MULTIPLICITY DISTRIBUTION IN UNITARIZED UNCORRELATED CLUSTER PRODUCTION MODELS

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It is shown that the unitarity corrections dramatically change the multiplicity distributions expected from uncorrelated cluster emission models.

1. *Introduction*

In this papei we discuss the multiplicity distribution in a class of models for multiple production, which satisfy exactly the multiparticle unitarity. Our aim is to investigate the effects of rescattering corrections on multiplicity distribution.

The analysis of the elastic pp scattering data in terms of impact parameters shows that, for a large range of the impact parameters, the elastic amplitude is close to the unitarity limit (see e.g. [1, 2]). Consequently, rather large rescattering corrections are expected. Since higher order corrections to the elastic amplitude are provided by inelastic processes with higher multiplicities, we expect a substantial increase of multiplicity as an effect of imposing the unitarity condition.

It is fairly obvious that these effects are strongly model-dependent. It is thus interesting to investigate different models from that point of view, in order to learn as much as possible on the structure of the unitarity condition. In this paper we discuss a class of unitarized uncorrelated cluster production models.

In the models we consider, the S-matrix is strictly unitary at each impact parameter:

$$S(b) = e^{iH(b)}, \tag{1.1}$$

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where $H(b)$ is a Hermitian operator. Operator H is taken in the form [3, 4]:

$$H(b) = t(b)S_c(b)+t^\dagger(b)S_c^\dagger(b), \tag{1.2}$$

where operator $S_c(b)$ describes the independent production of mesonic clusters and operator $t(b)$ describes the scattering and excitation of leading clusteıs.

We found that the multiplicity distribution implied by the unitary S-matrix (1.1) is indeed dramatically diffeıent from the one expected from the non-unitarized version of the model where the T-matrix is given by Eq. (1.2). Furthermoıe, we show that the resulting multiplicity distribution cannot be reconciled with the existing data.

A serious drawback of the models we consider is that energy and momentum conservation is not satisfied. This limits strongly the possible general significance of our results. Nevertheless, we feel that these results are of some interest, because they show that rescattering corrections can be very important for the multiplicity distribution and should be seriously taken into account in any model of multiple production.

In Section 2 we define the models considered. In Sections 3 and 4 the formulae for the multiplicity distribution are derived. Section 5 contains the comparison with experimental data. Our conclusions are summarized in Section 6.

2. *The model*

To define the model described by Eqs (1.1) and (1.2) we have to give explicit forms of the operators $S_c(b)$ and $t(b)$. Since the operator S_c should describe independent production of mesonic clusters, we take it in the form [3]

$$S_c = \exp\{i(A+A^\dagger)\}, \tag{2.1}$$

where

$$A^\dagger = \int \varrho(k;\, b)a^\dagger(k)\,\frac{d^3k}{E}\,. \tag{2.2}$$

Here $\varrho(k; b)$ is the probability amplitude for emission of a mesonic cluster in scattering of the incident particle at impact parameter b. $a^\dagger(k)$ is the cluster creation operator satisfying the usual commutation relations:

$$[a^\dagger(k'), a(k)] = -E\delta^{(3)}(k-k'). \tag{2.3}$$

Consequently we obtain

$$[A, A^\dagger] = \int |\varrho(k, b)|^2\,\frac{d^3k}{E} \equiv V(b). \tag{2.4}$$

The function $V(b)$ defined by this formula may be interpreted as the effective phase-space accessible to a cluster.

The operator $t(b)$ describes pp scattering without production of mesonic clusters. It may, however, lead to the excitation of leading clusters.

We assume that this operator satisfies the condition

$$[t, t^\dagger] = \Omega(b), \tag{2.5}$$

where $\Omega(b)$ is a function of the impact parameter of the collision. In the following we will consider two models:

(a) no excitation of leading clusters

In this case we assume

$$\Omega(b) = 0, \tag{2.6}$$

i.e. that operators t and $t^\dagger$ commute with each other.

