfers higher than $2\,(\mathrm{GeV}/c)^2$. In particular the recoil factors increase the cross-section in this region by several orders of magnitude, as compared to the calculation without recoil. They seem thus to be necessary to get rid of discrepancy [13] between the data and the composite models without recoil. We like to emphasize that these recoil corrections appear only if the colliding objects consist of *finite* number of constituents.

2. Multiple scattering terms are very important. In fact, to obtain a reasonable accuracy in the disscussed momentum transfer range, it is necessary to consider multiple scattering expansion up to (at least) fifth order.

3. With the assumptions listed at the beginning of this Section we were unable to fit the model to the existing data. In particular, it seems impossible to describe simultaneously the position of the dip in the differential cross-section and the value of the total cross-section. Furthermore, using the experimental value of the total cross-section, the calculated differential elastic cross-section behind the dip is too large and a secondary minimum appears at $|t| \sim 3(\mathrm{GeV}/c)^2$ contrary to the experimental observations.

4. These discrepancies between the model and the data indicate that the model overestimates the multiple scattering constributions. Indeed, the position of the dip is determined mainly by interference between single and double scattering. Consequently, by suppressing the multiple scattering terms, the dip is moved to higher values of momentum transfer, as required by the data. At the same time the magnitude of the secondary maximum is reduced and the second dip shifted to high values of momentum transfer. Let us add that also the well-established additivity rules [6] indicate that the multiple scattering corrections should not be too large. This last observation gives further support to our conclusion.

---

[7] It should be emphasized that this situation is quite different from that in the Van Hove glue model [2], where the change of the same function (glue-glue amplitude) affects simultaneously elastic and inelastic diffraction, whereas the corresponding density function (distribution of the glue position) is rather irrelevant for the results. This is so because in the Van Hove model there is only one active constituent — the glue, and its size is not much smaller than the size of the proton.

We would like to close our discussion by few speculations about possibilities of further investigation along the lines presented in this paper.

Since our numerical estimates indicated that multiple scattering corrections are too large, it is necessary to search for mechanisms which would reduce them. In the multiple scattering model we consider, a natural way to suppress these multiple scattering terms would be to introduce the repulsive short-range correlations (in the impact parameter plane) between the quarks in the proton. We thus feel that our calculation strongly suggests the existence of such correlations. It remains to be checked, however, if it is possible to obtain a quantitative agreement with the data in this approach. This is an interesting question to study.

The origin of such correlations between quarks in the impact parameter plane may be two-fold. They may be either the genuine dynamical correlations, or they may appear as the result of integration over the longitudinal variables of an otherwise uncorrelated (or weakly correlated) distribution (the momentum conservation constraints i.e. recoil factors should always be included). Longitudinal variables were neglected altogether in the present paper. We thus feel that the next step is to repeat our calculation with longitudinal variables taken properly into account.

Another possibility of reducing the multiple scattering corrections is to accept that the qq cross-section is not a fixed number but may depend on other parameters (e.g. x value of the quarks) [3]. This may occur, e.g. if the amount of glue which dresses a given quark fluctuates and consequently changes from one quark to another. It would certainly be interesting to formulate a specific model taking this possibility into account and to study its consequences.

We thank Professor W. Czyż for help in understanding the multiple scattering model and for constant encouragement. One of us (A.B.) thanks Professor L. Van Hove for illuminating discussions.

APPENDIX

In order to compute the elastic amplitude in impact parameter representation $\Gamma(\vec{b})$ (Eq. (2.3)) one has to average all terms of the multiple scattering expansion (Eq. (2.1)) over quark density distributions in particles A and B. Many terms produced by Eq. (2.1) are equivalent, i.e. after averaging they become equal. Thus, for each order of scattering, elastic profile $\Gamma(\vec{b})$ is the sum of only several different contributions. In the case of p$-$p scattering there are altogether 25 different terms contributing to $\Gamma(\vec{b})$ (Eq. (A.1)); the corresponding diagrams are presented in Fig. 2. The lines show, which pairs of quarks interact during collision. Each diagram is multiplied by the number c_k indicating how many multiple scattering expansion terms of this type are equivalent. We are going now to describe a simple algebraic scheme which solves the problem of integration over transverse quark positions $\{\vec{s}_i\}$ (Eq. (2.3)) when the quark density distributions D_A, D_B and q−q scattering amplitude γ are Gaussian or superpositions of Gaussians, e.g. (3.5), (3.9)[8].

