Application of "Travelling Salesman Problem" in Dynamic Programming for Efficient and Cost Effective Route Design for Distribution: Case of Chifles Distribution of ORFI Company, Ecuador

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https://riiopenjournals.com/index.php/Supply-Chain-Logistics-review/issue/view/5

Citation: Murillo, M.F.M., Murillo, W.A.M., Bermeo, M.A.C., Hinojosa, L.X.O., Ruiz, A.S.M., Pandey, D. & Pandey, B.K. (2020). Application of "Travelling Salesman Problem" in Dynamic Programming for Efficient and Cost Effective Route Design for Distribution: Case of Chifles Distribution of ORFI Company, Ecuador. *Logistics & Supply Chain Review,* 1(1), 1-11.

Research Article

Abstract

Purpose: *The present study is designed to optimize a route for the distribution of chifles of the ORFI Company in school bars within the Río Verde parish of the Santo Domingo City, Republic of Ecuador.*
Methodology: *Time and distance data were collected regarding the route of the vendors, then distance matrices were developed between distribution points, and a graph was designed to find the best route using the Bellman-Hell-Karp algorithm.*
Results: *The sector graph had 46 nodes and the optimal route was found by applying dynamic programming with the Held-Karp mathematical algorithm, the optimal route for the ORFI company chifles distribution is: 1-2-3-4-5-8-7-6-10-9-12-11-13-14-15-16-18-17-19-20-21-22-23-24-25-26-31-27-28-29-30-32-33-34-38-35-36-37-39-40-41-42-43-44-45-46-1, with 23089 meters of distance, optimizing in travel time 15% and travel distance 11%.*
Implications: *The application of the findings is expected to reduce the cost and time of distribution expended by the OFRI Company. Similar approaches also can be applied by other companies operating in the city.*

Keywords: Symmetric Travelling Salesman Problem, Dynamic Programming, Deductive analysis, Distribution, Symmetric distance matrices.

1. Introduction

The cost of distribution adds to the total cost of business. Therefore, in order to remain competitive, the business organizations, irrespective of their size and nature, should look for ways to minimize their cost of distribution. There is no exception to food processing firms. The present study aims to analyze and determine the cost-effective route for the distribution of a food processing company, ORFI operating in the Santo Domingo de Los Tsa'chilas province of the Republic of Ecuador. It is a manufacturer and distributor of snack foods. It distributes snack foods derived from the banana and plantain barraganete, and mature salt and sweet plantains. These types of food are very popular in the town since they are typical of the Ecuadorian coast. The distribution points of these snacks have grown exponentially, due to the guarantee of processing, safety, and especially the good taste. The distribution points include large and small stores, supermarkets, and, most importantly, in schools, colleges, and universities. Being carbohydrate-rich foods, they are considered as a nutritional supplement for children and young people.

There are 100 distribution points covered by the ORFI sellers daily according to the type of supply center. To supply to the school bars, the salesmen travel 40 to 60 points weekly between local schools and colleges that add to the distribution expenses for the company. Keeping the cost of distribution in mind, the present study is designed to optimize a route for the distribution of chifles of the ORFI Company in school bars within the Río Verde parish of Santo Domingo City, using the mathematical model of the Symmetric Traveler Agent Problem. It can be considered as an NP-Complex type problem since the sellers must leave the factory, pass through each supply point, and return to the starting point again. Our study variables logically are the time and distance traveled with the conventional route and with the new route to be designed.

This problem can be understood as the intersection of two known combinatorial optimization problems. The first as that of the street vendor problem (TSP), considering the capacity of each car as infinite"(Applegate, Bixby, & Chvátal, 2006). Then the other as "compartment packaging (BPP, container packaging problem)" according to (Toth, 1990),"This has led many researchers to explore various methods to address them. Most of these methods can be broadly classified as exact or optimization algorithms"(Aarts, 2003).

