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Some Types of Continuous Function Via N-Neutrosophic Crisp Topological Spaces

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Abstract

The aim of this article is to introduce a new type of continuous functions such as N-neutrosophic crisp γ continuous and weakly N-neutrosophic crisp γ continuous functions in a N-neutrosophic crisp topological space and also discuss a relation between them in N-neutrosophic crisp topological spaces. We also investigate some of their properties in N-neutrosophic crisp γ continuous function via N-neutrosophic crisp topological spaces. Further, a contra part of continuity called N-neutrosophic crisp γ-contra continuous map in a N-neutrosophic crisp topology is also initiated. Finally, an application based on neutrosophic score function of medical diagnosis is examined with graphical representation.

Keywords: N-neutrosophic crisp γ open set; N-neutrosophic crisp γ continuous; weakly N-neutrosophic crisp γ continuous; N-neutrosophic crisp γ contra continuous; Neutrosophic score function

MSC 2010 No.: 03E72, 54A05, 54C05, 54C10

1. Introduction

In mathematics, a idea of a fuzzy set between the intervals was first presented by Zadeh (1965) in order of logic and set hypothesis. The general topology has been system with fuzzy set was attempted by Chang (1968) as fuzzy topological space. Adlassnig (1986) applied fuzzy set theory to formalize medical relationships and fuzzy logic to mechanized analysis framework. This hypothesis (Innocent and John (2004); Roos (1994); Sugeno (1985)) has been utilized in the fields of man-made brainpower, likelihood, science, control frameworks and financial aspects. In 1983, Atanassov (1986) started intuitionistic fuzzy set which contains a membership and non-membership values. Coker (1997) made intuitionistic fuzzy set in a topology entitled as intuitionistic fuzzy topological spaces. De et al. (2001) was the first to built up the uses of intuitionistic fuzzy sets in medical diagnosis. Several researchers (Biswas et al. (2014); Khatibi and Montazer (2009); Szmidt and Kacprzyk (2001)) further examined intuitionistic fuzzy sets in medical diagnosis.

In our daily routine, we have used crisp sets in most of our life. The concepts of neutrosophy and neutrosophic set by Smarandache (1999) are the recent tools in a topological space. It was first introduced by Smarandache (2002) in the beginning of 21^{th} century. Salama et al. (2014) has provided the basic concept of neutrosophic crisp set in a topological space. After that Al-Omeri (2016) also investigated some fundamental properties of neutrosophic crisp topological spaces. Al-Hamido et al. (2018) explore the possibility of expanding the concept of neutrosophic crisp topological spaces into N-topology and investigate some of their basic properties in N-terms. By using N-terms of topological spaces, we can define $1_{nc}ts$, $2_{nc}ts$, $\cdots$, $N_{nc}ts$.

Andrijevic (1996) introduced b-open sets and develop some of their works in general topology. The notion of γ-open set in topological spaces was introduced by Min (2002) and worked in the field of general topology. Vadivel and Sundar (2020) presented γ-open sets in neutrosophic crisp topological spaces via N-terms of N-neutrosophic crisp topology.

Smarandache (2002) described the single valued Neutrosophic set on three portion (T-Truth, F-Falsehood, I-Indeterminacy) neutrosophic sets and further concentrated by Wang et al. (2010). Majumdar and Samanta (2014) characterized some similarity measures of single valued neutrosophic sets in decision making problems. Recently many researchers (Abdel et al. (2019), Broumi and Smarandache (2013), Nabeeh et al. (2019), Thanh and Ali (2017), Ye and Zhang (2014), and Ye and Ye (2015)) presented several similarity measures and single-valued neutrosophic sets in medical diagnosis. Applications based on neutrosophic score function by authors in various fields are Smarandache (2020), Vadivel and Sundar (2021), Vadivel et al. (2022), etc.

In this present work, we extend the open sets into a continuous functions such as N-neutrosophic crisp γ continuous and weakly N-neutrosophic crisp γ continuous functions in $N_{nc}ts$. Also, we study and investigate some of their fundamental properties in $N_{nc}ts$. Also, N-neutrosophic crisp γ contra continuous function in a N-neutrosophic crisp topological space is also discussed here. Finally, an application based on neutrosophic score function of medical diagnosis is examined with graphical representation.

