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Rapid and Accurate INS Transfer Alignment for Air Launched Tactical Missile Using Kalman Filter

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ABSTRACT

An Inertial Navigation System (INS) independently measures the Position, Velocity, and Attitude (PVA) of the vehicle to navigate it towards the target. Since INS is a dead-reckoning system, it requires accurate initialization to provide the navigation (PVA) solution. In the case of an air-launched tactical missile, the aircraft navigation system (Master INS) information is used to initialize accurately the missile INS (Slave INS). Rapid transfer alignment is needed in today's combat operation to converge slave INS initialization in the shortest possible time using aircraft navigation information. The transfer alignment consists of first initializing the missile INS and establishing a navigation solution (PVA) using the missile IMU rates and accelerations, then a Kalman filter is used to, estimate the errors between the Slave INS and Master INS. The proposed method's simulation results show that a tactical missile INS can be aligned to an acceptable accuracy in a very short time based on the aircraft's attitude information and with natural maneuvers experienced during aircraft take-off.

Keywords: Inertial Measurement Unit (IMU); Inertial Navigation System (INS); Position Velocity Attitude (PVA); Aircraft navigation system (Master INS); Missile navigation system (Slave INS); Kalman Filter (KF); Transfer Alignment (TA)

1. INTRODUCTION

Inertial Navigation System (INS) integrated into the air-launched tactical missile is normally a low-accuracy strap-down INS. The missile INS (Slave INS) can be initialized[4] using the Position, Velocity, and Attitude (PVA) of the Master INS (Aircraft)as measurements for the estimation filter. After initialization, the slave INS uses measured rates and accelerations along three orthogonal axes to propagate the weapon's Position, Velocity, and Attitude (PVA) independently.

As shown in Fig. 1, in a fighter aircraft Master INS is located at a location where the effect of the lever arm and vibration effect is minimum. Normally this location is along the aircraft axis.

Slave INS is located inside the missile and mostly mounted on the wing of the aircraft. Depending on the location of the Aircraft station, slave INS will experience the effect of the lever arm and environmental effects like vibration. Typically distance between Master INS and Slave INS is assumed to be 3 to 5m.

Initializing the Slave INS, the Master INS information is communicated to the slave INS with "nominal lever arm" compensation as shown in Fig. 2. The lever arm vector is the relative distance between the Master INS and the Slave INS. The lever arm compensation is necessary to account for the centripetal and vibration effects when aircraft performs

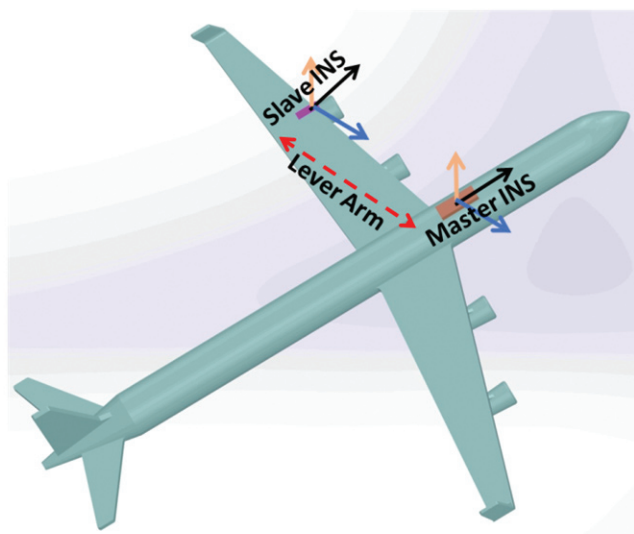

Figure 1. Location of Master INS and Slave INS in a typical aircraft-missile configuration (Lever arm).

a maneuver during alignment. This one-shot[1] transfer of information is used to initialise the slave INS.

