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Distance Measures of Complex Fermatean Fuzzy Number and Their Application to Multi-criteria Decision-making Problem

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Abstract

Multi-criteria decision-making (MCDM) is the most widely used decision-making method to solve many complex problems. However, classical MCDM approaches tend to make decisions when the parameters are imprecise or uncertain. The concept of a complex fuzzy set is new in the field of fuzzy set theory. It is a set that can collect and interpret the membership grades from the unit circle in a plane instead of the interval [0,1]. CFS cannot deal with membership and non-membership grades, while complex intuitionistic fuzzy set and complex Pythagorean fuzzy set works only for a limited range of values. The concept of a complex Fermatean fuzzy set (CFFS) is proposed to deal with these problems. This paper presents the main ideas of CFFN and its properties are studied. The proposed new distance measures for real-world problems are also discussed. A comparative study of the proposed new work is also conducted.

Keywords: Multi-criteria decision-making; Complex fuzzy set; Complex intuitionistic fuzzy set; Complex Pythagorean fuzzy set; Complex Fermatean fuzzy set

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1. Introduction

The study of uncertainty was considered to be unscientific until the introduction of fuzzy sets (FS) by Zadeh (1965). Atanossov (1986) presented the idea of intuitionistic FS (IFS), which formed another generalization of fuzzy sets. Xu (2010) developed the concept of Choquet integrals of weighted operator on IFS. Yager (1988) introduced the idea of weighted aggregation operators in decision making problems. Garg (2017) extended the concept of distance measure (DM) and similarity measure (SM) for IFS. Garg and Kumar (2018) presented DM idea for decision making problems. Singh and Garg (2017) further introduced the notion of DM between type-2 IFS and discussed some of its properties. Peng and Yang (2015) presented the concept of Pythagorean FS (PFS) and discussed some of its properties. Li and Zeng (2018) defined the notion of DM using PFS. Further, Zeng et al. (2018) established the properties of PFS and explained the concept with a real-life application. Senapati and Yager (2020; 2019) introduced the concept of Fermatean FS (FFS). They discussed some of its properties and applied the concepts of FFS in multi-criteria decision making (MCDM) problems.

Ramot et al. (2003) defined the concept of complex FS (CFS) and studied its properties. Hu et al. (2019) explained the notion to determine the distance between two CFSs and established its properties. Chinnadurai et al. (2020b) defined the concept of complex cubic set (CCS) and established some of its properties. Zhou et al. (2020) introduced the idea of aggregation operators in CCS with an application in MCDM problems. Alkouri and Salleh (2012) presented the concept of complex IFS (CIFS) and studied its fundamental properties. Later, Garg and Rani (2020; 2019) used aggregation operators and T-norm operations on CIFS and applied the concepts of CIFS in multi-criteria decision making (MCDM) problems. Kumar and Bajaj (2014) extended the concept of DM and entropies in CIFS using parameters.

Chinnadurai et al. (2020a) introduced the concept of complex cubic IFS (CCIFS) and discussed its properties. They also presented the notion of complex cubic interval valued IFS (CCIVIFS) with a case study. Akram et al. (2020) developed the concept of complex Pythagorean fuzzy Yager aggregation operator and applied it in real-life situations. Chinnadurai et al. (2020c) discussed the idea of complex interval-valued PFS (CIVPFS) and its properties. Ullah et al. (2020) studied the properties of complex PFS (CPFS) and discussed its DM with a pattern recognition application. Chinnadurai et al. (2021a) introduced the concept of complex FFS (CFFS) and established its properties. The study aims to explain the limitation of existing complex theories and discuss the superiority of CFFS. The concept of CFFS is proposed to address the information that could not be processed using CFS, CIFS and CPFS.

2. Distance Measures of CFFNs

The motive of this section is to develop some distance measures (DMs) and weighted distance measures (WDMs) for CFFNs.

Definition 2.1.

For a CFFS $\mathbf{A}$, we define $\mathbb{A} = (P_{\mathbb{A}}.e^{i2\pi\theta_{P_{\mathbb{A}}}}, Q_{\mathbb{A}}.e^{i2\pi\theta_{Q_{\mathbb{A}}}})$ is called complex Fermatean fuzzy number (CFFN), where $P_{\mathbb{A}}, Q_{\mathbb{A}} \in [0,1]$, $(P_{\mathbb{A}})^3 + (Q_{\mathbb{A}})^3 \leq 1$ and $\theta_{P_{\mathbb{A}}}, \theta_{Q_{\mathbb{A}}} \in [0,1]$, $(\theta_{P_{\mathbb{A}}})^3 + (\theta_{Q_{\mathbb{A}}})^3 \leq 1$. In order to apply CFFNs to practical problems, it is essential to rank these numbers.