This scheme can be useful in analytical and/or numerical calculations. The full elastic amplitude $\Gamma(\vec{b})$ can be written as follows

$$\Gamma(\vec{b}) = \sum_{k=1}^{25} (-1)^{n+1} c_k \Gamma_k(\vec{b}) \tag{A.1}$$

where the amplitude Γ_k corresponding to k-th diagram (which belongs to n-th order of scattering) has the form

$$\Gamma_k(\vec{b}) = \int d^2 s_1^A \dots d^2 s_3^B D_A(\{\vec{s}_i^A\}) D_B(\{\vec{s}_j^B\}) \prod_{(i,j)} \gamma_{ij}(\vec{b}+\vec{s}_i^A-\vec{s}_j^B). \tag{A.2}$$

The product $\prod_{(i,j)}$ involves all quark pairs (i,j) which undergo interaction in the process described by k-th diagram (see Fig. 2.). Thus, in general case, we have to calculate for each diagram an integral of the following type (the index k is not written explicitly)

$$I(\vec{b}) = \int d^2 s_1^A \dots d^2 s_3^B \exp(-w(\vec{s}_1^A, \dots, \vec{s}_3^B; \vec{b})), \tag{A.3}$$

where

$$w(\{\vec{s}_i\}; \vec{b}) = \sum_{i=1}^{3} \frac{(\vec{s}_i^A)^2}{A_i} + \sum_{j=1}^{3} \frac{(\vec{s}_j^B)^2}{B_j} + \sum_{i,j=1}^{3} m_{ij}(\vec{b}+\vec{s}_i^A-\vec{s}_j^B)^2. \tag{A.4}$$

The form (A.4) of w corresponds to Gaussian density distributions with individual radius squared (A_1, A_2, A_3 in particle A, B_1, B_2, B_3 in particle B) for each quark and to Gaussian quark profiles γ_{ij} with different radii r_{ij} for each interacting quark pair (i,j). The radii r_{ij} determine the matrix m_{ij}

$$m_{ij} = \begin{cases} 0 & \text{if quark pair } (i,j) \text{ does not interact in the process described} \\ & \text{by the considered diagram,} \\ \dfrac{1}{r_{ij}^2} & \text{otherwise.} \end{cases} \tag{A.5}$$

Writing explicitly the exponent w (Eq. (A.4)) as a second-order form of (2-dimensional) variables $\vec{s}_i (\vec{s}_1^A = \vec{s}_1, \dots, \vec{s}_3^B = \vec{s}_6)$ we have

$$w = \sum_{i,j=1}^{6} G_{ij} \vec{s}_i \cdot \vec{s}_j + 2\vec{b} \sum_{i=1}^{6} V_i \vec{s}_i + Z b^2. \tag{A.6}$$

[8] In the latter case each of the amplitudes Γ_k contributing to $\Gamma(\vec{b})$ becomes superposition of the terms which can be calulated in frames of the presented scheme (all of these terms are of the form described by Eqs (A.3), (A.4)). An example of the calculation with functions being superpositions of Gaussians is presented at the end of the Appendix. The δ-factors responsible for recoil effects can be easily introduced into the presented scheme using the standard method described e.g. in Ref. [9].

