Chapter

The CAPM is Not Dead: It Works Better for Average Daily Returns

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Abstract

In a series of papers, Fama and French showed that the CAPM failed to explain U. S. stock returns. Subsequently, they declared the CAPM dead. In its place, they proposed a number of different factors to better explain stock returns. Given that Sharpe received the Nobel Prize in Economics for the CAPM, their conclusion that it is dead is worth further investigation. This paper revisits cross-sectional tests of the Capital Asset Pricing Model (CAPM). To mitigate problems with noise in realized stock return series, we use a smoothed data series of average daily returns per month. Based on U.S. stock returns from 1965 to 2015, we find that, confirming Fama and French, beta is not priced for realized monthly returns. However, contrary to their findings, when we use average daily returns per month, we find that beta is significantly priced for average daily returns in equal-weighted stock portfolios. Further analyses reveal that popular factors proposed by Fama and French and others are more significantly priced for average daily returns also. Given that the CAPM is better supported by average daily returns per month than realized monthly returns, we conclude that it is not dead.

Keywords: asset pricing, average daily returns, beta risk, CAPM, cross-sectional regression

1. Introduction

One of the most controversial subjects in financial economics is the Capital Asset Pricing Model (CAPM) of Treynor [1, 2], Sharpe [3], Lintner [4], and Mossin [5]. Sharpe won a Nobel Prize in Economics in 1990 for his theoretical CAPM contributions to equilibrium asset pricing. The CAPM posits a linear relation between expected asset returns and beta (β) associated with the market portfolio. Unfortunately, based on extensive empirical tests in a series of research papers, Fama and French [6–10] concluded that, "In our view, the evidence that β does not suffice to explain expected return is compelling. The average return anomalies of the CAPM are serious enough to infer that the model is not a useful approximation." [9, p. 1957][1] Due to the CAPM's failure, a large number of multi-factors have been proposed by researchers to help

[1] Even early evidence raised suspicions about the CAPM. Seminal studies by Black et al. [11] and Fama and MacBeth [12] found a flatter relation between average returns and β than predicted by theory. See Levy [13] for an excellent survey of CAPM theory and empirical evidence.

explain cross-sectional stock returns (e.g., see Cochrane [14], Fama and French [10], Harvey et al. [15], Stambaugh and Yuan [16], Kolari et al. [17], and Kolari and Pynnonen [18]). According to Fama and French [9], these multi-factors add more to the explanation of average stock returns than CAPM beta.[2]

Why is it so difficult to find empirical evidence to support the CAPM? Elton [22] has argued that realized returns used in empirical asset pricing tests are poor estimates of expected returns. His main point is that occasional, large information surprises can dominate the estimate of mean returns even for long observation intervals such that unexpected returns do not tend to mean zero. This noise problem causes realized returns to be biased estimates of expected returns.[3] As anecdotal evidence, Elton observed that realized returns in the stock market have been less than the riskless rate for periods longer than 10 years (e.g., 1973 to 1984). Upon conducting simulation tests using individual U.S. stock returns, he found that information surprises could lead to the rejection of an asset pricing model.

Given the existence of information surprises, the present study seeks to not only use portfolios to mitigate information surprises but data smoothing to further remove noise from return series. To smooth data, we compute the average daily return in each month for stocks as an estimate of their expected daily return. Rather than a moving average that drops the last observation as it rolls forward in time, we compute independent average daily returns for each separate month. This simple mean daily return per month reduces the number of daily observations from an average of 252 trading days per year to 12 data points per year. Hence, the resultant smoothed series of average daily returns is much less sensitive to information surprises than daily returns.[4] By contrast, as observed by Elton, realized monthly returns compound daily returns and therefore multiply daily information surprises that leads to potentially biased estimates of expected monthly returns. Our main research question is: Do average daily returns work better than realized returns in cross-sectional tests of the CAPM?

Like Fama and French and many others, using realized monthly returns for U.S. common stocks from 1965 to 2015, no relation to market beta is evident based on Fama and MacBeth [12] cross-sectional tests. However, consistent with CAPM theory, our empirical results document a significant positive relation between mean daily returns and market beta. For equal-weighted portfolios, cross-sectional *t*-values associated with beta risk range from 1.66 to 2.61 across different test asset portfolios. As in

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[2] It should be recognized that some evidence supports the conditional CAPM. For example, Ang and Chen [19], and Bali and Engle [20] have found that conditional time-varying measures of market beta (either stochastic or GARCH-based estimates) can predict cross-sectional variation in equity portfolio returns. Also, Kothari et al. [21] found evidence to support the static CAPM using annual returns from 1927 to 1990; however, further beta tests failed in the shorter period 1963 to 1990.

[3] Black [23] observed that past average returns used in testing asset pricing models provide highly inaccurate estimates of expected returns. Numerous studies have attempted to utilize analyst forecasts to proxy expected returns, including Ang and Peterson [24], Shefrin and Statman [25], Botosan and Plumlee [26], Brav et al. [27], and others. Another study by Campello et al. [28] used corporate bond yields to construct estimated expected stock return series for firms. More recently, Wu [29] employed analysts consensus stock price targets to estimate the expected returns of stocks. These studies have provided some evidence in support of the CAPM.

[4] See Madan [30] for discussion on estimating expected stock returns using daily data. They used approximately 10 years of daily returns to estimate mean returns for the entire sample period.

Bali et al. [31], for comparison purposes with realized monthly returns, we scale up daily returns to monthly percentage returns assuming 21 trading days in a month. In this respect, estimated market premia range from 0.44 percent to 0.86 percent per month for different test asset portfolios, which are economically meaningful. For Fama and French's [6, 7] widely-used size-BM (book-to-market) sorted test assets, the R-squared value indicates strong goodness-of-fit at 0.87. We conclude that CAPM is not dead as beta is better supported by average daily returns than realized monthly returns.

Further analyses augment the market factor with the size, value, and momentum factors. Due to multi-collinearity between these factors, we compute market-risk-adjusted excess returns as the dependent variable for test asset portfolios in cross-sectional regression tests. The results show that size and value are more significantly priced using average daily returns than realized returns. Also, pricing errors are driven to zero using average daily returns but not necessarily so for realized returns. When momentum is added to the three-factor model, it is significantly priced and supplants the size factor which becomes insignificant. We interpret these results to mean that zero-investment portfolios are needed to augment the CRSP market index to approximate the cross-section of average returns.

The next section discusses the problem of using realized monthly returns to proxy expected returns in testing CAPM beta. We propose average daily returns per month as an alternative choice for testing CAPM beta. Section 3 conducts empirical tests based on U.S. CRSP stock returns data. Further empirical tests of popular size, value, and momentum factors are provided. Section 4 concludes.

2. The problem of realized returns in testing the CAPM

The CAPM posits the following equilibrium relationship for the expected return on the ith asset:

$$E(R_i) - R_f = \beta_i \left[E(R_m) - R_f \right], \tag{1}$$

where R_f is the riskless rate of return, R_m is the market index rate of return, and β_i is the systematic market risk of the ith asset. Testing the above equation with real world data is nontrivial due to fact that investors' expected returns cannot be directly observed. Instead, as noted by Elton [22], researchers typically utilize realized returns to proxy expected returns in tests of the CAPM, which introduces a gap between empirical tests and CAPM theory. In an efficient market, all assets should move with the general market simultaneously, such that the length of the period to estimate expected returns is very small. Of course, for the smallest time scale, realized returns converge to expected returns. Unfortunately, the CAPM does not specify the length of the time period to compute expected return (see Kothari et al. [21], p. 190).

