5-2020

The Impact of Social Constructivist Mathematics Learning Experiences on Latino, Middle School Students' Proportional Reasoning

Victor M. Vizcaino
The University of Texas Rio Grande Valley

THE IMPACT OF SOCIAL CONSTRUCTIVIST MATHEMATICS LEARNING EXPERIENCES ON LATINO, MIDDLE SCHOOL STUDENTS' PROPORTIONAL REASONING

A Dissertation

by

VICTOR M VIZCAINO

Submitted to the Graduate College of
The University of Texas Rio Grande Valley
In partial fulfillment of the requirements for the degree of

DOCTOR OF EDUCATION

May 2020

Major Subject: Curriculum and Instruction

THE IMPACT OF SOCIAL CONSTRUCTIVIST MATHEMATICS LEARNING EXPERIENCES ON LATINO, MIDDLE SCHOOL STUDENTS' PROPORTIONAL REASONING

A Dissertation
by
VICTOR M VIZCAINO

COMMITTEE MEMBERS

Dr. James A. Telese
Chair of Committee

Dr. Laura Jewett
Committee Member

Dr. Jair J. Aguilar
Committee Member

May 2020

ABSTRACT

Vizcaino, Victor M., The Impact of Social Constructivist Mathematics Learning Experiences on Latino, Middle School Students' Proportional Reasoning. Doctor of Education (Ed.D.), May 2020, 98 pp., 14 tables, 7 figures, references, 82 titles.

Due to its centrality in the curriculum, research on proportional reasoning (PR) has seen a resurgence of interest. Recent research examined students' patterns, strategies, and achievement when solving PR problems. Other studies explored PR learning trajectories. Additional studies focused on instructional interventions such as the effect of number structure, continuous, discrete, and intensive quantities to promote the development of proportional reasoning skills (PRS). The subjects of these studies have been United States white middle school students, Icelandic grade 5 girls, Spanish secondary school students, Turkish middle school students, as well as Scottish and Korean upper primary grades' students. Few studies examined the factors that explain how instructional experiences influence Latino, middle school students' PRS proficiency. This study aims to contribute to the literature by examining the impact of instruction that involves manipulatives, representations, and discourse to foster the strategic competence of Latino middle school students on proportional reasoning tasks (PRT).

Subjects were 80 sixth-grade Latino students with low socioeconomic background. A quasi-experimental, comparison group design was used involving two mathematics classrooms as the treatment and two classrooms as the comparison group. Each group was administered a pretest of mathematics ability and PR posttest. The teacher was a Mexican-heritage public school educator. Students varied in their mathematics ability and engaged in distinct curricula and pedagogical approaches to promote their PR development. Two treatment classrooms experienced tasks involving concrete manipulatives, multiple representations, and negotiation of meaning and two comparison classrooms received teacher-centered textbook instruction that consisted on a teacher explaining definitions and examples from the notes from the textbook, followed by individual problem-solving of worksheets from the textbook.

Analysis of Covariance and Pearson Chi-Square tests reported statistically significant differences in performance and use of representational reasoning when solving proportional reasoning tasks. A significant impact was also observed on the performance in PR tasks involving one-step partitioning and non-integer relationships.

Social constructivist experiences resulted in a meaningful understanding of PRS empowering Latino middle school students with stronger connections between the central ideas of PR. Participants receiving social constructivist instruction used significantly greater representational reasoning that resulted in statistically significantly higher PR strategic competence.

DEDICATION

Thank you, Lord, for all the blessings received. I dedicate this to my kids, Andrea Stephanie, Francisco Javier, and Juan Pablo. My purpose was always to motivate you. Thank you, Carmen, for being by my side in this dream, and always. Thank you, Mom and Dad, for your daily prayers, and for knowing that my absence was for a transcendental motive, I had to increase the value of the talents entrusted.

ACKNOWLEDGMENTS

Thank you, Dr. James A. Telese, for your countless guidance and encouragement was decisive for my success. Thank you, Dr. Jewett and Dr. Aguilar, for your enlightening advice.

TABLE OF CONTENTS

LIST OF TABLES

Page

LIST OF FIGURES

Page

CHAPTER I

INTRODUCTION

Proportional reasoning is fundamental for the development of mathematics literacy (Adjiage & Pluvinage, 2007; Boyer & Levine, 2015; Lamon, 2012; Lobato, Ellis, & Zbiek, 2010; National Council of Teachers of Mathematics [NCTM], 2000; Özgün-Koca & Altay, 2009). The students' ability to reason proportionally is central to many areas of the mathematics curriculum, these include ratios, proportions, percent, similarity, scaling, linear equations, slope, relative-frequency, histograms, and probability (Lobato et al., 2010; NCTM, 2000; Ojose, 2015). Proportional reasoning is necessary for solving a variety of real-life problems (Lamon, 2012). Research findings suggest that middle-school students frequently struggle with proportional reasoning tasks (Adjiage, & Pluvinage, 2007; Ayan & Bostan, 2018; Boyer & Levine, 2015; Cox, 2013; Howe, Nunes, & Bryant, 2011; Lamon, 2012; Lobato et al., 2010; Matney, Jackson, & Bostic, 2013; Ojose, 2015; Özgün-Koca & Altay, 2009; Pelen & Artut, 2016; Steinthorsdottir & Sriraman, 2009).

Statement of the Problem

Due to its centrality in the curriculum, research on proportional reasoning has seen a resurgence of interest (Ojose, 2015). Recent research examined middle school students' patterns, strategies, and achievement when solving missing value problems (Ayan & Bostan, 2018; Che, Wiegert, & Threlkeld, 2012). Steinthorsdottir and Sriraman, (2009) studied 5^{th}-grade Icelandic girls' learning trajectories in proportional reasoning. Additional studies focused on instructional interventions such as the effect of number structure, continuous, discrete, and intensive quantities to promote the development of proportional reasoning skills [PRS] (Fernández, Llinares, Van Dooren, De Bock, & Verschaffel, 2011; Howe et al., 2011; Jeong, Levine, & Huttenlocher, 2007). The subjects of these studies have been United States white, middle school girls and boys, Icelandic grade 5 Girls, Spanish secondary school students, Turkish middle school students, as well as Scottish and Korean upper primary grades' students. Although few studies examined the factors that explain how teachers influence Latino, middle school students' conceptual understanding of proportional reasoning skills, there are some studies that explored the effects of social-cultural activity and contextual tasks on Latino middle school students' proportional reasoning performance (Dominguez, LópezLeiva, & Khisty, 2014; Matney et al., 2013). This study aims to contribute to the literature by examining the impact of instruction that involves manipulatives, representations, and discourse to foster the proficiency of Latino middle school students on proportional reasoning tasks.

Purpose of the Study

The purpose of this study is to examine the impact of social constructivist learning experiences on the proportional reasoning proficiency of sixth grade Latino middle school students.

Research Questions

This study sought to answer the following research questions:

1. What is the difference in performance, when solving proportional reasoning tasks, of Latino, middle school students who experience instruction involving manipulatives, representations, and discourse and those who receive teacher-centered textbook instruction?

2. What are the differences between Latino, middle school students' ability to use integer relationships, partitioning, non-integer relationships, and multi-step relationships participating in experiences involving manipulatives, representations, and discourse and those who teacher-centered textbook instruction?
3. What are the differences in strategies used for solving proportional reasoning tasks between Latino, middle school students experiencing instruction that involves manipulatives, representations, and discourse, and those who receive teacher-centered textbook instruction?

Significance of the Study

Though the development of proportional reasoning skills is drawing increased attention among mathematics scholars and educators (Ojose, 2015), from 2016 to 2018, the performance of sixth and seventh-grade students persists at around 50 percent on items related to proportionality in the State Texas Assessments of Academic Readiness (STAAR). Disaggregated data from a high performing School District in Texas predominantly serving Latino students shows that the average performance of sixth- and seventh-grade Latino students prevail at 57 percent in the knowledge and skills related to proportional reasoning in the STAAR (Texas Education Agency [TEA], 2018). These data align with research findings suggesting that middle-school students encounter multiple difficulties understanding PRS concepts (Lamon, 2012; Lobato et al., 2010; Matney et al., 2013; Pelen & Artut, 2016). Even with increased attention, sixth- and seventh-grade students have a limited proficiency in proportional reasoning skills which results in low PR performance.

Lamon (2012) claimed that instruction of proportional reasoning focuses on “a simplistic, mechanical treatment of ratios and proportions, highlighting the algebraic representation of a proportion and the manipulation of symbols" (p.VI). These teaching strategies cause a shallow and narrow conceptual understanding of proportional reasoning (Ayan & Bostan, 2018) and hinder the development of proportional reasoning skills (Dominguez et al., 2014; Lamon, 2012; Steinthorsdottir & Sriraman, 2009). A limited conceptual understanding impacts students' procedural fluency (NCTM, 2014), which results in a limited strategic competence to solve everyday life problems, diminished capacity for representing PRS problems, as well as lessen the ability to make informed judgments and justifications. In consequence, limitations of PR proficiency affect students' success in higher-level mathematics and related disciplines (Lamon, 2012; Lobato et al., 2010).

Although the NCTM (2000), CCSS (2010), and TEA (2012) redesigned mathematics standards to provide all students with a more focused and coherent sequence of concepts while promoting the use of rich tasks centered about problem-solving, results of the National Assessment of Educational Progress (NAEP, 2017) reported mathematics performance differences, from the years 1990 to 2017, between White and Hispanic eighth-grade students in the average national mathematics scale score. White students scored 26 points higher than Hispanic students did on average on the eighth-grade NAEP mathematics scale score, and 21 points higher on average on the fourth-grade NAEP mathematics scale score. Similarly, the percentage at or above proficient for White students in 2017 was 44%, compared to 20% for Hispanic students. Thus, the mathematics performance of Latino students is significantly lower than that of White students.

A similar trend occurs in Texas. The statewide item analysis summary of grade 6 of the State of Texas Assessment of Academic Readiness (STAAR) underscores a lower performance of Latino students compared to White students (TEA, 2018). Table 1 presents disaggregated data from the standardized examination showing that White students performed 18 percent higher at the approaching level, 20 percent higher at the meets level, and 15 percent higher at the master's level than Latino students. This indicates that White students performed higher in all three ranges of scores than Latino students did.

Table 1

Average Performance on Fifth-Grade STAAR Administered in the spring of 2018

	Approaching Level 56% Correct	Meets Level 69% Correct	Masters Level 86% Correct
White	78%	49%	27%
Latino	60%	29%	12%

This study is significant because it may provide knowledge of the use of social constructivist learning theory to frame learning experiences to foster Latino, middle school students' PR performance and the efficiency of strategies implemented when solving PRT. The development of PRS is a cornerstone for the mathematics curriculum (Lobato et al., 2010), and is transcendental for developing students' mathematics literacy (Adjiage & Pluvinage, 2007; Lamon, 2012; NCTM, 2000). This dissertation can provide educators with learning experiences to improve all students', including Latino students, critical thinking skills to use, represent, and connect PRT, as well as to communicate solutions to applications of PR. Thus, maximizing teachers' impact on Latino, middle school students' PRS can contribute to narrowing the achievement gap between Latino and White middle-school students.

Definition of Terms

Proportional Reasoning. Lamon (2012) defined proportional reasoning as the competence to "scale up or down" in specific circumstances and to justify claims regarding relationships in problems that involve direct and inverse proportions (p.3). The development of proportional reasoning skills is a long-term learning process (Lamon, 2012; Riehl & Steinthorsdottir, 2019; Steinthorsdottir & Sriraman, 2009) that involves seven complex and interwoven mathematics concepts: (1) the five interpretations of rational numbers, (2) measurement, (3) sharing and comparing, (4) relative thinking, (5) reasoning up or down, (6) unitizing, and (7) quantities and covariation (Lamon, 2012). These mathematics concepts are challenging for learners and posit multiple obstacles for developing a meaningful understanding of proportionality (Adjiage, & Pluvinage, 2007; Ayan & Bostan, 2018; Cox, 2013; Howe et al., 2011; Lamon, 2012; Lobato et al., 2010; Matney et al., 2013; Ojose, 2015; Steinthorsdottir & Sriraman, 2009).

PRS are the conceptual link between elementary and high school forms of reasoning (Pelen & Artut, 2016). These complex concepts bridge early and advanced numeric, algebraic, and geometric concepts (Ayan & Bostan, 2018; Cox, 2013; DeJarnette, Walczak, & González, 2014; Morton, 2014). The standards adopted by the TEA (2012), the NCTM (2000), and the Common Core State Standards for School Mathematics ([CCSS], 2010) positioned the development of proportional reasoning skills in the elementary and middle grades. In particular, the TEA (2012) standards require learners in fourth- and fifth grade to solve problems using representational reasoning and proportional reasoning, including the use of multiple representations to analyze and create patterns and relationships. In sixth grade, the state standards focus on the understanding of proportional relationships in problem situations, including verbal, numerical, graphical, and symbolical representations, and involve ratios and rates using scale factors, tables, graphs, and proportions, as well as percents using concrete and pictorial models. In seventh grade, the standards extend the concepts of proportional reasoning to applications of probability and statistics, percent increase and decrease, financial literacy, conversions between measurement systems, similar shapes, and scale drawings. These include tasks involving pictorial, tabular, verbal, numeric, graphical, and algebraic representations. Additionally, students in seventh grade solve contextualized tasks using data represented in bar graphs, dot plots, and circle graphs, including part-to-whole and part-to-part comparisons and equivalents.

Integer Relationships. Unit scaling up or down by an integer is the ability to manipulate the length or amount of a small unit and to think of the length or amount a certain number of times to make up the length or amount of the larger unit without gaps. Lobato et al., (2010) defined integer relationships as the ability to anticipate the number of groups needed to reach a specific target number and maintain a proportional relationship. These researchers envisioned integer relationships as “the ability to truncate the work of iterating a composed unit by using the arithmetic operation of multiplication” (p.38). Riehl and Steinthordottir (2019) suggested that this skill consists of determining a scale factor and apply it to the given ratio to attain the target value.

One-Step Partitioning. A fundamental idea of proportional reasoning of partitioning is the ability to divide the unit into smaller parts. Lamon (2012) defined partitioning as “the act of dividing a set or a unit into nonoverlapping and nonempty parts” (p.49). It involves transforming a ratio using a multiplicative relationship less than one whole to produce a scaled version of the ratio with an invariant value (Riehl & Steinthorsdottir, 2019). Less sophisticated strategies used to partition the quantities of the attributes in the ratio can be build-down and more sophisticated involve multiplicative strategies involving fractions or decimals (Riehl & Steinthorsdottir, 2019). This skill, however, posits challenges for learners (Matney et al., 2013). It requires learners to reason proportionally instead of simply execute arithmetic computations (Riehl & Steinthorsdottir, 2019).

Non-Integer Relationships. A process in which learners apply multiplication and division by a non-integer to the quantities in a ratio. Lamon (2012) perceived that the operator defines a proportional relationship and that this way of reasoning proportionally involves the ability to use non-integer operators as the scale factor for shrinking or enlarging the quantities that define the attributes in a ratio. The ratio is seen as a single unit; however, scaling occurs by **Non-integers** multiplicative reasoning.

Multi-Step Relationships. Multi-step partitioning. The National Research Council (NRC, 2001), defined multi-step partitioning as a mental activity of slicing up an object into the same-sized units and then build down to the target number. This ability requires learners to understand that the whole can be cut up and then scaled down to attain the target value. It involves the construction of a reference unit from a given ratio relationship and then to operate with it (Steinthordottir & Sriraman, 2009).

CHAPTER II

REVIEW OF LITERATURE

In this chapter, the researcher explored the literature related to the achievement of Latino students in the mathematics classroom. Then moved to examine the uses, challenges, and significance of social constructivist theory for the development of proportional reasoning skills was provided. Toward these ends, the chapter started with a discussion of social constructivism as a broad theory of learning. The discussion included the educational possibilities of this theory, highlighting the type of results it has led to and how the theory has been used in the mathematics classroom, as well as the challenges or critiques. This inquiry included the main findings across investigations of proportional reasoning skills. A synthesis of the problems students encounter for developing PRS as well as learning advancements. Then an exploration of the significance of incorporating social constructivist theory for promoting an adequate development of PRS proficiency in terms of 21st-century educational practice in the mathematics curriculum is provided. The chapter concluded with an interpretation of the transcendence of incorporating social constructivist theory toward the teaching and learning of proportionality.

Latino Students and Mathematics Achievement

Although there is a continuous increase in the Latino population (United States Census Bureau, 2011), recent studies reported that educators experience difficulties engaging Latino students in mathematics (Dominguez et al., 2014). Low engagement levels result in low achievement (Short, Echevarría, & Richards-Tutor, 2011). In Texas, 17% of the total student population are English Language Learners (ELL); Spanish is their first language for 87% of them

(United States Census Bureau, 2011). Overall, the NAEP (2017) and the TEA (2018) show that Latino students continue to perform significantly lower than their White counterparts. Both organizations showed a performance difference of about 20% favoring White students. Therefore, it is imperative to design curricula and pedagogical approaches akin to the learning needs of Latino students, and in doing so, increase their mathematical proficiency.

Aiming to alleviate the learning difficulties that Latino students experience, recent investigations suggest that the engagement of Latino youth should be based on multiple relations across social, cultural, and mathematical dimensions (Dominguez et al., 2014; Montemayor, Kupczynski, & Mundy, 2015). They proposed a transformation framed on a characterization of learning as inherently cultural and social; with that, center curricula around socially arranged mathematical tasks. Other researchers suggested that mathematics learning of culturally and linguistically diverse increases when tasks provide greater language familiarity, including opportunities for "explanation, generalization, and justification of mathematical concepts and procedures" (Wong Fillmore, 2007, p.343). Including open-ended tasks involving everyday lived experiences, providing opportunities for intensive collaboration, multimodal communication, and including multiple mathematical representations fosters the engagement of ELLs (Dominguez et al., 2014; Moschkovich, 2007).

Orosco (2013) suggested that the mathematical proficiency of Latino English language learners depends on factors such as learning English as a second language, limited mathematics vocabulary, and limited strategies to solve problems. A focus on comprehension strategies may facilitate the conceptual development of mathematics skills for ELLs at risk (Orosco, 2013). Another study of Hispanic immigrant students by Montemayor et al., (2015) found that these learners utilized a social network to counter obstacles and have positive school experiences.

Their findings showed that participants rely on a strong safety net composed of family, teachers, and classmates that kept them on the right path to success. Latino students can succeed to high standards when exposed to curricula and pedagogies based on multiple relations, valuing their bilingualism, and is composed of open-ended scenarios using their own cultural funds of knowledge (Dominguez et al., 2014; Mejia Colindres, 2015). Hence, this study explores the benefits of social constructivist theory to foster engagement of Latino students, and thus, increase their mathematical proficiency.

Social Constructivist Learning Theory

Social constructivist theory is at the intersection of the individualistic cognitive equilibration regulated by the processes of assimilation and accommodation proposed by Piaget (1977) and the socio-cultural processes of learning advocated by Vygotsky (1978). On one hand, learning is an active and ongoing reorganization of ideas constructed by a process of cognitive structuring (Piaget, 1977). Amineh and Asl (2015) envisioned Piagetian perceptions of knowledge development as a progressive cognitive equilibration that includes a dynamic mechanism of assimilation, conflict, and accommodation in which knowledge proceeds from successive constructions. Similarly, Fosnot (2013) viewed Piaget’s concept development as cognitive mental systems characterized by three properties: wholeness, transformation, and self-regulation. Thus, knowledge acquisition goes through a dynamic and progressive equilibrium-disequilibrium of adaptation-organization and growth-change at the individual cognition level (Piaget, 1977). On the other hand, Amineh and Asl (2015) regarded Vygotsky’s theory as the process of cognitive development that first occurs at the social level, and then at the individual level. Learners construct knowledge within the classroom community, and then internally (Amineh & Asl, 2015). In a like manner, Fosnot (2013) suggested that Vygotsky viewed the formation of concepts through social interaction, language, and the culture of learners. For Vygotsky (1978), cognitive development is a sequential growth process that occurs within the Zone of Proximal Development. Learning experiences should facilitate students' cognitive development on the basis of present understandings and social interactions (Amineh & Asl, 2015). This extends individualistic reflections on every-day experience into "culturally-agreed upon, more formalized concepts" (Fosnot, 2013, p.18).

This study used the perspectives of Simon (1995) to position a social constructivist theory and mathematics knowledge development:

- Knowledge is actively constructed from the learners' perceptions and experiences from their own world,
- the acquisition of mathematics knowledge occurs by participating in meaningful and contextualized investigations in the classroom community,
- these center on non-routine problem-solving involving learners in informed explorations, small-group interactions, justification, negotiation of meaning, and manipulative materials.