The measurement of the lever arm is a highly time-intensive process for an aircraft with multiple missile stations. The requirements for the accuracy of measurement of the lever arm increase when the missile is mounted on outbound stations of the aircraft wing. In this paper, the authors tried to align the slave INS without the requirement of lever arm parameters measurement and compensation.

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After initialisation of slave INS, the Navigation outputs (PVA) of master INS and slave INS are dynamically compared and errors obtained in the process need to be compensated through Transfer Alignment (TA) for accurate initialization of slave INS. The TA procedure requires aircraft maneuver when both INS are in navigation mode. During the maneuver, the differences in the navigation of both INS are a function of the following errors:

- Misalignment errors
- Gyro sensor errors
- Accelerometer sensor errors and
- Physical acceleration and rates sensed by slave INS due to rotation-induced acceleration ($\omega^2 r$) produced by the physical separation (Lever arm).

The proposed transfer alignment scheme uses the aircraft take-off maneuver for observability of errors between master and slave INS and does not require any specific maneuver as required in other transfer alignment schemes.

The rapid transfer alignment was first studied by James E Kain and James R. Cloutier[2] and proposed a scheme for air-to-ground missiles for pre-defined targets. The scheme uses velocity and attitude measurements to estimate the slave misalignments. The rapid transfer alignment was further studied by scientists and engineers all over the world[3,6,8,12,13]. Robert M. Rogers[10] carried out the testing of transfer alignment algorithms using aircraft position information. In the simulation model for the design and evaluation of transfer alignment is presented[7,9]. This simulation model consists of a system model, measurement model, maneuver model, and evaluation model. Zhilan[6,14] gives the Observability analysis for rapid transfer alignment and estimation of errors. G. Satheesh Reddy[4] talked about the details of features to improve the Master INS accuracies and different types of alignment schemes proposed for Master INS.

Hongzhong[12] talks about the rapid transfer alignment using velocity plus attitude matching. The convergence is achieved assuming there is no lever arm effect. But, in the case of wing-mounted missiles, the ω^2r component introduces errors in accelerometer measurements and is not preferred for the wing-mounted missile.

Wei Gao[13] compares two types of filter approaches for rapid transfer alignment, the first method is based on the Kalman filter which takes 5 minutes and the second method is based on the H∞ filter which takes 3 min. The convergence time of the filter is very high for an air-launched tactical missile. The proposed research also assumes that the Lever arm parameters are pre-loaded, which is not a practical solution for air-launched tactical missiles, where multiple missiles are mounted on the aircraft wings.

However, the above works clearly brought out the accuracy of alignment and maneuver requirements. But, the minimum time required for launching the missile in the shortest possible time was not addressed.

In all the papers, maneuver and time required for alignment are not taken as the prime objective. The time taken for TA is of the order of a few minutes[13], which is critical for tactical missile navigation.

2. TRANSFER ALIGNMENT FORMULATION

2.1 The Kalman Filter

The Kalman Filter is a best linear unbiased recursive filter that estimates the states of the linear system from noisy measurements[1]. In Kalman Filter, most weight is given to the measurement/value with the lowest uncertainty. Kalman filter estimation is closer to the true parameter values.

A Kalman Filter (9 states) is considered for design, to estimate attitude misalignment between Master INS and Slave INS.

A random process (Gauss Markov) can be represented in the state space form as

$$X_{K+1} = A_k X_k + w_k$$

The observations on the process are associated with the state of the linear transformation:

$$Z_k = H_k X_k + v_k$$

where,

A_K= State transition model

X_K= States of the system

Z_K= Observation of the system states

H_K = Observation model

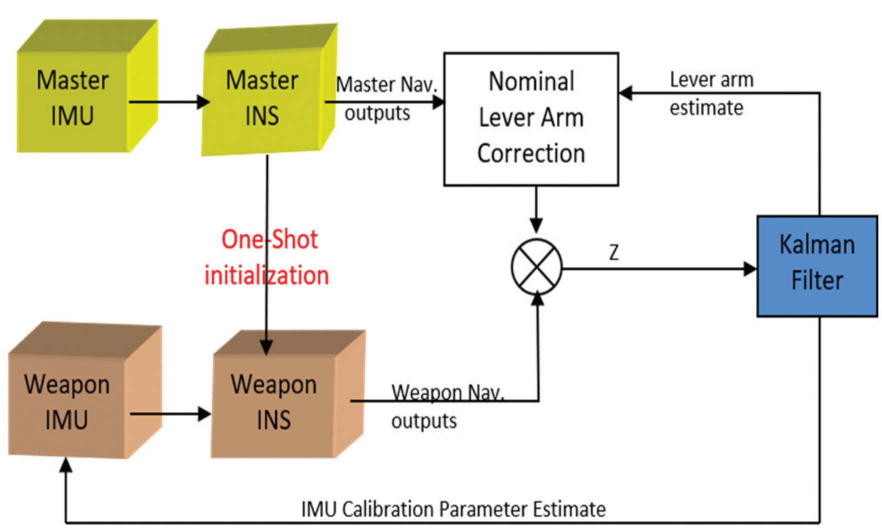

Figure 2. Schematic block diagram of transfer alignment scheme.

W_K = Process Noise with covariance Q_k
ν_K =Measurement Noise with covariance R

2.2 Kalman Recursions

$$K_k = P_{k-1} H_k^T (H_k P_{k-1} H_k^T + R_k)^{-1}$$

$$\hat{X}_k = \hat{X}_{k-1} + K_k (Z_k - H_k \hat{X}_{k-1})$$

$$P_k = (1 - K_k H_k) P_{k-1}$$

More elaborate treatment on the Kalman filter and its recursions are readily available in Gelb[11].

2.3 State Transition Matrix (A_k)

The states contain three attitude errors between Slave and navigation frame, three unaccounted gyro biases of the Slave, and three Attitude errors between Master and Slave INS. To build the state transition matrix, the dynamics of these 9 quantities to be known. Overall state dynamics

$$\dot{X} = \begin{pmatrix} -\omega_{ie}^e \times & -C_b^{ecef} & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & 0_{3x3} \end{pmatrix} X$$

$$\dot{X} = AX$$

2.4 Misalignment Model

The effect of misalignment is amplified during the initial launch phase of a missile due to large accelerations. This led to higher miss distance. Hence, misalignments are to be estimated correctly and with high accuracy.

The navigation frame of reference for both Navigation systems (two locations) can be considered to be the same. Therefore, the relation between the body to NED frame Direction Cosine Matrices (DCM) is defined for the master INS and slave system. It can be represented as follows:

$$C_s^n = C_M^n C_s^M \tag{1}$$

Differentiating the above equation, gives:

$$\dot{C}_s^n = \dot{C}_M^n C_s^M + C_M^n \dot{C}_s^M \tag{2}$$

where,

$$\dot{C}_s^n = C_s^n \omega_{ns}^s X \tag{3}$$

$$\dot{C}_M^n = C_M^n \omega_{nM}^M X \tag{4}$$

$$\dot{C}_b^M = C_s^M \omega_{Ms}^s X \tag{5}$$

ω_{ns}^s - Angular rate of s frame (Slave INS) with respect to n frame (Navigation Frame)

ω_{nM}^M - Angular rate of M frame (Master INS) with respect to n frame (Navigation Frame)

ω_{MS}^S - Angular rate of s frame (Slave INS) with respect to M frame (Master INS)

These angular rates are:

$$\omega_{ns}^s = \omega_{is}^s - C_n^s \omega_{in}^n \tag{6}$$

$$\omega_{nM}^M = \omega_{is}^s + \varepsilon - C_n^M \omega_{in}^n \tag{7}$$

$$\omega_{MS}^S = \dot{\psi} \tag{8}$$

Where, ε as gyro drift, ψ is the misalignment between the Slave INS and virtual Master INS, ω_{is}^s which is the angular rate vector of the true coordinate system with respect to an inertial frame.