Definition 2.2.

For two CFFNs $\mathbb{A} = (P_{\mathbb{A}}.e^{i2\pi\theta_{P_{\mathbb{A}}}}, Q_{\mathbb{A}}.e^{i2\pi\theta_{Q_{\mathbb{A}}}})$ and $\mathbb{B} = (P_{\mathbb{B}}.e^{i2\pi\theta_{P_{\mathbb{B}}}}, Q_{\mathbb{B}}.e^{i2\pi\theta_{Q_{\mathbb{B}}}})$ we define
(i) $\mathbb{A} \subseteq \mathbb{B}$ if $P_{\mathbb{A}} \leq P_{\mathbb{B}}, Q_{\mathbb{A}} \geq, Q_{\mathbb{B}}$ and $\theta_{P_{\mathbb{A}}} \leq \theta_{P_{\mathbb{B}}}, \theta_{Q_{\mathbb{A}}} \geq \theta_{Q_{\mathbb{B}}}$.
(ii) $\mathbb{A} = \mathbb{B}$ if $P_{\mathbb{A}} = P_{\mathbb{B}}, Q_{\mathbb{A}} =, Q_{\mathbb{B}}$ and $\theta_{P_{\mathbb{A}}} = \theta_{P_{\mathbb{B}}}, \theta_{Q_{\mathbb{A}}} = \theta_{Q_{\mathbb{B}}}$.

Definition 2.3.

A DM_1 for CFFN is defined as

$$\begin{aligned}\mathsf{DM}_1(\mathbb{A},\mathbb{B}) =& \frac{1}{2}\sum_{i=1}^{n}\Big[\alpha_1.\left|P^3_{\mathbb{A}_i} - P^3_{\mathbb{B}_i}\right| + \beta_1.\left|Q^3_{\mathbb{A}_i} - Q^3_{\mathbb{B}_i}\right| + \gamma_1.max\Big(\left|P^3_{\mathbb{A}_i} - P^3_{\mathbb{B}_i}\right|, \left|Q^3_{\mathbb{A}_i} - Q^3_{\mathbb{B}_i}\right|\Big) \\ &+ \frac{1}{2\pi}\Big(\alpha_2.\left|\theta^3_{P_{\mathbb{A}_i}} - \theta^3_{P_{\mathbb{B}_i}}\right| + \beta_2.\left|\theta^3_{Q_{\mathbb{A}_i}} - \theta^3_{Q_{\mathbb{B}_i}}\right| + \gamma_2.max\Big(\left|\theta^3_{P_{\mathbb{A}_i}} - \theta^3_{P_{\mathbb{B}_i}}\right|, \left|\theta^3_{Q_{\mathbb{A}_i}} - \theta^3_{Q_{\mathbb{B}_i}}\right|\Big)\Big)\Big].\end{aligned}$$

Then, $\mathbb{A}, \mathbb{B}, \mathbb{C}$ are three CFFNs; moreover, $\alpha_i, \beta_i, \gamma_i \in [0,1]$ such that $\alpha_i + \beta_i + \gamma_i = 1$. The above $\mathsf{DM}_1(\mathbb{A},\mathbb{B})$ satisfies the following properties:
(i) $0 \leq \mathsf{DM}_1(\mathbb{A},\mathbb{B}) \leq 1$.
(ii) $\mathsf{DM}_1(\mathbb{A},\mathbb{B}) = \mathsf{DM}_1(\mathbb{B},\mathbb{A})$.
(iii) $\mathsf{DM}_1(\mathbb{A},\mathbb{B}) = 1$ if $\mathbb{A} = \mathbb{B}$.
(iv) If $\mathbb{A} \subseteq \mathbb{B} \subseteq \mathbb{C}$, then $\mathsf{DM}_1(\mathbb{A},\mathbb{C}) > \mathsf{DM}_1(\mathbb{A},\mathbb{B})$ and $\mathsf{DM}_1(\mathbb{A},\mathbb{C}) > \mathsf{DM}_1(\mathbb{B},\mathbb{C})$.

Definition 2.4.