The problem of this investigation demands certain details such as the peddler sellers. Before applying any kind of mathematics, it is important to make some observations that form a case of the problem. First of all, it is looking for a closed route through all the places that want to visit, this means that the route will end where it started. Secondly, each place is visited exactly once on the route. As all lengths are positive, visiting a place twice would not give us a better result. As each place is visited exactly once, there is an infinite number of possible routes through each place. It calls such a possible route a solution for the TSP, as there are many solutions, it is known that there is a route with minimal travel costs and it requires for an optimal solution if there is no other solution with a strictly lower travel cost.

The main activity of the research will be to design a route for the distribution of ORFI chifles for school bars within the Río Verde parish, in Santo Domingo, Ecuador.

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To do this, we characterize the current system of distribution of chifles in ORFI, we will apply the mathematical model of the symmetric problem of the travel agent for the distribution of routes, and we will evaluate the new route of distribution of chifles of the ORFI company for school bars within the Río Verde parish.

2. Literature Review

2.1. Vehicle routing problem

Broadly speaking, a vehicle routing problem (VRP) consists of, given a set of geographically dispersed clients and warehouses and a fleet of vehicles, determining a set of minimum cost routes that start and end in warehouses, so that vehicles visit customers maximum once." (Julio Mario Daza, 2009)

Considering that definition, this problem could present the following variants:

- CVRP (Capacitated VRP) (Ralphs & Hartman, 2001)
- MDVRP (Multi-Depot VRP) (Hjorring, 1995)
- PVRP (Periodic VRP) (Baptista & Oliveira, 2002)
- SDVRP (Split Delivery VRP) (Archetti & Mansini, 2001)
- SVRP (Stochastic VRP) (Laporte, Gendreau, & Potvin, 1998)
- VRPB (VRP with Backhauls) (Ralphs & Hartman, 2001)

2.2. The Travel Agent Problem

The problem was first formulated in 1930 and is one of the most studied optimization problems. It is used as a test for many optimization methods. Although the problem is computationally complex, a lot of heuristics and exact methods are known, so some instances from one hundred to thousands of cities can be solved (Wikipedia, 2019). Held & Karp, (1971)present a branching and dimensioning procedure that starts from the study of the relationship between TSP and the tree of minimum expansion problem. Along with the first study in 1970, this second paper introduces proven Lagrangian relaxation in up to 64 cities. Cerný (1985) proposes the solution of combinatorial optimization problems through an analogy with the field of thermodynamics, tested in the TSP, randomly swapping the stations of the traveler with a probability depending on the size of the route. This work, although not credited as the sole proponent, introduces the Simulated Annealing method, obtaining satisfactory results in instances of up to 200 cities; in some cases throwing the optimal solution and, in others, approaching it. Angéniol, De La Croix, and Le Texier (1988) present an algorithm to approach TSP based on Kohonen's self-organized maps, validating this neural network in an instance of 1,000 cities.

The first reported solution to the Travel Agent problem was in 1954, when George Dantzig, Ray Fulkerson, and Selmer Johnson published a description of a PAV (Travel Salesman Problem) solution method entitled "Solutions of a large scale traveling salesman problem" to solve an instance of 49 cities where a travel agent wants to visit a set of cities, assigning them a cost for visiting contiguous cities (travel distance between two cities). Two conditions were proposed for this solution: return to the same city from which you left and not repeat cities to find a route or a road with the lowest possible cost". (Penna, 2013)

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There are several nodes (cities, localities, shops, companies, etc.) that must be visited by an entity (person, travel agent, automobile, airplane, bus, etc.), without visiting the same node twice. If we have 3 nodes (a, b and c) to visit, then we would have a function of combinations without repetition $c\,(3,2)$, that is to say, we would have 6 possible solutions: $abc, acb, bac, bca, cab, cba$, for the case of 4 nodes we would have 12 combinations, for 10 nodes we would have 90 combinations, for 100 cities we would have 9,900 combinations and so on. As an example, in the problem of Ulysses Homer trying to visit the cities described in the Odyssey exactly once (16 cities) where there are multiple connections between different cities, Grötschel and Padberg (1993) concluded that there are 653,837’184,000 different routes for the solution of this problem. (Penna, 2013)

2.3. Modeling of the Travel Agent Problem in a network with 4 nodes

To find an optimal route through a given set of places, we must first build a suitable model. When it comes to mathematics, within the field of combinatorial optimization, many have developed theories about the TSP. So, we’ll take a look at graph theory, because then we’ll apply for our new route of chifles distribution.