In practice, the following empirical form of the CAPM known as the *market model* by Markowitz [32] and Sharpe [33] is employed to estimate beta for common stocks:

$$R_{it} - R_{ft} = \alpha_i + \beta_i \left(R_{mt} - R_{ft} \right) + \varepsilon_{it}, \quad t = 1, \cdots T, \tag{2}$$

where $\hat{\alpha}$ is a mispricing term equal to zero in theory, ε_{it} is a random error term that represents idiosyncratic behavior of the ith stock's rate of return with $\varepsilon_{it} \sim N\left(0, \sigma_{i,Id}^2\right)$, and t is a time subscript. In cross-sectional asset pricing tests, it is common to use realized monthly returns for 60 months to estimate the market model. Subsequently, stocks are formed into (for example) 25 test asset portfolios based on their ranked $\hat{\beta}_i$ values and portfolio returns are computed for the next month (or next 12 months). In step one of cross-sectional Fama and MacBeth [12] tests, these monthly portfolio returns are used in a 60-month time-series regression to get $\hat{\beta}_p$ estimates for the 25 test assets. In step two, the 25 estimated portfolio betas are employed as independent variables in the following cross-sectional regression:

$$R_{pt+1} - R_{ft+1} = \alpha + \lambda\hat{\beta}_p + \varepsilon_{pt+1}, \quad t = 1, \cdots T, \tag{3}$$

where $R_{pt+1} - R_{ft+1}$ is the realized excess return of the pth test asset in the 61st month, and λ is the market price of risk associated with market beta. If the estimate $\hat{\lambda}$ is significantly different from zero, we infer that beta loadings associated with the market factor are priced in the cross-section of stocks. Also, the estimated intercept term $\hat{\alpha}$ should be statistically zero. A significant positive $\hat{\alpha}$ would imply mispricing in the sense that other factors are needed to fully price stocks.

As mentioned earlier, a potential problem in implementing the market model to test the CAPM is the common usage of realized rates of return rather than expected rates of return. In his AFA presidential address, Elton [22] defined realized returns as the sum of expected and unexpected returns, or

$$R_t = E(R_t) + I_t + \varepsilon_t, \tag{4}$$

where the unexpected return contains a systematic information component I_t plus a firm-specific component ε_t.[5] Due to information surprises, the unexpected return does not converge to zero over time. As such, realized returns do not equal expected returns. Information can have a large, permanent effect on unexpected returns and, in turn, realized returns. Elton argued that I_t follows a jump process that can influence factors in asset pricing models. In his words, "I believe that developing better measures of expected return and alternative ways to testing asset pricing theories that do not require realized returns have a much better payoff than any additional developent of statistical tests that continue to rely on realized returns as a proxy for expected returns."[6]

Elton's specification in Eq. (4) can be described as a random component model in which systematic component I_t consists of information surprises that potentially exist in stock returns. Instead of using realized monthly returns to estimate monthly expected returns, the alternative use of daily averages per month yields a model wherein I_t is the average surprise in month t and ε_t becomes a shrinked idiosyncratic random component with occasional daily noise effects averaging out. Replacing $E(R_t)$ in eq. (4) with the CAPM representation in Eq. (1), and denoting α_t as the daily average of I_t in month t, we can interpret the alpha term in Eq. (3) as the time-varying

[5] The existence of a noise component in realized returns is well recognized. See discussions in Brav et al. [27] and Wu [29] as well as citations therein, including Blume and Friend [34], Sharpe [35], Elton [22], Lewellen and Shanken [36], and Lundblad [37].

[6] See also Campbell [38] and Vuolteenaho [39] on decomposing unexpected returns into different systematic and firm-specific components. Vuolteenaho documented that unexpected returns primarily drive firm-specific realized returns.

mean of systematic information surprises. Consistent with Elton's model, we can redefine Eq. (3) as

$$\overline{R}_{pt+1} - R_{ft+1} = \alpha_{t+1} + \lambda\hat{\beta}_p + \overline{\varepsilon}_{pt+1}, \quad (5)$$

where the average daily return in month $t + 1$ for the pth portfolio is

$$\overline{R}_{pt+1} = \frac{1}{M_{t+1}} \sum_{u=1}^{M_{t+1}} R_{pu}^{daily}, \quad (6)$$

with M_{t+1} = number of trading days in the month (e.g., $M_{t+1} = 21$ for most months), and $\overline{\varepsilon}_{pt+1}$ = average daily noise term in month $t + 1$. The above specification with monthly averages of daily returns makes it possible to efficiently filter out the information surprise component I_t from the relation of stock returns and market risks in cross-sectional regression testing of the CAPM.

It is clear that average daily returns per month $\overline{R}_{pt}$ differ from realized monthly returns R_{pt} computed by cumulatively compounding daily returns for M_t days within each month, i.e., $\prod_{u=1}^{M_t}\left(1 + R_{iu}^{daily}\right) = \left(1 + R_{iu=1}^{daily}\right)\left(1 + R_{iu=2}^{daily}\right) \dots \left(1 + R_{iu=M_t}^{daily}\right)$. However, given that the CAPM is based on expected returns, the average daily return within 1 month is a potential proxy for the expected return that has not previously been considered in asset pricing tests. Consistent with Elton's [22] arguments, even though realized returns capture actual investor returns in each month, they are potentially biased estimates of expected returns as the additive daily effects of systematic I_t and occasional noise term ε_t become compounded into monthly returns. While average daily returns per month $\overline{R}_{pt}$ do not match realized monthly returns R_{pt}, the average daily return filters out most daily noise effects ε_t leaving occasional systematic component I_t essentially intact. This filtering makes it possible to single out its effect in modeling the expected return. Our specification [31] accomplishes this outcome by accounting for the I_t component via time-varying intercept term α_{t+1}, thereby reducing its biasing effect in our testing of the pricing relation predicted by the CAPM.

3. Cross-sectional tests using actual U.S. stock returns

The standard approach for testing the validity of asset pricing models is the two-step Fama and MacBeth [2] procedure. In this section, based oon U.S. stock returns, we conduct cross-sectional Fama-MacBeth regression tests by means of Eq. (3) in the previous section. First, we perform cross-sectional tests using a variety of beta-sorted test assets. Second, we repeat these tests using size and book-to-market (BM) sorted as well as industry portfolios downloaded from Kenneth French's website. Third, and last, we provide tests of popular multi-factors, including size, value, and momentum.

3.1 Beta-sorted test assets

Our data consist of all U.S. common stocks (share codes 10 and 11) in both CRSP and COMPUSTAT databases with at least 60 consecutive months of returns from January 1960 to December 2015. In any given month, we drop stocks in the bottom 10 percent market capitalization of this population to avoid liquidity issues associated with very small stocks. From these data we construct four sets of test assets comprised of 25 portfolio returns from January 1965 to December 2015:

1. Test assets I are value-weighted realized monthly returns for 25 test asset portfolios based on estimated betas using market model Eq. (2) for individual stocks and the CRSP market index. For all stocks with 60 monthly returns available from January 1960 to December 1964, 25 beta-sorted portfolios are formed and value-weighted one-month realized returns for these portfolios are computed in the subsequent month of January 1965. This procedure is repeated by rolling forward 1 month at a time until the end of the sample period to generate a time series of value-weighted realized monthly returns for each portfolio.