After changes of variables in w (Eq. (A.6)) — translation to obtain a (positively defined) quadratic form and subsequent diagonalisation of this form — the integrand in (A.3) becomes the product of independent Gaussians and integration is then trivial. In terms of matrix G, vector V, and scalar Z the result reads

$$I(\vec{b}) = \frac{\pi^6}{\det G} \exp\left[-b^2(Z - V^T \cdot G^{-1} \cdot V)\right]. \tag{A.7}$$

The explicit forms of Z, V, and G can be written in the following way:

$$Z = \sum_{i,j=1}^{3} m_{ij}, \tag{A.8}$$

$$V_i = \begin{cases} \sum\limits_{j=1}^{3} m_{ij}, & i = 1, 2, 3 \\ -\sum\limits_{j=1}^{3} m_{j,i-3}, & i = 4, 5, 6 \end{cases}, \tag{A.9}$$

$$G_{ii} = \begin{cases} V_i + \dfrac{1}{A_i}, & i = 1, 2, 3 \\ -V_i + \dfrac{1}{B_{i-3}}, & i = 4, 5, 6 \end{cases}, \tag{A.10}$$

$$G_{ji} = G_{ij} = \begin{cases} -m_{i,j-3}, & i = 1, 2, 3,\ j = 4, 5, 6 \\ 0 & \text{for other } i < j \end{cases}, \tag{A.11}$$

$$G = \left[\begin{array}{ccc|ccc} G_{11} & 0 & 0 & & & \\ 0 & G_{22} & 0 & & -\hat{m} & \\ 0 & 0 & G_{33} & & & \\ \hline & & & G_{44} & 0 & 0 \\ & -\hat{m}^T & & 0 & G_{55} & 0 \\ & & & 0 & 0 & G_{66} \end{array}\right].$$

The generalization of the presented scheme for different numbers of quarks in colliding hadrons is straightforward. Now, we list explicit forms of amplitudes corresponding to first seven diagrams (three orders of scattering) for our choices of functions D (Eq. (3.1)) and γ (Eq. (3.2)) ($A_i = A$, $B_j = B$, $r_{ij}^2 = r^2$, $i, j = 1, 2, 3$). For the sake of simplicity we introduce dimensionless parameters $X = A/r^2$, $Y = B/r^2$.

$$\Gamma_1(\vec{b}) = \frac{1}{1+X+Y} \exp\left[-\frac{b^2}{r^2}\,\frac{1}{1+X+Y}\right], \tag{A.12}$$

$$\Gamma_2(\vec{b}) = [\Gamma_1(\vec{b})]^2, \tag{A.13}$$

$$\Gamma_3(\vec{b}) = \frac{1}{1+2(X+Y)+(2X+Y)Y} \exp\left[-\frac{b^2}{r^2}\,\frac{2(1+Y)}{1+2(X+Y)+(2X+Y)Y}\right], \tag{A.14}$$

$$\Gamma_4(\vec{b}) = [\Gamma_1(\vec{b})]^3, \tag{A.15}$$

$$\Gamma_5(\vec{b}) = \Gamma_1(\vec{b})\Gamma_3(\vec{b}), \tag{A.16}$$

$$\Gamma_6(\vec{b}) = \frac{1}{1+3(X+Y)+2XY(3+X+Y)+2(X^2+Y^2)}$$
$$\times \exp\left[-\frac{b^2}{r^2}\frac{3+4(X+Y+XY)}{1+3(X+Y)+2XY(3+X+Y)+2(X^2+Y^2)}\right], \tag{A.17}$$

$$\Gamma_7(\vec{b}) = \frac{1}{1+3(X+Y)+3XY(2+Y)+Y^2(3+Y)}$$
$$\times \exp\left[-\frac{b^2}{r^2}\frac{3(1+Y)^2}{1+3(X+Y)+3XY(2+Y)+Y^2(3+Y)}\right]. \tag{A.18}$$

The corresponding amplitudes $T_k(\vec{\Delta})$ in momentum transfer representation can be easily obtained as Fourier transforms of $\Gamma_k(\vec{b})$ (compare Eq. (2.2)). The amplitudes $T_{k,\text{recoil}}(\vec{\Delta})$ with recoil corrections taken into account are simply connected with these functions T_k:

$$T_{k,\text{recoil}}(\vec{\Delta}) = \exp\left[\frac{\Delta^2(A+B)}{12}\right] T_k(\vec{\Delta}) \tag{A.19}$$

(see Ref. [9]).