Substituting Eqns (3)-(8) in Eqn (2), we get:

$$\dot{\psi} = (I - C_M^S)\omega_{is}^s - C_M^S \varepsilon \tag{9}$$

We know that,

$$C_M^S = I - \psi X \tag{10}$$

Substituting Eqn (10) in (9), we get

$$\dot{\psi} = \omega_{is}^s X \psi - \varepsilon + \psi X \varepsilon$$

Since, $\psi X \varepsilon$ is very small and will not have any impact on $\dot{\psi}$. Hence, Neglecting $\psi X \varepsilon$ gives:

$$\dot{\psi} = -\omega_{is}^s X \psi - \varepsilon \tag{11}$$

Eqn. (11) is the misalignment model of INS transfer alignment. This $\dot{\psi}$ is the most important parameter of the transfer alignment. It is a combination of :

- The physical geometrical misalignment of the Slave INS axes with Master INS axes set and

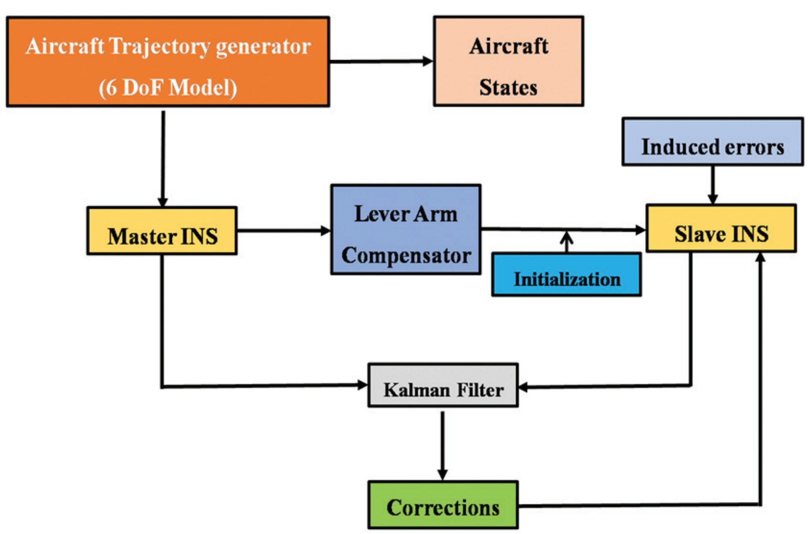

Figure 3. Transfer alignment simulation environment and initialisation flow chart.

Table 1. Master INS sensor specifications[4]

Parameter	Unit	Specifications
Gyro:		
Bias stability	Deg/Hr	0.01
Input range	Deg/sec	±300
Scale factor stability	ppm	10
Random walk	Deg/√Hr	0.003
Accelerometer:		
Dynamic range	‘g’	50
Bias stability	µg	30
Scale factor stability	ppm	30

Table 2. Slave INS sensor specifications

Parameter	Unit	Specifications
Gyro:		
Bias stability	Deg/Hr	5-15
Input range	Deg/sec	±300
Scale factor stability	ppm	100
Random walk	Deg/√Hr	0.3
Accelerometer:		
Dynamic Range	‘g’	50
Bias Stability	mg	1
Scale factor stability	ppm	50

- The drift of the computed slave INS navigation frame during the transfer alignment process.