A graph $G = (V;\ E)$ is a pair of sets V and E where V is a set of vertices and E is a set of edges. Each element $e = \{v, u\} \in E$ is an edge exactly between two vertices, u in V.

It has to $V = \{1,2,3,4\}$ and $E = \{1,2\};\ \{1,3\};\ \{2,3\};\ \{2,4\};\ \{3,4\}\}$, we represent it in the following figure.

Fig. 1: Graph with 4 nodes and 5 edges

For $v, u \in V$, we say that u is a neighbor of v if $\{v, u\} \in E$. We denote $N\,(v)$ as the set of all neighbors of v.

Using a graph is the most common way to model the TSP. Let’s tone the example where a seller wants to travel through n cities in minimum time. For each city, we make a vertex that belongs to V. Whenever possible to travel from city v to city or directly, we make the edge $e = \{v, u\} \in E$. We now have a graph $G = (V;\ E)$ representing the space that our seller can travel.

As it has noticed, this model does not give us any information about which route is short and which is not. To solve a TSP, it needs a weight function that captures the distances between each pair of vertices.

A weight function or a distance function in a graph $G = (V;\ E)$ is a function $w: E \rightarrow R$ that gives each edge of E a real number.

Now it can give a weight to each edge, which denotes the distance or travel costs between two vertices, therefore, the travel costs to get from v to u are denoted with $w\,(v;\ u)$. To avoid some

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calculation problems, we generally use the edges with Q constants to ensure optimal and well-defined value. It will assume that for all instances of TSP we will have w (e) $> 0 \, \forall e \in E$, since negative distances are not represented in reality. It is now considered that this weight function can be all sorts of things. A logical choice would be the Euclidean distance between two vertices, this assumes that the vertices are in a Euclidean space, which is an assumption we don't always want to make, and Euclidean distances aren't rational numbers in general. Now let's suppose that it wants to measure the travel costs (on time) between Amsterdam and Rotterdam by car. The geometric distance in kilometers is a simple measure, but not necessarily the most accurate. Taking the length of a road from Amsterdam to Rotterdam can be a more accurate measure. As not all roads have the same speed limit, a shorter route does not always represent a faster route. There are all sorts of ways to define a weight function, and its structure is important for modeling.

Before continue, it takes a look at some definitions that help us identify the characteristics of a graph.

A graph $G = (V; E)$ we have to:

- $(v_1, v_2 \ldots\ldots v_{k-1}, v_k,)$ is a G path from $v_1$to $v_k$if $\{v_i, v_{i+1}\} \in E, \ \forall 1 \le i \le k-1$.
- A $G -$ path is as such if it is only used once.
- $(v_1, v_2 \ldots\ldots v_{k-1}, v_k,)$is a cycle or a G circuit if $v_1 = v_k$, $\{v_i, v_{i+1}\} \in E$ and $v_1 \neq v_j \ \forall 1 \le i \le k-1, i \neq j$.
- A path or cycle of G is Hamiltonian if it contains all the vertices of V.
- A $G -$ chart is Hamiltonian if there is a Hamiltonian cycle of G.

The graph $G = (V; E)$ it has to:

- Gis connected if there is a path between each pair of vertices.
- Gis complete if there is a border between each pair of vertices.

Note that a complete (unweighted) graph with n vertices is unique. We denote k_n for the complete network with $|V| = n$.