A social constructivist theory envisioned the teaching and learning of mathematics in terms of a collection of fundamental mathematical ideas (Brooks, 1999; Herbst, Fujita, Halverscheid, & Weiss, 2017). The development of mathematical proficiency occurs on the basis of active construction of understanding, sense-making, and negotiation of meaning (Clements & Battista, 1990; van Oers, 1996; Voigt, 1996). In doing so, learners attain deeper and more meaningful mathematical understandings, more effective problem-solving strategies, and increased skills to communicate and interact with others (Clements & Battista, 1990; Ernest, 1996; Forman, 1996; Fosnot, 2013; Simon & Schifter, 1993). This promotes an adaptive way of reasoning to actively dialogue and collaborate with others (Brooks, 1999; Fosnot, 2013). Thus, social constructivist learning theory suggests that learning is not passive or should not isolate learners engaged in developing isolated skills rather they should be engaged in active construction of fundamental mathematical ideas.

Furthermore, social constructivist theory shifts historical views of infallible mathematical knowledge into a fallible and socially adapted perspective, building on preexisting mental structures from both, the social contexts and the internal construction of self, beliefs, and cognitions (Ernest, 1996). As a social process, Simon (1995) claimed that students' prior experiences mediate the construction of knowledge as it resides in the learners' culture and emanates from their world, perceptions, and experiences. Learners' collaboratively solve problems using preconceptions and context to construct meaning (Clements & Battista, 1990). This taken-as-shared mathematics knowledge "constructed or modified during problem-solving attain the status of knowledge in the classroom community" (Simon, 1995, p.120).

An active engagement in contextualized tasks promotes sense-making and refine understandings of mathematical ideas; in doing so, students' thinking becomes more complex and abstract (Clements & Battista, 1990). Learning mathematics is a meaningful activity intrinsically connected to real-life. van Oers (1996) noted:

> Real mathematics is conceived here as the historically developing human pursuit of making sense of the quantitative and relational aspects of our physical and cultural world by way of reasoning, discourse, and judgments regarding to what is or is not to be accepted as mathematical. (p. 95)

Cobb (2007) contended that experiences that integrate socio-mathematical norms, discourse, and learning tools promote a meaningful learning of mathematics concepts. He maintained that the focus of social constructivist theory is on the "physical, social, and symbolic classroom environment" as elements of reasoning systems to support mathematics learning (Cobb, 2007, p.27). Similarly, in his exploration of constructivism, Ernest (1996) stressed the importance of tools, language, and the social context for understanding mathematics. He asserted that learning involving tools "takes place in a social context of meaning and is always mediated by language and the associated socially negotiated understanding" (p.343).

Under the umbrella of a social constructivist theory "social activity and discourse play important roles for understanding to occur" (Telese, 1999, p.2). In these environments, students are accountable to negotiate within their groups the efficiency of their heuristics, solutions, and representations (Ernest, 1996; Fosnot, 2013; Simon, 1995; Voigt, 1996). Morton (2014) engaged students in proportional reasoning tasks involving negotiation of meaning. Students communicated mathematical principles to explain and justify solutions to problems. Doing this involves students in critically thinking about their solutions and those of their peers, which promotes a more meaningful conceptual understanding (Morton, 2014). Classroom discourse also provides evidence of student strategies and teachers can use it to examine the correctness and robustness of arguments (Morton, 2014). Additionally, the discussion of different strategies enables students to move to higher levels of proportional reasoning (Steinthorsdottir & Sriraman, 2009). Here, language, tools, and the social context are significant to derive meaning (Ernest, 1996; Telese, 1999).

Furthermore, students' mathematical critical abilities develop by gathering and presenting credible and relevant evidence, a more compelling form of justification, and better-integrated representations for evaluating and defending a point of view. Research suggests implementing classroom discourse to support students' procedural and conceptual knowledge (Fuchs, Schumacher, Long, Namkung, Hamlett, Cirino, & Changas, 2013; Hamdan & Gunderson, 2017). Tasks that involve the discussion of heuristics and justification of claims promote mathematical proficiency.

In alignment with the perspectives of social constructivist theory, the NCTM in their publication, *Principles to Actions* (2014) recommends that to improve mathematics education there should be less time devoted to learning meaningless procedures without understanding and more time spent on the applicability of concepts. The NCTM (2014) proposes that mathematics teachers should systematically build procedural fluency on the basis of a meaningful conceptual understanding. Recent research suggests that the consistent use of manipulatives fosters conceptual understanding and enables learners to represent and explain their thinking in higher levels and meaningful ways (Cramer, Ahrendt, Monson, Wyberg, & Miller, 2016; Reed, 2008; Richardson, 2012; Moyer-Packenham, Bolyard, & Tucker, 2014; Utley & Reeder, 2012). Thus, learning tasks founded in a social constructivist theory foster mathematical proficiency.

A crucial component of social constructivist theory is the use of manipulative materials. Using manipulatives to represent and solve problems fosters a deep comprehension of mathematical concepts. Reed (2008) suggested that the implementation of meaningful mathematical tasks involving concrete and virtual tools for the construction of knowledge develops an adaptive way of thinking and support representational communication. Reed (2008) noted that “the mathematical materials, which seem to embody many of the features of mathematical concepts and procedures that are deemed important in understanding mathematics, provide representations that may help a child to construct these concepts and procedures for themselves” (p.8). Hence, an active use of manipulatives fosters the learners’ adaptive reasoning and strategic competence.

The NCTM (2014) advocates for using and connecting mathematical representations as necessary attributes of high-quality mathematics education. Mathematics instruction framed under the umbrella of social constructivist theory advocates for the use of multiple representations for exploring the properties and attributes of mathematics concepts. Telese (1999) suggested, "Mathematical modeling can be viewed as a mathematical language that describes certain aspects of empirical and social realities" (p.12). Using mathematical models, participants of a mini-society negotiate taken-as-shared meanings through agreed rules and conventions (Cobb, Jaworsky, & Presmeg, 1996; Ernest, 1996; Fosnot, 2013; Hennig, 2010; Telese, 1999). This process aids learners to identify relationships, derive meaning, refine their understandings, and meaningfully organize their thinking (Clements & Battista, 1990). Thus, the use of models to explore the underlying properties of mathematical concepts and identify relationships results in a more meaningful comprehension of mathematics concepts.

Moreover, the use of multiple representations fosters strategic competence and aids learners to solve non-routine problems. Richardson (2012) found that students meaningfully utilized mental models to create meaning and internalized concepts to transfer to symbolic mathematics. These constructions served as entry points for students' thinking, schematic representations, generalizations, and properties and processes involved. Also, using applicable and relevant representations helps students generalize; in so doing, it promotes a deeper understanding of abstract concepts applicable to their life-experiences (Cramer et al., 2016; Moyer-Packenham et al., 2014; Reed, 2008; Richardson, 2012). Social constructivist theory envisions learners actively constructing a variety of models for negotiating meaning and solving problems (Hennig, 2010).

An example of this is the construction of a number line model to developing a more meaningful understanding of fractions (Cramer et al., 2016; Fonger, Tran, & Elliott, 2015; Hamdan & Gunderson, 2017). A number line model bridges numerical and spatial attributes for deepening understanding of magnitude. Because an understanding of magnitude is necessary when representing, comparing, and ordering fractions. Hence, using a number line in a social constructivist classroom allows learners to draw meaning from abstractions by identifying critical attributes and relationships of fractions. Thus, teaching and learning under a social constructivist theory foster students' procedural fluency and conceptual understanding.

In sum, research used learning interactions framed under a social constructivist theory to promote a deeper and more meaningful understanding of mathematics concepts, support procedural fluency, and foster students' strategic competence (Cramer et al., 2016; Hamdan & Gunderson, 2017; Morton, 2014; Reed, 2008; Steinthorsdottir & Sriraman, 2009). These researchers found that involving learners' life-experiences, concrete manipulative materials, multiple representations, and classroom discourse result in greater levels of mathematical proficiency. A social constructivist theory prioritizes the use of informed exploration, manipulative materials, models, discussions, and sense-making to negotiate meaning and in doing so, instills an adaptive way of reasoning for learning mathematics (Clements & Battista, 1990). The students' explanations are more efficient and robust when familiar life-situations are represented and critically discussing attributes in the classroom. Therefore, a socio-constructivist philosophy of teaching and learning represents the theoretical framework for the long-desired mathematical reform.

Proportional Reasoning Learning Difficulties

The learners' maturation and experience complemented with current instruction in PR are not enough for developing sophisticated PRS (Lamon, 2012). The development of proportionality proportional reasoning ability requires conceptual understanding, procedural fluency to carry out procedures efficiently, and strategic competency to represent, solve, and communicate PR problems in a broad range of life applications (Lamon, 2012; Matney et al., 2013). However, recent investigations emphasized that K-12 youth encountered multiple difficulties as they develop proportional reasoning skills. Even college students struggled to solve unusual tasks involving PRS resulting from conceptual misunderstandings (Ojose, 2015). Some of the most significant struggles related to the use of additive thinking (Lamon, 2012; Lobato et al., 2010; Matney et al., 2013; Pelen & Artut, 2016), the use of inadequate heuristics (Lamon, 2012), difficulties to discern proportional from non-proportional relationships (Ayan & Bostan, 2018; Cox, 2013; Lobato et al., 2010), difficulties with tasks involving shrinking scale (Matney et al., 2013; Riehl & Steinthorsdottir, 2019), and limited proficiency with rational numbers (Boyer & Levine, 2015; Howe et al., 2011).

The most salient problem related to the incorrect use of strategies centered on additive thinking. Researchers found that the use of additive approaches hinder students' ability to reason multiplicatively (Lamon, 2012; Lobato et al., 2010; Matney et al., 2013; Pelen & Artut, 2016). A build-up strategy, such as using additive thinking, prevented students from recognizing the multiplicative relationships to scale up and down in problem situations. Learners perceive this as absolute counting, this impeded learners to comprehend more sophisticated forms of multiplicative or relative thinking that are critical for an adequate development of PRS (Lamon, 2012; Lobato et al., 2010; Matney et al., 2013; Pelen & Artut, 2016). Learners should move from building up to more sophisticated forms of proportional reasoning (Ayan & Bostan, 2018).

Another issue is the implementation of inefficient heuristics to solve problems involving proportional reasoning. Historically, the most implemented heuristic to solve PRS problems is cross-multiplication (Lamon, 2012; Lobato et al., 2010; Steinthorsdottir & Sriraman, 2009). However, algorithms for symbolic operations such as cross-multiplication do not necessarily represent reasoning proportionally; it may only reflect the students' application of procedural knowledge (Lamon, 2012). Then, the use of inadequate heuristics affects students' conceptual understanding and strategic competence to formulate, represent, and solve PRS problems.

Moreover, students experience difficulties distinguishing between proportional relationships and non-proportional relationships (Ayan & Bostan, 2018; Cox, 2013; Lobato et al., 2010; Matney et al., 2013). Ayan and Bostan (2018) found that learners use the scale factor to calculate the area or volume of enlarged or shrunken figures instead of enlarging or shrinking the figure first and then calculating the area or volume. They noted, "Most of the students implied that they just needed to multiply the area and volume by 2 when all the sides are enlarged by a scale factor of 2" (Ayan & Bostan, 2018, p.513). This process requires first to enlarge or shrink the figure and then calculate the area or volume.

In addition, recent research found that students struggle to solve problems involving shrinking scale (Matney et al., 2013; Riehl & Steinthorsdottir, 2019). Shrinking figures requires learners to distinguish that the scale factor needs to be less than one. Matney et al., (2013) revealed that learners asked more questions and took more than twice as long to solve problems going from a large figure to a similar smaller figure. Similarly, Riehl and Steinthorsdottir (2019) noted, "Students had a lower overall success rate on shrink problems compared to enlarge problems" (p.9). Hence, students experience greater difficulties to conceptually understand and carry out procedures efficiently when PRS tasks require transforming an object from large to small.

Another source of difficulty related to solving proportional reasoning tasks is a weak foundation related to rational numbers. Poor understanding of rational numbers negatively affects the students' proportional reasoning skills. Boyer and Levine (2015) stressed that a deep conceptual understanding of fractions and decimals is of transcendental importance for developing PRS. In the same way, Lamon (2012) highlighted that rational numbers encompass a set of broad and intertwined interpretations that influence the development of PRS. Lamon (2012) noted:

> Unfortunately, fraction instruction has traditionally focused on only one interpretation of rational numbers, that of part-whole comparisons, after which the algorithms for symbolic operation are introduced. This means that student understanding of a complex structure (the rational number system) is teetering on a small, shaky foundation". (p.32)

Lamon (2012) suggested that learners should develop a deep understanding of fractions as part-whole comparisons with unitizing, as measurement or magnitudes, as operators, as quotients, and as ratios. Similarly, research by Howe et al., (2011) claimed that traditionally teachers introduce the concept of fractions via partitioning of food such as cakes and pizza; they suggested, however, that sharing provides a more attainable context, as sharing eases the introduction of ratios, another crucial concept for the development of PRS. Therefore, inadequate instruction of rational numbers resulted in a limited proficiency with rational numbers and negatively influences the development of PRS.

Proportional Reasoning and Social Constructivist Theory

A social constructivist theory aligns with the principles for effective mathematics teaching proposed by the NCTM (2014). It envisions the development of mathematical knowledge as actively constructed from the learners' perceptions and experiences, learners participating in informed exploration, small-group interactions, challenging non-routine problem-solving, justification, negotiation of meaning, including manipulative materials, and implementing meaningful and contextualized investigations in the classroom community (Simon, 1995). Cobb (2007) suggested that the focus of social constructive theory is on the use of the physical, social, and symbolic classroom environment as reasoning systems to support a deeper understanding. Thus, a social constructivist theory aligns with the principles to action that the NCTM (2014) proposes to promote an effective teaching and learning of mathematics.

In addition, under the perspectives of social constructive theory, students refine their mathematical proficiency by actively construct knowledge, use manipulatives, and multiple representations to present relevant evidence as well as more compelling justifications. An active engagement incentivizes sense-making and more refined understandings (Clements & Battista, 1990). Thus, curricula and pedagogical approaches in which learners actively participate in the construction of knowledge, use tools, create representations, collaborate with peers, and include social-cultural experiences promote a more meaningful development of proportional reasoning skills.

Proportional Reasoning Learning Trajectory

Steinthorsdottir and Sriraman, (2009) portrayed the learning trajectory of proportional reasoning skills in a rubric of four developmental levels. Level zero of development of proportional reasoning defined on the PRS rubric involves the use of an incorrect strategy involving random calculations and additive strategies to solve PRT. This limited ratio knowledge highlights the lack of understanding of the multiplicative relationship. It also involves conceptual or computational errors that produce an unreasonable answer. The first level of development of PR defined by the rubric involves integer relationships. It consists of the ability to identify the factor needed to enlarge a composed unit in order to attain the target quantity with an invariant value of the ratio and without gaps (Lobato et al., 2010; Riehl & Steinthordottir, 2019). PR tasks involving integer relationships do not pose higher levels of proportional reasoning (Riehl & Steinthorsdottir, 2019). Recent PR research claimed that students are more successful in solving tasks that involve integer ratios (e.g., Carney, Smith, Hughes, Brendefur, Crawford, & Totorica, 2015; Fernández et al., 2011; Riehl & Steinthorsdottir, 2019). Carney et al. (2015) stressed, however, that although students can correctly solve integer scaling PR problems, it might not be indicative of a meaningful PR understanding, nor indicative of success with PR problems with increased difficulty. They suggested that using random calculations involving additive or multiplicative strategies does not signify an adequate understanding of this level of PRS.

The second level of development of PR defined by the rubric involves one-step partitioning. It is the ability to divide the composed unit into nonoverlapping and nonempty parts (Lamon, 2012). It consists on the ability to transforming a ratio using a factor less than one to produce a shrunken scaled version of the ratio with an invariant value (Riehl & Steinthorsdottir, 2019). This skill posits greater challenges for students (Matney et al., 2013) because it requires learners to reason proportionally (Moyer-Packenham et al., 2014; Riehl & Steinthorsdottir, 2019).

The third level of development of PR defined by the rubric involves non-integer relationships. It involves scaling a composed unit by a non-integer in order to shrink or enlarge a ratio with an invariant value (Lamon, 2012). PR research stressed that students encounter greater difficulties with non-integer ratios (Carney, et al., 2015; Fernández et al., 2011; Hilton, A., Hilton, G., Dole, & Goos, 2016; Riehl & Steinthorsdottir, 2019).

The fourth and higher level of development of PR defined by the rubric involves multi-step relationships. It consists on the ability to slicing or enlarging a composed unit into the same-sized units and then build down or up to attain the target quantity with an invariant value of the ratio. Multi-step relationships involve unitizing, multi-step partitioning, and covariation. Level four of PR is a long-term process the requires a greater effort (Lobato et al., 2010; Steinthorsdottir & Sriraman, 2009).

Learning Advancements Toward 21st Century Educational Practice

The teaching and learning of mathematics should be integrated from a social constructivist learning perspective around fundamental ideas (Herbst et al., 2017; Ernest, 1996; Telese, 1999). Good candidates for these fundamental ideas are the proportional reasoning concepts of "symmetry, invariance, number, exhaustion, algorithm, functional thinking, and so forth" (Herbst et al., 2017, p.41). In addition, the development of mathematical proficiency should be based on five components or strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and predictive disposition (NRC, 2001).

Furthermore, proponents of the mathematical reform advocate for instructional approaches blending geometric and algebraic concepts and giving greater emphasis to mathematical reasoning (Herbst et al., 2017). This can be accomplished by continuously adjusting instruction in ways that support and extend conceptual understanding and associated procedures (Gleason, Livers, & Zelkowsky, 2017; Greeno, 2006; Schoenfeld, 2007; Telese, 1995). The implementation of student-centered interactions, involving exploration, the use of manipulatives, and visual reasoning promote the development of the mathematical skills to necessary to represent, investigate, and justify algebraic and geometric relationships (Herbst et al., 2017). Thus, implementing student-centered explorations constructing meaning using manipulatives, making connections among multiple representations, and negotiation of meaning promotes proportional reasoning proficiency.

Manipulatives and Representations

In response to the multiple hurdles that youth encounters for appropriately developing proportional reasoning skills, researchers emphasized the importance of using mathematical tools such as concrete manipulatives. Studies examining the development of PRS suggest implementing mathematical tasks that involve different types of manipulatives to construct knowledge (Cramer et al, 2016; Jitendra, Star, Dupuis, & Rodriguez, 2013; Moyer-Packenham et al., 2014; Richardson, 2012; and Utley & Reeder, 2012). These researchers advocated for the use of concrete manipulative materials and multiple representations to enhance visual-spatial abilities, improve students' ability to efficiently approach, represent, and communicate relationships and solutions, deepen understanding and flexibility, and support mathematical justifications, and generalizations.

An example of this is the use of concrete manipulatives such as unit blocks, puzzles, and tangrams to develop higher visual-spatial skills (Clements & Sarama, 2014; Herbst et al., 2017; NRC, 2001). Clements and Sarama (2014) highlight that the use of "tactile kinesthetic tasks ask children to identify, name, and describe objects and shapes placed in a box… improve spatial perception" (p.133). These researchers also suggested that generating discussions emphasizing the properties of the shapes builds learners' spatial abilities. Thus, the use of manipulative materials aids the students' visual-spatial thinking, which is critical for the development of numeric and geometric proportional reasoning skills.

In addition to that, researchers contend that the use of multiple representations to communicate relationships and solutions develops a flexible way of thinking (Cramer et al., 2016; Jitendra et al., 2013; Moyer-Packenham et al., 2014; Richardson, 2012; and Utley & Reeder, 2012). Also, using schematic representations highlights the spatial relationships relevant to a context, and thus support problem-solving (Clements & Sarama, 2014). Using models gives learners a tool to solve and evaluate their numerical and conceptual work (Matney et al., 2013). Hence, using representations promotes the development of geometric and numeric proportional reasoning, and aids learners when solving problems and communicating solutions (Clements & Sarama, 2014).

Similarly, Moyer-Packenham and colleagues (2014) found that learners utilized different representations of fractions and created a variety of additional models on their own to model mathematical relationships. They asserted, “Students used both empirical evidence and analytical thinking to construct arguments” (p.77). Thus, because of the use of manipulatives and multimodal communication, learners become empowered to efficiently approach, solve, represent, and communicate solutions to everyday-life problems involving PRS.

Complementing these views, the NCTM (2014) stressed the importance of using and interconnecting mathematical representations to deepen understanding and flexibility as necessary attributes of “effective teaching of mathematics” (p.10). In support of this idea, recent scholarship stressed that the consistent use of manipulatives fosters conceptual understanding and enables learners to represent and explain their thinking in higher levels and meaningful ways (Cramer et al., 2016; Moyer-Packenham et al., 2014; Richardson, 2012; Utley & Reeder, 2012). Richardson (2012) found that students utilized mental models to create meaning and internalize concepts to transfer to symbolic mathematics. He argued that constructions resulting from models served as entry points for students' thinking, schematic representations, generalizations, properties, and processes involved. Therefore, models resulting from representational reasoning support mathematical justifications and generalizations, in so doing deepen students' mathematical proficiency.