2. Preliminaries

Some basic definitions and properties of N-neutrosophic crisp topological spaces are discussed in this section. A neutrosophic crisp set (briefly, ncs), $\emptyset_N$, Y_N, subset, union, intersection and complement (briefly, ${K_o}^c$) are defined by Salama and Smarandache (2015). A neutrosophic crisp topology (briefly, $_{nc}t$) and neutrosophic crisp topological space (briefly, $ncts$) are defined by Salama et al. (2014). A N-neutrosophic crisp sets (briefly, $N_{nc}s$), N-neutrosophic crisp (briefly, N_{nc}) topology, N-neutrosophic crisp topological space (briefly, $N_{nc}ts$), N-neutrosophic crisp open set (briefly, $N_{nc}os$), N-neutrosophic crisp closed set (briefly, $N_{nc}cs$), N-neutrosophic crisp interior of K_o (briefly, $N_{nc}int(K_o)$), N-neutrosophic crisp closure of K_o (briefly, $N_{nc}cl(K_o)$), N-neutrosophic crisp pre open set (briefly, $N_{nc}\mathcal{P}os$), N-neutrosophic crisp semi open set (briefly, $N_{nc}\mathcal{S}os$), N-neutrosophic crisp α-open set (briefly, $N_{nc}\alpha os$) and their respective closed sets are defined by Al-Hamido et al. (2018). A N-neutrosophic crisp regular open set (briefly, $N_{nc}ros$), N-neutrosophic crisp γ-open set (briefly, $N_{nc}\gamma os$) and their respective closed sets are defined by Vadivel and Sundar (2020). The family of all $N_{nc}os$ of X is denoted by $N_{nc}OS(X)$ and respective other open sets and closed sets. A N-neutrosophic crisp continuous (briefly, $N_{nc}Cts$) is defined by Thivagar et al. (2018).

For an application part, some basic definitions such as neutrosophic sets, empty set, whole set, union and intersection of neutrosophic set, neutrosophic topological spaces (briefly, NTS) are defined by Salama and Smarandache (2015) and Vadivel and Sundar (2021).

3. N-neutrosophic crisp γ continuous maps

Here we study about N-neutrosophic crisp γ continuous maps and their properties in $N_{nc}ts$. Throughout Sections 3, 4 and 5, let $(X_1, N_{nc}\Gamma)$, $(X_2, N_{nc}\Psi)$ and $(X_3, N_{nc}\Phi)$ be any $N_{nc}ts$'s. Let h and g be a maps defined as $h : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ and $g : (X_2, N_{nc}\Psi) \to (X_3, N_{nc}\Phi)$ in a $N_{nc}ts$. Let K_o and M_o be a $N_{nc}s$'s in $N_{nc}ts$.

Definition 3.1.

A map h is said to be N-neutrosophic crisp γ (respectively, α, semi , pre and regular) continuous (briefly, $N_{nc}\gamma Cts$ (respectively, $N_{nc}\alpha Cts$, $N_{nc}\mathcal{S}Cts$, $N_{nc}\mathcal{P}Cts$ and $N_{nc}rCts$)) if the inverse image of every $N_{nc}os$ in $(X_2, N_{nc}\Psi)$ is a $N_{nc}\gamma os$ (respectively, $N_{nc}\alpha os$, $N_{nc}\mathcal{S}os$, $N_{nc}\mathcal{P}os$ and $N_{nc}ros$) in $(X_1, N_{nc}\Gamma)$.

Example 3.1.

Let $X = \{l_o, m_o, n_o, o_o, p_o\}$, $_{nc}\tau_1 = \{\phi_N, X_N, L, M, N\}$, $_{nc}\tau_2 = \{\phi_N, X_N\}$. $L = \langle\{n_o\}, \{\phi\}, \{l_o, m_o, o_o, p_o\}\rangle$, $M = \langle\{l_o, m_o\}, \{\phi\}, \{n_o, o_o, p_o\}\rangle$, $N = \langle\{l_o, m_o, n_o\}, \{\phi\}, \{o_o, p_o\}\rangle$, then $2_{nc}\tau = \{\phi_N, X_N, L, M, N\}$.

Let $h : (X, 2_{nc}\tau) \to (X, 2_{nc}\tau)$ be an identity function. Then, h is $2_{nc}Cts$, $2_{nc}\alpha Cts$, $2_{nc}\mathcal{S}Cts$, $2_{nc}\mathcal{P}Cts$ and $2_{nc}\gamma Cts$.

Theorem 3.1.

The statements are true for a map h but not converse.

(i) Every $N_{nc}rCts$ is a $N_{nc}Cts$.

(ii) Every $N_{nc}Cts$ is a $N_{nc}\alpha Cts$.

(iii) Every $N_{nc}\alpha Cts$ is a $N_{nc}\mathcal{S}Cts$.

(iv) Every $N_{nc}\alpha Cts$ is a $N_{nc}\mathcal{P}Cts$.

(v) Every $N_{nc}\mathcal{S}Cts$ is a $N_{nc}\gamma Cts$.

(vi) Every $N_{nc}\mathcal{P}Cts$ is a $N_{nc}\gamma Cts$.

Proof:

(i) Let $h : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be a $N_{nc}rCts$ and K_o is a $N_{nc}os$ in X_2. Then $h^{-1}(K_o)$ is $N_{nc}ros$ in X_1. Since every $N_{nc}ro$ is $N_{nc}o$ by Proposition 3.2 in Vadivel and Sundar (2020), $h^{-1}(K_o)$ is $N_{nc}os$ in X_1. Therefore, h is $N_{nc}Cts$.