(b) excitation of leading clusters

In this case $\Omega(b) \neq 0$, but we assume that the operator t, when acting on states with two nucleons $|0\rangle$, gives zero

$$t|0\rangle = 0. \tag{2.7}$$

Thus the operator $t^\dagger$ can be interpreted as an operator which creates the nucleon excitation. Accordingly, t is the operator which deexcites the excited state. The condition (2.7) follows from the observation that the nucleon itself cannot be deexcited (since there are no lower--lying baryon states).

The conditions (2.5) — (2.7) are assumed for technical reasons. Using them, we can find analytic expressions for multiplicity distributions and sum them up to all orders. Thus we can determine exactly the rescattering corrections even in the case of full absorption. They restrict significantly, however, the general form of the model.

We would like to emphasize again that the S-matrix defined by the formulae (1.1), (1.2) and (2.1), (2.2) does not conserve energy and momentum. Nevertheless, we feel that it can be used as an interesting laboratory for studying the effects of rescattering corrections.

In the next section we present the formulae for multiplicity distributions in model (a), in which no leading clusters are excited. In Section 4 corresponding formulae for model (b), which incorporates the excitation of leading clusters, are given.

3. *Model with no excitation of leading clusters*

Let us denote by $|N\rangle$ a state with N mesonic clusters and two nucleons present:

$$|N\rangle = a^\dagger(k_1) \dots a^\dagger(k_N)\,|0\rangle, \tag{3.1}$$

where $|0\rangle$ is the state with only two nucleons present. Further indices, denoting e.g. impact parameter, cluster momenta, nucleon momenta, etc. are omitted to simplify the notation.

Using conditions (2.5) and (2.6) and the relation

$$S_c^\dagger S_c = S_c S_c^\dagger = 1 \tag{3.2}$$

which follows immediately from Eq. (2.1), we can expand the exponents in Eq. (2.1). After some rearrangements one obtains

$$iT|0\rangle = (S-1)\,|0\rangle = [J_0(2\sqrt{t^\dagger t})-1]\,|0\rangle$$

$$+\sum_{p=1}^{\infty} i^p (t^\dagger t)^{-p/2} J_p(2\sqrt{t^\dagger t}) \times \sum_{N=0}^{\infty} \frac{p^N}{N!}\, e^{-p^2V/2} \times \prod_{j=1}^{N} \varrho(k_j)\,[t^p+(-1)^N (t^\dagger)^p]\,|N\rangle, \tag{3.3}$$

where $J_\nu(z)$ are the Bessel functions.

We show further that the phase-space volume $V \sim \overline{N}$, so that at high energies it is a large parameter, increasing with energy typically as $\log s$. Consequently, the exponentials in Eq. (3.3) decrease like powers of energy and we get the following picture. The amplitude consists of a diffractive and a non-diffractive piece. The diffractive piece, (described by the first term in Eq. (3.3)) with little energy dependence, contains no production of mesonic clusters (the reason for that was explained in Ref. [3]). In our model, due to condition (2.6) it corresponds to elastic scattering. The cross-section for any channel with $N \neq 0$ goes to zero as a power of s multiplied by a slowly (e.g. logarithmically) varying function of s.

In order to obtain the cross-sections it is necessary to multiply Eq. (3.3) by its Hermitian conjugate and integrate over the momenta of the final particles. Of course, only terms diagonal in N contribute. It is, however, easy to see also that terms non-diagonal in p give contributions, which are negligible at high energies. The reason is essentially that

$$\frac{(pp'V)^N}{N!}\, e^{-(p^2+p'^2)V/2} = \frac{(pp'V)^N}{N!}\, e^{-pp'V} e^{-(p-p')^2V/2}. \tag{3.4}$$

For given N each term goes to zero like a power of energy. Summing over N, however, one finds a finite high-energy limit if and only if $p = p'$.