Negotiation of Meaning

Recent studies claimed that discussing heuristics and solutions of problems fosters procedural and conceptual understanding of proportional reasoning skills (Morton, 2014; Moyer-Packenham et al., 2014; Richardson, 2012). Morton (2014) suggested that the use of classroom discourse to communicate mathematical principles, explain, and justify solutions to problems fosters the conceptual understanding of PRS. It enhances students' understanding by allowing them to think critically about their solutions and of those of their peers (Morton, 2014). Therefore, providing learners with ample opportunities to communicate and assess the heuristics implemented to solve proportion problems promotes the students' proportional reasoning proficiency.

Additionally, identifying the most efficient approach when solving PRS problems fosters the students' ability to make well-founded judgments and justifications (Cox, 2013; Herbst et al., 2017; Lamon, 2012; Matney et al., 2013). Learning interactions that involve discussing strategies for solving problems from different perspectives promote a deeper understanding of proportional reasoning concepts (Cramer et al., 2016; Jitendra et al., 2013; Utley & Reeder, 2012). Lamon (2012) suggested that to promote flexibility in the children's thinking, educators should "encourage multiple solution strategies and discuss which strategies are easier, faster, and more reasonable" (p.107). These experiences highlight the need to pay attention to the structure and essential features of the relationships addressed by the task. Thus, the implementation of mathematical tasks that involve negotiating different types of tools and strategies to construct knowledge promotes the development of proportional reasoning skills.

Visual Reasoning and Contextualized Tasks

Recent research on proportional reasoning emphasizes the use of visual reasoning to support students' ability to solve PR tasks (Cox, 2013; DeJarnette et al., 2014; Lobato et al., 2010; Herbst et al., 2017; Matney et al., 2013). Familiar and contextual experiences aided learners with the visualization of the context and helped them create an adequate depiction for solving the problem (Matney et al., 2013). In the same way, DeJarnette and colleagues (2014) indicated that PR, and in particular similarity, allowed learners to shift from visual perceptions to more advanced symbolic approaches. Thus, visual perceptions foster students' strategic competence for tasks involving proportional reasoning concepts.

Likewise, Clements and Sarama (2014) note that imagery and spatial visualization are important aspects for developing an understanding of the concepts of scale and similar figures. They described special visualization skills as "processes involved in generating and manipulating mental images of two- and three-dimensional objects, including moving, matching, and counting them" (p.127). Students' initial images are static and should be purposefully developed by implementing instructional approaches that promote the use of schematic representations in context (Clements & Sarama, 2014). Hence, the use of schematic images allows learners to visualize relevant relationships of the context, and thus, aids in their ability to solve PRS tasks.

Another strategy to promote success with proportional reasoning tasks is the use of minute contextual experiences (MCE). Implementing MCEs promoted students' visualization of the context and assists them in solving the problem (Matney et al., 2013). A mundane familiarity with a situation does not result in deeper levels of understanding; to the contrary, familiarity with the context “may still lack the heightened cognitive experience necessary for connecting the context during problem solving” (Matney et al., 2013, p.59). Their investigation showed that implementing MCE regenerated cognitive connections among students’ knowledge structures and visualization. This allowed learners to link academic knowledge and mathematics, thus, enhancing the transfer of mathematics skills and their ability to solve scale and similar figures problems.

Summary

This chapter envisioned incorporating social constructivist learning theory for promoting proportional reasoning proficiency of Latino, middle school students. The researcher discussed how a social constructivist theory provides the best hopes for a high-quality mathematical reform. As a broad philosophy of learning, under a social constructivist theory knowledge is actively constructed via the implementation of meaningful and contextualized investigations within the classroom community, explorations using manipulative materials, small-group interactions, non-routine problem-solving, justification, and negotiation of meaning. A discussion of the Vygotskyan views of taken-as-shared knowledge integrates Piagetian views of internally cognitive development. The discussion of the educational possibilities highlighted how this epistemological perspective integrates the central strands for mathematical practices oriented to foster conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and predictive disposition. In addition, this paper reported that social constructivist theory promotes mathematical proficiency and a deeper understanding of mathematical concepts applicable to life experiences. It highlighted why is important in mathematics to implement meaningful and purposeful tasks to support the student’s development of PRS. This can be accomplished by providing learners with tasks that “make them think strategically, tactically, and technically” and promoting a deep understanding of relationships (Burkhardt, 2007, p.84). From this perspective, learners should be empowered to interchangeably use multiple heuristics and representations to solve and communicate PRS problems (Lamon, 2012; Steindottir & Sriraman 2009).

A discussion of the development of PRS and its place in the mathematics curriculum highlighted the transcendental relevance of PRS toward mathematics literacy. The discussion of the challenges for developing students’ PRS centered around the use of incorrect strategies based on additive thinking that hinders more sophisticated multiplicative thinking skills, the implementation of inefficient heuristics such as cross-multiplication that only promotes procedural knowledge, the difficulties to distinguish between proportional relationships and non-proportional relationships that impede students to correctly calculate the area and volume of transformed figures, the struggles to solve problems that involve shrinking scale, the students' inability to utilize a scale factor less than one that results in shrunken figures, and students’ weak foundations of rational numbers that negatively affects their PRS overall performance.

The significance of adequately developing proportional reasoning skills remarked the need for redesigning instructional approaches blending geometric and algebraic concepts and giving greater emphasis to mathematical reasoning. This highlights the need for integrating a social constructivist learning theory to support and extend PRS proficiency. It blends geometric and algebraic reasoning, and it centers around the use of manipulatives to construct knowledge, the use of multiple representations, and discussions of relationships and solutions. Social constructivist theory revolves around the five strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and predictive disposition (NRC, 2001). These educational practices deepen proportional reasoning proficiency. Therefore, this chapter proposed a social constructivist theory for the design curricula and pedagogical approaches to promote Latino students’ proportional reasoning proficiency.

CHAPTER III

METHODOLOGY

Research Design

This study took place during October and November of 2019 over four weeks in *Si Puedo (I can do it)* Middle School, a pseudonym for a public middle school located along the Mexican border, predominantly serves Mexican-American or Mexican born youth with low socio-economic background. A quasi-experimental, comparison group design was used. There was a treatment group and a comparison group. An experiment commonly involves randomly selecting individuals two either an experimental group or a control group (Mills & Gay, 2016). In a quasi-experimental design, there is a comparison group and a treatment group without random selection (Mills & Gay, 2016).

Two mathematics classrooms received the treatment and two classrooms served as the comparison group. Each group was administered a proportional reasoning pretest and posttest. A pretest was administered to determine that groups were similar in mathematics ability at the start of the study as well as other comparisons of demographic data (Creswell, 2015). Jitendra et al., (2013) implemented a similar research design to examine the effects of schema-based instruction on the mathematical problem-solving performance of seventh-grade students. A study from Özgün-Koca and Altay (2009) used a convenience sample of sixth- and seventh-grade students to examine proportional reasoning performance. In educational settings, it is necessary to use intact groups because of the availability of participants.

Sample

Students. The subjects selected for this study were 72 sixth-grade Latino students from four mathematics classrooms. Participants' age ranged from 11 years 6 months to 12 years 11 months. All students participating in this investigation received free lunch. These students attended school in the United States since Kindergarten and the language of instruction was English. Participants were not randomly assigned, as the control and intervention groups were selected on the basis of their accessibility. The process of selecting participants from the population was a convenience sample, as each teaching method was carried out in four preexisting classrooms of students (Mills & Gay, 2016).

Power analysis was used to establish an appropriate number of participants. Power analysis is a systematic approach that researchers use to determine the appropriate sample size for group comparisons; and it involves the level of statistical significance, the amount of power in the study, as well as the effect size (Creswell, 2015). Lipsey (1990) suggested that for educational research, 35 students are sufficient for each of the two conditions when researchers use an alpha level of .05, a standard for power of .80, and an effect size of .70. A standard power of .80 means that 80% of the times the null hypothesis will be rejected, a .70 effect size is the standard difference in the means between the control and experimental groups determined in standard deviation units. Thus, 40 participants for each of the two conditions was an adequate sample size for this study.

Teacher. The teacher was a public-school educator and taught sixth-grade mathematics for three years. She is of Mexican heritage. At the time of the study, she was 26 years old. Also, she is fluent in Spanish and English languages. She graduated from the University of Texas Rio Grande Valley.

Conditions

Two mathematics classrooms participated in the treatment group and two mathematics classrooms participated in the control group. There were 36 students in each of the groups. Students in both conditions, treatment, and control, engaged in distinct curricula and pedagogical approaches to promote their development of PRS. Participants from both groups— intervention and control—vary in mathematics ability. Learning interactions were scaffolded during the regular mathematics class with their usual mathematics teacher. Each session was 45-minutes long two times per week. In total, participants in both conditions participated in eight sessions of proportional reasoning instruction. Students in both groups solved the same number and type of problems. Xin, Jitendra, and Deatline-Buchman (2005) used this procedure for exploring the effects of two different problem-solving instructional approaches on middle school students with learning disabilities.

Two treatment classrooms used familiar real-life tasks involving concrete manipulatives, multiple representations, and negotiation of meaning to support students' PR conceptual understanding, procedural fluency, and strategic competence (Burkhardt, 2007; Shannon, 2007; Schoenfeld, 2007; Telese & Avalos, 2013; Telese 1999). The manipulatives consisted of pattern blocks, 2-color counters, and snap blocks. The mathematical representations included tables, coordinate planes, and number lines. And learning experiences engaged groups of three students in exploration, representation, and solution of PRS tasks that concluded in math congresses in which students discussed their mathematical thinking. A math congress continuously engages students in dialogue for exchanging thoughts, solutions, problems, justifications, and generalizations (Fosnot & Dolk, 2001).

During the four weeks of the instructional unit in PR, participants collaborated in groups to construct learning. An initial step of the learning experience engaged participants in using concrete manipulatives to highlight the relationship described in a PRT. Then, participants identified and discussed within their group the underlying properties and attributes of the relationship involved in the task. Next, students constructed a mathematical representation of the real-life situation followed by a discussion with peers highlighting the multiplicative relationship. At this step, students clarified misunderstandings that any of the team participants could have. Once all participants had developed a meaningful understanding of the proportional relationship, they deepened their understandings by representing the relationship using a different mathematical model. This was followed by a discussion within their group to compare the underlying mathematical attributes and properties of the context and multiplicative relationship using the different representations. During the activity, the teacher walked around the classroom asking guiding questions to address misconceptions and capitalize on strengths to assure that all participants developed a meaningful understanding of the concepts. The lesson concluded with a math congress in which students discussed their mathematical thinking, highlighting the multiplicative relationship involved in the task using multiple representations, and evaluating the efficiency of the strategies utilized to solve and represent the task.

In each of the four weeks of the proportional reasoning unit, the tasks focused on one of the four levels of proportional reasoning as defined by the proportional reasoning rubric. This allowed an adequate scaffolding of the complexity of the tasks according to the levels of proportional reasoning. As the complexity of the tasks increased, participants identified the proportional reasoning, compared, and contrasted the properties and attributes of each of the four levels of proportional reasoning. This included a variety of constructions involving pictorial and schematic representations using concrete manipulatives and representations. These facilitated the creation of meaning and internalization of concepts to transfer to symbolic mathematics (Richardson, 2012). At the conclusion of each task, participants presented their findings to their peers and compared and evaluated strategies and solutions. Students utilized representational reasoning to support mathematical justifications and generalizations, and thus deepen mathematical understanding and flexibility. Thus, mathematical models allow participants of a mini-society to negotiate taken-as-shared meanings through agreed rules and conventions (Ernest, 1996; Fosnot, 2013; Telese, 1999).

In a similar way, two control classrooms received teacher-centered textbook instruction. Students solved the same number and type of problems than the participants receiving social constructivist instruction. This involved the teacher in explaining definitions and examples from the notes provided from the textbook, followed by students individually solving problems on worksheets from the textbook. These curricula and instructional approaches focused on the development and mastery of a series of isolated skills. The NRC (2001) noted that traditional instruction in the United States of America "continues to emphasize the execution of paper and pencil skills in arithmetic through demonstrations of procedures followed by repeated practice" (p.4). In alignment with these perceptions, the NCTM in their publication, *Principles to Actions* (2014) described traditional curricula and pedagogical approaches implemented in mathematics classrooms in the United States of America as instruction that devotes "Too much focus on learning procedures without any connection to meaning, understanding, or the application that require these procedures" (p.3). Furthermore, Lobato et al., (2010) suggested that traditional instruction on ratio and proportion "shows students different ways to write ratios and then introduces a proportion as two equivalent ratios. Next, students usually encounter the cross-multiplication algorithm as a technique for solving a proportion" (p.7).

Data Collection

Research instrument

A posttest was used to assess students' performance on proportional reasoning tasks (see Appendix A). This assessment instrument was a modified version of the tasks used by Riehl and Steinthorsdottir (2019). It matched the number structure used by Riehl and Steinthorsdottir (2019) but with differences in the life-contexts, as they included experiences familiar for the students participating in this study. The focus was to assess students' performance on the concepts of PRS multiplicative reasoning, unitizing, and partitioning.

According to the NRC (2001), "competence in an area of inquiry depends upon knowledge that is not merely stored but represented mentally and organized (connected and structured) in ways that facilitate appropriate retrieval and application" (p.4). Student performance on the posttest was scored using the scale from an adapted rubric of strategy types implemented for solving proportion problems created by Steinthorsdottir and Sriraman (2009). This grading rubric defined the strategy type and the description of the strategy implemented to solve the PRS tasks (see Appendix B).

Pilot Study

In March 2019, the researcher conducted a pilot study to measure the posttest content validity and internal consistency. It was necessary to examine the students' performance on an assessment of proportional reasoning skills to determine if the test interpretation matches its proposed use and the reliability to produce consistent results over time. The findings from the pilot study confirmed the validity and reliability of the research instrument used in this dissertation.

Instrument's validity. The pilot study involved two mathematics experts in reviewing the instrument's content validity. Evidence-based on the test content is useful to assess achievement tests in education, that is, to involve a panel of experts for providing evidence of whether the test's content relates to what the test is intended to measure (Creswell, 2015). First, a mathematics education professor evaluated the appropriateness of the posttest. This expert validated that the problems on the instrument are relevant and representative of proportional reasoning skills. That is, how well the posttest measures the core concepts of PRS multiplicative reasoning, unitizing, and partitioning. Second, a middle school mathematics teacher provided his professional judgment and validated that the items included in the posttest adequately measure or represent PRS. Hence, two experts interpreted that the questions are well known and easily identifiable to measure the mathematics constructs that it is assumed to measure, thus the test scores can be generalized overtime (Creswell, 2015).

Instrument's reliability. It was also necessary to examine the instrument's reliability or the internal consistency of the posttest to produce consistent results over time. Forty Latino sixth-grade students solved the posttest during a 45-minute class period during March of 2019. Two mathematics teachers separately graded all the students' responses and strategies. Then, the researcher used a post-hoc analysis of Cronbach's alpha to assess the instrument's reliability using the Statistical Package for Social Sciences (SPSS) version 25. The results yielded a Cronbach's alpha value of .70. This means that the posttest shows acceptable reliability to measure the students' proportional reasoning performance.

Data Analysis

An analysis of covariance (ANCOVA) was used to test associations between instructional approach and proportional reasoning performance, while statistically adjusting the scores on the PRS posttest to remove initial advantages, so that, the results can be fairly compared as if the two groups started equally (Mills & Gay, 2016, p. 247). An ANCOVA combines two independent categorical variables and one dependent variable as the covariate to evaluate the scores on a quantitative outcome variable (Warner, 2012). A study of Bostic, Pape, and Jacobbe, (2016) used an ANCOVA to examine the differences between an intervention and comparison groups' performance on a unit test. This study analyzed quantitative data using an ANCOVA with the covariate as the scores on a pretest of mathematics ability—the 5th-grade STAAR test administered in the spring of 2018. The two predictor variables corresponded to two different instructional approaches and the outcome variable was the score on a posttest of proportional reasoning tasks. Because the differences between the intervention and control groups on the proportionality assessment vary, there is some confound between the mean level of PRS ability prior to the intervention in each group. The ANCOVA adjusted for differences in preexisting participant characteristics (Mills & Gay, 2016).

Furthermore, the researcher used a Pearson Chi-square statistic to measure any differences in the students' performance when solving PRS tasks in the posttest between instruction methods. A Pearson Chi-Square test compared how well an observed breakdown of observations over various categories fits some expected breakdown (Aron, Coups, & Aron, 2011). It evaluates how likely the distribution of scores is due to chance if the variables are independent. Because the posttest scores of participants from the comparison group were not normally distributed, the researcher used a non-parametric alternative, Pearson Chi-Square test, to evaluate the groups' performance and to increase confidence in the results. In addition to that, the researcher used a Pearson Chi-square statistic to measure any differences in the strategies that were implemented when solving PRS tasks in the posttest between groups receiving different instruction methods.

Validity and Reliability Measures

Internal validity. The threats to internal validity for a quasi-experimental design were: maturation, mortality, history, threats of testing, selection of participants, statistical regression, and differential instrumentation (Creswell, 2015; Mills & Gay, 2016). History and maturation relate to the length of the experiment, in this case, the study lasted four weeks, thus both factors might influence the results. Testing did not likely influence the results because the pretest and posttest were administered in a four-month range; testing influences studies when the testing time is short (Mills & Gay, 2016). The inability to assign participants to treatment and comparison groups randomly adds validity threats to this study, thus results were be interpreted with caution (Mills & Gay, 2016).

When is not possible to randomly assign participants to groups, a quasi-experimental design provides adequate control of threats to validity (Mills & Gay, 2016). To reduce the threats and strengthen the validity of the study, the groups of students participating were selected as equivalent as possible (Mills & Gay, 2016). In addition, an ANCOVA is a statistical instrument that allows for controlling extraneous variables (Mills & Gay, 2016). ANCOVA equated the groups by controlling the students' initial mathematics ability, as it adjusted the posttest scores for initial differences between the two groups of students. Lipsey noted, "ANCOVA makes use of all the variability among subjects on the covariate without requiring that they be distributed evenly over each value of that variable" (1990, p.130). ANCOVA eliminated from subject heterogeneity that portion of the scores predictable by that covariate, leaving only the uncorrelated residuals to analyze for treatment versus control differences (Lipsey, 1990).

ANCOVA serves as an instrument to increase the power of a statistical test, as it reduces the within-group variance (Mills & Gay, 2016). Therefore, by adjusting participants' initial mathematics ability, greater levels of posttest variance can be attributed to the treatment (Mills & Gay, 2016).

External validity. The threats to external validity for this quasi-experimental design were: pretest-treatment interaction, selection-treatment interaction, treatment diffusion, and experimenter effects (Creswell, 2015; Mills & Gay, 2016). The pretest-treatment interaction might sensitize participants of the nature of the treatment and potentially makes the treatment effect different than it would be had participants not being pretested. Mills and Gay (2016) suggested that a pretest on algebraic algorithms would have little effects on a group's responsiveness to a new teaching method. Thus, in this study, the pretest-treatment interaction did not play a significant role in the effect of the assignments to different groups. Similarly, selection-treatment interaction refers to nonrepresentative participants of the population. This study used Latino middle school students; therefore, participants' reactions to treatment reflected those of the population. In a like manner, participants had differences in mathematics ability, but the ANCOVA controlled for these differences. Then, the threats for selection-treatment interaction did not affect the inferences made in this study.

Nevertheless, treatment diffusion appears as one of the biggest threats to this study. As one can expect to occur in a small middle school, participants from both conditions can communicate and learn from each other. By having treatments overlapping and diffused, the inferences might not represent the initially designed instructional approaches. Mills and Gay (2016) recommended using only one teacher in each school to reduce treatment diffusion and to implement the treatment and control groups on different campuses. This study took place in one middle school and only one teacher carried out both intervention and control groups. Thus, there was a possibility that treatment diffusion affected the inferences of this study, therefore, results were interpreted with caution (Mills & Gay, 2016). Lastly, the experimenter effects, such as the characteristics of the teacher race, gender, and age also present threats to this study, as well as the active experimenter bias effects. To ameliorate these effects, the researcher discussed the experimenter's effects with the teacher participating in this study to avoid communicating emotions and expectations differently to participants of both research conditions.

CHAPTER IV

RESULTS

This study used a comparison group pretest-posttest design to explore the impact of social constructivist learning experiences on the performance and strategies employed by sixth grade Latino middle school students when solving PRT. The Statistical Package for Social Sciences (SPSS, version 25) was used for computing all tests at a .05 significance level.