2. Test assets II are equal-weighted realized monthly monthly returns for 25 test asset portfolios based on estimated betas using market model Eq. (2) for individual stocks and the CRSP market index. The one-month-ahead rolling procedure used for test assets I is repeated.

3. Test assets III are value-weighted average daily returns per month for portfolios based on estimated betas using market model Eq. (2) by replacing R_{it} and R_{mt} with their daily average returns per month for individual stocks and the CRSP market index, respectively. The one-month-ahead rolling procedure used for test assets I is repeated.

4. Test assets IV are equal-weighted average daily returns per month for portfolios based on estimated betas using the market model [24] by replacing again R_{it} and R_{mt} with their daily average returns per month for individual stocks and the CRSP market index, respectively. The one-month-ahead rolling procedure used for test assets I is repeated.

Statistical properties of these test assets in the sample period are shown in **Table 1**. For value-weighted returns in test assets I and III, the mean CRSP index realized return and mean average daily return are the same at 0.89 percent per month. For comparative purposes with respect to monthly realized returns and later monthly risk premiums, we scale up average daily returns in each month as well as estimated daily risk premiums in cross-sectional tests by the number of trading days per month (i.e., $M = 21$) to obtain monthly return estimates.[7] Notice that the mean equal-weighted realized return in the sample period at 1.15 percent per month is approximately 30 percent greater than the mean value-weight realized return at 0.89 percent. Equal-weighted returns are clearly larger than value-weighted returns in our sample period. Turning to equal-weighted returns in test assets II and IV, the mean CRSP index

[7] Bali et al. [31] similarly scale up daily returns and risk premiums to monthly values assuming 21 trading days.

realized return is lower at 1.15 percent per month compared to 1.60 percent per month for average daily returns.[8]

Not only are (scaled) average daily returns consistently higher than realized returns (especially for equal-weighted returns), but the Sharpe ratios for test assets using equal-weighted returns are larger than those for value-weighted returns. As an example, the CRSP index has a Sharpe ratio of 0.21 for equal-weighted average daily returns compared to only 0.11 for both value-weighted realized and average daily returns. We interpret these results to mean that the equal-weighted market portfolio dominates the value-weighted portfolio and, therefore, is closer to the efficient frontier.[9] (Kothari et al. [21], p. 189) have commented that the question of whether the value-weighted stock index is preferred over an equal-weight stock index in CAPM beta tests is not settled. According to CAPM theory, they noted that the stock index most correlated with the true market portfolio is efficient.

The patterns of monthly returns in **Table 1** are particularly noteworthy. Average daily returns based on value-weighted portfolios increase from 0.79 percent per month for the low beta portfolio to 1.21 percent for the highest beta portfolio. However, portfolio returns do not linearly increase as beta risk increases for portfolios, and most portfolios have returns in a fairly narrow range (e.g., 0.90 and 1.10 percent). These patterns are evident in value- and equal-weighted realized monthly returns also. By contrast, based on equal-weighted portfolios, average daily returns increase in a fairly steady linear manner as beta risk increases for portfolios from 0.99 percent per month for the lowest beta portfolio to 1.60 percent for the highest beta portfolio. This descriptive pattern between equal-weighted, average daily returns and portfolio betas tends to support CAPM beta.

Monthly rolling cross-sectional Fama-MacBeth regressions are estimated for the test assets defined above. As described in Cochrane [41], we implement the following sequence of steps:

1. Estimate either market model Eq. (2) with realized monthly returns or with proposed daily average monthly returns to obtain a $\hat{\beta}_p$ estimate (factor loading) for each portfolio using monthly excess returns for the entire sample period January 1965 to December 2015.[10]

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[8] The relatively high average daily return of 1.60 percent per month can be explained by the definition of realized returns. Assuming a stock's rate of return has average rate $\overline{R}$ and standard deviation σ_{AVG}, the realized rate of return in period T (i.e., 1 month) can be derived by means of a Taylor series expansion of the natural log function of each period's gross return $(1 + R_t)$ to obtain (after dropping higher order terms):

$$R_{1\to T} = \prod_{t=1}^{T}(1 + R_t) - 1 \approx \exp\left(\overline{R}T - \sigma_{AVG}^2 T/2\right) - 1, T = 1, \cdots, 21 \tag{6a}$$

where $\overline{R} = \frac{1}{T}\sum_{t=1}^{T} R_t$, and $\sigma_{AVG}^2 = \frac{1}{T-1}\sum_{t=1}^{T}\left(R_t - \overline{R}\right)^2$. Over time only one path of realized returns is observed by the investor. This path represents one draw from a distribution of many potential paths for realized returns. It is impossible for investors to know the true values of expected returns and volatilities. Investors may well perceive that $\overline{R}$ computed as the average of daily returns in a month provides a reasonable estimate of daily expected returns.

[9] Value-weighted portfolio returns introduce a size effect in the computation of returns that weights large stocks more heavily than small stocks. Since the equal-weighted CRSP index places more emphasis on smaller stocks and has a much larger Sharpe ratio than the value-weighted CRSP index, it appears that small stocks are needed to better approximate efficient portfolios.

[10] This approach is not feasible for individual stocks due to difficulties in gathering a sample of stocks with return series over the entire sample period.

Portf	Realized returns						Average daily returns					
	I. Value-weighted			II. Equal-weighted			III. Value-weighted			IV. Equal-weighted		
	μ	σ	$\frac{\mu-R_f}{\sigma}$	μ	σ	$\frac{\mu-R_f}{\sigma}$	μ	σ	$\frac{\mu-R_f}{\sigma}$	μ	σ	$\frac{\mu-R_f}{\sigma}$
1	0.69	4.15	0.07	0.81	3.10	0.13	0.79	4.09	0.09	0.99	3.13	0.19
2	0.83	3.77	0.11	0.90	2.98	0.16	0.92	3.56	0.15	0.96	2.95	0.19
3	0.88	3.69	0.13	1.00	3.11	0.19	0.85	3.57	0.12	1.06	3.09	0.21
4	0.95	3.70	0.15	1.04	3.39	0.19	0.99	3.71	0.16	1.21	3.37	0.24
5	0.80	3.90	0.10	1.06	3.68	0.18	0.91	3.91	0.13	1.21	3.70	0.22
6	0.96	3.92	0.14	1.13	3.85	0.19	0.94	3.90	0.14	1.25	3.88	0.22
7	0.91	4.06	0.13	1.16	4.08	0.18	1.03	4.27	0.15	1.29	4.06	0.22
8	0.97	4.39	0.13	1.11	4.28	0.16	1.05	4.34	0.15	1.26	4.23	0.20
9	0.95	4.39	0.13	1.14	4.48	0.16	0.93	4.35	0.12	1.31	4.51	0.20
10	0.90	4.47	0.11	1.18	4.69	0.17	1.09	4.38	0.16	1.30	4.64	0.19
11	1.01	4.58	0.13	1.22	4.86	0.17	0.94	4.64	0.12	1.32	4.80	0.19
12	0.88	4.74	0.10	1.11	5.01	0.14	1.01	4.72	0.13	1.36	4.96	0.19
13	0.87	4.87	0.10	1.21	5.14	0.16	0.85	5.01	0.09	1.30	5.13	0.17
14	0.80	5.04	0.08	1.28	5.37	0.16	0.96	4.96	0.11	1.42	5.38	0.19
15	1.02	5.12	0.12	1.26	5.57	0.15	0.98	5.29	0.11	1.43	5.49	0.19
16	0.94	5.38	0.10	1.13	5.75	0.13	1.03	5.33	0.12	1.38	5.64	0.17
17	0.94	5.70	0.09	1.23	6.03	0.14	0.87	5.68	0.08	1.34	5.94	0.16
18	0.83	5.93	0.07	1.21	6.36	0.13	1.12	5.78	0.12	1.39	6.20	0.16
19	0.95	5.86	0.09	1.11	6.55	0.11	1.04	6.18	0.10	1.43	6.41	0.16
20	0.94	6.38	0.08	1.21	6.95	0.12	1.03	6.36	0.10	1.42	6.67	0.15
21	0.79	6.68	0.06	1.16	7.22	0.10	0.88	6.59	0.07	1.50	7.18	0.15
22	0.93	7.23	0.07	1.14	7.81	0.09	0.89	7.10	0.07	1.50	7.63	0.14
23	0.81	7.69	0.05	1.13	8.36	0.09	1.16	7.92	0.10	1.45	8.15	0.13
24	0.99	8.36	0.07	0.95	9.23	0.06	0.99	8.41	0.07	1.56	8.92	0.13
25	0.94	9.42	0.06	1.00	10.71	0.06	1.21	9.54	0.08	1.72	10.46	0.13
CRSP	0.89	4.49	0.11	1.15	5.68	0.13	0.89	4.45	0.11	1.60	5.60	0.21