Another interesting term in the formula for the cross-section is the bracket containing t^p. Multiplying it by its Hermitian conjugate, we find

$$2(t^\dagger t)^p + (-1)^N [t^{2p} + (t^\dagger)^{2p}]. \tag{3.5}$$

When calculating at high energies quantities summed over N, like moments of the multiplicity distribution, the second term is unimportant, because

$$\sum (-1)^N \frac{(p^2V)^N}{N!}\, e^{-p^2V} = e^{-2p^2V}. \tag{3.6}$$

The multiplicity distribution, however, can be strongly affected. E.g. if t were a Hermitian operator, the probability of producing an odd number of mesonic clusters would vanish. This nicely illustrates the fact that high energy limits of the moments are not enough to determine the multiplicity distribution. Experimentally, the probabilities of producing odd numbers of particles interpolate smoothly the probabilities for producing even numbers ot particles. This can be accomodated for instance by assuming multiplicity fluctuations in cluster decays.

Furthermore, we assume that

$$\langle 0|\,(tt^\dagger)^p|0\rangle = (\langle 0|tt^\dagger|0\rangle)^p = [c(b)]^p. \tag{3.7}$$

Experimentally, $c(b)$ depends also on energy of the collision. This energy dependence can be summarized in the property of geometrical scaling [5, 6], i.e. $c(b, E) = c(b')$, where $b' = b/R(E)$. In the following we will consider all functions at fixed b'.

We thus obtain

$$\sigma_{\mathrm{tot}}(b') = 2\{1-J_0(2\sqrt{c(b')})\}, \tag{3.8}$$

$$\sigma_{\mathrm{inel}}(b') = 1-J_0^2(2\sqrt{c(b')}), \tag{3.9}$$

$$\sigma_{\mathrm{el}}(b') = \tfrac{1}{4}\,\sigma_{\mathrm{tot}}^2(b') \tag{3.10}$$

for total, inelastic and elastic cross-sections at fixed impact parameter. These formulae allow us to determine $c(b')$, since both $\sigma_{\mathrm{tot}}(b')$ and $\sigma_{\mathrm{inel}}(b')$ are relatively well known from experiment [1, 2].

The multiplicity distribution at fixed impact parameter is given by an incoherent superposition of the Poisson distributions:

$$\sigma_N(b') = 2\sum_{p=1}^{\infty} J_p^2(2\sqrt{c(b')})\,\frac{[p^2V(b')]^N}{N!}. \tag{3.11}$$

Here $J_p(z)$ are the Bessel functions and indeed we have:

$$\sum_N \sigma_N(b') = \sigma_{\mathrm{inel}}(b'). \tag{3.12}$$

To obtain the observed cross-sections, it is necessary to integrate over the impact parameter. With our normalization we have:

$$\sigma = \int d^2b\sigma(b'). \tag{3.13}$$

It is also useful to determine the multiplicity moments. Using the well-known identity for the Poisson distribution with the average p^2V:

$$\langle N(N-1)\,\ldots\,(N-m+1)\rangle = (p^2V)^N, \tag{3.14}$$

we obtain:

$$\sigma_{\mathrm{inel}}\langle N(N-1)\,\ldots\,(N-m+1)\rangle = 2V^m\sum_{p=0}^{\infty} p^{2m}J_p^2(2\sqrt{c}). \tag{3.15}$$

After some algebra, using known identities for the Bessel functions we have again for fixed b':

$$\sigma_{\mathrm{inel}}\langle N(N-1)\,\ldots\,(N-m+1)\rangle = V^m\sum_{k=0}^{m}\frac{c^k}{(k!)^2}\sum_{l=0}^{k}(-1)^l\binom{2k}{l}(k-l)^{2m}. \tag{3.16}$$