- For $S \subseteq V$ we denote δ (S) as the set of edges that have exactly a final vertex in S.
- To denoted (v) as the degree (number of adjacent edges) of a vertex $v \in V$.
- To denote δ (G) as the minimum degree of G.

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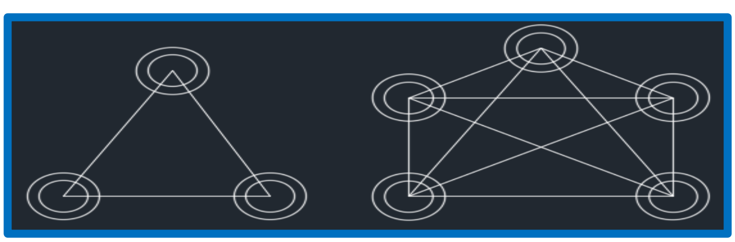

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Fig. 2: Two graphs with 3 and 5 nodes respectively

Now it can be defined the problem of the street vendor and it will define it in the following way: given a graph G and a weight function w over G, determine the minimum weight of the Hamiltonian cycle in G. The weight of the cycle is the sum of all the weights of the edges in the cycle. To find a minimum weight Hamiltonian cycle will say that G is Hamiltonian, otherwise, there is no such route. To know if a graph is Hamiltonian can sometimes be quite difficult. There are several ways to find out whether a graph is Hamiltonian or not, but in many cases, these methods will leave the question unclear.

3. Methodology

In the present investigation, we apply the quantitative method, because it raises information using the deductive analysis and interpretation of information, to reach real data that facilitate the solution of the problematic, contrast of hypothesis, and verification of the veracity in answers obtained in the present study.

In the same way, it applies research techniques such as bibliographic, field and descriptive research, because its take bibliographic sources to conceptualize and deepen in judgments made by different authors in solution edges for the Travel Agent Problem, at the same time when it collects information in the field (time and distance between nodes) of the routes followed by the ORFI company's whistleblowers within the Rio Verde parish, later with objective and truthful observations it develops the application of the Travel Agent Problem method for the new route.

The ORFI Company does not have technically established routes to supply its products to points of sale such as school bars and others.

Within the Rio Verde parish there are the following schools and colleges:

Carlos Rufino Marín, Calazacón, Darío Kanyat, Seis de Octubre, Laura Flores de Paz y Miño, Ramón Gallegos, Batallón Montufar, Instituto Tecnológico Superior Pichincha, Alessandro Volta, Manuel Agustín Aguirre, Alfredo Pérez Chiriboga, Nicolás Gómez Tobar, Republica de Francia, Pablo Enrique Albornoz, Héroes de Paquisha, Chiguilpe, Stephen Hawking, Ernesto Albán Mosquera, Augusto Arias, Federico Gonzáles Suárez.

For the survey of information in symmetrical distance matrices, it proceeded to take measurements using coordinates in the map of the city of Santo Domingo on a scale of 1:1, there was considered the distances between nodes to feed our matrix of distances and then consider the graph with edges between all points of supply within the parish Rio Verde.

It was considered as edges of union between nodes only streets in good condition and skillful for pedestrian or vehicular crossing.

Fig. 3: Río Verde Parish Plan

Next, we have in figure 4 the graph to generate the new route of distribution of chifles of the company ORFI with the 46 nodes.

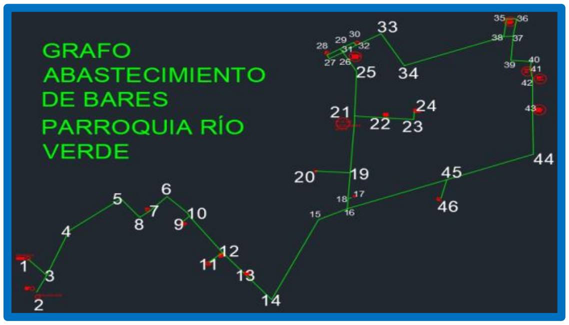

Fig. 4: Rio Verde Parish graph with 46 nodes

Exemplification of the method of the Problem of the Traveling Agent with 4 nodes.