Based on CRSP data, test assets are created by estimating individual stock betas using the market model with U.S. common stocks' monthly excess returns for the 51-year sample period January 1965 to December 2015 and then forming the stocks into 25 beta-sorted portfolios, i.e., portfolio 1 [40] has the lowest (highest) beta. Both value- and equal-weighted, monthly excess returns are used for individual stocks and the CRSP market index, where excess returns are either realized or average daily excess returns. For comparison purposes to monthly realized returns, average daily returns are scaled up to monthly returns.

Table 1.
Statistical properties of test asset portfolios sorted on market beta.

2. Estimate cross-sectional OLS regressions using monthly excess returns in January 1965 as the dependent variable for all portfolios and $\hat{\beta}_p$ factor loadings from time-series regressions in step 1 as the independent variables for portfolios. The estimated coefficient denoted $\hat{\lambda}$ measures the beta risk factor price or risk premium.

3. Roll step 2 forward 1 month at a time to the end of the sample period December 1965.

4. Retain the monthly time series of 612 estimated factor prices of risk $\hat{\lambda}$ from January 1965 to December 2015. Use these series to compute the average value denoted $\hat{\bar{\lambda}}$ in percent per month.

5. Estimate t-statistics for the difference of mean values $\hat{\bar{\lambda}}$ from zero, which are corrected for errors-in-variables as in Shanken [42].

We evaluate the goodness-of-fit of the cross-sectional regressions using the adjusted R^2 statistic as suggested by Jagannathan and Wang [40] and Lettau and Ludvidgson [43]. This R-squared value can be computed via the pure cross-section approach in which a single cross-sectional regression is run:

1. As in the Fama-MacBeth approach, estimate either market model eq. (2) with realized monthly returns or proposed daily average monthly returns to obtain $\hat{\beta}_p$ factor loadings for each portfolio using monthly excess returns for the entire sample period January 1965 to December 2015.

2. Compute the time series average of 612 monthly excess returns for portfolios.

3. Estimate a single cross-sectional OLS regressions using the time series average monthly excess returns for all portfolios and the $\hat{\beta}_p$ factor loadings as the independent variables. The estimated coefficient denoted $\hat{\lambda}$ is the same as the time-series average value $\hat{\bar{\lambda}}$ in the Fama-MacBeth approach above.[11]

Fama-MacBeth test results using value-weighted monthly realized excess returns for both test asset portfolios and the CRSP market index are provided in Panel A of **Table 2**. We obtain $\hat{\bar{\lambda}} = 0.05$ percent per month with $t = 0.18$, which is far from significance. Also, $\hat{\bar{\alpha}} = 0.45$ with $t = 2.43$, which indicates highly significant mispricing errors. Goodness-of-fit is very low with R-squared at only 0.04. These results are comparable to those of Fama and French [6–8] and many other studies that fail to accept the CAPM based on empirical tests. Similar findings are obtained with equal-weighted monthly realized excess returns in Panel B as well as value-weighted average daily excess returns in Panel C. The insignificance of beta with both value- and equal-weighted returns suggests that small and illiquid firms do not have an impact on the pricing evidence of market beta.

Using equal-weighted average daily excess returns for both test asset portfolios and the CRSP index in Panel D of **Table 2**, the market price of risk becomes marginally significant (at the 10 percent level) at $\hat{\bar{\lambda}} = 0.44$ percent per month with $t = 1.66$. Here the average estimated market premium is economically meaningful. Moreover,

[11] As Cochrane [41] observes, the Fama-MacBeth estimator for the standard error of $\hat{\lambda}$ adjusts for the cross-correlation of residuals and equals the pure cross-section OLS standard error of $\hat{\lambda}$ after correction for cross-sectional correlation. Hence, the two methods generate numerically equivalent cross-sectional results.

Panel A. Value-weighted realized returns				
$\hat{\bar{\alpha}}$	*t*-value	$\hat{\bar{\lambda}}$	*t*-value	R-squared
0.45	2.43	0.05	0.18	0.04
Panel B. Equal-weighted realized returns				
$\hat{\bar{\alpha}}$	*t*-value	$\hat{\bar{\lambda}}$	*t*-value	R-squared
0.65	4.45	0.07	0.25	0.05
Panel C. Value-weighted average daily returns				
$\hat{\bar{\alpha}}$	*t*-value	$\hat{\bar{\lambda}}$	*t*-value	R-squared
0.43	2.31	0.15	0.57	0.27
Panel D. Equal-weighted average daily returns				
$\hat{\bar{\alpha}}$	*t*-value	$\hat{\bar{\lambda}}$	*t*-value	R-squared
0.54	3.78	0.44	1.66	0.87

*Using 25 beta-sorted test assets described in **Table 1** with monthly excess returns for the 51-year sample period January 1965 to December 2015, we conduct cross-sectional Fama-MacBeth tests. In the first time-series regression step, the market model with monthly excess returns is used to estimate the market betas for the 25 test asset portfolios over the 51-year sample period. Here both equal-weighted and value-weighted returns are computed for test asset portfolios as well as the CRSP market index. Treasury bill rates are used to compute excess returns. Also, both realized excess returns and average daily excess returns are used. In the second cross-sectional step, the market price of risk denoted $\hat{\lambda}$ is estimated by regressing the excess returns for test assets in the first month of the sample period on their estimated full-sample betas. Subsequently, rolling forward 1 month at a time, this second step is repeated until the last month of the sample period to generate a time series of $\hat{\lambda}$ estimates, which are then averaged to obtain $\hat{\bar{\lambda}}$ for tests of their significant difference from zero. Using this monthly rolling approach, the portfolio beta for each test asset is the same in each monthly cross-sectional regression. For comparison purposes to results using monthly realized returns, risk premia based on average daily returns are scaled up to monthly premia. Standard errors are computed with Shanken [42] corrections to estimate t-values. R-squared values are based on the pure cross-section approach with a single cross-sectional OLS regression (see Jagannathan and Wang [40] and (Lettau and Ludviegson [43], footnote 17, p. 1254)).*

Table 2.
Cross-sectional Fama-MacBeth tests for the market factor using realized and average daily U.S. stock market returns for beta-sorted test asset portfolios.

the *R*-squared value indicates strong goodness-of-fit at 0.87.[12] Nonetheless, significant mispricing exists (i.e., $\hat{\bar{\alpha}} = 0.54$ with $t = 3.78$[13]), which suggests that other factors are needed to more fully price test assets. In forthcoming Subsection 3.3, we conduct further tests by augmenting the market factor with size, value, and momentum multi-factors in an attempt to mitigate mispricing errors.