The first two moments calculated from (3.16) are

$$\sigma_{\text{inel}}\langle N\rangle = 2Vc, \tag{3.17}$$

$$\sigma_{\text{inel}}\langle N(N-1)\rangle = 2V^2c(1+3c). \tag{3.18}$$

Thus we see that indeed $V(b)$ is proportional to the average multiplicity at a given impact parameter. However, since $\bar{N}(b)$ cannot be directly measured, we cannot determine easily $V(b)$, which should then be left as an essentially arbitrary function of the model. This function is to be chosen in such a way as to describe satisfactorily the experimental data. In Section 5 we will show that this condition is a very severe one and cannot be exactly satisfied, at least at presently available energies.

4. *Model with excitation of leading clusters*

Using the conditions (2.5) and (3.2) we can write the S-matrix in the from

$$S = \exp\left[-\tfrac{1}{2}\,\Omega\right]\exp\left[it^\dagger S_c^\dagger\right]\exp\left[itS_c\right], \tag{4.1}$$

where $\Omega = \Omega(b)$ is given by Eq. (2.5).

Consequently, when the S-matrix acts on the initial state with only two nucleons present, the result is

$$S|0\rangle = \exp\left[-\tfrac{1}{2}\,\Omega\right]\exp\left[it^\dagger S_c^\dagger\right]|0\rangle, \tag{4.2}$$

where we have used explicitly Eq. (2.7).

Expanding $\exp\left[it^\dagger S_c^\dagger\right]$ in a power series and using Eq. (2.4) we thus obtain

$$iT|0\rangle \equiv (S-1)\,|0\rangle = \left\{\exp\left[-\tfrac{1}{2}\,\Omega\right]-1\right\}|0\rangle$$

$$+\exp\left[-\tfrac{1}{2}\,\Omega\right]\sum_{p=1}^{\infty}\frac{(-it^\dagger)^p}{p!}\,e^{-p^2V/2}\sum_{N=0}^{\infty}\frac{p^N}{N!}\prod_{j=1}^{N}\varrho(k_j)\,|N\rangle. \tag{4.3}$$

Furthermore, we note that the matrix elements $\langle N|(t)^p(t^\dagger)^p|N\rangle$ can be calculated using the conditions (2.5) and (2.7).

We obtain

$$\langle N|t^p(t^\dagger)^p|N\rangle = p!N!\Omega^p. \tag{4.4}$$

Consequently, by squaring Eq. (4.3) and using arguments similar to those in Section 3 (in particular the relation (3.4)) we obtain the cross-section for non-diffractive production of N mesonic clusters

$$\sigma_N(b') = e^{-\Omega}\sum_{p=1}^{\infty}\frac{\Omega^p}{p!}\,\frac{e^{-p^2V}(p^2V)^N}{N!}\,. \tag{4.5}$$

The total inelastic cross-section is thus

$$\sigma_{\text{inel}} = \sum_{N=0}^{\infty}\sigma_N = 1-e^{-\Omega}. \tag{4.6}$$

For elastic scattering one obtains

$$\sigma_{el} = (e^{-\frac{1}{2}\Omega}-1)^2. \tag{4.7}$$

Finally, for the total cross-section we have

$$\sigma_{tot} = \sigma_{el}+\sigma_{inel} = 2(1-e^{-\frac{1}{2}\Omega}). \tag{4.8}$$

Formulae (4.6) — (4.8) show that the commutator $\Omega(b')$ is just the eikonal. Thus $\Omega(b')$ is easily determined from the existing impact parameter analyses of the pp elastic scattering. To find the physical meaning of the commutator $V(b')$ we will calculate the moments of multiplicity distributions. Using the identity (3.14) we can perform the summation over N in Eq. (4.5) and find:

$$\sigma_{inel}\langle N(N-1)\ldots(N-m+1)\rangle = e^{-\Omega}\sum_{p=1}^{\infty}\frac{\Omega^p}{p!}(p^2V)^m = V^m e^{-\Omega}\left(\Omega\frac{d}{d\Omega}\right)^{2m} e^{\Omega}. \tag{4.9}$$