A DM_2 for CFFN is defined as

$$\begin{aligned}\mathsf{DM}_2(\mathbb{A},\mathbb{B}) =& \frac{1}{2}\sum_{i=1}^{n}\Big[\alpha_1.\left|P^3_{\mathbb{A}_i} - P^3_{\mathbb{B}_i}\right| + \beta_1.\left|Q^3_{\mathbb{A}_i} - Q^3_{\mathbb{B}_i}\right| + \delta_1.\left|T^3_{\mathbb{A}_i} - T^3_{\mathbb{B}_i}\right| \\ &+ \gamma_1.max\Big(\left|P^3_{\mathbb{A}_i} - P^3_{\mathbb{B}_i}\right|, \left|Q^3_{\mathbb{A}_i} - Q^3_{\mathbb{B}_i}\right|, \left|T^3_{\mathbb{A}_i} - T^3_{\mathbb{B}_i}\right|\Big) \\ &+ \frac{1}{2\pi}\Big(\alpha_2.\left|\theta^3_{P_{\mathbb{A}_i}} - \theta^3_{P_{\mathbb{B}_i}}\right| + \beta_2.\left|\theta^3_{Q_{\mathbb{A}_i}} - \theta^3_{Q_{\mathbb{B}_i}}\right| + \\ &\delta_2.\left|\theta^3_{T_{\mathbb{A}_i}} - \theta^3_{T_{\mathbb{B}_i}}\right| + \gamma_2.max\Big(\left|\theta^3_{P_{\mathbb{A}_i}} - \theta^3_{P_{\mathbb{B}_i}}\right|, \left|\theta^3_{Q_{\mathbb{A}_i}} - \theta^3_{Q_{\mathbb{B}_i}}\right|, \left|\theta^3_{T_{\mathbb{A}_i}} - \theta^3_{T_{\mathbb{B}_i}}\right|\Big)\Big)\Big].\end{aligned}$$

The above $\mathsf{DM}_2(\mathbb{A},\mathbb{B})$ satisfies the following properties:
(i) $0 \leq \mathsf{DM}_2(\mathbb{A},\mathbb{B}) \leq 1$.
(ii) $\mathsf{DM}_2(\mathbb{A},\mathbb{B}) = \mathsf{DM}_2(\mathbb{B},\mathbb{A})$.
(iii) $\mathsf{DM}_2(\mathbb{A},\mathbb{B}) = 1$ if $\mathbb{A} = \mathbb{B}$.
(iv) If $\mathbb{A} \subseteq \mathbb{B} \subseteq \mathbb{C}$, then $\mathsf{DM}_2(\mathbb{A},\mathbb{C}) > \mathsf{DM}_2(\mathbb{A},\mathbb{B})$ and $\mathsf{DM}_2(\mathbb{A},\mathbb{C}) > \mathsf{DM}_2(\mathbb{B},\mathbb{C})$.

Currently, the intended distance measure are extended to next level of weighted distance measure, which can be used in many real life decision-making problems.

Definition 2.5.

A WDM_1 for CFFN is defined as

$$\begin{aligned}\mathsf{WDM}_1(\mathbb{A},\mathbb{B}) =& \frac{1}{2}\sum_{i=1}^{n}\Omega_i\Big[\alpha_1.\left|P^3_{\mathbb{A}_i}-P^3_{\mathbb{B}_i}\right|+\beta_1.\left|Q^3_{\mathbb{A}_i}-Q^3_{\mathbb{B}_i}\right|+\gamma_1.max\Big(\left|P^3_{\mathbb{A}_i}-P^3_{\mathbb{B}_i}\right|,\left|Q^3_{\mathbb{A}_i}-Q^3_{\mathbb{B}_i}\right|\Big)\\ &+\frac{1}{2\pi}\Big(\alpha_2.\left|\theta^3_{P_{\mathbb{A}_i}}-\theta^3_{P_{\mathbb{B}_i}}\right|+\beta_2.\left|\theta^3_{Q_{\mathbb{A}_i}}-\theta^3_{Q_{\mathbb{B}_i}}\right|+\gamma_2.max\Big(\left|\theta^3_{P_{\mathbb{A}_i}}-\theta^3_{P_{\mathbb{B}_i}}\right|,\left|\theta^3_{Q_{\mathbb{A}_i}}-\theta^3_{Q_{\mathbb{B}_i}}\right|\Big)\Big)\Big].\end{aligned}$$

$\Omega_i=(\Omega_1,\Omega_2,...,\Omega_n)^T$ represents the weight vector, where $\Omega_i\in[0,1]$, $\sum_{i=1}^{n}\Omega_i=1$. Let $\mathbb{A},\mathbb{B},\mathbb{C}$ be any three CFFNs and $\alpha_i,\beta_i,\gamma_i,\delta_i\in[0,1]$, such that $\alpha_i+\beta_i+\gamma_i+\delta_i=1$. The above $\mathsf{WDM}_1(\mathbb{A},\mathbb{B})$ satisfies the following properties:
(i) $0\le \mathsf{WDM}_1(\mathbb{A},\mathbb{B})\le 1$.
(ii) $\mathsf{WDM}_1(\mathbb{A},\mathbb{B})=\mathsf{WDM}_1(\mathbb{B},\mathbb{A})$.
(iii) $\mathsf{WDM}_1(\mathbb{A},\mathbb{B})=1$ if $\mathbb{A}=\mathbb{B}$.
(iv) If $\mathbb{A}\subseteq\mathbb{B}\subseteq\mathbb{C}$, then $\mathsf{WDM}_1(\mathbb{A},\mathbb{C})>\mathsf{WDM}_1(\mathbb{A},\mathbb{B})$ and $\mathsf{WDM}_1(\mathbb{A},\mathbb{C})>\mathsf{WDM}_1(\mathbb{B},\mathbb{C})$.