The analysis of the data was organized in accordance with the research questions. The two groups were: the social constructivist method of teaching of proportional reasoning was the treatment group, and the teacher-centered textbook instruction was the comparison group. The students' overall performance on the PRS posttest was analyzed using descriptive statistics, a one-way ANCOVA, as well as a Pearson Chi-Square test. The students' ability to solve tasks involving partitioning, unitizing, and multiplicative reasoning as defined by the PRS rubric was computed using a series of Pearson Chi-Square tests. Descriptive statistics are presented. The Pearson Chi-Square test was used for evaluating any differences in the students' performance and the strategies that were implemented when solving PRS tasks in the posttest comparing instructional approaches.

Pre-, and Post-test Performance Differences Between Treatment Conditions

An Analysis of Variance (ANOVA) revealed no statistically significant differences in the pretest scores between the social constructivist and the control groups (see Table 2).

Table 2

Pre-test One Way ANOVA Results Comparing Teaching Method

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.014	1	.014	.001	.991
Within Groups	8413.972	70	120.200		
Total	8413.986	71			

Table 3 presents the pre-, and post-test means and standard deviations. Students in each group performed at similar levels on the pretest. However, statistically significant results were found on the posttest, as reported by an ANOVA. The mean scores where the social constructivist group were 19.8 points higher than the mean scores of the control group.

Table 3

Pre- and Post-test Means and Standard Deviations by Method of Instruction

	Pretest			Posttest		
	N	Mean	SD	N	Mean	SD
Teacher Centered Group	36	79.72	9.28	36	45.83	27.22
Social Constructivist Group	36	79.75	12.42	36	65.63	22.83

$p < 0.05$

Analysis of Covariance

A one-way ANCOVA was computed to assess whether scores on the posttest of PRS significantly differ across levels of the independent variable while statistically controlling for initial mathematics ability as measured by the pretest. An ANCOVA provides a way to assess whether mean outcome scores differ across treatment groups when a statistical adjustment is made to control for the effect of different participant characteristics across groups (Warner, 2012).

An ANCOVA has several assumptions that affect the interpretation of results (Warner, 2012). The following analyses were conducted to evaluate the assumptions for using an ANCOVA. First assumption: The scores on the posttest need to be independent of each other. These data consisted of the posttest scores from each participant of the social constructivist and control groups given the second week of May of 2019. As mandated by the Texas Education Agency, participants were not allowed to communicate with each other or share answers with their classmates during testing time. Thus, the independence of observations assumption is met. Second assumption: The covariate or pretest was measured prior to the administration of the treatment. The pretest was used to establish the extent to which social constructivist and control groups initially differed in their mathematics ability. It is assumed that the mean levels of the posttest across groups do not significantly differ (Warner, 2012). An ANOVA yielded non-significant differences across instructional groups, $F(1, 70) < .001$, $p = .991$ at the conventional $\alpha = .05$ alpha level. Thus, there was a non-significant difference in mathematics ability between the comparison groups prior to the implementation of the study. Third assumption: The variances on the pretest across levels of the independent variable, method of instruction, need to be homogeneous. Homogeneity of variance assumes that both groups have equal error variances

(Warner, 2012). Levene's test of standardized residuals yielded a non-significant statistical difference, thus, the variances of the two groups are homogeneous, F(1, 70) = .896, p = .347 at an alpha level of .05.

Fourth assumption: ANCOVA assumes that there is a linear relationship between the covariate and the dependent variable for each level of the independent variable (Warner, 2012). The scores on the pretest must be linearly related to the scores on the posttest. In this case, the relationship between the scores on the pretest of mathematics ability and posttest scores are similar for both the social constructivist and teacher-centered textbook instruction groups. A visual inspection of the scatterplot for each level of the independent variable appears to be fairly scattered in a linear fashion, hence, there exists a linear relationship between the participants' initial mathematics ability and their PRS performance (see Figure 1). To corroborate this, a Pearson correlation coefficient was calculated. The value of R for participants from the teacher-centered textbook instruction group was 0.5679. The value of R for participants from the social constructivist group was also positively moderated, 0.5154. This represents a moderate positive correlation for participants from both teaching methods, which means that there is a tendency for students who attained high pretest scores to get high posttest scores, and students with low pretest scores to obtain low posttest scores. Thus, there is a linear relationship between the pretest and the posttest for each level of the independent variable.

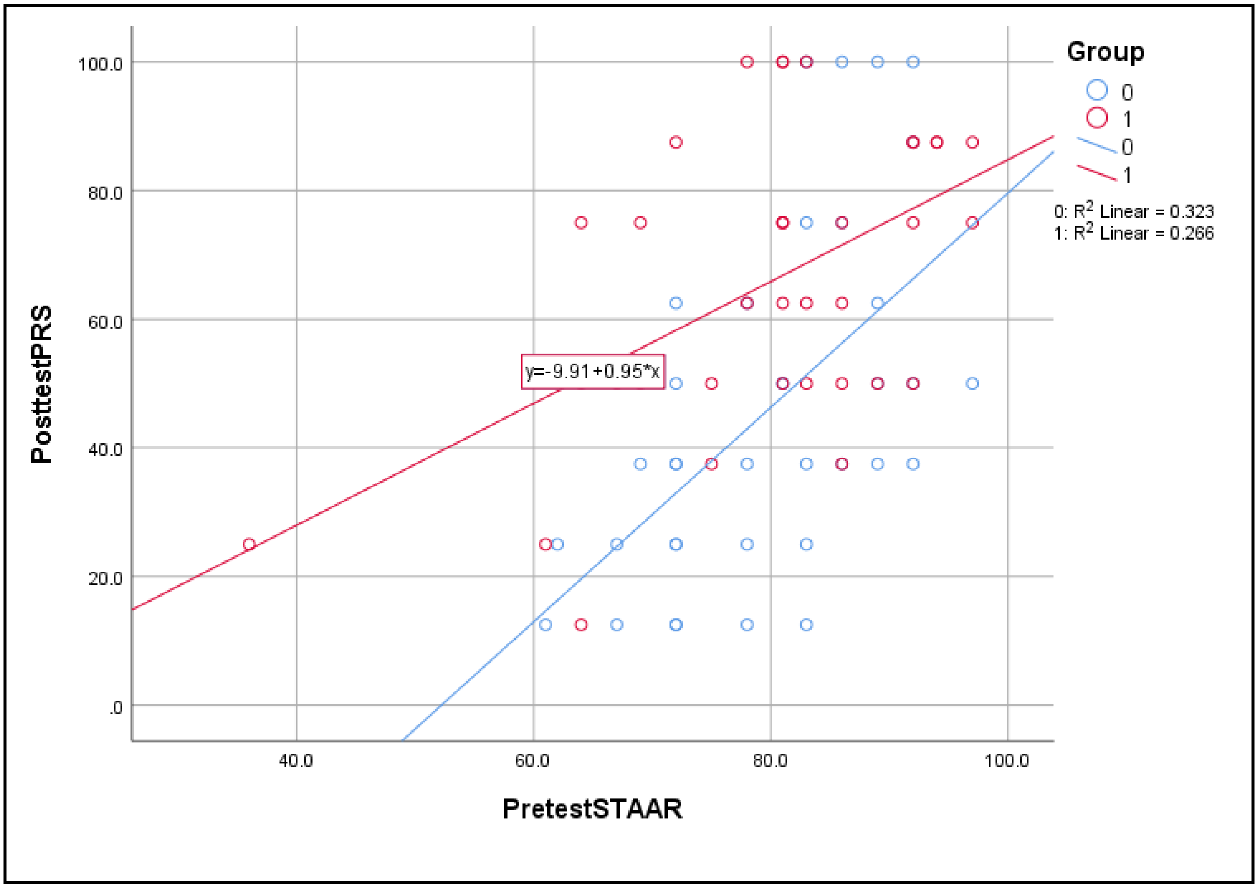


Figure 1. Pretest STAAR and PRS Posttest by Method of Instruction

Fifth assumption: The regression slopes must be homogeneous, there must be no treatment by covariate (Warner, 2012). A test of between-subjects effects indicated that the group*pretest integration was non-significant, F(1, 68) = 2.20, *p* = .031 at an alpha level of .05 (see Table 4). This test of treatment-by-covariate interaction yielded a non-statistically significant result; hence, the homogeneity of regression slopes assumption is met.

Table 4

Test of Homogeneity of Slopes for Treatment Groups

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	20262.384	3	6754.128	14.831	.000	.396
Intercept	2810.371	1	2810.371	6.171	.015	.083
Group	1778.031	1	1778.031	3.904	.052	.054
PretestSTAAR	13210.535	1	13210.535	29.008	.000	.299
Group* PretestSTAAR	1000.035	1	1000.035	2.196	.143	.031
Error	30968.085	68	455.413			
Total	274843.750	72				
Corrected Total	51230.469	71				

Note: a. R Squared = .396 (Adjusted R Squared = .369)

Sixth assumption: there should not be outliers on the posttest scores for each level of the independent variable (Warner, 2012). A visual inspection of the box plots constructed using the posttest scores for each level of the independent variable confirmed the absence of outliers. No data pieces are plotted below the bottom whisker, nor above the top whisker, therefore, this assumption is also met (see Figure 2).

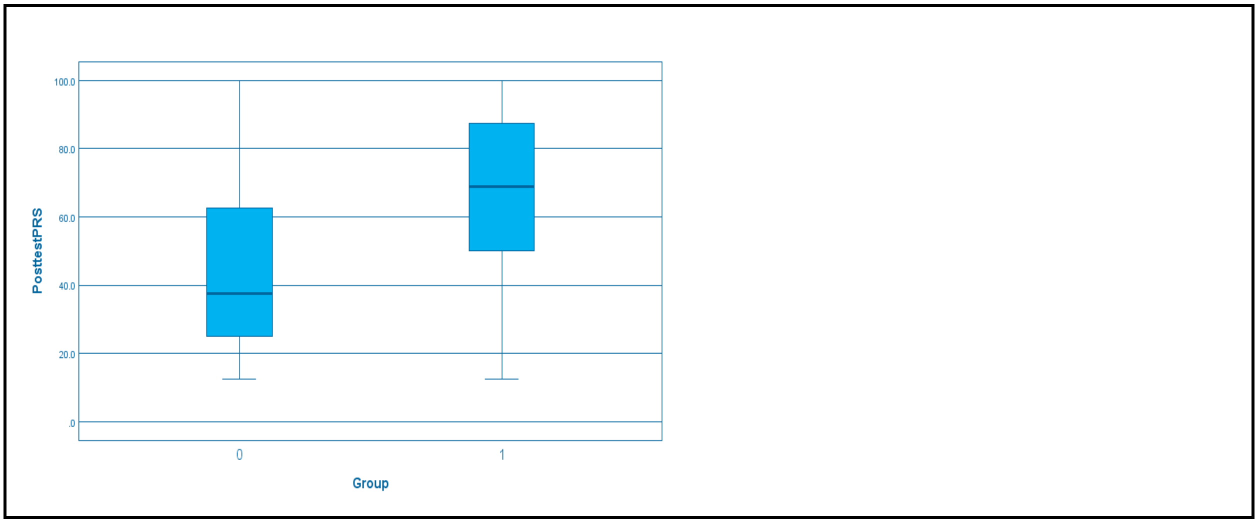


Figure 2: Box Plots of Posttest PRS Scores by Treatment Group.

Seventh assumption: The covariate and the dependent variable have distribution shapes that are fairly close to normal (Warner, 2012). A visual inspection of the q-q plots for each level of the dependent variable showed that the probability distributions of the posttest scores appear to fall fairly close to a standard normal distribution, the scores were somewhat close to the normal curve. An inspection of the histograms created with the posttest scores confirms that for the control group the data appears skewed right and for the social constructivist group we can almost see a normal curve (see Figure 3).

Further tests of normality were carried out and indicated that the posttest scores are not normally distributed for the teacher-centered textbook instruction group. The Shapiro-Wilk test resulted in a statistically significant difference, $p = .003$ at an alpha level of .05, thus, the posttest scores of the control group are not fairly similar to a normal curve. The assumption of normality is not met for the posttest scores of the teacher-centered textbook instruction group. In contrast, the posttest scores are normally distributed for the social constructivist group. The Shapiro-Wilk test yielded a non-statistically significant difference, $p = .071$ at an alpha level of .05, thus, the test of normality is met for this level of the independent variable.

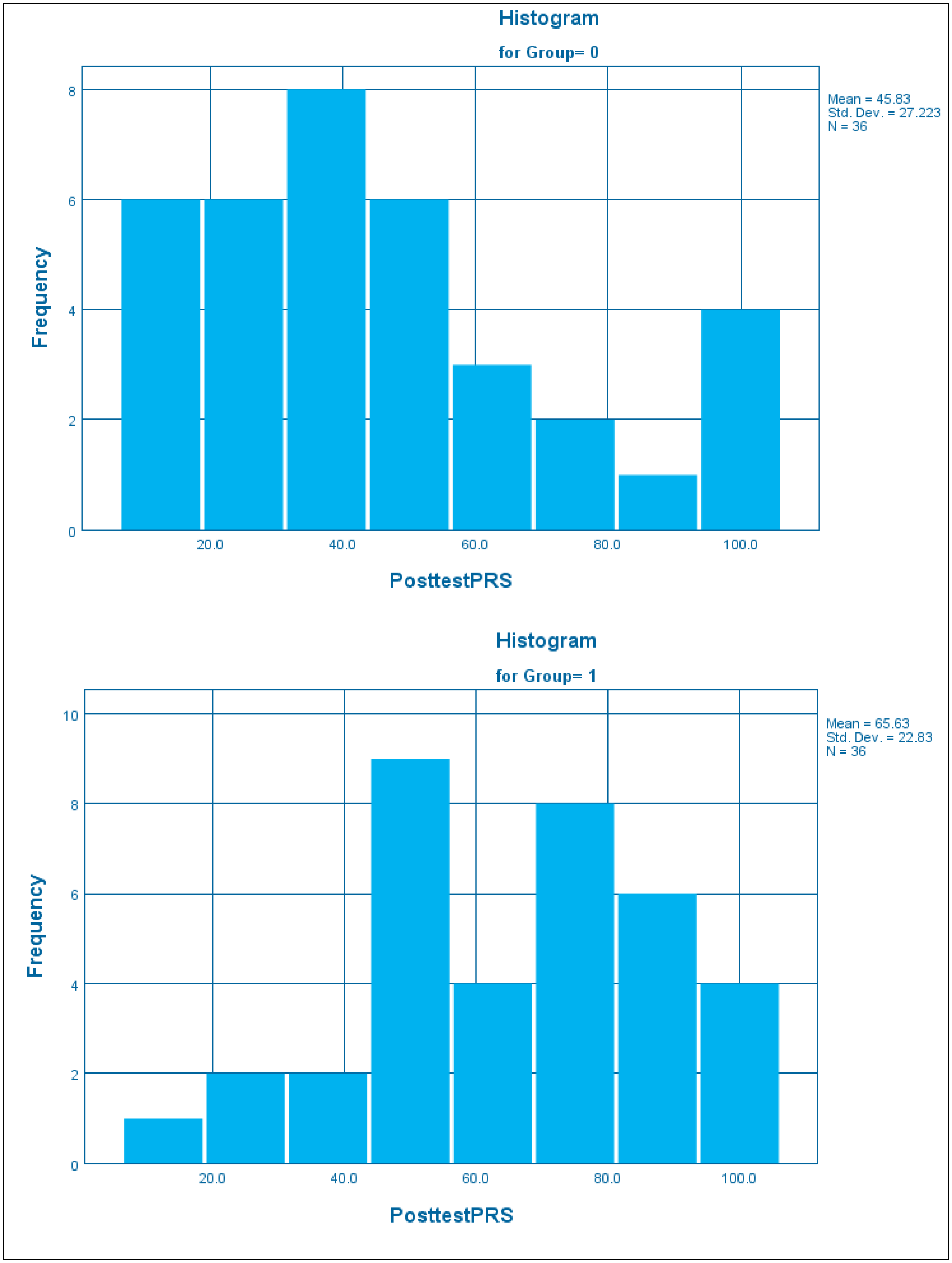


Figure 3: PRS Posttest Scores by Treatment Group

An ANCOVA is a robust test of associations, even when the assumption of normality is not met. Warner (2012) recommended using parametric statistics “when only one or two of the requirements for a parametric statistic are violated” (p.23). She suggested that violations of normality are problematic when the group sizes are small and unequal, neither of which applies to the data from this study. In addition, Lipsey (1990) recommended using an ANCOVA in pretest-posttest designs because, even when certain assumptions might not hold, “ANCOVA will be more appropriate and more powerful than the attainable alternatives” (p.133). Three constraints were used for using parametric statistics to assess for statistical significance: (1) all assumptions were met, but the assumption of normality, (2) the data set is not small, 36 participants per group are adequate for using an ANCOVA (Lipsey, 1990), and (3) for the control group the median score is 37.5 and the mean of the scores is 45.83. The median and mean are not too far apart, thus, both measures, median and mean fairly represent the center of the data distribution, hence, a diminished likelihood to make a Type-I error.

After validating the appropriateness of these data for using an ANCOVA statistic, the researcher proceeded to examine if there was a statistically significant difference in the PRS performance between students experiencing social constructivist and teacher-centered textbook instruction while controlling for initial mathematics ability. The mean scores on the posttest of PRS of the social constructivist and teacher-centered textbook instruction groups are significantly different (see Figure 4).

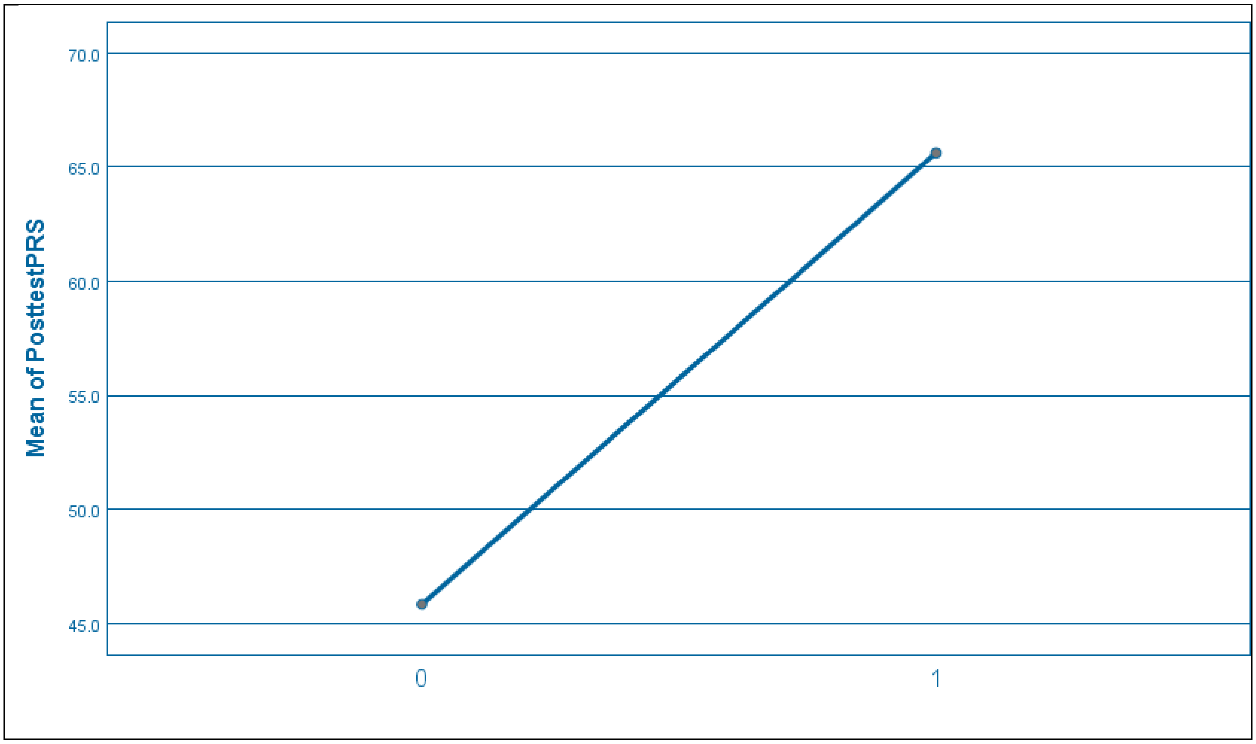


Figure 4: PRS Posttest Mean Scores by Treatment Group.

The results of the ANCOVA presented in Table 5 show a statistically significant difference. The performance of participants from the social constructivist and teacher-centered textbook instruction groups significantly differ at an alpha level of .05, while controlling for initial mathematics ability. The partial Eta Squared value indicates that the group effect size is small (.18). About 18 percent of the variance in the PRS performance is explained by the instructional approach.