For $m = 1$ and 2 we obtain

$$\sigma_{inel}\langle N\rangle = V\Omega(1+\Omega), \tag{4.10}$$

$$\sigma_{inel}\langle N(N-1)\rangle = V^2\Omega(1+7\Omega+6\Omega^2+\Omega^3). \tag{4.11}$$

5. *Comparison with experimental data*

We would like now to confront the multiplicity distributions derived in Sections 3 and 4 with the experimental data. This is, however, not directly possible because our formulae (3.11) and (4.5) give multiplicity distributions of clusters which are not easily accessible experimentally. To go around this problem we have taken advantage of the experimentally observed Koba–Nielsen–Olesen scaling [7] of the observed multiplicity distributions of charged particles. At asymptotic energies we should have

$$\langle N\rangle P(N) = \psi(z), \tag{5.1}$$

where

$$z = \frac{N}{\langle N\rangle}. \tag{5.2}$$

Since the approximate scaling is already satisfied for existing data, the shape of scaling function is reasonably well determined and known. Now, the important point is that the same scaling function $\psi(z)$ describes also the asymptotic shape of cluster distribution (provided the cluster decay properties are energy-independent). Consequently, we used the experimentally known function $\psi(z)$ in our discussion of cluster multiplicity distributions. We feel that this procedure is a sensible one and, although it cannot be rigorously justified, provides reasonable estimates of cluster multiplicity distributions.

In our models, the multiplicity distribution at each impact parameter is given by Eqs (3.11) and (4.5). As we already noted, the predictions of the model depend on two essentially arbitrary functions of the impact parameter, i.e. $c(b)$ or the eikonal $\Omega(b)$ and phase-space volume $V(b)$.

To compare these results with experimental data, it is necessary to determine c, Ω and V. The eikonal $\Omega(b)$ and $c(b)$ are easily accessible. Indeed, several fits were proposed which describe reasonably well the data on elastic and total pp cross-sections. We have taken the one used in Ref. [8]. This fit has the advantage of being consistent with the hypothesis of geometrical scaling [5]. It reads:

$$\sigma_{\text{inel}}(b) = 0.94 \exp[-1.03\beta] + 0.54\beta \exp[-2.68\beta], \tag{5.3}$$

where

$$\beta = \pi b^2 / \int \sigma_{\text{inel}}(b) d^2 b.$$

From $\sigma_{\text{inel}}(b)$, $c(b)$ and $\Omega(b)$ are easily calculated using Eqs (3.9) or (4.6).

The phase-space volume $V(b)$ cannot be determined in such a simple fashion. We therefore first try to determine the ratio of dispersion to the average multiplicity and to find whether it is possible to choose $V(b)$ in such a way as to get this ratio compatible with experiment. As is now well-known, the ratio of dispersion to the average multiplicity is a characteristic parameter of high-energy hadron-hadron collisions which is approximately independent of the energy and the nature of colliding hadrons [9], [10].

We will show below that it is in fact impossible to find $V(b)$ which would give the correct D/N ratio. The distributions predicted by both versions of the model are too broad. The effect is particularly strong for the model with cluster excitation. Thus, we will find that this version of the model fails rather badly and is unable to describe the very essential features of multiplicity distributions. The version considered in Section 3 is closer to data. Significant discrepancies are seen, however, also in this case.

The average multiplicity $\langle N \rangle$ is given by the formula

$$\langle N \rangle = \frac{1}{\sigma_{\text{inel}}} \int d^2 b \sigma_{\text{inel}}(b) \langle N(b) \rangle, \tag{5.4}$$

where $\langle N(b) \rangle$ is given by Eqs (3.17) or (4.11).