Definition 2.6.

A WDM_2 for CFFN is defined as

$$\begin{aligned}\mathsf{WDM}_2(\mathbb{A},\mathbb{B}) =& \frac{1}{2}\sum_{i=1}^{n}\Omega_i\Big[\alpha_1.\left|P^3_{\mathbb{A}_i}-P^3_{\mathbb{B}_i}\right|+\beta_1.\left|Q^3_{\mathbb{A}_i}-Q^3_{\mathbb{B}_i}\right|\\ &+\delta_1.\left|T^3_{\mathbb{A}_i}-T^3_{\mathbb{B}_i}\right|+\gamma_1.max\Big(\left|P^3_{\mathbb{A}_i}-P^3_{\mathbb{B}_i}\right|,\left|Q^3_{\mathbb{A}_i}-Q^3_{\mathbb{B}_i}\right|,\left|T^3_{\mathbb{A}_i}-T^3_{\mathbb{B}_i}\right|\Big)\\ &+\alpha_2.\left|\theta^3_{P_{\mathbb{A}_i}}-\theta^3_{P_{\mathbb{B}_i}}\right|+\beta_2.\left|\theta^3_{Q_{\mathbb{A}_i}}-\theta^3_{Q_{\mathbb{B}_i}}\right|\\ &+\delta_2.\left|\theta^3_{T_{\mathbb{A}_i}}-\theta^3_{T_{\mathbb{B}_i}}\right|\gamma_2.max\Big(\left|\theta^3_{P_{\mathbb{A}_i}}-\theta^3_{P_{\mathbb{B}_i}}\right|,\left|\theta^3_{Q_{\mathbb{A}_i}}-\theta^3_{Q_{\mathbb{B}_i}}\right|,\left|\theta^3_{T_{\mathbb{A}_i}}-\theta^3_{T_{\mathbb{B}_i}}\right|\Big)\Big].\end{aligned}$$

The above $\mathsf{WDM}_2(\mathbb{A},\mathbb{B})$ satisfies the following properties:
(i) $0\le \mathsf{WDM}_2(\mathbb{A},\mathbb{B})\le 1$.
(ii) $\mathsf{WDM}_2(\mathbb{A},\mathbb{B})=\mathsf{WDM}_2(\mathbb{B},\mathbb{A})$.
(iii) $\mathsf{WDM}_2(\mathbb{A},\mathbb{B})=1$ if $\mathbb{A}=\mathbb{B}$.
(iv) If $\mathbb{A}\subseteq\mathbb{B}\subseteq\mathbb{C}$, then $\mathsf{WDM}_2(\mathbb{A},\mathbb{C})>\mathsf{WDM}_2(\mathbb{A},\mathbb{B})$ and $\mathsf{WDM}_2(\mathbb{A},\mathbb{C})>\mathsf{WDM}_2(\mathbb{B},\mathbb{C})$.

Theorem 2.1.

Let $\mathbb{A},\mathbb{B},\mathbb{C}$ be three CFFNs. Then, $\mathsf{DM}(\mathbb{A},\mathbb{B})$ satisfies the following properties:
(i) $0\le \mathsf{DM}(\mathbb{A},\mathbb{B})\le 1$.
(ii) $\mathsf{DM}(\mathbb{A},\mathbb{B})=\mathsf{DM}(\mathbb{B},\mathbb{A})$.
(iii) $\mathsf{DM}(\mathbb{A},\mathbb{B})=1$ if $\mathbb{A}=\mathbb{B}$.
(iv) If $\mathbb{A}\subseteq\mathbb{B}\subseteq\mathbb{C}$, then $\mathsf{DM}(\mathbb{A},\mathbb{C})>\mathsf{DM}(\mathbb{A},\mathbb{B})$ and $\mathsf{DM}(\mathbb{A},\mathbb{C})>\mathsf{DM}(\mathbb{B},\mathbb{C})$.