3. SIMULATION RESULTS, ANALYSIS AND DISCUSSION

3.1 Attitude Initialisation

An assumption is made that the two INS systems ‘S’ (slave) and ‘M’ (Master) are mounted on a rigid body and an unknown angular separation C_S^M exists between them. At the start of transfer alignment, both the INS have initialized with identical Euler angles $\hat{C}_S^n(0) \equiv C_M^n(0)$.’^’ operator indicates the incorrect initialization of Slave to navigation DCM. The initial DCM for misalignment is initialized to $\tilde{C}_S^M \equiv I_{3x3}$. The DCM $\tilde{C}_S^M$ is the attitude misalignment between the Master INS and Slave INS. The superscript ‘~’ refers to ambiguity in that particular matrix. The error in the $\tilde{C}_S^M$ is estimated using the attitude matching information in Kalman Filter. The DCM relating the Slave INS frame to the navigation frame is initialized a $\tilde{C}_S^n = C_M^n \tilde{C}_S^M$.

Slave INS Position, Velocity, and Attitude (PVA) are different from Master PVA since both INS are located at different positions in the aircraft. The Master INS data with compensated Lever Arm (LA) error to be provided to the Slave INS. Since angular relations between true Master INS and Slave INS vary with time due to aircraft dynamics, we can design a virtual Master INS at the Slave location.

This paper aims to estimate the misalignment between the virtual Master INS and Slave INS in presence of Slave INS sensor noises which are unknown to the Master INS sensors.

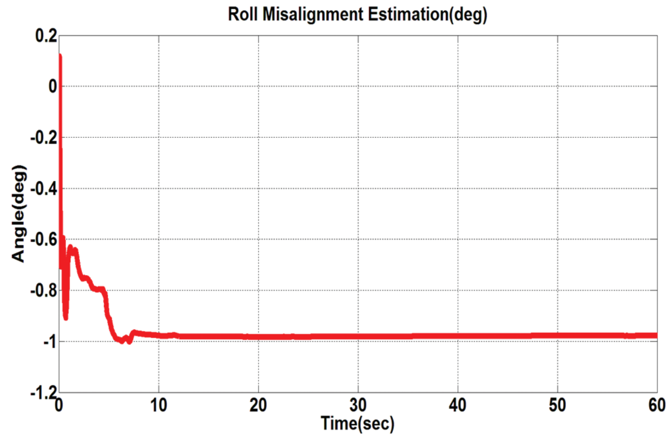

Figure 4. Roll misalignment -1 deg.

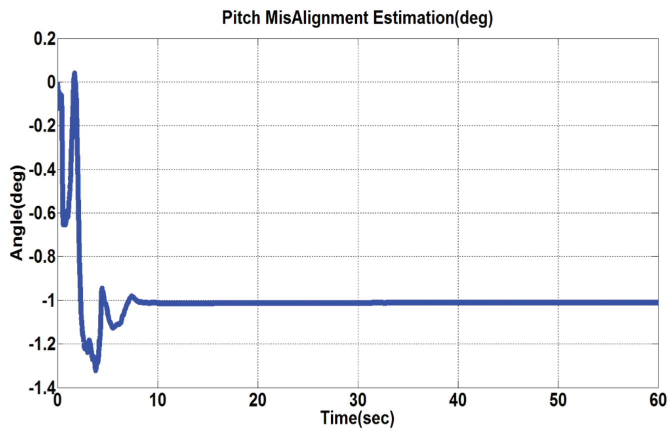

Figure 5. Pitch misalignment -1 deg.

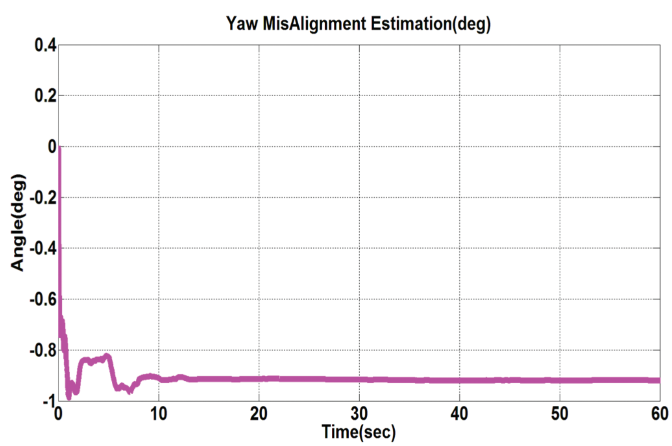

Figure 6. Yaw misalignment -1 deg.