Introducing the notation $w(b)$, $v(b)$ and $P(b)$ by

$$w(b) = \frac{\langle N(b) \rangle}{\langle N \rangle} \equiv \frac{V(b)}{\langle N \rangle} v(b) \tag{5.5}$$

and

$$P(b) = \frac{\sigma_{\text{inel}}(b)}{\sigma_{\text{inel}}} \tag{5.6}$$

we thus obtain

$$\int d^2 b P(b) w(b) = 1. \tag{5.7}$$

For the dispersion we obtain

$$D^2 = \langle N^2 \rangle - \langle N \rangle^2 = \int d^2 b P(b) D^2(b) + \int d^2 b P(b) [\langle N(b) \rangle^2 - \langle N \rangle^2], \tag{5.8}$$

where

$$D^2(b) = \langle N^2(b)\rangle - \langle N(b)\rangle^2 \equiv V^2(b)\gamma^2(b) + \langle N(b)\rangle. \tag{5.9}$$

$\langle N^2(b)\rangle$ and $\gamma^2(b)$, which is defined by this formula, can be calculated from formulae (3.18) and (4.12). Now we can ask what is the minimum value of the ratio $D^2/\langle N\rangle^2$, for given $\langle N\rangle$, $P(b)$ and $H(b) = \gamma^2(b)/v^2(b)$. This amounts to a minimization problem with the subsidiary condition (5.7). Its solution is straightforward and gives

$$w(b) = \frac{1+\mu^2}{1+H(b)}, \tag{5.10}$$

where

$$\mu^2 = \int d^2b \, \frac{P(b)H(b)}{1+H(b)} \bigg/ \int d^2b \, \frac{P(b)}{1+H(b)}. \tag{5.11}$$

The minimal value of the integral $D^2/\langle N\rangle^2$ is

$$D^2/\langle N\rangle^2 = \mu^2 + \frac{1}{\langle N\rangle}. \tag{5.12}$$

We have calculated μ^2 for both types of models discussed in this paper. The results are

$$\mu^2 = 0.49 \text{ for models (a) without excitation of leading clusters} \tag{5.13}$$

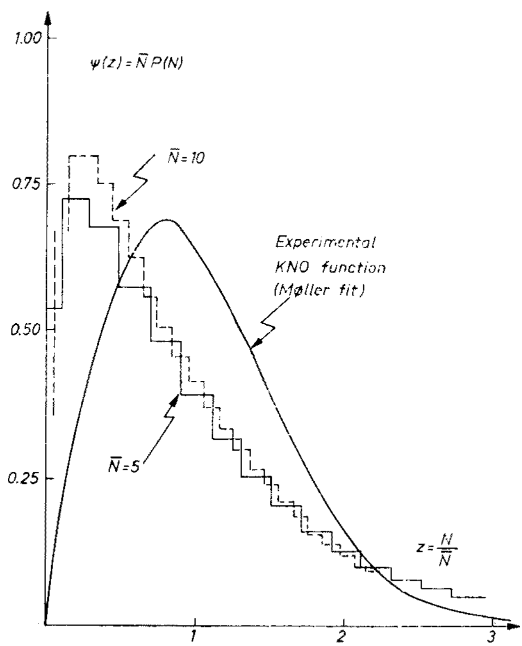

Fig. 1. Multiplicity distribution in the model with excitation of leading clusters. The curve is the KNO function taken from Ref. [10]. The histograms represent the predicted multiplicity distributions for $\bar{N} = 5$ (solid line) and $\bar{N} = 10$ (dotted line), respectively

and

$$\mu^2 = 0.99 \text{ for models (b) with excitation of leading clusters.} \tag{5.13b}$$

These numbers are to be compared with the experimental value $D^2/\langle N\rangle^2 = 0.34$ [9].