Figure 1 plots average daily excess returns against estimated beta values for the 25 beta-sorted assets. The left-hand graph contains the results using value-weighted realized excess returns as in Panel A of **Table 2**. By casual inspection, there is little

[12] Levy [13] has observed that R-squared values in cross-sectional regressions with CAPM beta are typically low at around 0.04 with monthly data and 0.20 for annual data with individual stocks.

[13] It is interesting to note that, as the standard error of $\hat{\lambda}$ is 0.27, a similar size market price of risk estimate, i.e., $\hat{\bar{\lambda}} = 0.54$ (6.48 p.a) would result in only a borderline statistically significant *t*-value of $t = 2.00$, which suggests much lower power of the *t*-test of $\hat{\bar{\lambda}}$ compared to that of $\hat{\bar{\alpha}}$. Thus, the weak significance of $\hat{\bar{\lambda}}$ is attributable to the weak power of the standard *t*-test produced by the traditional Fama-MacBeth approach. The development of more powerful testing procedures is recommended that take into account highly correlated portfolios.

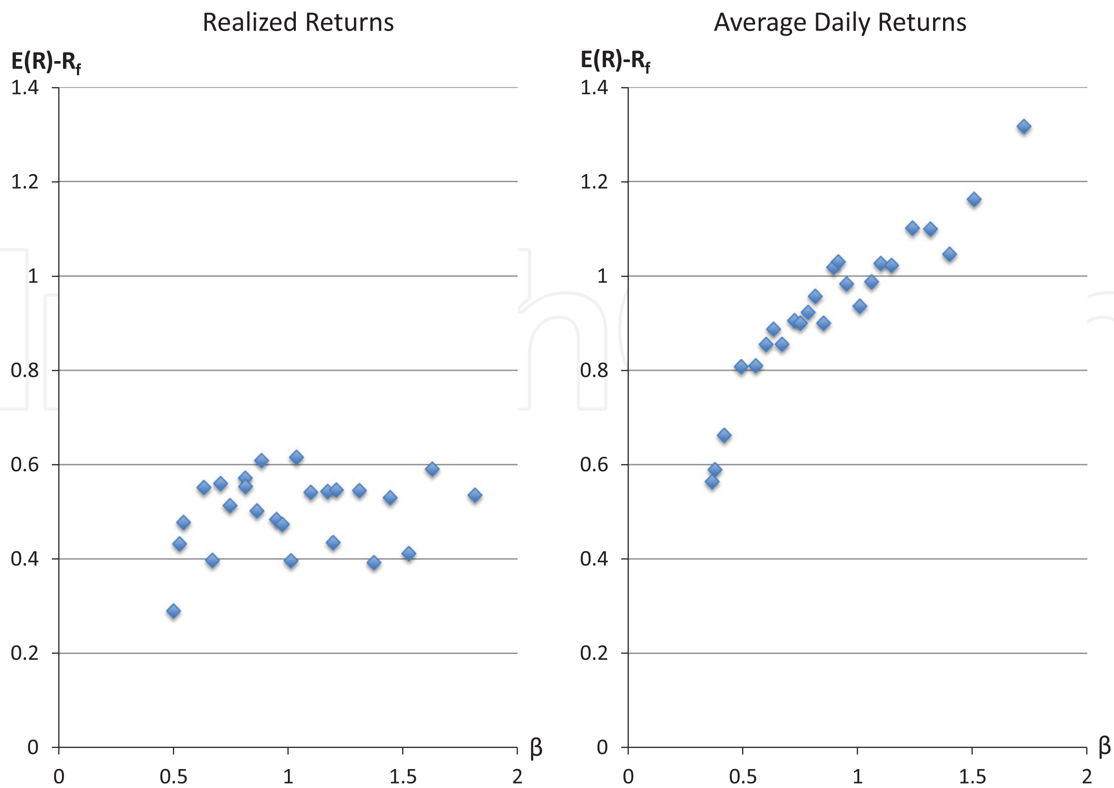

Figure 1.
The relation between average excess returns and market beta. The graph on the left shows the results for 25 beta-sorted test asset portfolios with the time-series average of market betas for each portfolio on the X-axis and the time-series average of value-weighted realized excess returns on the Y-axis. The graph on the right shows the results for the time-series mean of market betas for each portfolio on the X-axis and the time-series mean of equal-weighted average daily excess returns on the Y-axis (scaled to monthly returns). The graphs are based on results in Panels A and D of ***Table 2*** *using the pure cross-section approach for estimating R-squared values in the sample period January 1965 to December 2015.*

or no discernable relationship evident. By contrast, using equal-weighted average daily excess returns (scaled to monthly returns for comparison purposes) as in Panel D of **Table 2**, the right-hand graph illustrates a positive relation between average beta values and average daily excess returns. The linear relation shown there is almost a perfect depiction of CAPM theory. Also, it is consistent with earlier descriptive evidence in **Table 1** for equal-weighted average daily returns and portfolio betas.

3.2 Fama: French test assets and industry portfolios

Table 3 repeats the above cross-sectional tests of realized and average daily returns for widely-used test asset portfolios. From Kenneth French's website, we downloaded value- and equal-weighted returns for 25 portfolios sorted on quintiles of size and book-to-market (BM) as well as 30 industry portfolios. Average daily excess returns are computed from daily realized returns. Traditional test results in Panel A for value-weighted realized returns with 25 size-BM sorted portfolios reveal the following results: $\hat{\bar{\lambda}} = -0.33$ percent per month with $t = -0.77$. As in **Table 2**, the market factor is not significantly priced in the cross section of average stock returns. Again, mispricing errors are highly significant at the 1 percent level, and R-squared is relatively low at 0.06. The estimated market prices of risk for equal-weighted realized returns with 25 size-BM portfolios in Panel B are again insignificant but the coefficient signs switch from negative to positive. As before in **Table 2**, this evidence implies that small and illiquid firms do not affect market beta

Panel A. Value-weighted realized returns					
Portfolios	$\hat{\hat{\alpha}}$	t-value	$\hat{\hat{\lambda}}$	t-value	R-squared
25 Size-BM	1.06	2.76	−0.33	−0.77	0.06
30 Industry	0.66	2.84	−0.07	−0.23	0.01
25 Size-BM + 30 Industry	0.75	3.08	−0.10	−0.34	0.01
Panel B. Equal-weighted realized returns					
Portfolios	$\hat{\hat{\alpha}}$	t-value	$\hat{\hat{\lambda}}$	t-value	R-squared
25 Size-BM	0.71	2.49	0.06	0.17	0.00
30 Industry	0.72	3.31	0.05	0.17	0.00
25 Size-BM + 30 Industry	0.72	3.10	0.05	0.18	0.00
Panel C. Value-weighted average daily returns					
Portfolios	$\hat{\hat{\alpha}}$	t-value	$\hat{\hat{\lambda}}$	t-value	R-squared
25 Size-BM	1.23	2.44	−0.53	−1.32	0.17
30 Industry	0.69	3.05	−0.12	−0.41	0.03
25 Size-BM + 30 Industry	0.84	3.53	−0.21	−0.71	0.05
Panel D. Equal-weighted average daily returns					
Portfolios	$\hat{\hat{\alpha}}$	t-value	$\hat{\hat{\lambda}}$	t-value	R-squared
25 Size-BM	0.16	0.57	0.87	2.56	0.16
30 Industry	0.69	3.33	0.53	1.79	0.24
25 Size-BM + 30 Industry	0.34	1.54	0.79	2.61	0.21