Proof:

The condition $0 \leq \mathsf{DM}(\mathbb{A},\mathbb{B})$ holds true. So consider

$$\begin{aligned}\mathsf{DM}(\mathbb{A},\mathbb{B}) = \frac{1}{2}\sum_{i=1}^{n}\Big[&\alpha_1.\left|P^3_{\mathbb{A}_i} - P^3_{\mathbb{B}_i}\right| + \beta_1.\left|Q^3_{\mathbb{A}_i} - Q^3_{\mathbb{B}_i}\right| + \gamma_1.max\Big(\left|P^3_{\mathbb{A}_i} - P^3_{\mathbb{B}_i}\right|, \left|Q^3_{\mathbb{A}_i} - Q^3_{\mathbb{B}_i}\right|\Big) + \\ &\alpha_2.\left|\theta^3_{P_{\mathbb{A}_i}} - \theta^3_{P_{\mathbb{B}_i}}\right| + \beta_2.\left|\theta^3_{Q_{\mathbb{A}_i}} - \theta^3_{Q_{\mathbb{B}_i}}\right| + \gamma_2.max\Big(\left|\theta^3_{P_{\mathbb{A}_i}} - \theta^3_{P_{\mathbb{B}_i}}\right|, \left|\theta^3_{Q_{\mathbb{A}_i}} - \theta^3_{Q_{\mathbb{B}_i}}\right|\Big)\Big]\\ = \frac{1}{2}\sum_{i=1}^{n}\Big[&\alpha_1.1 + \beta_1.1 + \gamma_1.max\big(1,1\big) + \alpha_2.1 + \beta_2.1 + \gamma_2.max\big(1,1\big)\Big]\\ = \frac{1}{2}\sum_{i=1}^{n}\Big[&\alpha_1 + \beta_1 + \gamma_1 + \alpha_2 + \beta_2 + \gamma_2\Big].\end{aligned}$$

As $\alpha_1 + \beta_1 + \gamma_1 = 1$ and $\alpha_2 + \beta_2 + \gamma_2 = 1$. Consequently, $\mathsf{DM}(\mathbb{A},\mathbb{B}) = 1$.

Therefore, $0 \leq \mathsf{DM}(\mathbb{A},\mathbb{B}) \leq 1$. Then the conditions (ii) and (iii) are straightforward.

To prove (iv), using Definition 2.2, we have $1 \geq P^3_{\mathbb{A}_i} \geq P^3_{\mathbb{B}_i} \geq P^3_{\mathbb{C}_i} \geq 0$ and $1 \leq \theta^3_{P_{\mathbb{A}_i}} \leq \theta^3_{P_{\mathbb{B}_i}} \leq \theta^3_{P_{\mathbb{C}_i}} \leq 0$. then, $\left|P^3_{\mathbb{A}_i} - P^3_{\mathbb{B}_i}\right| \leq \left|P^3_{\mathbb{A}_i} - P^3_{\mathbb{C}_i}\right|$, $\left|\theta^3_{P_{\mathbb{A}_i}} - \theta^3_{P_{\mathbb{B}_i}}\right| \leq \left|\theta^3_{P_{\mathbb{A}_i}} - \theta^3_{P_{\mathbb{C}_i}}\right|$ and $\left|Q^3_{\mathbb{A}_i} - Q^3_{\mathbb{B}_i}\right| \leq \left|Q^3_{\mathbb{A}_i} - Q^3_{\mathbb{C}_i}\right|$, $\left|\theta^3_{Q_{\mathbb{A}_i}} - \theta^3_{Q_{\mathbb{B}_i}}\right| \leq \left|\theta^3_{Q_{\mathbb{A}_i}} - \theta^3_{Q_{\mathbb{C}_i}}\right|$. Similarly, $\mathsf{DM}(\mathbb{A},\mathbb{B}) \leq \mathsf{DM}(\mathbb{A},\mathbb{C})$, $\mathsf{DM}(\mathbb{B},\mathbb{C}) \leq \mathsf{DM}(\mathbb{A},\mathbb{C})$. ■

3. Decision Making in Complex Fermatean Fuzzy Environment (CFFE)

We present a real life situations using CFFN. We define DM between two CFFNs with same set of parameters. An algorithm is given and illustrated with examples. Also, we propose a method to rank the alternatives in CFFE.

3.1. Case study

We present two case studies to show the reliability of the proposed method. In the first case study, we evaluate the banks by using CFFNs and weight vectors for each attribute. In the second case study, we evaluate the banks by using same CFFNs and use different weight vectors for each attribute. We show that the rank is the same in both cases and we present the result that the rank is based on the CFFNs and not based on the weight vector of each attribute.