Now let us apply the results (A.12)–(A.18) in the case of quark distributions given by Eq. (3.5) and Gaussian q–q amplitude Eq. (3.2). The functions D_A, D_B are the superpositions of Gaussians

$$D_A(\{\vec{s}_i\}) = \int_0^\infty dx\, w(x) D_A(\{\vec{s}_i\}, x) \tag{A.20}$$

with normalization conditions

$$\int_0^\infty dx\, w(x) = 1, \tag{A.21}$$

$$\int d^2 s_1 \dots d^2 s_3 D_A(\{\vec{s}_i\}, x) = 1. \tag{A.22}$$

The explicit form of D_A reads

$$D_A(\{\vec{s}_i\}; x) = \frac{1}{(\pi x A)^3} \exp\left(-\frac{\sum_{i=1}^{3} (\vec{s}_i)^2}{xA}\right) \tag{A.23}$$

and the choice

$$w(x) = x \exp(-x) \tag{A.24}$$

leads to dipole proton formfactor.

Substituting Eq. (A.20) into Eq. (A.2) we obtain

$$\Gamma_k(\vec{b}) = \int_0^\infty dx\, w(x) \int_0^\infty dy\, w(y) \Gamma_k(\vec{b}; x, y), \tag{A.25}$$

where

$$\Gamma_k(\vec{b}; x, y) = \int d^2 s_1^A \dots d^2 s_3^B D_A(\{\vec{s}_i^A\}, x) D_B(\{\vec{s}_i^B\}, y) \prod_{(i,j)} \gamma_{ij}. \tag{A.26}$$

The previously obtained results (Eqs (A.12)–(A.18)) hold for $\Gamma_k(\vec{b}; x, y)$ with substitutions $X = xA/r^2$, $Y = yB/r^2$.

REFERENCES

[1] S. Pokorski, L. Van Hove, *Acta Phys. Pol.* **B5**, 229 (1974); L. Van Hove, S. Pokorski, *Nucl. Phys.* **B86**, 243 (1975).
[2] L. Van Hove, *Acta Phys. Pol.* **B7**, 339 (1976).
[3] L. Van Hove, K. Fiałkowski, *Nucl. Phys.* **B107**, 211 (1976).
[4] L. Van Hove, CERN preprint — TH-2278, 1977.
[5] H. De Kerret et al., *Phys. Lett.* **62B**, 363 (1976).
[6] H. J. Lipkin, F. Scheck, *Phys. Rev. Lett.* **16**, 71 (1966); E. M. Levin, L. L. Frankfurt, *Zh. Eksp. Teor. Fiz.* **2**, 105 (1965).
[7] W. Królikowski, *Acta Phys. Pol.* **B8**, 237 (1977) and references quoted therein.
[8] R. J. Glauber, *Lectures in Theoretical Physics*, Vol. 1. Interscience, New York 1959.
[9] W. Czyż, L. C. Maximon, *Ann. Phys.* (USA) **52**, 59 (1969).
[10] N. W. Dean, *Nucl. Phys.* **B4**, 534 (1968); *Phys. Rev.* **182**, 1695 (1969); *Phys. Rev.* **D1**, 2703 (1970); *Phys. Rev.* **D5**, 124 (1972); A. Deloff, *Nucl. Phys.* **B2**, 597 (1967); V. Franco, *Phys. Rev. Lett.* **18**, 1159 (1967).
[11] D. Harrington, A. Pagnamenta, *Phys. Rev. Lett.* **18**, 1147 (1967); *Phys. Rev.* **173**, 1599 (1968).
[12] A. Białas, K. Fiałkowski, unpublished.
[13] U. P. Sukhatme, *Phys. Rev. Lett.* **38**, 124 (1977).
[14] T. T. Chou, C. N. Yang, *Phys. Rev.* **170**, 1591 (1968).
[15] H. I. Miettinen, *Acta Phys. Pol.* **B6**, 625 (1975) and private communication.