From Kenneth French's website, we downloaded daily and monthly returns for Fama and French's [6–8] 25 size and book-to-market (BM) portfolios as well as 30 industry portfolios for the 51-year sample period January 1965 to December 2015. Value- and equal-weighted portfolio and CRSP market index returns as well as Treasury bill rates are downloaded. Implementing the Fama-MacBeth test procedure, in the first time-series regression step, the market model with monthly excess returns is used to estimate the market betas for the test asset portfolios over the 51-year sample period. Here both equal-weighted and value-weighted returns are used for test asset portfolios as well as the CRSP market index. Also, both realized returns and average daily returns per month are used. In the second cross-sectional step, the market price of risk denoted as $\hat{\lambda}$ is estimated by regressing the excess returns for test assets in the first month of the sample period on their estimated full-sample betas. Subsequently, rolling forward 1 month at a time, this second step is repeated until the last month of the sample period to generate a time series of $\hat{\lambda}$ estimates, which are then averaged to obtain $\hat{\hat{\lambda}}$ that is tested for significant difference from zero. Using this monthly rolling approach, the portfolio beta for each test asset is the same in each monthly cross-sectional regression. For comparison purposes to results using monthly realized returns, risk premia based on average daily returns are scaled up to monthly premia. Standard errors are computed with Shanken [42] corrections to estimate t-values. R-squared values are based on the pure cross-section approach with a single cross-sectional OLS regression (see Jagannathan and Wang [40] and (Lettau and Ludviegson [43], footnote 17, p. 1254)).

Table 3.
Cross-sectional Fama-MacBeth tests for the market factor using realized and average daily returns for size-BM sorted and industry test asset portfolios.

pricing. For value-weighted average daily returns in Panel C, the results are insignificant with negative coefficient signs similar to value-weighted realized returns in Panel A. The results for the 30 industry portfolios and combined 25 size-BM plus 30 industry portfoliios in Panels A to C are similar those for the 25 size-BM portfolios.

Using equal-weighted average daily excess returns in Panel D of **Table 3**, the estimated market prices of risk are statistically significant and economically substantial: $\hat{\hat{\lambda}} = 0.87$ percent per month with $t = 2.56$ for the 25 size-BM sorted portfolios,

$\hat{\hat{\lambda}} = 0.53$ percent per month with $t = 1.79$ for 30 industry portfolios, and $\hat{\hat{\lambda}} = 0.79$ percent per month with $t = 2.61$ for the combined 25 size-BM plus 30 industry portfolios. A recent study by Harvey et al. [15] argues that, due to the large number of proposed multi-factors, an asset pricing factor should be evaluated against a t-statistic threshold of 3.0 or more. Our t-values are somewhat less than this hurdle rate but market beta has the advantage of CAPM theoretical support. In this regard, Harvey et al. conclude that a lower threshold t-ratio may be justifiable for a factor with strong theoretical underpinnings.

In Panel D of **Table 3**, notice that the mispricing error terms $\hat{\hat{\alpha}} = 0.16$ and $\hat{\hat{\alpha}} = 0.34$ are relatively small and insignificant for the size-BM and size-BM plus industry portfolios, respectively. For these test assets, the magnitudes of the market factor prices of risk $\hat{\hat{\lambda}}$ are relatively high at about 0.80 percent or more per month and the associated t-values are significant at the 1 percent level. Hence, at higher estimated market factor prices of risk, mispricing error tends to disappear. In view of our earlier discussion of Elton's random component model, average information surprises embedded in $\hat{\hat{\alpha}}$ are diminished by significant market factor loadings. Finally, the estimated R-squared values in the range of 0.16 to 0.24 across test assets are relatively higher in Panel D using equal-weighted average daily returns than in other panels. These results are encouraging in terms of supporting the CAPM.

Some researchers (e.g., Lewellen et al. [44] and Daniel and Titman [45]) have recommended using industry portfolios as test assets (as well as combining them with Fama-French portfolios) due to their exogeneity with respect to model factors. However, empirical tests using industry portfolios have typically not supported the CAPM. Based on equal-weighted average daily returns in Panel D, the market price of risk is marginally significant (at the 10 percent level) for 30 industry portfolios with an economically meaningful risk premium of 0.53 percent per month. These results based on exogenous test assets support CAPM beta.

3.3 Cross-sectional tests of multi-factors

We next augment the market model with Fama and French's size and value factors plus the momentum factor. In unreported results, for both value-weighted and equal-weighted average daily returns, we find that the market factor is not significantly priced in cross-sectional tests models containing these factors. The primary reason for this insignificance appears to be multicollinearity between the market factor and the multi-factors. In our sample period the estimated correlations for value-weighted realized returns between the market factor and both size and value factors are 0.31 and -0.31, respectively. The size and value factors have an estimated correlation of -0.23. Correlations using average daily returns are similar or higher. For example, the market and size factors have a correlation coefficient of 0.56. Due to this collinearity issue, we tested the size and value factors as follows:

- Using the entire sample period, the market model is estimated by regressing test asset portfolios' monthly excess returns on CRSP market index excess returns.

- Risk-adjusted excess returns for (zero-cost investment) portfolios are computed for the entire sample period as

$$R_{pt}^{ex} = R_{pt} - R_{ft} - \hat{\beta}_p\left(R_{Mt} - R_{ft}\right). \tag{7}$$

- Risk-adjusted excess returns R_{pt}^{ex} are regressed on the *SMB* and *HML* factors to obtain $\hat{\beta}_{SMB}$ and $\hat{\beta}_{HML}$ as

$$R_{pt}^{ex} - R_{ft} = \alpha_p + \beta_{pSMB}SMB_t + \beta_{pHML}HML_t + \varepsilon_{pt}. \tag{8}$$

- Based on the second step in the Fama-MacBeth testing method, the market prices of risk $\hat{\lambda}_{SMB}$ and $\hat{\lambda}_{HML}$ are estimated.

Table 4 reports the results of this risk-adjusted excess return approach for testing the size and value factors. In Panel A using realized excess returns, size is marginally significant in the 30 industry portfolios, and the value factor is significant in the 25 size-BM portfolios and combined 25 size-BM and industry portfolios. In Panel B using average daily returns, both size and value factors are significant in the 25 size-BM portfolios and combined 25 size-BM and industry portfolios but not industry portfolios alone. Notice that $\hat{\hat{\alpha}}$s equal zero for all three test asset portfolios. We infer from this evidence that the size and value factors are better supported via our risk-adjusted excess return procedure than the traditional three-factor model.