3.2. Description of the problem

Consider the following situation. Let $\mathbb{A}_t = \{\mathbb{A}_1, \mathbb{A}_2, ..., \mathbb{A}_n\}$ be a universal set and $E_j = \{e_1, e_2, ..., e_m\}$ be the common set of parameters. Consider that each alternative $\mathbb{A}_t$, where $t = (1, 2, ..., n)$ is given in CFFNs with reference to each parameter e_j, where $j = (1, 2, ..., m)$. Similarly let us consider another alternative $\mathbb{A}$ with same set of parameters. Assume that for the alternative, $\mathbb{A}$ is given in CFFNs with reference to each parameter e_j. Now the problem is to identify an alternative $\mathbb{A}_t$ which is similar to a selected alternative $\mathbb{A}$. Thus, the DMs between $\mathbb{A}_t$ and $\mathbb{A}$ can be derived by using Definitions 2.5 or 2.6.

3.3. Method for evaluating banks

Let U represent the set of banks given as universal set. Let E_j denote well defined set of parameters for evaluating banks. Let $\mathbb{A}_t$ denotes an alternative with data constructed with the help of banking experts. Assume that there are number of banks which need evaluation based on the given parameters. Then, by using Definitions 2.5 or 2.6, determine an alternative $\mathbb{A}_t$ which is similar to the selected alternative $\mathbb{A}$ that represents the data for a selected bank. Finally, arrange and rank the alternatives for each bank under consideration.

3.4. Algorithm

Step 1: Construct CFFN over U for a specific problem with experts opinion.

Step 2: Construct CFFN for an alternative that provides the related information for the given problem.

Step 3: Calculate the DMs between $\mathbb{A}_t, t = (1, 2, ..., m)$ using Definition 2.5 or 2.6.

Step 4: Arrange and rank the alternatives.

Example 3.1.

A fund manager is assessing four banks $\mathbb{A}_t = \{\mathbb{A}_1, \mathbb{A}_2, \mathbb{A}_3, \mathbb{A}_4\}$ based on $E_j = \{e_1, e_2, e_3, e_4\}$, where e_1 = Interest rate, the amount of interest due every period, expressed as a percentage of the amount lent, deposited or borrowed. e_2 = check on bank's accessibility, customer to cheek if there are any branches or ATMs close to his/her house or office. e_3 = Other benefits some may choose to keep all of their financial transactions in the same bank. If the customer wants to do more than just keep the funds at same bank, check to see if it also offers mortgages, loans, credit cards and financial planning, and e_4 = check on extra charges, an automated teller machine (ATM) allows customer to conduct simple transactions without the assistance of a teller or branch personnel. Cash withdrawal fees are frequently charged by the bank where the account is held. The weight vector of attribute is (Ω = 0.3,0.4,0.2,0.1) and the result obtained using Definition 2.5 with $\{\alpha_1 = 0.3, \beta_1 = 0.4, \gamma_1 = 0.3\}$ and $\{\alpha_2 = 0.2, \beta_2 = 0.6, \gamma_1 = 0.2\}$ are given in Table 2. The result obtained using Definition 2.6 with $\{\alpha_1 = 0.2, \beta_1 = 0.3, \gamma_1 = 0.1, \delta_1 = 0.4\}$ and $\{\alpha_2 = 0.4, \beta_2 = 0.2, \gamma_1 = 0.3, \delta_2 = 0.1\}$ are given in Table 3.

Step 1: Based on the CFFNs data received from the fund manager over U and the related data $\mathbb{A}$ for the given problem are given in Table 1.

Table 1. Shows the observation values

	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$
e_1	$(0.8.e^{i2\pi(0.6)}, 0.7.e^{i2\pi(0.8)})$	$(0.71.e^{i2\pi(0.61)}, 0.85.e^{i2\pi(0.91)})$	$(0.85.e^{i2\pi(0.69)}, 0.72.e^{i2\pi(0.85)})$
e_2	$(0.4.e^{i2\pi(0.5)}, 0.9.e^{i2\pi(0.94)})$	$(0.84.e^{i2\pi(0.74)}, 0.72.e^{i2\pi(0.83)})$	$(0.91.e^{i2\pi(0.87)}, 0.4.e^{i2\pi(0.66)})$
e_3	$(0.7.e^{i2\pi(0.73)}, 0.8.e^{i2\pi(0.84)})$	$(0.69.e^{i2\pi(0.81)}, 0.87.e^{i2\pi(0.77)})$	$(0.75.e^{i2\pi(0.76)}, 0.83.e^{i2\pi(0.82)})$
e_4	$(0.6.e^{i2\pi(0.8)}, 0.9.e^{i2\pi(0.78)})$	$(0.59.e^{i2\pi(0.65)}, 0.88.e^{i2\pi(0.77)})$	$(0.59.e^{i2\pi(0.66)}, 0.92.e^{i2\pi(0.89)})$