Thus, we see that the model with excitation of leading clusters badly violates the data. With such a value of μ^2 it is indeed impossible to obtain a reasonable fit to the multiplicity distribution. To illustrate this, we have plotted in Fig. 1 the multiplicity distributions in this model corresponding to the minimal solution (5.13b) and average multiplicities $\langle N\rangle = 5$ and $\langle N\rangle = 10$. In the same plot also the experimentally observed KNO function is plotted (fit taken from Ref. [10]). We see that the shapes of theoretical and experimental distributions are very different.

The model (a) without excitation of leading clusters gives the $D/\bar{N}$ ratio closer to the experimental value, although also too large. In Fig. 2 we plotted the multiplicity distributions in this model. It is seen that they do not describe very well the data. However,

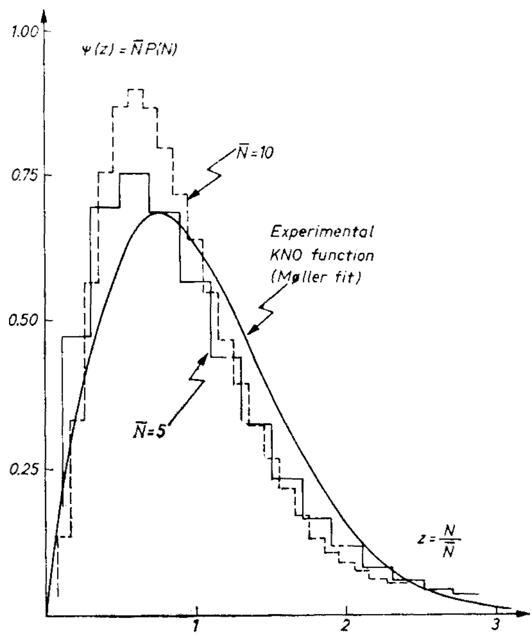

Fig. 2. Multiplicity distribution in the model without excitation of leading clusters. The curve is the KNO function taken from Ref. [10]. The histograms represent multiplicity distributions for $\bar{N} = 5$ (solid line) and $\bar{N} = 10$ (dotted line), respectively

qualitatively they are not very bad, particularly for $\langle N\rangle = 5$ which is a more realistic value. It should be remarked here that the minimizing procedure we used is not a good one for obtaining the best fit to the data. It is conceivable that a better description could be found. However, the perfect agreement is certainly impossible.

Generally, it is clear that the reason of the discrepancy is the long tail of the theoretical distribution, not observed in the data. This tail results from the terms with $p > 1$, which give very large multiplicities. It is thus likely that energy and momentum conservation effects should reduce the discrepancy particularly for the model (a). However, the discussion of these effects goes beyond the scope of this paper.

5. Conclusions

We have analyzed the multiplicity distribution from a class of unitarized uncorrelated cluster production models.

The main qualitative conclusion from this analysis is that unitarity (rescattering corrections) modifies dramatically the multiplicity distribution expected from the simple, non-unitary uncorrelated cluster production models. Although the models we considered are not entirely realistic (the energy-momentum conservation is neglected and diffraction dissociation is not present) we feel that this conclusion is of importance, because it indicates that

a) the analysis of multiplicity distributions may be a useful tool in studying the magnitude of unitarity corrections and

b) the rescattering corrections must be seriously considered in theoretical estimates of multiplicity distributions even in more realistic models.

The comparison with experiment did not turn out to be very satisfactory for the model. Although the models can reproduce the most important qualitative features of the data (like e.g. asymptotic KNO scaling), the quantitative comparison shows a serious discrepancy. The predicted multiplicity distribution has too large tail which is not observed in the data. Consequently, the dispersion of the distribution is greater than the experimental one, and it is impossible to reconstruct the experimental data even with the optimal choice of the parameters.

Consequently, the model cannot be used directly for a quantitative analysis of the data. However, it still remains, in our opinion, a good "theoretical laboratory" which is certainly useful for qualitative considerations. It is also quite likely that energy and momentum conservation effects would reduce the discrepancy. This suggests clearly the direction of further investigations.

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