3.2 Simulation Environment

The following assumption was considered for simulating the proposed scheme:

- Complete error model is required to design an optimal Kalman filter. There are many environmental error sources that cannot be modeled as mathematical models. Hence, a sub-optimal filter is used by simplifying the state dynamics by neglecting certain states
- No specific maneuver requirement, the scheme uses natural maneuvers sensed during the aircraft take-off.

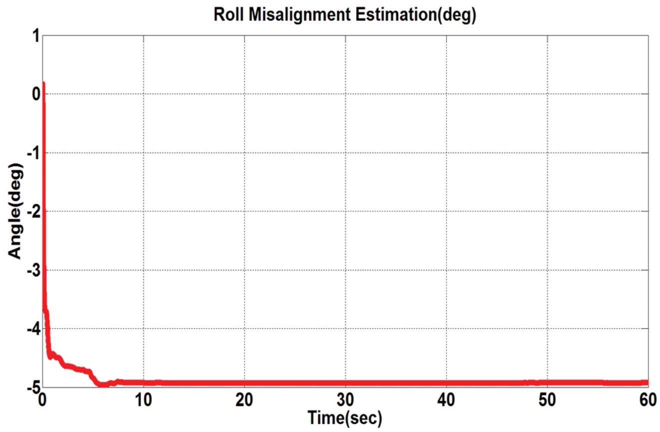

Figure 7. Roll misalignment-5 deg.

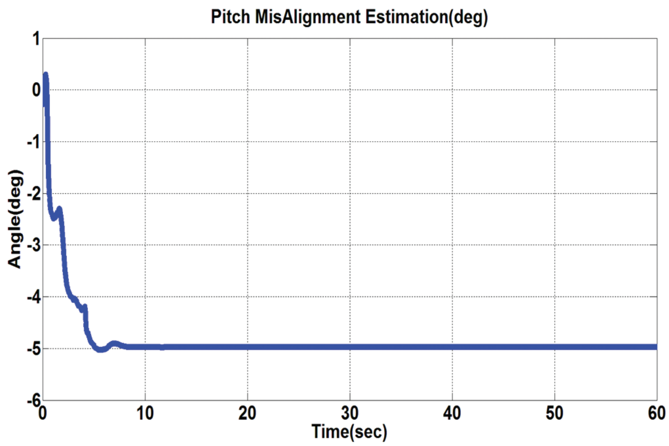

Figure 8. Pitch misalignment-5 deg.

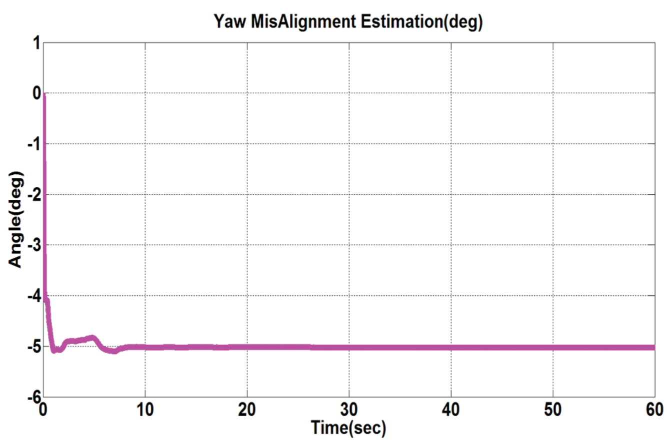

Figure 9. Yaw misalignment-5 deg.

To evaluate the performance and estimation accuracy of the proposed method and its convergence time. A 6-DoF simulation platform has been used to evaluate performance under various test conditions. The simulation system is developed accordingingly[7,9], this simulation system consists of Master INS, Slave INS, and aircraft motion simulator model (6-DoF trajectory generator), where the aircraft motions are simulated using Aircraft trajectory Generator as shown in Fig. 3.