Table 5

ANCOVA Results of PRS Performance by Teaching Method Controlling for Mathematics Ability

Source	Type III Sum of Squares	df	Mean Square	F	Sig	Partial Eta Squared
Corrected Model	19262.348	2	9631.174	20.788	.001	.376
Intercept	2113.703	1	2113.703	4.562	.036	.062
PretestSTAAR	12211.567	1	12211.567	26.357	.001	.276
Group	7026.946	1	7026.946	15.167	.001	.180
Error	31968.120	69	463.306			
Total	274843.750	72				
Corrected Total	51230.469	71				

Note: R Squared = .376 (Adjusted R Squared = .358) *p < .05

Pearson Chi-square

Since the posttest scores of participants from the teacher-centered textbook instruction group were not normally distributed, the researcher performed a non-parametric alternative to increase confidence in the results. A Pearson Chi-Square test was conducted to assess "whether group membership on the DV is predictable from group membership on the IV" (Warner, 2012, p.28). A Pearson Chi-Square test compares how well an observed breakdown of observations over various categories fits some expected breakdown (Aron, Coups, & Aron, 2011). It evaluates how likely an observed distribution is due to chance or how well the observed distribution of data fits with the distribution that is expected if the variables are independent. The expected count is what is expected if there is no relationship between the two variables. In this case, the researcher assessed whether there is a relationship between instruction affiliation group, social constructivist and teacher-centered textbook instruction, and their attainment for solving PRS problems, correct and wrong. Both variables are dichotomous, the independent variable relates to the group membership, 0 = teacher-centered textbook instruction, and 1 = social constructivist, and the outcome variable is composed of 1 = correct and 0 = wrong answers on the posttest.

The results of the Pearson Chi-Square test are presented in Table 6. The interaction, Group by Pass crosstabulation shows the students' wrong and correct responses, as well as the expected count for each level of the independent variable. The posttest results of students from the social constructivist group generated 99 wrong responses with an expected count of 127.5 and 189 correct responses with an expected count of 160.5. That is 28.5 fewer wrong answers and 28.5 more correct answers than expected. While students from the teacher-centered textbook instruction group generated 156 wrong responses with an expected count of 127.5 and 132 correct responses with an expected count of 160.5. That is 28.5 more wrong answers and 28.5 less correct answers than expected.

Table 6 shows that the within group percent for attaining correct responses of the participants in social constructivist instruction is 65.6 percent, whereas the percent for attaining correct responses of the participants who received teacher-centered textbook instruction is 45.8 percent. The within group percent for attaining wrong responses of the participants in social constructivist instruction is 34.4 percent, whereas the percent for attaining correct responses of the participants who received teacher-centered textbook instruction is 54.2 percent. Overall, a higher rate of correct responses was observed from participants of the social constructivist group, 19.8 percent more than participants from the control group, and participants from the control group obtained 19.8 percent more wrong responses than those participating in the social constructivist group. Thus, participants from the social constructivist group performed better than those receiving teacher-centered textbook instruction.

Table 6

Crosstabs Table of PRS Posttest Performance by Teaching Method

Group		Wrong	Correct	Total
Teacher-centered	Count	156	132	288
	Expected Count	127.5	160.5	288.0
	% within Group	54.2%	45.8%	100.0%
	Standardized Residual	2.5	-2.2	
Social Constructivist	Count	99	189	288
	Expected Count	127.5	160.5	288.0
	% within Group	34.4%	65.6%	100.0%
	Standardized Residual	-2.5	2.2	
Total	Count	255	321	576
	Expected Count	255.0	321.0	576.0
	% within Group	44.3%	55.7%	100.0%

The results of the Pearson Chi-Square test presented in Table 7 show that there was a statistically significant relationship between group of instruction affiliation and attainment for solving PRS problems, $\chi^2(1) = 22.863$, $P < .05$, at an alpha level of .05. The proportions are not homogeneous across the categories, thus, there exists a significant relationship. A Cramer's V value was used to indicate the effect size, Cramer's V = .199. This Cramer's V value of .199 corresponds to a small effect size (Cohen, 1988).

Table 7

Pearson Chi-Square Results PRS posttest

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)
Pearson Chi-Square	22.863[a]	1	.000	
Continuity Correction[b]	22.068	1	.000	
Likelihood Ratio	23.025	1	.000	
Fisher's Exact Test				.001
Linear-by-Linear Association	22.823	1	.000	
N of Valid Cases	576			

About 20 percent of the difference in the PRS performance is explained by the instructional approach. This finding aligns with the ANCOVA results both in the significance level and the effect size level, there is a statistically significant difference in performance levels between those students who experienced social constructivist teaching and those who experienced teacher-centered textbook instruction (see Figure 5). Then, a Pearson Chi-Square test confirmed that participants from the social constructivist group have a significantly higher probability of attaining a correct response in the PRS posttest (about 65.6 percent) than participants from the teacher-centered textbook instruction group (about 45.8 percent).

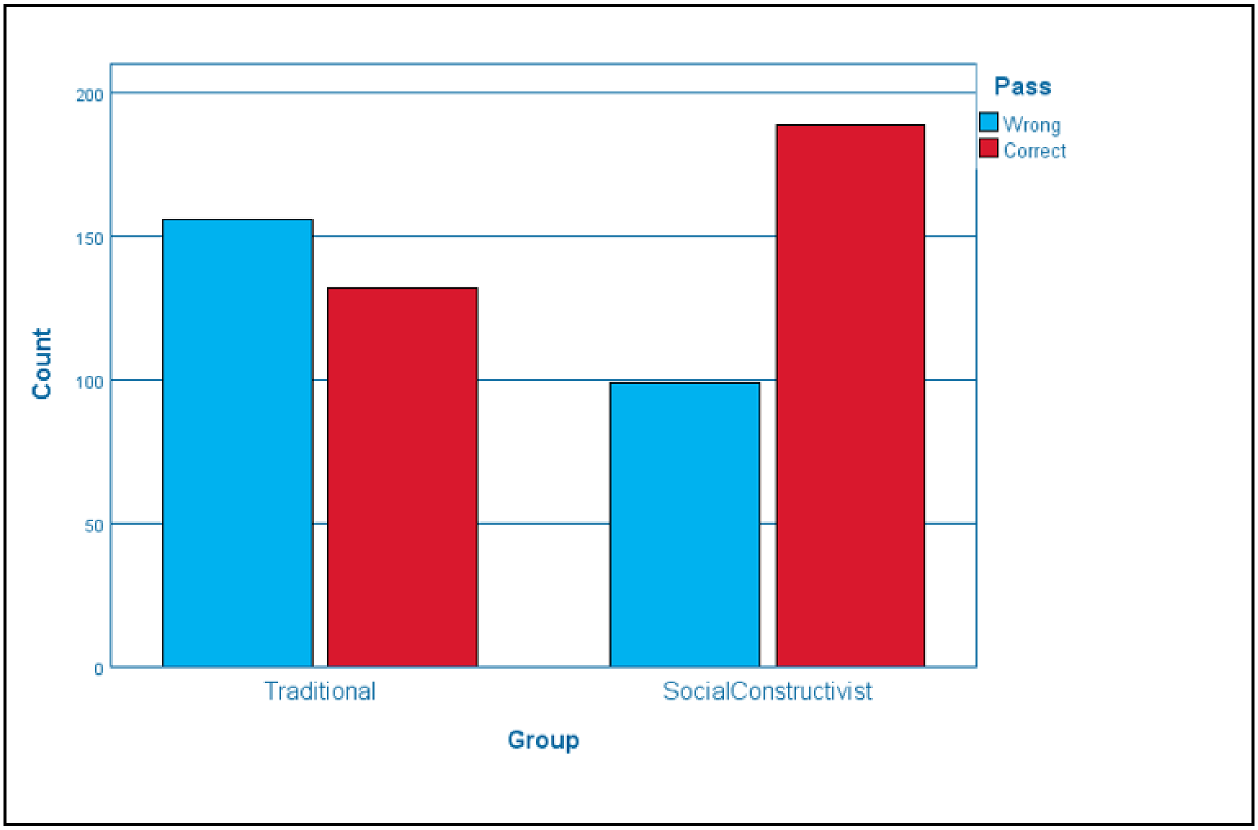


Figure 5: PRS Posttest Attainment by Group of Instruction

Proportional Reasoning Levels

Pearson Chi-Square tests were used to assess the performance differences between groups using proportional reasoning strategies of (1) integer scaling, (2) one-step partitioning, (3) non-integer scaling, and (4) multistep strategies involving partitioning, unitizing, and covariation. Using non-parametric tests, the researcher explored the participants' PRS differences for solving problems involving each of the PRS levels as defined by the PRS rubric. Both variables involved in these analyses are dichotomous. The independent variable relates to the group membership, coded 0 = control and 1 = treatment, and the dependent variable is was coded 1 = correct and 0 = wrong answers on the posttest. This allowed the researcher to determine statistical significance between the groups, treatment and control for each of the PRS levels.

Table 8

Pearson Chi-Square Posttest Results by PRS Level According to the Rubric

	PRS Level 1: Integer Relationships	PRS Level 2: One-Step Partitioning	PRS Level 3: Non-Integer Relationships	PRS Level 4: Multistep Relationships
Pearson Chi-Square	2.372	20.202	6.464	3.273
df	1	1	1	1
p-value	.124	.000	.011	.070

Integer Scaling. The first analysis evaluated performance differences of students using the strategy, integer scaling. Although participants in the social constructivist group performed better than those receiving teacher-centered textbook instruction, a Pearson Chi-Square test at an alpha level of .05 showed that there was not a statistically significant relationship between the group of instruction affiliation and attainment for solving PRS tasks involving integer relationships, $\chi^2(1) = 2.37$, $p = .12$ (see Table 8). Participants that received social constructivist instruction attained 87.5 percent correct responses, whereas those who received teacher-centered textbook instruction attained 77.8 percent. The PRS performance of both groups is fairly equivalent, as does not exist a significant relationship.

A higher number of incorrect responses were obtained from participants receiving teacher-centered textbook instruction and a higher number of correct responses from participants receiving social constructivist instruction. A Cramer's V value was .13, this indicated a small effect size (Cohen, 1988). About 13 percent of the difference in the PRS performance is explained by the instructional approach. This finding diverts from the overall Pearson Chi-

Square results in the significance level, although the overall performance of participants from the social constructivist and teacher-centered textbook groups are statistically significantly different, the performance is not statistically significantly different for the PRS level of scaling up by an integer.

Table 9

Strategies Implemented on PRT Involving Integer Relationships by Teaching Method

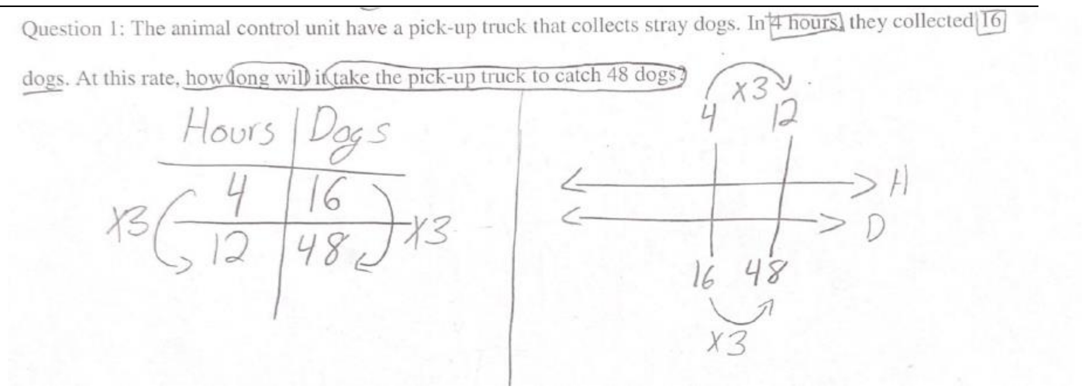
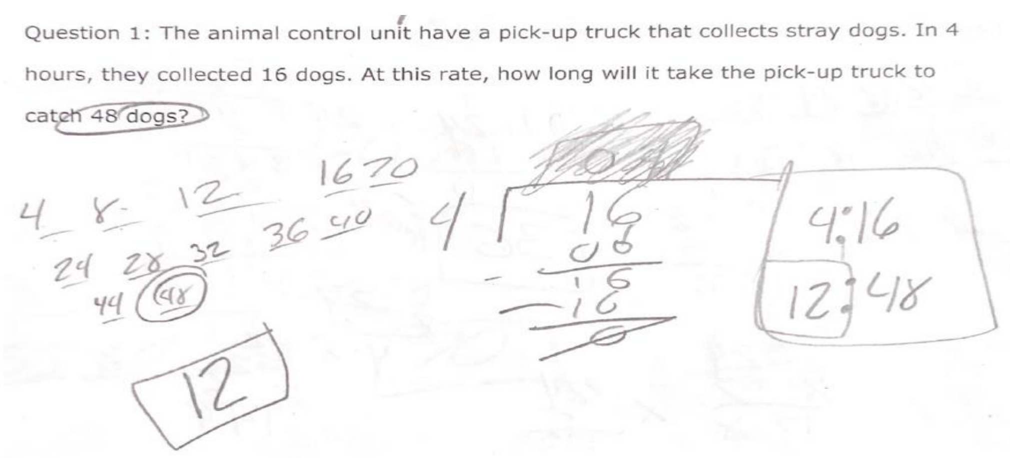
Teaching Method	Strategy Used to Solve the PRT Involving an Integer Relationship
Social Constructivist	
Teacher-Centered	

Table 9 shows the strategies implemented when solving a PRT involving an integer multiplicative relationship by students from the social constructivist group and the teacher-centered group. It can be noted that both students successfully solved the task. There is a noticeable difference in the strategies used to solve the task. While the participant from the social constructivist group used representational reasoning to identify the multiplicative relationship to solve the task, a table of values and a double number line, the participant from the teacher-centered group used a less efficient approach involving a build-up strategy to find the target quantity. Though both students solved the task, the strategies used show that the participant receiving social constructivist instruction identified the integer multiplicative relationship and used proportional reasoning to solve the task while the participant receiving the teacher-centered instruction might not have identified the multiplicative relationship by an integer and used a less sophisticated approach that evidences early levels of development of PR.

One-step Partitioning. The second analysis consisted of evaluating the performance differences for using one step partitioning between Latino, middle school students participating in experiences involving manipulatives, models, and discourse, and those who receive teacher-centered textbook instruction. Participants in the social constructivist group performed better than those receiving teacher-centered textbook instruction, a Pearson Chi-Square test at an alpha level of .05 showed that here was a significant relationship between the group of instruction affiliation and attainment for solving PRS problems involving one-step partitioning, $\chi^2(1) = 20.20$, $p < .05$ (see Table 8). Participants that received social constructivist instruction attained 86.1 percent correct responses, while those who received teacher-centered textbook instruction attained 51.4 percent. The PRS performance of both groups is not equivalent, as does exist a significant relationship.

A higher number of incorrect responses were obtained from participants receiving teacher-centered textbook instruction and a higher number of correct responses from participants receiving social constructivist instruction. The Cramer's value was .38, this corresponds to a medium effect size (Cohen, 1988). About 38 percent of the difference in the PRS performance is explained by the instructional approach. This finding aligns with the overall Pearson Chi-Square results in the significance level, the overall performance of participants from the social constructivist and teacher-centered textbook groups are statistically significantly different and the performance is statistically significantly different for the PRS level of one-step partitioning.

Table 10

Strategies Implemented on PRT Involving One-Step Partitioning by Teaching Method

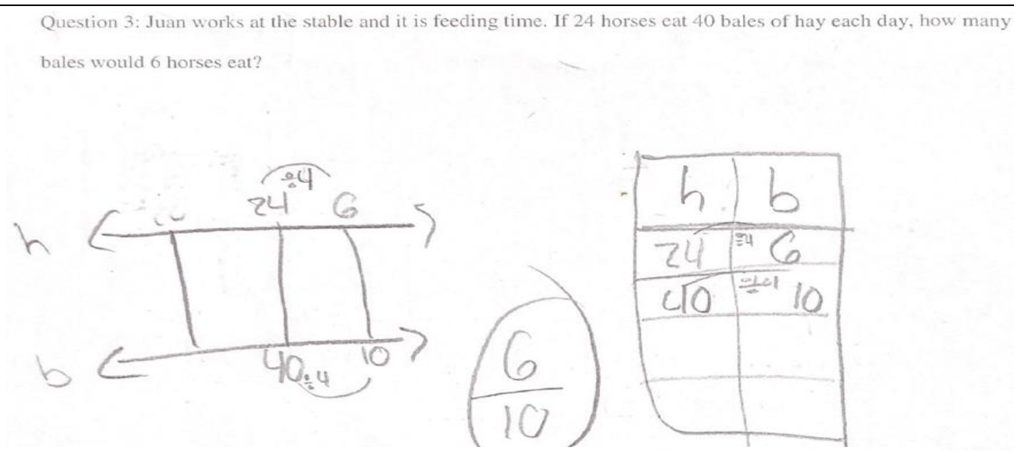
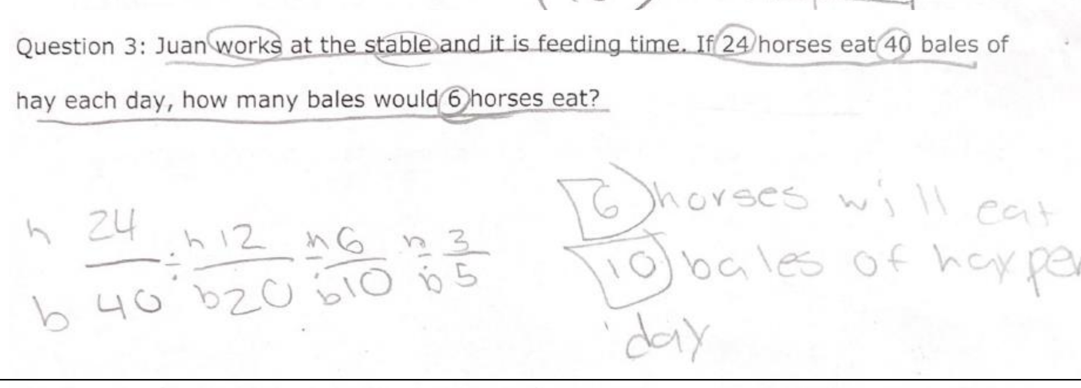
Teaching Method	Strategy Used to Solve the PRT Involving One-Step Partitioning
Social Constructivist	
Teacher Centered	

Table 10 shows examples of the strategies implemented for solving a PRT involving one-step partitioning by students from the social constructivist group and the teacher-centered group. It can be noted that both students successfully solved the task. There is a noticeable difference in the strategies used to solve the task. While the participant from the social constructivist group used representational reasoning to identify the multiplicative relationship to solve the task, a table of values and a double number line, the participant from the teacher-centered textbook group used a build-down strategy to find the target quantity. Though both students solved the task, the strategies used show that the participant receiving social constructivist identified the multiplicative relationship to solve the task while the participant receiving the teacher-centered instruction used a less sophisticated approach that evidences early levels of PR development.

Non-integer Scaling. The third analysis consisted of evaluating the performance differences for using non-integer scaling. Participants in the social constructivist groups performed better than those receiving teacher-centered textbook instruction, a Pearson Chi-Square test of independence at an alpha level of .05 showed that there was a significant relationship between the group of instruction affiliation and attainment for solving PRS problems involving non-integer relationships, $\chi^2(1) = 6.46$, $p = .01$ (see Table 8). Participants that received social constructivist instruction attained 51.4 percent correct responses, whereas those who received teacher-centered textbook instruction attained 30.6 percent. The PRS performance of both groups is not equivalent, as does exist a significant relationship.

A higher number of incorrect responses were obtained from participants receiving teacher-centered textbook instruction and a higher number of correct responses from participants receiving social constructivist instruction. The Cramer's V value was .21, this indicated a small effect size (Cohen, 1988). About 21 percent of the difference in the PRS performance is explained by the instructional approach. This finding aligns with the overall Pearson Chi-Square results in the significance level, the overall performance of participants from the social constructivist and teacher-centered groups are statistically significantly different and the performance is statistically significantly different for the PRS level of non-integer relationships.