	$\mathbb{A}_4$	$\mathbb{A}$
e_1	$(0.66.e^{i2\pi(0.79)}, 0.87.e^{i2\pi(0.78)})$	$(1.e^{i2\pi}, 0)$
e_2	$(0.77.e^{i2\pi(0.51)}, 0.81.e^{i2\pi(0.91)})$	$(1.e^{i2\pi}, 0)$
e_3	$(0.81.e^{i2\pi(0.41)}, 0.7.e^{i2\pi(0.93)})$	$(1.e^{i2\pi}, 0)$
e_4	$(0.93.e^{i2\pi(0.83)}, 0.45.e^{i2\pi(0.75)})$	$(1.e^{i2\pi}, 0)$

Step 2: Calculate WDM_1 by using Definition 2.5 and tabulate the results as in Table 2.

Table 2. WDM_1 for CFFNs

$\mathsf{WDM}_1(\mathbb{A}_1, \mathbb{A})$	$\mathsf{WDM}_1(\mathbb{A}_2, \mathbb{A})$	$\mathsf{WDM}_1(\mathbb{A}_3, \mathbb{A})$	$\mathsf{WDM}_1(\mathbb{A}_4, \mathbb{A})$
0.6799	0.5823	0.4372	0.6001

Step 3: Calculate WDM_2 by using Definition 2.6 and tabulate the results as in Table 3.

Table 3. WDM_2 for CFFNs

$\mathsf{WDM}_2(\mathbb{A}_1, \mathbb{A})$	$\mathsf{WDM}_2(\mathbb{A}_2, \mathbb{A})$	$\mathsf{WDM}_2(\mathbb{A}_3, \mathbb{A})$	$\mathsf{WDM}_2(\mathbb{A}_4, \mathbb{A})$
0.5624	0.4561	0.3595	0.4901

Step 4: Arrange the alternatives and tabulate the results as in Table 4.

Table 4. Ranking of distance measures

	Ranking
Using Definition 2.5	$\mathbb{A}_1 > \mathbb{A}_4 > \mathbb{A}_2 > \mathbb{A}_3$
Using Definition 2.6	$\mathbb{A}_1 > \mathbb{A}_4 > \mathbb{A}_2 > \mathbb{A}_3$

From the above result, we conclude that alternative $\mathbb{A}_1$ is more similar to the chosen alternative $\mathbb{A}$ than others.

Example 3.2.

Let us consider the same values given in Case I with different weight values. The weight vector of attribute is (Ω = 0.1,0.2,0.3,0.4) and the result obtained using Definition 2.5 with

$\{\alpha_1 = 0.2, \beta_1 = 0.6, \gamma_1 = 0.2\}$ and $\{\alpha_2 = 0.3, \beta_2 = 0.4, \gamma_1 = 0.3\}$ are given in Table 5. The result obtained using Definition 2.6 with $\{\alpha_1 = 0.2, \beta_1 = 0.3, \gamma_1 = 0.1, \delta_1 = 0.4\}$ and $\left\{\alpha_2 = 0.4, \beta_2 = 0.2, \gamma_1 = 0.2, \delta_2 = 0.2\right\}$ are given in Table 6.

Step 1: Calculate WDM_1 by using Definition 2.5 and tabulate the results as in Table 5.

Table 5. WDM_1 for CFFNs

$\mathsf{WDM}_1(\mathbb{A}_1, \mathbb{A})$	$\mathsf{WDM}_1(\mathbb{A}_2, \mathbb{A})$	$\mathsf{WDM}_1(\mathbb{A}_3, \mathbb{A})$	$\mathsf{WDM}_1(\mathbb{A}_4, \mathbb{A})$
0.6436	0.6049	0.6102	0.4975

Step 2: Calculate WDM_2 by using Definition 2.6 and tabulate the results as in Table 6.

Table 6. WDM_2 for CFFNs

$\mathsf{WDM}_2(\mathbb{A}_1, \mathbb{A})$	$\mathsf{WDM}_2(\mathbb{A}_2, \mathbb{A})$	$\mathsf{WDM}_2(\mathbb{A}_3, \mathbb{A})$	$\mathsf{WDM}_2(\mathbb{A}_4, \mathbb{A})$
0.4829	0.4552	0.4116	0.3945

Step 3: Arrange the alternatives and tabulate the results as in Table 7.

Table 7. Ranking of distance measures

	Ranking
Using Definition 2.5	$\mathbb{A}_1 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_4$
Using Definition 2.6	$\mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_3 > \mathbb{A}_4$

From the above result, we conclude that alternative $\mathbb{A}_1$ is more similar to the chosen alternative $\mathbb{A}$ than others, even when the weight vector of attributes and $\alpha_i, \beta_i, \gamma_i, \delta_i$ are changed from Case I. So we conclude from Case I and II that the change in rank is based on CFFNs.