We repeated the above analyses for the four-factor Carhart [46] model with market, size, value, and momentum factors in **Table 5**. Most of the results for the size and value factors are similar to those in **Table 4**, with the exception that size is no longer priced in Panels A and B. It appears that, in this model, the momentum factor supplants size, as momentum is normally priced in Panel A to B.[14] Due to low estimated correlations in the sample period between size and momentum (i.e., -0.00 and $-$ 0.06 for realized and average daily excess returns, respectively), multicollinearity does not appear to be the reason for size dropping out.[15] Like the three-factor model, no mispricing error exists for tests using average daily returns, but mispricing occurs in the test assets using realized returns.

Overall, these results suggest that zero-investment portfolios are needed to augment the CRSP market index in order to better approximate the cross-section of average stock returns.

3.4 Robustness tests

We conducted a number of robustness checks. For example, we repeated cross-sectional tests in **Tables 2** and **3** of the CRSP market index using average daily returns in each quarter rather than each month. Smoothing over a longer period may further suppress information surprises. Also, further tests of average daily returns using a

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[14] Recent work by Asness et al. [47] has shown that the size effect is masked to some extent by the quality of firms. Upon controlling for quality, they found that the returns to size are more stable and significant than otherwise. Also, momentum did not explain the returns to size-quality portfolios. Interactions between multi-factors were recommended for further study by the authors.

[15] The estimated correlations between momentum and the excess market returns are -0.13 and $-$ 0.27 for realized and average daily returns, respectively.

Panel A. Value-weighted realized returns							
Portfolios	$\hat{\alpha}$	t-value	$\hat{\lambda}_{SMB}$	t-value	$\hat{\lambda}_{HML}$	t-value	R-squared
25 Size-BM	−0.00	−0.00	0.06	1.42	0.51	4.05	0.67
30 Industry	0.15	4.07	−0.39	−1.88	−0.14	−0.21	0.25
25 Size-BM + 30 Industry	0.07	2.26	−0.02	−0.17	0.28	2.16	0.17
Panel B. Equal-weighted average daily returns							
Portfolios	$\hat{\alpha}$	t-value	$\hat{\lambda}_{SMB}$	t-value	$\hat{\lambda}_{HML}$	t-value	R-squared
25 Size-BM	−0.03	−1.00	0.35	2.18	0.71	5.20	0.67
30 Industry	0.03	0.99	−0.23	−0.78	−0.12	−0.62	0.03
25 Size-BM + 30 Industry	−0.02	−0.97	0.35	2.09	0.49	3.41	0.38

From Kenneth French's website, we downloaded daily and monthly returns for Fama and French's [6–8] 25 size and book-to-market (BM) portfolios as well as 30 industry portfolios for the 51-year sample period January 1965 to December 2015. Value- and equal-weighted portfolio and CRSP market index returns are downloaded in addition to the Fama-French size ((SMB) and value (HML) factors and one-month Treasury bill rates. Implementing the Fama-MacBeth test procedure, in the first time-series regression step, the market model with monthly excess returns is used to estimate the market betas $\hat{\beta}_p$ for the test asset portfolios over the 51-year sample period. Here both value- and equal-weighted returns are used for test asset portfolios as well as the CRSP market index. Also, both realized returns and average daily returns per month are used. Excess monthly returns for each portfolio are computed as follows: $R^{ex}_{pt} - R_{ft} = R_{pt} - R_{ft} - \hat{\beta}_p R_{mt} - R_{ft}$, where R_{mt} and R_{ft} are the CRSP index return and Treasury bill rate, respectively. Subsequently, these excess returns are regressed on the SMB and HML factors to estimate the multi-factors' loadings for the full sample period. In the second cross-sectional step, the market price of risk denoted as $\hat{\lambda}$ for each multi-factor is estimated by regressing the excess returns for test assets in the first month of the sample period $R^{ex}_{pt=1}$ on their estimated full sample multi-factor betas. Rolling forward 1 month at a time, this second step is repeated until the last month of the sample period to generate a time series of $\hat{\lambda}$ estimates for each multi-factor, which are then averaged to obtain $\hat{\bar{\lambda}}$ that is tested for significant difference from zero. Using this monthly rolling approach, the estimated portfolio SML and HML betas for each test asset are the same in each monthly cross-sectional regression. For comparison purposes to results using monthly realized returns, risk premia based on average daily returns are scaled up to monthly premia. Standard errors are computed with Shanken [42] corrections to estimate t-values. R-squared values are based on the pure cross-section approach with a single cross-sectional OLS regression (see Jagannathan and Wang [40] and (Lettau and Ludviegson [43], footnote 17, p. 1254)).

Table 4.
Cross-sectional Fama-MacBeth tests for the size and value factors using realized and average daily market beta risk-adjusted returns for size-BM sorted and industry test asset portfolios.

longer period provide some validation of our monthly results. Based on equal-weighted returns, **Table 6** shows that market beta is again significantly priced for 25 beta-sorted portfolios (viz., t-value = 1.91), 25 size-BM portfolios (viz., t-value = 2.88), 25 size-BM plus 30 industry portfolios (viz., t-value = 2.47), and 25 size-BM plus 30 industry plus 25 beta-sorted portfolios (viz., t-value = 2.37). For industry portfolios, market beta is not significantly priced (viz., t-value = 1.40), which contrasts with the nominal significance of market beta in Panel D of **Table 3** (viz., t-value = 1.79) As before, mispricing error is insignificant for the size-BM and size-BM plus 30 industry portfolios (i.e., average information surprises are eliminated). In general, these results using average daily returns per quarter corroborate our findings using average daily returns per month.

In unreported tests, we found that market beta was not significantly priced using median (rather than mean) returns per month. Also, we repeated the realized return tests in Panels A and B of **Tables 2** and **3** using daily (rather than monthly) realized

Panel A. Value-weighted realized returns									
Portfolios	$\hat{\alpha}$	t-val	$\hat{\lambda}_{SMB}$	t-val	$\hat{\lambda}_{HML}$	t-val	λ_{MOM}	t-val	R-squared
25 Size-BM	0.06	2.37	−0.08	−0.54	0.64	4.97	3.25	4.85	0.77
30 Industry	0.15	3.92	−0.36	−1.61	−0.01	−0.07	0.27	0.44	0.27
25 Size-BM + 30 Industry	0.10	2.84	−0.05	−0.34	0.36	2.65	0.97	2.08	0.31
Panel B. Equal-weighted average daily returns									
Portfolios	$\hat{\alpha}$	t-val	$\hat{\lambda}_{SMB}$	t-val	$\hat{\lambda}_{HML}$	t-val	λ_{MOM}	t-val	R-squared
25 Size-BM	0.02	0.74	−0.11	−0.53	1.03	6.52	3.08	3.94	0.71
30 Industry	0.04	1.32	−0.39	−1.17	0.07	0.32	1.53	2.11	0.27
25 Size-BM + 30 Industry	−0.01	−0.25	0.08	0.39	0.67	3.99	1.74	2.52	0.47