Table 11

Strategies Implemented on PRT Involving Non-Integer Relationships by Teaching Method

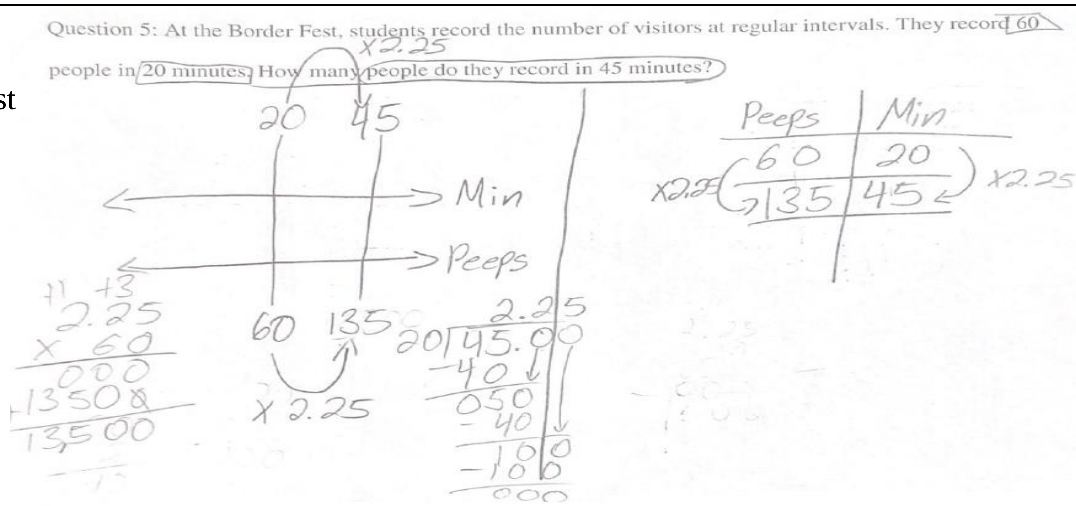
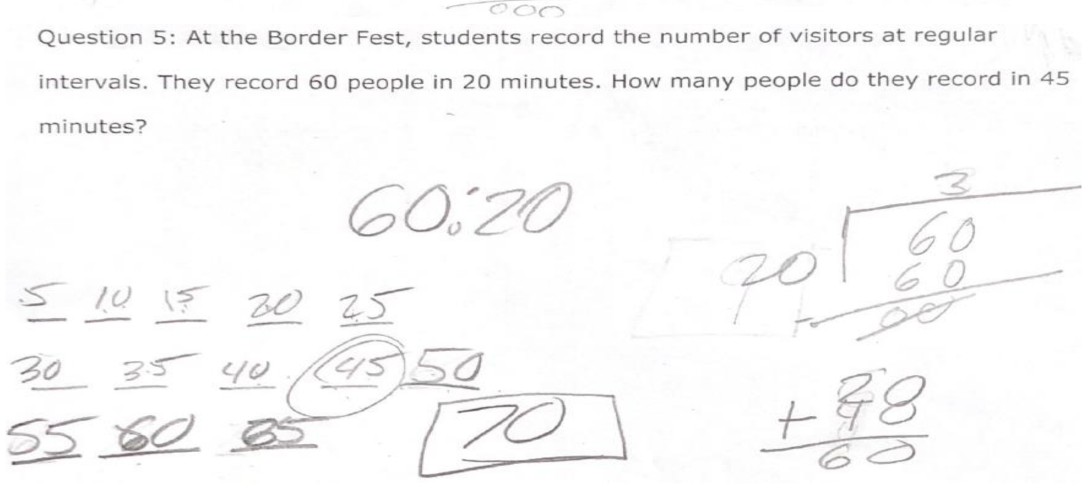
Teaching Method	Strategy Used to Solve the PRT Involving Non-Integer Relationships
Social Constructivist	
Teacher Centered	

Table 11 shows examples of the strategies implemented for solving a PRT involving a non-integer relationship by students from the social constructivist group and the teacher-centered group. It can be noted that only the student receiving social constructivist instruction successfully solved the task. There is a noticeable difference in the strategies used to solve the task. While the participant from the social constructivist group used representational reasoning to identify the multiplicative relationship to solve the task, a table of values and a double number line, the participant from the teacher-centered group used a less efficient approach involving build-up and failed to find the target quantity. The strategies used show that the participant receiving social constructivist identified the non-integer multiplicative relationship and used proportional reasoning to solve the task while the participant receiving the teacher-centered instruction did not identify the multiplicative relationship and used a less sophisticated approach that evidences early levels of development of PR. It is evident that the use of representational reasoning allowed students receiving social constructivist instruction with the skills to solve non-integer relationships. It is also manifest that inefficient approaches, such as the use of build-up strategies, limits the students' ability to solve PRT involving higher levels of difficulty.

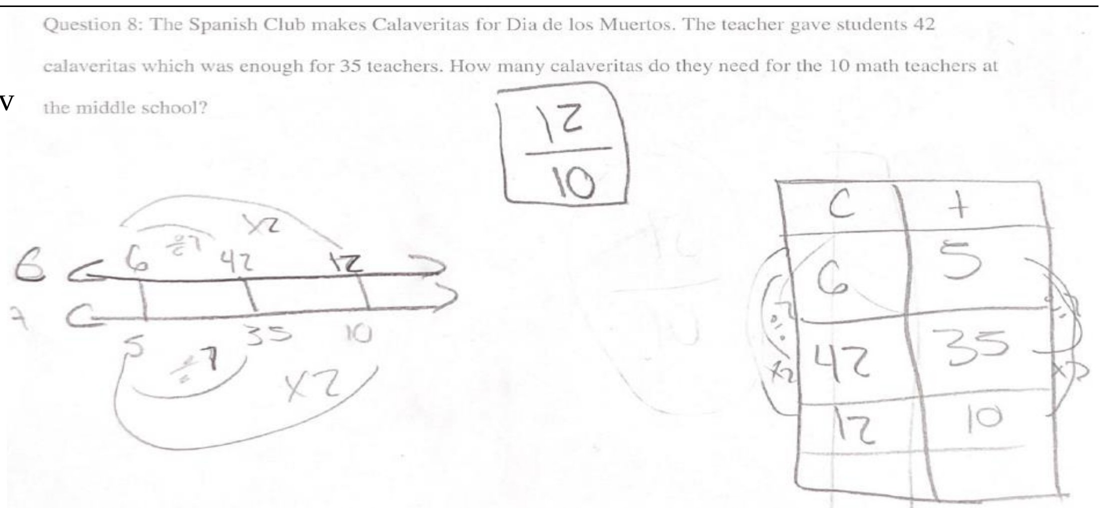
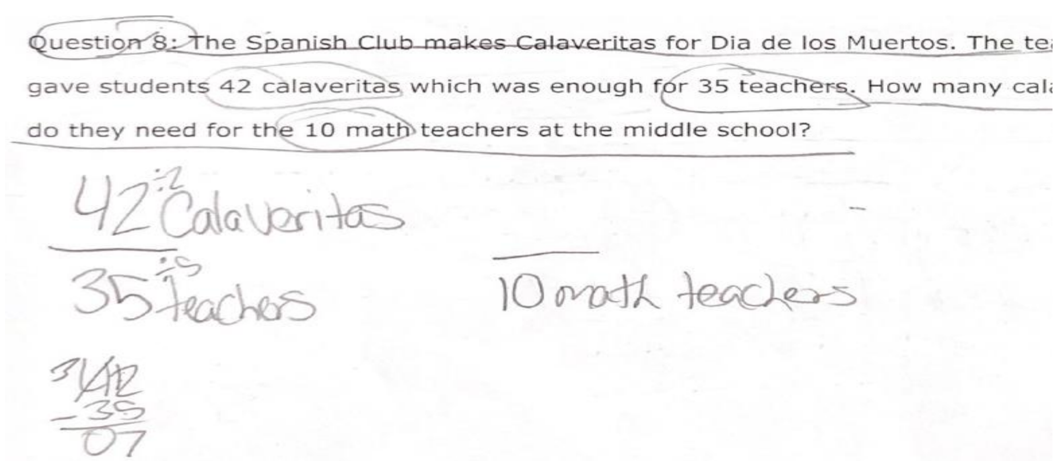
Multi-step Solution Methods. The fourth analysis involved evaluating the performance differences for using multistep PRS strategies involving multi-step partitioning, unitizing, and covariation. Although participants in the social constructivist groups performed better than those receiving teacher-centered textbook instruction, a Pearson Chi-Square test at an alpha level of .05 showed that there was not a statistically significant relationship between the group of instruction affiliation and attainment for solving PRS problems involving multi-step relationships, $\chi^2(1) = 3.27$, $p = .07$ (see Table 8). Participants that received social constructivist instruction attained 37.5 percent, whereas those who received teacher-centered textbook instruction attained 23.6 percent. The PRS performance of both groups is fairly equivalent, as does not exist a significant relationship.

A higher number of incorrect responses were obtained from participants receiving teacher-centered textbook instruction and a higher number of correct responses from participants receiving social constructivist instruction. The Cramer's V value was .15, this corresponds to a small effect size (Cohen, 1988). About 15 percent of the difference in the PRS performance is explained by the instructional approach. This finding diverts from the overall Pearson Chi-Square results in the significance level, although the overall performance of participants from the social constructivist and teacher-centered are statistically significantly different, the performance for the PRS level of multi-step relationships is not statistically significantly different.

Table 12 shows examples of the strategies implemented for solving a PRT involving a multi-step relationship by students from the social constructivist group and the teacher-centered group. It can be noted that only the student receiving social constructivist instruction successfully solved the task. There is a noticeable difference in the strategies used to solve the task. While the participant from the social constructivist group used representational reasoning to identify the multiplicative relationships to solve the task, a table of values and a double number line, the participant from the teacher-centered group used a less efficient approach involving random calculations in an attempt to find the target quantity. The strategies used show that the participant receiving social constructivist instruction partitioned the ratio first, and then used a multiplicative relationship to attain the target quantity. This evidences the use of high levels of proportional reasoning to solve the task. Contrarily, the participant receiving the teacher-centered textbook instruction did not identify the multiplicative relationship and unsuccessfully used a less sophisticated approach that evidences early levels of development of PR. It is evident that the use of representational reasoning allowed students receiving social constructivist instruction with the skills to solve multi-step relationships. It is also manifest that inefficient approaches, such as the use of random calculations limits the students' ability to solve PRT involving higher levels of difficulty.

Table 12

Strategies Implemented on PRT Involving Multi-step Relationships by Teaching Method

<table>
<tr><th>Teaching Method</th><th>Strategy Used to Solve the PRT Involving Multi-step Relationships</th></tr>
<tr><td>Social Constructivist</td><td>
</td></tr>
<tr><td>Teacher Centered</td><td>
</td></tr>
</table>

In summary, students who experienced learning how to solve proportional reasoning tasks in a social constructivist teaching environment performed better than those receiving teacher-centered textbook instruction for all four levels of proportional reasoning (see Table 13). For PRS tasks involving one-step partitioning and non-integer relationships students in the social constructivist teaching group correctly solved a greater number of tasks and the performance differences were statistically significant. Of note, students in the social constructivist teaching group solved a greater number of tasks at the level of proportional reasoning that involve integer relationships and multi-step relationships although the result was not statistically significant. The results of a series of Pearson Chi-Square tests align with prior findings of this study in that participants from the social constructivist group have a higher probability of attaining correct responses on the posttest by the level of PRS.

Table 13

Posttest Performance by PRS Level by Teaching Method

PRS Rubric	*Description of PRS Level*	Social Constructivist Percent Performance	Teacher-Centered Percent Performance
Level 1	Integer Relationships	87.5	77.8
Level 2	One-Step Partitioning	86.1	51.4
Level 3	Non-Integer Relationships	51.4	30.6
Level 4	Multistep Relationships	37.5	23.6

Analysis of the Use of Proportional Reasoning Strategies

Table 14 shows the differences in the strategies implemented to solve PRS problems on the posttest by students from the treatment and comparison groups. First, 141 problems were solved using multiple representations from students of the social constructivist group while only 8 problems from students of the control group did. Of those, 84.94 percent of the responses from the participants from the social constructivist group were correct and 100 percent from the control group were. Second, 14 problems were solved using a double number line from students of the social constructivist group while none of the problems from students of the control group did. Of those, 66.67 percent of the responses from the participants from the social constructivist group were correct and 0 percent from the control group were. Third, 26 problems were solved using an X-Y table of values from students of the social constructivist group while only 5 of the problems from students of the control group did. Of those, 52 percent of the responses from the participants from the social constructivist group were correct and 45.45 percent from the control group were.

A marked contrast was observed with strategies that did not involve mathematical representations for solving PRT. Students from the social constructivist group solved 21 tasks using equivalent ratios in comparison to 148 tasks from students of the control group. Of those, 28.57 percent of the responses from the participants from the social constructivist group were correct and 50.68 percent from the control group were. In a like manner, 14 tasks were solved using random calculations from students of the social constructivist group while 102 tasks from students of the control group did. Of those, 14.29 percent of the responses from the participants from the social constructivist group were correct and 43.14 percent from the control group were.

Last, students of the social constructivist group did not provide a response to 16 of the tasks, while students of the control group did not provide a response to 19 tasks.

Table 14

Posttest PRS Differences in Implemented Strategies by Teaching Method

Strategy	Social Constructivist Group			Teacher Centered Group		
	Correct	Incorrect	Percent Correct	Correct	Incorrect	Percent Correct
Multiple Representations	141	25	84.94	8	0	100
Double Number Line	14	7	66.67	0	0	0
X-Y Table of Values	26	24	52.00	5	6	45.45
Equivalent Ratios	6	15	28.57	75	73	50.68
Random Calculations	2	12	14.29	44	58	43.14
No Response	0	16	0	0	19	0
Total	189	99	65.63	132	156	45.83

These findings were corroborated using a Pearson Chi-Square test. The analysis consisted of evaluating if there was a significant relationship for using representational reasoning to solve PRT between Latino, middle school students participating in experiences involving manipulatives, models, and discourse and those who receive teacher-centered textbook instruction. Participants in the social constructivist groups used significantly more mathematical representations for solving PRT than those receiving teacher-centered textbook instruction, a Pearson Chi-Square test of independence at an alpha level of .05 showed that here was a significant relationship between the group of instruction affiliation and the use of representational reasoning for solving PRS problems, $\chi^2(1) = 334.153$, $p < .05$.

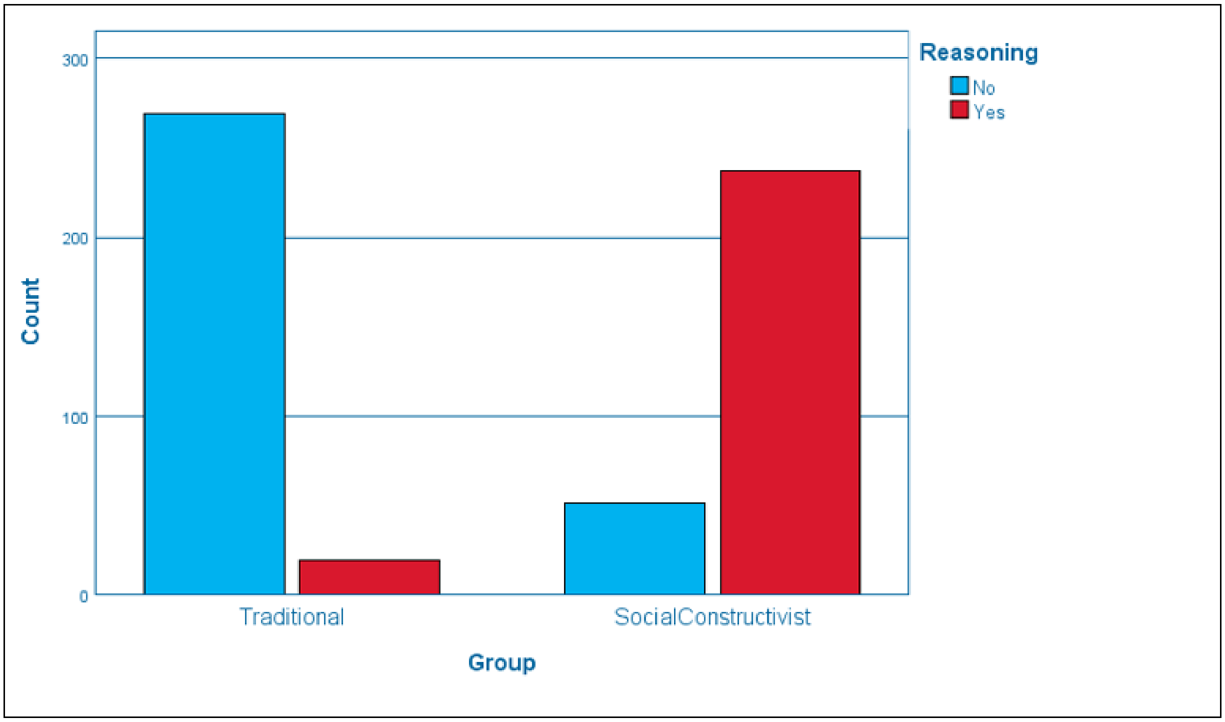


Figure 6: Use of Representational Reasoning by Group of Instruction

The percent for using representations of the participants in social constructivist instruction is 82.3, while that of the participants who received teacher-centered textbook instruction is 6.6 (see Figure 6). The proportions are not homogeneous across the categories, which means, the percent of correct responses from both groups are not equivalent, as there exists statistical significance. The Cramer's V value was .762, this indicated a large effect size (Cohen, 1988). About 76 percent of the difference in representations used for solving PRS tasks is explained by the instructional approach. This finding aligns with the overall Pearson Chi-Square results in the significance level, the overall performance of participants from the social constructivist and teacher-centered groups are statistically significantly different and the use of representational reasoning is statistically significantly different.

In addition, the researcher computed a Pearson Chi-Square test to evaluate if there was a statistically significant performance difference using representational reasoning to solve PRT between Latino, middle school students participating in social constructivist experiences and those who receive teacher-centered textbook instruction. Participants in the social constructivist group performed better on problems on which they used mathematical representations than those receiving teacher-centered textbook instruction (see Figure 7). A Pearson Chi-Square test of independence at an alpha level of .05 showed that here was not a significant relationship between the group of instruction affiliation and the performance on PRS problems, $\chi^2(1) = .606$, $p > .05$. The proportions are homogeneous across the categories, which means, the percent of correct responses from both groups is equivalent, as the relationship is not significant. The Cramer's V value was .049, this indicated a small effect size (Cohen, 1988). About 4.9 percent of the performance difference for solving PRS tasks is explained by the instructional approach. This finding diverts with the overall Pearson Chi-Square results in the significance level, the overall performance of participants from the social constructivist and teacher-centered groups are statistically significantly different and the performance using representational reasoning is not statistically significantly different.

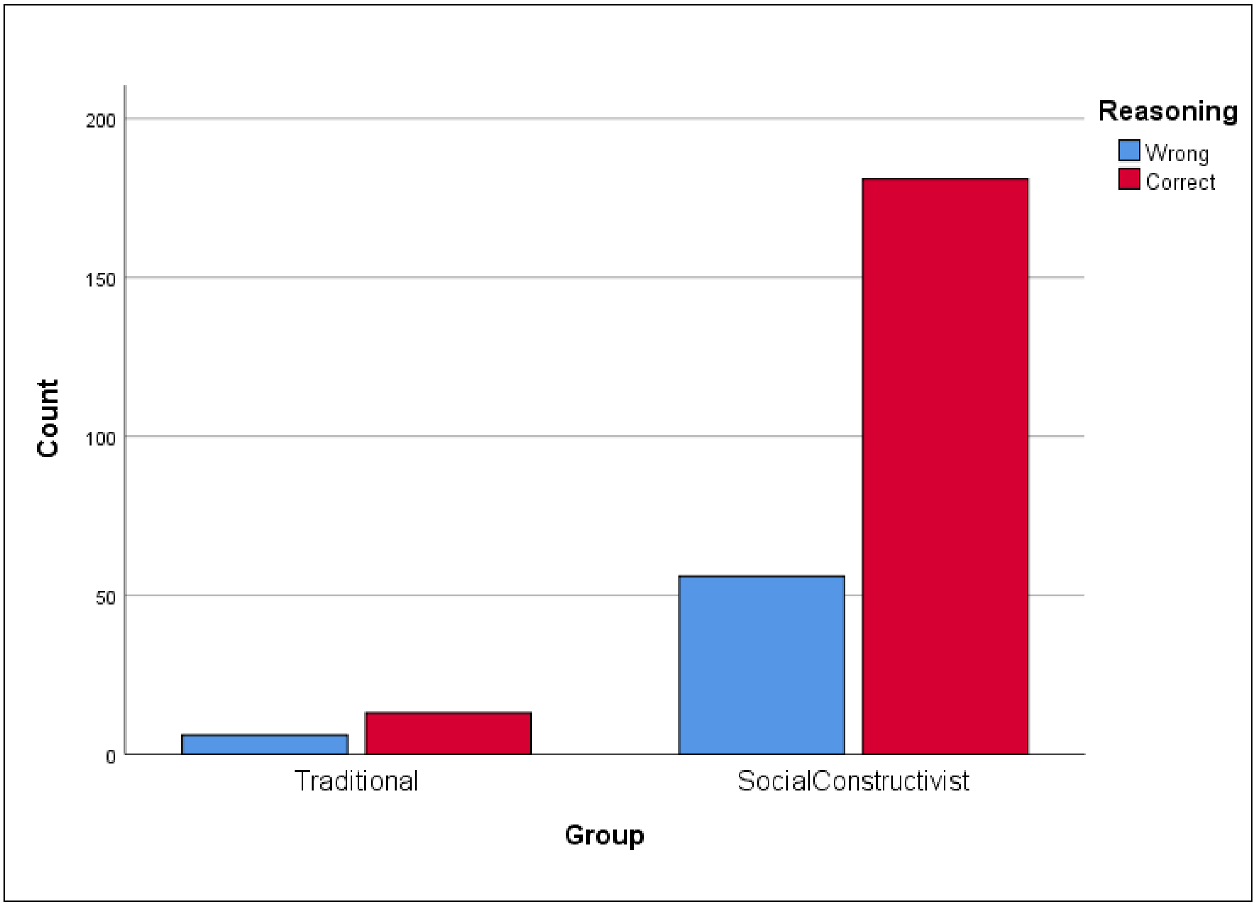


Figure 7: Group Performance Using Representational Reasoning on PRS Tasks

Participants of the social constructivist used more representational reasoning and performed better than those receiving teacher-centered textbook instruction. For PRS tasks in which students used mathematical representations, the participants from the social constructivist group used significantly more representations and performed better than students receiving teacher-centered textbook instruction. Pearson Chi-Square tests align with prior findings of this study in that participants from the social constructivist group have a higher probability of using representational reasoning and attaining more correct responses on tasks involving PRS than those receiving teacher-centered textbook instruction.