4. Comparative Study

In this section, we compare with existing complex theories (Ullah et al. (2020)) and show the superiority of the proposed method.

Let us consider the values in CPFN form as below for each alternative.

Step 1: Calculate WDM_1 by using Ullah et al. (2020) and tabulate the results as in Table 9.

From Table 9, it is clear that $\mathsf{WDM}_1(\mathbb{A}_1, \mathbb{A})$= $\mathsf{WDM}_1(\mathbb{A}_2, \mathbb{A})$ = 0.5305. In this case, it is not possible to choose the alternative which is similar to A.

Step 2: Calculate WDM_1 by using Definition 2.5 and tabulate the results as in Table 10.

From Table 10, it is evident that $\mathsf{WDM}_1(\mathbb{A}_1, \mathbb{A}) > \mathsf{WDM}_1(\mathbb{A}_2, \mathbb{A})$. This shows the superiority of proposed method over the existing method.

Table 8. Shows the values for CPFNs

	$\mathbb{A}_1$	$\mathbb{A}_2$	$\mathbb{A}_3$
e_1	$(0.7.e^{i2\pi(0.4)}, 0.6.e^{i2\pi(0.3)})$	$(0.5.e^{i2\pi(0.6)}, 0.4.e^{i2\pi(0.7)})$	$(0.8.e^{i2\pi(0.9)}, 0.5.e^{i2\pi(0.4)})$
e_2	$(0.4.e^{i2\pi(0.5)}, 0.1.e^{i2\pi(0.5)})$	$(0.7.e^{i2\pi(0.5)}, 0.6.e^{i2\pi(0.7)})$	$(0.9.e^{i2\pi(0.8)}, 0.3.e^{i2\pi(0.4)})$
e_3	$(0.6.e^{i2\pi(0.4)}, 0.3.e^{i2\pi(0.7)})$	$(0.4.e^{i2\pi(0.7)}, 0.6.e^{i2\pi(0.5)})$	$(0.5.e^{i2\pi(0.8)}, 0.8.e^{i2\pi(0.3)})$
e_4	$(0.5.e^{i2\pi(0.3)}, 0.7.e^{i2\pi(0.8)})$	$(0.8.e^{i2\pi(0.5)}, 0.37.e^{i2\pi(0.8)})$	$(0.3.e^{i2\pi(0.6)}, 0.7.e^{i2\pi(0.8)})$

	$\mathbb{A}_4$	$\mathbb{A}$
e_1	$(0.8.e^{i2\pi(0.9)}, 0.4.e^{i2\pi(0.3)})$	$(1.e^{i2\pi}, 0)$
e_2	$(0.7.e^{i2\pi(0.8)}, 0.3.e^{i2\pi(0.4)})$	$(1.e^{i2\pi}, 0)$
e_3	$(0.9.e^{i2\pi(0.5)}, 0.2.e^{i2\pi(0.3)})$	$(1.e^{i2\pi}, 0)$
e_4	$(0.7.e^{i2\pi(0.4)}, 0.51.e^{i2\pi(0.9)})$	$(1.e^{i2\pi}, 0)$

Table 9. WDM_1 for CPFNs

$\mathsf{WDM}_1(\mathbb{A}_1, \mathbb{A})$	$\mathsf{WDM}_1(\mathbb{A}_2, \mathbb{A})$	$\mathsf{WDM}_1(\mathbb{A}_3, \mathbb{A})$	$\mathsf{WDM}_1(\mathbb{A}_4, \mathbb{A})$
0.5305	0.5305	0.3384	0.3057

Table 10. WDM_1 for CFFNs

$\mathsf{WDM}_1(\mathbb{A}_1, \mathbb{A})$	$\mathsf{WDM}_1(\mathbb{A}_2, \mathbb{A})$	$\mathsf{WDM}_1(\mathbb{A}_3, \mathbb{A})$	$\mathsf{WDM}_1(\mathbb{A}_4, \mathbb{A})$
0.5488	0.5406	0.3542	0.3382

5. Conclusion

We have presented the concept of CFFS to deal with the limitations of existing complex theories of CFS, CIFS and CPFS. We have also developed DMs and WDMs for CFFN. We have provided a solution to solve multi-criteria decision making problems using CFFN. We have given an algorithm and illustrated the same with two case studies. Initially, we computed the rank for each bank by using CFFNs and weight vectors for each attribute. Secondly, we computed the rank for each bank by using the same CFFNs and by using different weight vectors for each attribute. We have shown that the rank is same in both cases and concluded that the result is based on the CFFNs and not based on the weight vector of each attribute. Finally, we have compared with existing studies and shown the superiority of the proposed method.

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