From Kenneth French's website, we downloaded daily and monthly returns for Fama and French's [6–8] 25 size and book-to-market (BM) portfolios as well as 30 industry portfolios for the 51-year sample period January 1965 to December 2015. Value- and equal-weighted portfolio and CRSP market index returns are downloaded in addition to the Fama-French size ((SMB) and value (HML) factors, momentum factor, and Treasury bill rates. Implementing the Fama-MacBeth test procedure, in the first time-series regression step, the market model with monthly excess returns is used to estimate the market betas $\hat{\beta}_p$ for the test asset portfolios over the 51-year sample period. Here both equal- and value-weighted excess returns are used for test asset portfolios as well as the CRSP market index. Also, both realized excess returns and average daily excess returns per month are used. Excess monthly market returns for each portfolio are computed as follows: $R^{ex}_{pt} - R_{ft} = R_{pt} - R_{ft} - \hat{\beta}_p(R_{mt} - R_{ft})$, where R_{mt} and R_{ft} are the CRSP index return and Treasury bill rate, respectively. Subsequently, these excess returns are regressed on the SMB, HML, and MOM factors to estimate the multi-factors' loadings for the full sample period. In the second cross-sectional step, the market price of risk denoted as $\hat{\lambda}$ for each factor is estimated by regressing the excess returns for test assets in the first month of the sample period, or $R^{ex}_{pt=1} - R_{ft=1}$, on their estimated full sample factor betas. Rolling forward 1 month at a time, this second step is repeated until the last month of the sample period to generate a time series of $\hat{\lambda}$ estimates for each factor, which are then averaged to obtain $\hat{\hat{\lambda}}$ that is tested for significant difference from zero. Using this monthly rolling approach, the estimated portfolio SML, HML, and MOM betas for each test asset are the same in each monthly cross-sectional regression. For comparison purposes to results using monthly realized returns, risk premia based on average daily returns are scaled up to monthly premia. Standard errors are computed with Shanken [42] corrections to estimate t-values. R-squared values are based on the pure cross-section approach with a single cross-sectional OLS regression (see Jagannathan and Wang [40] and (Lettau and Ludviegson [43], footnote 17, p. 1254)).

Table 5.
Cross-sectional Fama-MacBeth tests for the size, value, and momentum factors using realized and average daily market beta risk-adjusted returns for size-BM sorted and industry test asset portfolios.

returns to estimate market betas. However, market beta again was not significantly priced in these cross-sectional tests.

4. Conclusion

Based on empirical evidence using U.S. stock returns, Fama and French [6–10] concluded that Sharpe's [3] famous CAPM is dead. Since the Sharpe received the Nobel Prize in Economics for the CAPM, this conclusion is worth further investigation. Elton [22] has argued that empirical tests of the CAPM are flawed due the gap between theory based on expected returns and real world tests using realized returns. The main problem is daily informational surprises that can bias monthly compounded daily realized returns as proxies for expected returns. To address this problem, this paper sought to mitigate information surprises in realized returns by smoothing data series using the average of daily returns per month.

Equal-weighted average daily returns per quarter					
Portfolios	$\hat{\bar{\alpha}}$	***t*-value**	$\hat{\bar{\lambda}}$	***t*-value**	**R-squared**
25 Beta	1.28	2.75	1.77	1.91	0.94
25 Size-BM	0.09	0.12	3.12	2.88	0.25
30 Industry	2.24	3.34	1.43	1.40	0.22
25 Size-BM + 30 Industry	0.93	1.40	2.51	2.47	0.26
25 Size-BM + 30 Industry+25 Beta	1.09	2.09	2.23	2.37	0.34

*This table repeats equal-weighted return tests in Panel D of **Tables 3** and **4** using average daily returns in each quarter rather than each month. The 25 beta-sorted portfolios are described in the text and **Table 1**. From Kenneth French's website, we downloaded equal-weighted daily returns for Fama and French's [6–8] 25 size and book-to-market (BM) portfolios (i.e., 25 Size-BM) as well as 30 industry portfolios (i.e., 30 Industry) for the 51-year sample period January 1965 to December 2015. Equal-weighted CRSP market index returns as well as Treasury bill rates are downloaded. Average daily excess returns per quarter are computed for all data series. Beta-sorted test asset portfolios are created by estimating CRSP stocks' betas as described in Subsection 3.1 (i.e., test assets IV using average daily returns per quarter rather than per month). Implementing the Fama-MacBeth test procedure, in the first time-series regression step, the market model with average daily excess returns in each quarter is used to estimate the market betas for the test asset portfolios over the 51-year sample period. In the second cross-sectional step, the market price of risk denoted as $\hat{\lambda}$ is estimated by regressing the excess returns for test assets in the first quarter of the sample period on their estimated full-sample betas. Subsequently, rolling forward one quarter at a time, this second step is repeated until the last quarter of the sample period to generate a time series of $\hat{\lambda}$ estimates, which are then averaged to obtain $\hat{\bar{\lambda}}$ that is tested for significant difference from zero. Using this quarterly rolling approach, the portfolio beta for each test asset is the same in each quarterly cross-sectional regression. For comparison purposes to results using realized returns, risk premia based on average daily returns are scaled up to quarterly premia. Standard errors are computed with Shanken [42] corrections to estimate t-values. R-squared values are based on the pure cross-section approach with a single cross-sectional OLS regression (see Jagannathan and Wang [40] and (Lettau and Ludviegson [43], footnote 17, p. 1254)).*

Table 6.
Further cross-sectional Fama-MacBeth tests for the market factor using average daily returns for beta-sorted, size-BM sorted, and industry test asset portfolios on a quarterly basis.

Based on U.S. common stock returns from the CRSP database for the sample period 1965 to 2015, we conducted cross-sectional tests of the CAPM. A variety of test assets were employed, including beta-sorted, size-BM (book-to-market) sorted, and industry portfolios. Both value- and equal-weighted portfolio returns were computed for realized returns and average daily returns per month. Consistent with many previous studies, in the case of realized returns, the market price of beta risk was insignificant, pricing errors were highly significant, and R-squared values were relatively low. These lackluster findings have motivated researchers to conclude that the CAPM is dead. However, when we repeated the cross-sectional tests using average daily returns per month, the market price of beta risk became statistically significant. In these tests, cross-sectional t-values for beta risk ranged from 1.66 to 2.61 across different test asset portfolios. For beta-sorted test assets, the estimated R-squared value was relatively high at 0.87. And, even though most researchers cannot price the market factor using industry portfolios, we found some evidence that β is significantly priced in these exogenous portfolios. Lastly, for size-BM (book-to-market) sorted portfolios, mispricing error was insignificant, which suggested that average information surprises are diminished using average daily returns.

Subsequently, utilizing size-BM and industry portfolios as test assets, we tested multi-factor models by augmenting the market model with size, value, and momentum factors. For the three-factor model, the size and value factors were more significantly priced in different test assets using average daily excess returns than realized excess returns. Mispricing errors existed in both return cases. To take into account multicollinearity arising from the correlation between the market factor and multi-factors, we repeated the three-factor analyses by regressing risk-adjusted excess returns (after taking into account the market factor) on the size and value factors. Cross-sectional results for the size and value factors were similar to those for the non-orthogonalized three-factor model but now mispricing errors became insignificant (i.e., information surprises were diminished). Also, including the momentum factor in the latter risk-adjusted analyses revealed that it tends to supplant size as a priced factor in this model. We infer that multi-factors are important in helping to better explain the cross-section of expected returns.

Based on these findings, we conclude that CAPM is not dead as beta appears to be better supported by average daily returns per month than realized monthly returns. Using average daily returns, in Merton's [48] words, "… the capital market operates 'as if' the assumptions of the CAPM were satisfied." (Merton [37], p. 867) Further research is recommended on average daily returns, including other return smoothing approaches, different sample periods, asset classes, and countries.

JEL classification

C10, C20, G12

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