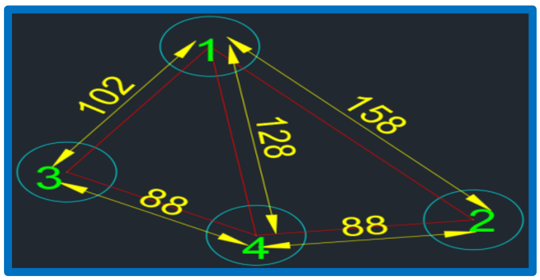

Fig. 5: Example of route calculation with 4 nodes

Let's exemplify by taking cities $1,2,\ldots,N$ and suppose we start in city 1, and the distance between city i and city j is $(d\,i,j)$. Consider subsets $S \subseteq \{2,\ldots,N\}$ of cities and, for $c \in S$, be $D(S,c)$ the minimum distance, starting in city 1, visiting all cities in S and ending in city c.

First phase: if $S = \{c\}$,then $D(S,c) = d_{1,\ c}$. Otherwise: $D(S,c) = \min_{x_\in} S_c\,(D(S-c,x) + d_{x,\ c})$.

Second phase: the minimum distance for a complete tour of all cities is $M = \min_{c \in \{2,\ldots\ldots,N\}}(D(\{2,\ldots,N\},c) + d_{c,\ 1})$.

A course $n_1,\ldots,n_N$ is the minimum distance just when it satisfies.

$M = D(\{2,\ldots,N\},n_N) + d_{n_{N,1}}$.

Successor of node 1: $p(1,\{2,3,4\}) = 3$

Successor of node 3: $p(3,\{2,4\}) = 4$

Successor of node 4: $p(4,\{2\}) = 2$

Go back the optimal TSP route reaches: $1 \rightarrow 3 \rightarrow 4 \rightarrow 2 \rightarrow 1$

The optimum distance is 436 meters.

4. Results

To find the optimal route in the distribution of chifles of the company ORFI we use dynamic programming with the mathematical algorithm of Held-Karp, the same one is an improvement of the TSP. The algorithm was applied in a computer with the following characteristics:

- Brand: Dell.
- Storage in C: 1 TB.
- RAM: 16 GB.
- Intel: i7.
- Hard disk: 250 GB SSD.

The optimal route for distribution of chifles of the company ORFI is:
1-2-3-4-5-8-7-6-10-9-12-11-13-14-15-16-18-17-19-20-21-22-23-24-25-26-31-27-28-29-30-32-33-34-38-35-36-37-39-40-41-42-43-44-45-46-1.
Optimum route 23089 meters.

Table 1: Route for distribution of ORFI Chifles Company

General matrix of the route for distribution of chifles in school bars by ORFI in the Parish River Green, Santo Domingo De Los Colorados								
Product	Graphs	Stage	Place	Optimal Route Start-End	Product	Graphs	Stage	Place
Chifles for school bars	Graph 1-46	1	A	1-2-3-4-5-8-7-6-10-9-12-11-13-1415-16-18-17-19-20-21-22-23-24-25-26-31-27-28-29-30-32-33-34-38-35-36-37-39-40-41-42-43-44-45-46-1.	23,089	0,835	1,796	196,350

Table 2: Collection of information in the field with the new route

Variables	Route Distribution of Chilfes ORFI: 20 Points							
	Month 1				Month 2			
	Monday	Monday	Monday	Monday	Monday	Monday	Monday	Monday
Total Time. (h)	4,5	4,8	4,7	4,3	4,8	4,8	4,5	4,5
Total. Distance (km)	22,9	23,1	23,0	22,9	23,1	23,1	22,9	22,9
Speed $\bar{\alpha}$ (km/h)	5,1	4,9	4,9	5,4	5,2	5,0	5,2	5,4
total time traveled only p. (h)	0,51	0,49	0,54	0,47	0,44	0,45	0,46	0,47
time $\bar{\alpha}$ per point (h)	0,2	0,21	0,21	0,19	0,2	0,21	0,2	0,19
total time in points (h)	4	4,2	4,2	3,8	4	4,2	4	3,8
speed $\bar{\alpha}$ travel only (km/h)	45	47	43	49	52	51	50	49