Summary

The results of this study revealed statistically significant differences in the performance and use of sophisticated strategies for solving PRS tasks favoring Latino middle school students that received social constructivist instruction. Though the initial performance on a mathematics ability test was equivalent, the overall PRS performance on the post-test between groups was statistically significantly different, as computed using an ANCOVA and Pearson Chi-Square tests. Similarly, students that received social constructivist instruction performed better for all four levels of PRS, as calculated using Pearson Chi-Square tests. The performance difference on proportional reasoning tasks involving one-step partitioning and non-integer relationship was statistically significant, while the performance on tasks involving integer and multi-step relationships was not statistically significantly different. In addition, participants in the Social constructivist group solved significantly more sophisticated PRS strategies using multiple representations, X-Y tables of values, and double number lines. Participants from the control group used equivalent ratios and random calculations to solve PRS tasks. Therefore, these results signify marked differences in performance and the use of sophisticated strategies between the students receiving social constructivist and teacher-centered textbook instruction.

CHAPTER V

SUMMARY AND CONCLUSION

In response to voices claiming that "mathematics classes are dull and routine" (Telese & Abete Jr., 2002, p.8) and that "current instruction is not serving many students" (Lamon, 2012, p.255), this study examined the impact of social constructivist learning experiences on the performance and the strategies employed when solving PRS tasks of sixth-grade Latino middle school students. PR's development is hindered by teacher-centered learning and that focuses on a symbolic and rote manipulation of algorithms (Ayan & Botan, 2018; Dominguez et al., 2014; Lamon, 2012; Steinthorsdottir & Sriraman, 2009). This study explored the impact of instructional practices based on intensive collaboration in the classroom community, multimodal communication, and multiple representations (Dominguez et al., 2014; Montemayor et al., 2015; Moschkovich, 2007; Simon, 1995; Ernest, 1996; Cobb, 2007) to alleviate the learning difficulties associated with developing proportional reasoning ability. The findings were interpreted using the lenses of a social constructivist theoretical framework and compared with relevant, recent research. In order to answer the three research questions, in this chapter, the researcher theorized the findings and connected them to the literature.

Interpretation of Findings and Connections with Theory and Research

Overall results of this study indicated that participants in the social constructivist group used statistically significantly more mathematical representations to solve PRS tasks and performed statistically significantly better than students who received teacher-centered textbook instruction. The findings provide evidence to support social constructivist learning theory when teaching proportions (e.g., Clements & Battista, 1990; Ernest, 1996; Forman, 1996; Fosnot, 2013; Simon & Schifter, 1993). These findings suggest that Latino middle school students participating in instructional experiences that involve manipulatives, representations, and discourse attained deeper and more meaningful mathematical understandings of proportional reasoning, and in doing so, developed a more sophisticated strategic competency for solving proportional reasoning tasks.

These findings build upon recent PR research advocating for instruction centered on social constructivist theory (Adjiage & Pluvinage, 2007; Cox, 2013; Cramer et al., 2016; Hamdan & Gunderson, 2017; Morton, 2014; Reed, 2008; Steinthorsdottir & Sriraman, 2009; Telese, 2002). Data from this study yielded statistical significance in the performance and use of mathematical representations to solve PRS tasks favoring participants from the social constructivist group. Thus, in line with the proponents of social constructivist theory and recent PRS research, the PRS performance of Latino middle school students is significantly impacted by participating in social constructivist learning experiences.

Performance Differences Between Treatment Conditions

Research question one. *What is the difference in performance, when solving proportional reasoning tasks, of Latino, middle school students who experience instruction involving manipulatives, representations, and discourse and those who receive teacher-centered textbook instruction?*

The first research question explored the PR performance differences using parametric and non-parametric statistics. ANOVA, ANCOVA, and Pearson Chi-Square statistics were used to assess the performance differences. A summary of key findings followed by a discussion of the students' PR performance and the intersections with social constructivist theory and recent research is presented below.

The students began with similar levels of mathematics ability as evidenced by the pre-test results. The implementation of instructional practices based on intensive collaboration, multimodal communication, and multiple representations resulted in the treatment group to gain a deeper understanding of proportional reasoning. The participants of the social constructivist outperformed the teacher-centered taught students. When controlling for initial mathematics ability of Latino middle school students, there was a positive learning experience that fostered a greater ability to solve proportional reasoning tasks. Students who experienced social constructivist teaching methods had more correct responses than those in the teacher-centered taught group. Both, parametric and non-parametric statistics yielded a significantly higher probability of attaining a correct response in the PRS posttest from participants of the social constructivist group than that of participants receiving teacher-centered textbook instruction. Thus, Latino middle school students receiving social constructivist theory instruction have a significantly higher probability of attaining a correct response in PRS tasks than participants receiving teacher-centered textbook instruction, while controlling for initial mathematics ability and about 20 percent of the group variance in PRS performance was explained by the method of instruction.

This finding aligns with that of Jitendra and colleagues (2013). Their pre- to posttest study reported that participants in the treatment classes outperformed those in the comparison classes on solving problems involving PRS. They found that instruction using an explicit problem-solving heuristic emphasizing the underlying problem structure via visual representations and encouraging procedural flexibility enhanced students' PRS performance. A similar result was obtained by Ceylan and Güler (2018). Their quasi-experimental pretest-study reported that teaching problem-solving strategies positively contributed to 6th-grade students' PRS. Thus, in concurrence with recent research findings (Ceylan & Güler, 2018; Jitendra, et al., 2013), data from this study highlighted significantly PRS performance differences between Latino middle school students receiving social constructivist instruction than those students experiencing teacher-centered PRS instruction, while controlling for initial mathematics ability.

This finding contributes to the development of a clearer understanding of previous research suggested instructional experiences to foster the development PRS (Adjiage & Pluvinage, 2007; Cramer et al., 2016; Hamdan & Gunderson, 2017; Morton, 2014; Reed, 2008; Steinthorsdottir & Sriraman, 2009). The strategies that previous research proposed were designed on individual components of social constructivist theory. This study provides new insight into the association between social relations (Dominguez et al., 2014; Montemayor et al., 2015) discourse (Wong Fillmore, 2007), and multimodal communication (Dominguez et al., 2014; Moschkovich, 2007) and Latino students' PR performance. Consequently, learning experiences that reflect multiple perspectives of a social constructivist theory significantly impact the proportional reasoning performance of Latino middle school students.

Differences by Proportional Reasoning Level

Research Question Two. *What are the differences between Latino, middle school students' ability to use integer relationships, partitioning, non-integer relationships, and multi-step relationships participating in experiences involving manipulatives, representations, and discourse and those who receive teacher-centered textbook instruction?*

The second research question explored the level of proportional reasoning of the students. Pearson Chi-Square tests assessed the group performance differences for each of the PR levels as defined by the revised PRS rubric proposed by Steinthorsdottir and Sriraman, (2009): integer scaling, one-step partitioning, non-integer scaling, and multistep PRS problems such as multi-step partitioning, unitizing, and covariation. A discussion of the students' performance and strategies implemented by PR level and the intersections with social constructivist theory and recent PR research is presented next.

Integer Relationships

Though a higher number of correct responses were obtained from participants receiving social constructivist instruction and a higher number of incorrect responses from participants receiving teacher-centered textbook instruction, the overall performance of participants from the social constructivist and teacher-centered textbook instruction on the PRS level of scaling up by an integer did not reveal a significant relationship. This result extends the claims of recent research that suggested that students are more successful with integer ratios (e.g., Carney, Smith, Hughes, Brendefur, Crawford, & Totorica, 2015; Fernández et al., 2011; Riehl & Steinthorsdottir, 2019). Students can correctly solve integer scaling PR problems using additive or multiplicative strategies, yet, this might not be indicative of a meaningful PR understanding, nor indicative of success with PR problems with increased difficulty (Carney et al., 2015).

In her examination of PR strategy levels comparing LEGO robotics and traditional mathematics experiences, Araceli Martínez Ortiz (2015) found a non-significant statistical difference for solving problems involving numerical computations between the two comparison groups with an effect size showing that only 2 percent of the variance was attributed to the instructional method. In alignment with her research, the findings of this study reported a non-significant difference and a low effect size, Latino students solve problems involving integer relationships with a comparable performance regardless of the teaching method. Thus, it can be interpreted that PR problems involving integer relationships did not pose higher levels of proportional reasoning resulting in non-statistically significant performance differences between the two comparison groups as suggested by recent PR research (Carney et al., 2015; Riehl & Steinthorsdottir, 2019).

One-Step Partitioning Relationships

The second PR level analyzed consisted of evaluating the participants' ability to solve problems that involve one-step partitioning. A higher number of correct responses from participants receiving social constructivist instruction was observed. Contrarily, a higher number of incorrect responses were obtained from participants receiving teacher-centered textbook instruction. This resulted in a significant relationship with a medium effect size between the performance of both groups for the PR level of partitioning.

This finding contradicts and extends past research claiming that students encounter greater challenges to solve problems involving one-step partitioning (Moyer-Packenham et al., 2014; Riehl & Steinthorsdottir, 2019). In their exploration of number structure characteristics on student thinking in solving proportion problems, Riehl and Steinthorsdottir (2019) found that shrink problems are harder than enlarge problems. They noted that the learning to partition ratios is fragile and "There is little middle ground between students who use a multiplicative solution method and those who do not use proportional reasoning" (p.66). Though tasks involving one-step partitioning pose greater difficulties, a significant relationship in the performance between the participants of the two comparison groups suggests that participants receiving social constructivist learning experiences recognize that a ratio can be partitioned and thus, solved significantly more correct PRS problems.

A possible reason for a greater strategic competence to solve PR tasks involving one-step partitioning relationships can be derived from the findings of Moyer-Packenham and colleagues (2014). They found that the use of manipulative materials and models to split the whole into equal parts allowed learners to conceptualize and make sense of PR tasks. The use of manipulatives and representations provided students with a deeper understanding, resulting in an advantage to execute procedures and solve these PR tasks. Thus, social constructivist theory experiences significantly impact Latino students' performance on proportional relationships involving one-step partitioning.

Non-Integer Relationships

A statistically significant relationship existed between participants from the social constructivist group and those receiving teacher-centered textbook instruction in their ability to solve PR problems involving non-integer relationships. This result contradicts and extends prior PR research suggesting that students were less successful with non-integer ratios (Carney, et al., 2015; Fernández et al., 2011; Hilton, A., Hilton, G., Dole, & Goos, 2016; Riehl & Steinthorsdottir, 2019). Hilton et al., (2016) found that students in lower middle grades experience greater difficulties than grade 8 and 9 students to solve tasks involving non-integer relationships. Similarly, Riehl and Steinthorsdottir (2019) found that the students encountered increased difficulties to scale by a non-integer and that these obstacles result in lower PRS performance. In contrast, data from this study showed that participants of social constructivist theory instruction solved significantly more correct PR problems involving non-integer relationships than those students receiving teacher-centered textbook instruction.

A possible explanation for greater attainment to solve these tasks can be attributed to the use of manipulative materials and multiple representations. These permit Latino middle school students to recognize the properties and processes involved resulting in a more meaningful PRS understanding (Moyer-Packenham, et al., 2014). More refined understandings allowed students to use manipulatives and representations strategically and successfully implement procedures for solving the tasks (Clements & Battista, 1990). It can be interpreted that the use of concrete manipulatives and representational reasoning empowered Latino middle school students with deeper PRS understanding so that their strategic competence to solve non-integer relationships is significantly greater.

Multi-Step Relationships

The participants' ability to solve PRS problems involving multi-step relationships such as unitizing, multi-step partitioning and covariation was not statistically significantly different between the comparison groups. Although a higher number of correct responses were obtained from participants receiving social constructivist instruction and a higher number of incorrect responses were obtained from participants receiving teacher-centered textbook instruction, the performance differences were not statistically significant. This finding can be explained by the claim of Steinthorsdottir and Sriraman, (2009) in that "The transition to Level 4, which involves explicit awareness of within and between multiplicative relationships, took greater time and effort" (p.6). In their study, only 12 percent of the participants reached level 4 of proportional reasoning, whereas, in this study, 37.5 percent of the participants receiving social constructivist instruction attained the correct answer compared to 23.6 percent of students receiving teacher-centered textbook instruction.

Thus, in line with past research, this data suggests that PRS problems involving multi-step relationships pose higher levels of proportional reasoning and that participants require more time to master these skills (Lobato et al., 2010; Steinthorsdottir & Sriraman, 2009). It can be interpreted that four weeks of social constructivist PRS instruction was not enough for developing a meaningful understanding of PRS on problems involving multi-step relationships, resulting in a non-statistically significant difference in group performance. Regardless of the instructional approach, social constructivist or teacher-centered, Latino middle school students perform at a comparable success rate in PRS problems involving multi-step relationships.

In sum, these results indicated than participants receiving social constructivist theory instruction performed better than those receiving teacher-centered textbook instruction in all four levels of proportional reasoning. Building on prior PRS research claiming that instruction that encouraged flexibility to use multiple representations and solution strategies, as well as the discussion of the efficiency of strategies, significantly impacted students' PRS performance (Lamon, 2012; Lobato et al., 2010; Matney et al., 2013). Latino middle school students participating in such experiences developed more robust strategic competence. A significant relationship between the performance for solving PRS tasks involving one-step partitioning and non-integer relationships provides a clearer understanding of how social constructivist instruction promotes a deeper understanding of PRS. Though for tasks involving integer relationships and multi-step relationships the students from the social constructivist group performed better, there was not a significant relationship between comparison groups. A possible explanation for this relates to the amount of exposure to social constructivist theory experiences involving integer and multi-step relationships. Four weeks of PRS instruction does not significantly promote PRS. Therefore, participants from the social constructivist group have a higher probability of attaining correct responses on the PRS levels of one-step partitioning and non-integer relationships than those receiving teacher-centered textbook instruction.

Analysis of Strategies Implemented in Posttest

Research Question Three. *What are the differences in strategies used for solving proportional reasoning tasks between Latino, middle school students experiencing instruction that involves manipulatives, representations, and discourse, and those who receive teacher-centered textbook instruction?*

The third research question explored strategies used to solve PRS tasks with an emphasis on the use of sophisticated strategies involving representational reasoning and native strategies involving equivalent ratios and random calculations. A Pearson Chi-Square statistic was used to assess if a relationship existed between the two comparison groups, social constructivist, and teacher-centered textbook instruction. In particular, the researcher explored if there was a relationship by group of affiliation and the use of strategies that involved representational reasoning consisting of ratio tables and a double number line and those strategies that involved algebraic algorithms such as equivalent ratios and random calculations for solving PRS problems. A discussion of the students' use of tables, number lines, and multiple representations and the intersections with social constructivist theory and recent research is presented beneath.

The analysis of the use of representational reasoning for solving PRS tasks yielded statistical significance between the comparison groups. Overall, participants of the social constructivist group solved a higher number of problems using representational reasoning such as a double number line and a ratio table. This finding aligned with the perspectives of Ernest (1996), who stressed the importance of tools, language, and the social context for understanding mathematics. A marked difference between participants from the two groups was observed in the strategies that did not involve mathematical representations for solving PRS tasks. Students receiving teacher-centered textbook instruction mainly used equivalent ratios and random calculations. Then, students receiving teacher-centered textbook instruction solved a significantly higher number of tasks without using sophisticated approaches involving representational reasoning.

This finding provides new insight into the relationship between the use of representational reasoning and the development of PRS of Latino middle school students. Participants receiving social constructivist learning experiences used significantly more tasks using representational reasoning than those receiving teacher-centered textbook instruction. As Cobb (2007) contended, learning experiences designed with a social constructivist view promote meaningful learning of mathematics concepts. In addition, Lamon (2012) claimed that using a ratio table, aids with the learners' organization of work and combines arithmetic operations that result in equivalent rations which in turn allow them to attain the desired quantity. In line with the findings of this study, research on PRS advocated for the implementation of experiences involving representational reasoning to promote PRS proficiency. Representations deepen conceptual understanding and procedural fluency, as they highlight the multiplicative relationships between context and cognitive experience (Clements & Sarama, 2014; Matney, et al., 2013). This allows students to create meaning and become empowered to transfer to symbolic mathematics (Richardson, 2012).

Möhring, Newcombe, Levine, & Frick, (2016) found that PRS understanding was associated with the use of multiple representations of fractions. They claimed that using multiple representations of fractions promoted fractions knowledge, which in turn helped students to make better judgments about proportions. Lamon (2012) also claimed that concrete and mathematical representations significantly impacted the students' ability to "think about the quantities and how they are related to each other in order to determine appropriate operations" (p.25). Similarly, Carney and colleagues (2015) contended that students used different strategies to correctly solve a problem involving PRS. This study found that the teaching method significantly impacts the use of sophisticated representational reasoning for solving PRS tasks. This process facilitates the identification of relationships and refines the comprehension of proportional reasoning concepts. Thus, in line with recent research (e.g., Clements & Battista, 1990; Cobb et al, 1996; Cramer et al., 2016; Ernest, 1996; Fosnot, 2013; Hennig, 2010; Reed, 2008; Richardson, 2012; Moyer-Packenham et al., 2014; NCTM, 2014; Telese, 1999; Utley & Reeder, 2012) it can be inferred that a statistically significantly greater use of mathematical representations significantly fosters Latino middle school students' proportional reasoning proficiency.

Conclusion

The main goal of this study was to evaluate the impact of social constructivist learning experiences on Latino middle school students' proportional reasoning. Though PRS research documented that PRS concepts are cognitively and mathematically complicated (Lamon, 2012; Lobato et al., 2010), data from this study, consistent with SCT proponents (e.g., Cobb, 2007; Burkhardt, 2007; Ernest, 1996; Simon, 1995; Telese, 1999) documented how using classroom discourse, concrete manipulatives, and multiple representations foster Latino middle school students PR proficiency. Participants receiving social constructivist instruction used significantly greater representational reasoning and a greater use of representational reasoning resulted in significantly higher PR strategic competence. Social constructivist experiences resulted in a more meaningful understanding empowering Latino middle school students with stronger connections between the central ideas of proportional reasoning. A significantly greater representational reasoning of proportional relationships using number lines, X-Y tables of values, and concrete manipulatives resulted in statistically significantly higher PR strategic competence. Similarly, the students' ability to solve tasks involving one-step partitioning and non-integer in PRS relationships was statistically significantly promoted. Therefore, social constructivist learning experiences significantly impacted Latino middle school students' strategic competence for solving proportional reasoning tasks.

Future Research

The most important recommendation for further research relates to the length of the intervention to evaluate the impact of social constructivist theory on students' strategic competence on multi-step proportional relationships. As documented by prior PRS research, the development of PRS is a long-term learning process (Lamon, 2012; Riehl & Steinthorsdottir, 2019; Steinthorsdottir & Sriraman, 2009). It would be noteworthy to assess the impact of social constructivist theory on a long-term intervention. In addition, it is interesting to explore the effect of social constructivist theory on the PRS of a more diverse population. Comparing the gains between the different ethnical groups in Texas and the United States is important. Measuring the gains by ethnical group can provide teachers with an important basis for designing PR learning experiences. Last, it would be paramount to compare the impact of social constructivist theory on middle school students with low- and high- mathematics ability.