The rates and acceleration sensed by the Master and Slave IMU are as per the actual specifications of the Master[4] and Slave

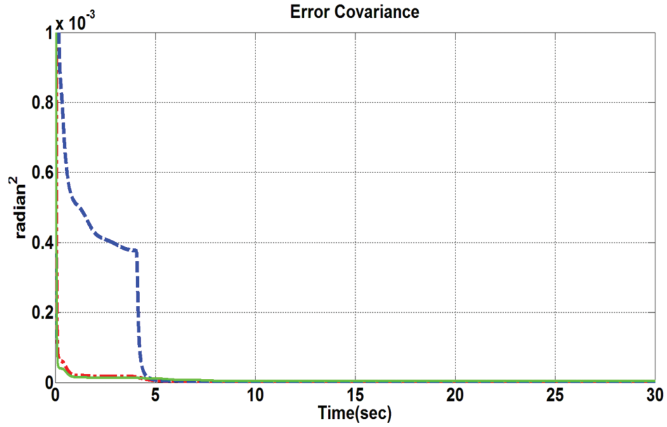

Figure 10. Error covariance.

IMU respectively, IMU contains gyros and accelerometers. The specifications of Master and Slave IMU are given in Table 1 and Table 2 respectively. The flexure[7,9] of the body and lever arm effects are simulated and sensed by the slave INS and not by Master INS, this makes the master INS highly accurate than the slave INS in dynamic conditions. By matching the Master and Slave navigation solutions PVA to obtain the measurement states of transfer alignment, the Kalman filter is used to estimate the misalignment.

The performance of the method is evaluated in both small (1 degree) and large (5 degrees) misalignment angle conditions.

3.3 Simulation Results

This section presents simulation results to demonstrate the performance of the proposed rapid transfer alignment method for required accuracy in a short time. Simulation results are shown in Fig. 4 to Fig. 9 and can be analysed in two cases. Case-1 is for small angle misalignment, normally observed between Master and Slave INS. Case-2 is for large angle misalignment. Large misalignments are, in general not expected between Master and Slave INS. However, it is considered to bring out the robustness of the algorithm.

Case 1: Misalignment of -1, -1, and -1 deg is given between roll, pitch, and yaw axes of Master and Slave INS.

Case 2: Misalignment of -5, -5, and -5 deg is given between roll, pitch, and yaw axes of Master and Slave INS.

To evaluate the algorithm, natural pitch and yaw maneuvers experienced by Master and Slave INS during take-off were provided. The roll and pitch angles are estimated immediately and accurately as expected, whereas yaw angle estimation is less accurate. The results are given as follows. Figure 4 to Fig. 6 shows the results in small misalignment angle conditions. Figure 7 to Fig. 9 show the results in large misalignment angle conditions.

Table 3. Misalignment error (ME) for -1 deg and -5 deg misalignment

	Yaw ME (mrad)	Pitch ME (mrad)	Roll ME (mrad)
Case 1	0.96	0.35	0.49
Case 2	1.15	0.6	0.8

The exact value of misalignment obtained for case 1 and case 2 are given in Table 3. These results show that the misalignment error estimation is within the design requirements of the missile in pitch, yaw, and roll channel.

Therefore, the results demonstrate that the proposed method is suitable for rapid and accurate transfer alignment with small and large misalignment errors.

4. CONCLUSIONS

- The main objectives of the proposed scheme are demonstrated for estimating the accurate misalignment between Master and Slave INS and initializing the Slave INS with corrected Position, Velocity, and Attitude from Master INS
- An innovative provision without the need for specific maneuver and Lever arm compensation, the accurate transfer of INS information demonstrated, which avoids the need for extra maneuver on combat operation by Aircraft pilot.

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