Table 3: Field information with the new route

Variables	Route Distribution of Chilfes by ORFI; 20 Points							
	Month 1				Month 2			
	Monday	Monday	Monday	Monday	Monday	Monday	Monday	Monday
Total Time. (h)	4,5	4,8	4,7	4,3	4,8	4,8	4,5	4,5
Total Distance. (km)	22,9	23,1	23,0	22,9	23,1	23,1	22,9	22,9

Table 4: Field information without the new route

Variables	Route Distribution of Chilfes by ORFI; 20 Points							
	Month 1				Month 2			
	Monday	Monday	Monday	Monday	Monday	Monday	Monday	Monday
Total Time. (h)	5,6	5,3	5,8	5,2	5,9	5,3	5,4	5,3
Total Distance. (km)	26,0	25,7	26,2	25,6	26,2	25,7	25,8	25,7

5. Discussion

Observing the information of tables 3 and 4 it has a reduction in time and distance with the new route for distribution of chifles in ORFI. An analysis was made of averages in these variables at the rate of having the percentages in tables 5 and 6, which correspond to both 15% and 11% respectively, where time is more favored because it avoids streets with high traffic to reach the points of supply for school bars in the parish Rio Verde of Santo Domingo Canton.

Table 5: Analysis of means in the distance variable

DISTANCE	
$\bar{\alpha}$ With Route	$\bar{\alpha}$ Without Route
23,0	25,9
88,9	100
% Óptimal	
11,1	

Table 6: Analysis of means in the time variable.

TIME	
$\bar{\alpha}$ With Route	$\bar{\alpha}$ Without Route
4,6	5,5
84,2	100
% Óptimal	
15,8	

6. Conclusions

The routes for distribution of chifles in the ORFI Company were formerly applied considering the proximity between points, but there was no support of a technical study for the whole canton in which they cover around 100 points between all the places of supply with their banana products. The mathematical model of the Travel Agent Problem is one of the exact methods that can be applied for the routing of vehicles. The accuracy of this method demands a large amount of mathematical computation, for which low-processor machines are not helpful. The new route of distribution of chifles for the ORFI in the parish Rio Verde optimizes in time and distance by 15% and 11% respectively in comparison to the type of resources that were invested before to cover this route. Now the company is expected to spend 0.8 gallons of fuel and this represents 1.79 USD.

Availability of data and material: This research contains specific information from the ORFI Company and also from the municipality of Santo Domingo. We cannot share information such as distance matrices or dynamic programming code without the consent of the authorities of the city and ORFI management.

Conflicts of Interest: The authors declare no conflict of interest.

Authors' contribution: MoisésFiliberto Mora Murillo conceives the idea, analyzes data, and runs the dynamic programming in Python; Walter Alfredo Mora Murillo collects the data and inputs the data, and the code in Python; Manuel Alejandro Cuenca Bermeocollects the data, monitors the route, surveys variables in the field such as time, fuel distance, load capacity; Luis Xavier Orbea Hinojosa analyzes the data, interprets the information in the SPSS software; Andrea SilvanaMorejón Ruiz collects the data and writes the paper; DigvijayPandey conceives the idea, input for the development of a statistical model and writes the paper; Binay Kumar Pandey conceives the idea, develops the model and writes the paper.

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Appendix

Abbreviations:
TSP: Travelling Salesman Problem
BHK: Bellman–Held–Karp algorithm
HDD: Hard Drive Disk
SSD: Solid-State Drive
CSV: Comma-Separated Values