Limitations

There are several limitations in this dissertation that require discretion when interpreting the results. This study was limited to a single ethnical group of students selected using non-random assignment of participants. Another potential limitation of this study is that the research design was not fully experimental. For comparing the two groups, the researcher utilized preexisting classrooms of students. This less rigorous design quasi-experimental design there involves more threats to validity and thus, impacts the effects on the teaching method.

Implications of the Findings for Instruction and Teaching PR

This study provided strong evidence that social constructivist theory impacts the proportional reasoning performance of Latino middle school students. It was statistically demonstrated that PRS are significantly impacted by instruction centered on discourse and representational reasoning. The principles for effective mathematics teaching proposed by the NCTM (2014) involving social interactions and tools facilitate Latino middle school students' PR proficiency. An active engagement in explorations for constructing meaning using manipulatives, making connections among multiple representations, as well as explaining and justifying approaches, resulted in more effective problem-solving strategies, a deeper conceptual understanding, and strengthened their strategic competence (Cobb, 2007; Simon, 1995) on proportional reasoning. Hence, understanding how social constructivist theory impacts the PR proficiency of Latino middle school students provides a foundation for increasing engagement and thus, narrowing the learning gaps between Latino students and those from other ethnical groups.

REFERENCES

Adjiage, R., & Pluvinage, F. (2007). An experiment in teaching ratio and proportion. *Educational Studies in Mathematics*, *65*(2), 149-175.

Amineh, R. J., & Asl, H. D. (2015). Review of constructivism and social constructivism. *Journal of Social Sciences, Literature and Languages*, *1*(1), 9-16.

Aron, A., Aron, E., & Coups, E. J. (2008). *Statistics for the behavioral and social sciences: A brief course*. Upper Saddle River, NJ: Pearson Prentice Hall.

Ayan, R., & Bostan, M. I. (2018). Middle school students' reasoning in nonlinear proportional problems in geometry. *International Journal of Science and Mathematics Education*, *16*(3), 503-518.

Bostic, J. D., Pape, S. J., & Jacobbe, T. (2016). Encouraging sixth-grade students' problem-solving performance by teaching through problem solving. *Investigations in Mathematics Learning*, *8*(3), 30–58.

Boyer, T. W., & Levine, S. C. (2015). Prompting children to reason proportionally: Processing discrete units as continuous amounts. *Developmental Psychology*, *51*(5), 615.

Brooks, J. G. (1999). *In search of understanding: The case for constructivist classrooms*. Alexandria, VA: ASCD.

Burkhardt, H. (2007). Mathematical proficiency: What is important? How can it be measured? In A. Schonefeld, (Eds.) *Assessing Mathematical Proficiency* (75-99). New York: Cambridge University Press.

Carney, M., Smith, E., Hughes, G., Brendefur, J., Crawford, A., & Totorica, T. (2015). *Analysis of students' proportional reasoning strategies*. Proceedings of the North American Chapter of the International Group for the Psychology of Mathematics Education. East Lansing, MI: Michigan State University.

Ceylan, Ş. E. N., & Güler, G. (2018). Effect of strategy teaching for the solution of ratio problems on students' proportional reasoning skills. *MOJES: Malaysian Online Journal of Educational Sciences*, *5*(2), 1-15.

Che, M., Wiegert, E., & Threlkeld, K. (2012). Problem solving strategies of girls and boys in single-sex mathematics classrooms. *Educational Studies in Mathematics*, *79*(2), 311-326.

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach*. New York, NY: Routledge.

Clements, D. H., & Battista, M. T. (1990). Constructivist learning and teaching. *Arithmetic Teacher*, *38*(1), 34-35.

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Hillsdale, NJ: Lawrence Earlbaum Associates.

Cobb, P. (2007). Putting philosophy to work: Coping with multiple theoretical perspectives. *Second handbook of research on mathematics teaching and learning,* pp. 3-39. Charlotte, NC: Information Age.

Cobb, P., Jaworsky, B., & Presmeg, N. (1996). Learning mathematics as a meaningful activity. In Steffe, L. P., Nesher, P., Cobb, P., Sriraman, B., & Greer, B. (Eds.). (1996). *Theories of mathematical learning*. (pp. 91-113). Mahwah, NJ: Laurence Erlbaum Publishers.

Cox, D. C. (2013). Similarity in middle school mathematics: At the crossroads of geometry and number. *Mathematical Thinking and Learning*, *15*(1), 3-23.

Cramer, K., Ahrendt, S., Monson, D., Wyberg, T., & Miller, C. (2016). Making sense of third-grade students' misunderstandings of the number line. *Investigations in Mathematics Learning*, *9*(1), 19–37.

Creswell, J. W. (2015). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research.* Upper Saddle River, NJ: Pearson

DeJarnette, A. F., Walczak, M., & González, G. (2014). Students' concepts-and theorems-in-action on a novel task about similarity. *School Science and Mathematics*, *114*(8), 405-414.

Dominguez, H., LópezLeiva, C. A., & Khisty, L. L. (2014). Relational engagement: Proportional reasoning with bilingual Latino/a students. *Educational Studies in Mathematics*, *85*(1), 143-160.

Ernest, P. (1996). Varieties of constructivism: A framework for comparison. In L. Steffe, P. Nesher, P. Cobb, G. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (335-350). Mahwah, NJ: Lawrence Erlbaum Associates.

Fernández, C., Llinares, S., Van Dooren, W., De Bock, D., & Verschaffel, L. (2011). Effect of number structure and nature of quantities on secondary school students' proportional reasoning. *Studia Psychologica*, *53*(1), 69.

Fonger, N. L., Tran, D., & Elliott, N. (2015). Variation in children's understanding of fractions: preliminary findings. *Grantee Submission*. Retrieved from http://ezhost.utrgv.edu:2048/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=ED566521&site=ehost-live

Forman, E.A. (1996). Learning mathematics as participation in classroom practice: Implications of sociocultural theory for educational reform. In L. P. Steffe, P. Nesher, P. Cobb, B. Sriraman, & B. Greer. (Eds.), *Theories of mathematical learning* (pp. 115-130). Mahwah, NJ: Laurence Erlbaum Publishers.

Fosnot, C. T. (2013). *Constructivism: Theory, perspectives, and practice*. New York, NY: Teachers College Press.

Fosnot, C. T., & Dolk, M. L. A. M. (2001). *Young mathematicians at work*. Portsmouth, NH: Heinemann.

Fuchs, L. S., Schumacher, R. F., Long, J., Namkung, J., Hamlett, C. L., Cirino, P. T., ... & Changas, P. (2013). Improving at-risk learners' understanding of fractions. *Journal of Educational Psychology*, *105*(3), 683.

Gleason, J., Livers, S., & Zelkowski, J. (2017). Mathematics classroom observation protocol for practices (MCOP2): A validation study. *Investigations in Mathematics Learning*, *9*(3), 111-129.

Greeno, J. G. (2006). Theoretical and practical advances through research on learning. In J. Green, G. Camilli, & P. B. Elmore (Eds.), *Handbook of complementary methods in educational research* (795-822). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Hamdan, N., & Gunderson, E. A. (2017). The number line is a critical spatial-numerical representation: Evidence from a fraction intervention. *Developmental Psychology*, *53*(3), 587.

Hennig, C. (2010). Mathematical models and reality: A constructivist perspective. *Foundations of Science*, *15*(1), 29-48.

Herbst, P., Fujita, T., Halverscheid, S., & Weiss, M. (2017). *The learning and teaching of geometry in secondary schools: A modeling perspective*. New York, NY: Routledge.

Hilton, A., Hilton, G., Dole, S., & Goos, M. (2016). Promoting middle school students' proportional reasoning skills through an ongoing professional development programme for teachers. *Educational Studies in Mathematics*, *92*(2), 193-219.

Howe, C., Nunes, T., & Bryant, P. (2011). Rational number and proportional reasoning: Using intensive quantities to promote achievement in mathematics and science. *International Journal of Science and Mathematics Education*, *9*(2), 391-417.

Jeong, Y., Levine, S. C., & Huttenlocher, J. (2007). The development of proportional reasoning: Effect of continuous versus discrete quantities. *Journal of Cognition and Development*, *8*(2), 237-256.

Jitendra, A. K., Star, J. R., Dupuis, D. N., & Rodriguez, M. C. (2013). Effectiveness of schema-based instruction for improving seventh-grade students' proportional reasoning: A randomized experiment. *Journal of Research on Educational Effectiveness*, *6*(2), 114-136.

Lamon, S. J. (2012). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*. New York, NY: Routledge.

Lipsey, M. W. (1990). *Design sensitivity: Statistical power for experimental research*. Thousand Oaks, CA: Sage.

Lobato, J., Ellis, A., & Zbiek, R. M. (2010). *Developing essential understanding of ratios, proportions, and proportional reasoning for teaching mathematics: Grades 6-8*. Reston, VA: National Council of Teachers of Mathematics.

Matney, G., Jackson, J. L., & Bostic, J. (2013). Effects of minute contextual experience on realistic assessment of proportional reasoning. *Investigations in Mathematics Learning*, *6*(1), 41-68.

Martínez Ortiz, A. (2015). Examining students' proportional reasoning strategy levels as evidence of the impact of an integrated LEGO robotics and mathematics learning experience. *Journal of Technology Education*, *26*(2), 46-69.

Mejia Colindres, C. A. (2015). *The role of translanguaging in Latino mathematics classrooms* (Unpublished doctoral dissertation, Texas State University). Retrieved from http://ezhost.utrgv.edu:2048/login?url=https://search-proquest-com.ezhost.utrgv.edu/docview/1712381070?accountid=7119

Mills, G. E., & Gay, L. R. (2016). *Educational research: Competencies for analysis and applications*. Upper Saddle Ridge, NJ: Pearson.

Montemayor, D., Kupczynski, L., & Mundy, M. A. (2015). Achievement and experiences of first and second-generation students in a rural south Texas high school. *Journal of Instructional Pedagogies*, *16*, 1.

Morton, C. H. (2014). A story of african american students as mathematics learners. *International Journal of Education in Mathematics, Science and Technology*, *2*(3), 234-245.

Moschkovich, J. (2007). Beyond Words to Mathematical Content: Assessing English Learners in the Classroom. In A. Schonefeld, (Eds.) *Assessing mathematical proficiency* (345-353). New York: Cambridge University Press.

Moyer-Packenham, P. S., Bolyard, J. J., & Tucker, S. I. (2014). Second-graders' mathematical practices for solving fraction tasks. *Investigations in Mathematics Learning*, *7*(1), 54-81.

Möhring, W., Newcombe, N. S., Levine, S. C., & Frick, A. (2016). Spatial proportional reasoning is associated with formal knowledge about fractions. *Journal of Cognition and Development*, *17*(1), 67-84.

Namkung, J. M., & Fuchs, L. S. (2016). Cognitive predictors of calculations and number line estimation with whole numbers and fractions among at-risk students. *Journal of Educational Psychology*, *108*(2), 214-228.

National Council of Teachers of Mathematics (NCTM). (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: Author.

National Council of Teachers of Mathematics (Ed.). (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Center for Education Statistics., National Assessment of Educational Progress (Project), Educational Testing Service., & United States. (2017). *NAEP ... mathematics report card for the nation and the states*. Washington, D.C: National Center for Education Statistics, Office of Educational Research and Improvement, U.S. Dept. of Education. Retrieved from https://www.nationsreportcard.gov/math_2017/nation/scores?grade=4

National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). *Common Core State Standards for mathematics.* Washington, DC: Authors. Retrieved from http://www.corestandards.org/wp-content/uploads/Math_Standards1.pdf

National Research Council (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: The National Academies Press.

Ojose, B. (2015). Proportional reasoning and related concepts: Analysis of gaps and understandings of middle grade students. *Universal Journal of Educational Research*, *3*(2), 104-112.

Orosco, M. J. (2013). The development of a math strategy in Spanish for Latino English language learners at risk for math disabilities. *International Journal for Research in Learning Disabilities, 1*(2), 86–109.

Özgün-Koca, S. A., & Altay, M. K. (2009). An investigation of proportional reasoning skills of middle school students. *Investigations in Mathematics Learning*, *2*(1), 26-48.

Pelen, M. S., & Artut, P. D. (2016). Seventh grade students' problem solving success rates on proportional reasoning problems. *International Journal of Research in Education and Science*, *2*(1), 30-34.

Piaget, J. (1977). *The development of thought: Equilibration of cognitive structures. (Trans A. Rosin)*. New York: The Viking Press.

Reed, M. K. (2008). Comparison of the place value understanding of montessori elementary students. *Investigations in Mathematics Learning*, *1*(1), 1-26.

Richardson, K. (2012). Signs and tools of algebraic reasoning: A study of models among fifth grade students. *Investigations in Mathematics Learning*, *5*(1), 21-37.

Riehl, S. M., & Steinthorsdottir, O. B. (2019). Missing-value proportion problems: The effects of number structure characteristics. *Investigations in Mathematics Learning*, *11*(1), 56-68.

Shannon, A. (2007). Task context and assessment. In A. Schonefeld, (Eds.) *Assessing mathematical proficiency* (177-193). New York: Cambridge University Press.

Schoenfeld, A. (2007). Issues and tensions in the assessment of mathematical proficiency. In Schoenfeld, A. (Eds.) *Assessing mathematical proficiency* (3-17). New York: Cambridge University Press.

Short, D. J., Echevarría, J., & Richards-Tutor, C. (2011). Research on academic literacy development in sheltered instruction classrooms. *Language Teaching Research*, *15*(3), 363-380.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, *26*(2), 114-145.

Simon, M. A., & Schifter, D. (1993). Toward a constructivist perspective: The impact of a mathematics teacher inservice program on students. *Educational Studies in Mathematics*, *25*(4), 331-340.

Steinthorsdottir, O. B., & Sriraman, B. (2009). Icelandic 5th-grade girls' developmental trajectories in proportional reasoning. *Mathematics Education Research Journal*, *21*(1), 6-30.

Texas Education Agency. (2018). *STAAR Statewide Summary Reports 2017-2018.* Retrieved from https://tea.texas.gov/Student_Testing_and_Accountability/Testing/State_of_Texas_Assessments_of_Academic_Readiness_(STAAR)/STAAR_Statewide_Summary_Reports_2017-2018/

Texas Education Agency. (2012). *Texas Essential Knowledge and Skills.* Retrieved from http://www.tea.state.tx.us/index2.aspx?id=6148.

Telese, J. A., & Avalos, B. (2013). *Coaching mathematics teachers to teach for understanding.* Paper presented at the Annual Convention of School Science and Mathematics Association, San Antonio, Tx. November 2013.

Telese, J. A., & Abete Jr, J. (2002). Diet, ratios, proportions: A healthy mix. *Mathematics Teaching in the Middle School*, *8*(1), 8.

Telese, J. A. (1999). *The role of social constructivist philosophy in the teaching of school algebra and in the preparation of mathematics teachers*. Paper presented at the Annual Meeting of the Association of Teacher Educators. Chicago, IL. February 1999. Retrieved from http://ezhost.utrgv.edu:2048/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=eric&AN=ED432469&site=ehost-live

United States Census Bureau (2011). Selected housing charactersitcs, 2007-2011 *American community survey 5-year estimates*. Retrieved from https://factfinder.census.gov/faces/tableservices/jsf/pages/productview.xhtml?pid=DEC_10_DP_DPDP1&src=pt

Utley, J., & Reeder, S. (2012). Prospective elementary teachers' development of fraction number sense. *Investigations in Mathematics Learning*, *5*(2), 1-13.

Van Oers, B. (1996). Learning mathematics as a meaningful activity. In Steffe, L. P., Nesher, P., Cobb, P., Sriraman, B., & Greer, B. (Eds.). (1996). *Theories of mathematical learning*. (pp. 91-113). Mahwah, NJ: Laurence Erlbaum Publishers.

Voigt, J. (1996). Negotiation of mathematical meaning in classroom processes: Social interaction and learning mathematics. In Steffe, L. P., Nesher, P., Cobb, P., Sriraman, B., & Greer, B. (Eds.). (1996). *Theories of mathematical learning*. (pp. 21-50). Mahwah, NJ: Laurence Erlbaum Publishers.

Vygotsky, L. S. (1978). *Mind in society: The development of higher mental process.* Cambridge: Harvard University Press.

Warner, R. M. (2012). *Applied statistics: from bivariate through multivariate techniques: from bivariate through multivariate techniques*. Thousand Oaks, CA: Sage.

Wong Fillmore, L. (2007). English learners and mathematics learning: Language issues to consider. In A. Schonefeld, (Eds.) *Assessing mathematical proficiency* (333-345). New York: Cambridge University Press.

Xin, Y. P., Jitendra, A. K., & Deatline-Buchman, A. (2005). Effects of mathematical word problem-solving instruction on middle school students with learning problems. *The Journal of Special Education*, *39*(3), 181-192.

APPENDIX A

APPENDIX A

PRETEST AND POSTTEST OF PROPORTIONAL REASONING SKILLS

Question 1: The animal control unit has a pick-up truck that collects stray dogs. In 4 hours, they collected 16 dogs. At this rate, how long will it take the pick-up truck to catch 48 dogs?

Question 2: A group of UIL students is planning their end-of-the-year trip to South Padre Island. For every 5 students, they need 7 chaperons in order to meet safety needs. If there are 40 UIL students traveling, how many chaperons will they need for the trip?

Question 3: Juan works at the stable and it is feeding time. If 24 horses eat 40 bales of hay each day, how many bales would 6 horses eat?

Question 4: Frank plays at the Middle School soccer team. If he scores 36 goals in 24 games, how many goals would he score in 8 games?

Question 5: At the Border Fest, students record the number of visitors at regular intervals. They record 60 people in 20 minutes. How many people do they record in 45 minutes?

Question 6: Grandma has some recipe to make guacamole for flautas. She uses 3 scoops of guacamole to serve 9 flautas. There are 11 scoops of guacamole left. Using the same amount, how many flautas can she serve?

Question 7: Cristina is making lemonade to sell at the park. She needs 18 ounces of sugar for 16 glasses of lemonade. La Michoacana Meat Market sells sugar in bags of 45 ounces. How many lemonade glasses can she make with 45 ounces of sugar?

Question 8: The Spanish Club makes Calaveritas for Dia de los Muertos. The teacher gave students 42 calaveritas which was enough for 35 teachers. How many calaveritas do they need for the 10 math teachers at the middle school?

APPENDIX B

APPENDIX B

RUBRIC OF PROPORTIONAL REASONING LEVELS

Strategy Type	Description of strategy
Level 0: *Incorrect Strategy* Erroneous Answer: Illogical or Additive Partially correct: Conceptual error and/or Computation error Strategy without reasoning proportionally: Cross-Multiplication.	Limited Ratio Knowledge – use of random calculations or additive thinking, clear evidence of lack of understanding the multiplicative relationship. A conceptual or computation error (e.g., incorrect reasoning, misinterpreting the remainder in a division) produces an unreasonable answer. Solving an algebraic algorithm: The numerator of one ratio is multiplied by the denominator of the other ratio and the result is divided by the denominator of the other ratio.
Level 1: *Integer Multiplicative Reasoning.*	The ratio is seen as a single unit; combining units together by repeated addition to itself or by multiplying that ratio by a whole number, but the ratio is not partitioned. Resulting in a build-up strategy with addition and multiplication, or a combination of the two operations.
Level 2: *Integer Multiplicative Reasoning with One-Step Partitioning.*	As above and involves one step partitioning.
Level 3: *Non-integer Multiplicative Reasoning.*	The ratio is seen as a single unit; however, scaling occurs by non-integers, solving integer, and non-integer multiplicative reasoning. There is effective scaling by using build-up strategies, either by adding or multiplying, but not unitizing, with a reduction of the given ratio.
Level 4: *Unitizing, Non-Integer Multiplicative Reasoning, and Multi-step Partitioning.*	Flexible and efficient use of heuristics. Ratios are seen as being more than unit quantities. There is a focus on numbers and numeric relationships to determine effective strategy.

BIOGRAPHICAL SKETCH

Victor Manuel Vizcaino received a bachelor's degree in Electronic Engineering from the Universidad Autonoma de Guadalajara, México in 1994. He received a master's degree in Science in Mathematics Teaching from The University of Texas Pan American in 2011. He received a doctor's degree in Curriculum and Instruction with a specialization in Mathematics Education from The University of Texas Rio Grande Valley in 2020. He taught mathematics in Lyford, Texas from 2007 to 2009 and middle school mathematics in Pharr, Texas from 2009 to 2020. From 2018 to 2020 he taught mathematics courses for in-service teachers and school administrators at The University of Texas Rio Grande Valley.

Email address: victor.vizcaino01@trgv.edu