(ii) Let $h : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be a $N_{nc}Cts$ and K_o is a $N_{nc}os$ in X_2. Then $h^{-1}(K_o)$ is $N_{nc}os$ in X_1. Since every $N_{nc}o$ is $N_{nc}\alpha o$ by Proposition 3.2 in Vadivel and Sundar (2020), $h^{-1}(K_o)$ is $N_{nc}\alpha os$ in X_1. Therefore, h is $N_{nc}\alpha Cts$.

(iii) Let $h : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be a $N_{nc}\alpha Cts$ and K_o is a $N_{nc}os$ in X_2. Then $h^{-1}(K_o)$ is $N_{nc}\alpha os$ in X_1. Since every $N_{nc}\alpha o$ is $N_{nc}\mathcal{S}o$ by Proposition 3.2 in Vadivel and Sundar (2020), $h^{-1}(K_o)$ is $N_{nc}\mathcal{S}os$ in X_1. Therefore, h is $N_{nc}\mathcal{S}Cts$.

(iv) Let $h : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be a $N_{nc}\alpha Cts$ and K_o is a $N_{nc}os$ in X_2. Then $h^{-1}(K_o)$ is $N_{nc}\alpha os$ in X_1. Since every $N_{nc}\alpha o$ is $N_{nc}\mathcal{P}o$ by Proposition 3.2 in Vadivel and Sundar (2020), $h^{-1}(K_o)$ is $N_{nc}\mathcal{P}os$ in X_1. Therefore, h is $N_{nc}\mathcal{P}Cts$.

(v) Let $h : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be a $N_{nc}\mathcal{S}Cts$ and K_o is a $N_{nc}os$ in X_2. Then $h^{-1}(K_o)$ is $N_{nc}\mathcal{S}os$ in X_1. Since every $N_{nc}\mathcal{S}o$ is $N_{nc}\gamma o$ by Proposition 3.2 in Vadivel and Sundar (2020), $h^{-1}(K_o)$ is $N_{nc}\gamma os$ in X_1. Therefore, h is $N_{nc}\gamma Cts$.

(vi) Let $h : (X_1, N_{nc}\Gamma) \to (X_2, N_{nc}\Psi)$ be a $N_{nc}\mathcal{P}Cts$ and K_o is a $N_{nc}os$ in X_2. Then $h^{-1}(K_o)$ is $N_{nc}\mathcal{P}os$ in X_1. Since every $N_{nc}\mathcal{P}o$ is $N_{nc}\gamma o$ by Proposition 3.2 in Vadivel and Sundar (2020), $h^{-1}(K_o)$ is $N_{nc}\gamma os$ in X_1. Therefore, h is $N_{nc}\gamma Cts$. ■

Example 3.2.

In Example 3.1, h is $2_{nc}Cts$ but not $2_{nc}rCts$, the set $h^{-1}(N) = N$ is a $2_{nc}os$ but not a $2_{nc}ros$.

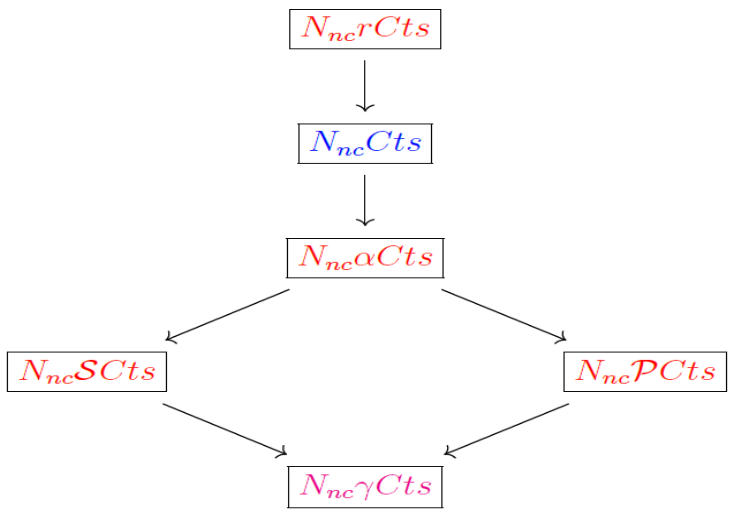

Figure 1. $N_{nc}\gamma Cts$ function in $N_{nc}ts$.

Example 3.3.

Let $X = \{l_o, m_o, n_o, o_o\}$, ${}_{nc}\tau_1 = \{\phi_N, X_N, \langle\{l_o, o_o\}, \{\phi\}, \{m_o, n_o\}\rangle\}$, ${}_{nc}\tau_2 = \{\phi_N, X_N\}$, then we have $2_{nc}\tau = \{\phi_N, X_N, \langle\{l_o, o_o\}, \{\phi\}, \{m_o, n_o\}\rangle\}$. Let $Y = \{w_o, x_o, y_o, z_o\}$, ${}_{nc}\Psi_1 = \{\phi_N, Y_N, W, X, Y\}$, ${}_{nc}\Psi_2 = \{\phi_N, Y_N\}$. $W = \langle\{x_o\}, \{\phi\}, \{w_o, y_o, z_o\}\rangle$, $X = \langle\{w_o, y_o\}, \{\phi\}, \{x_o, z_o\}\rangle$, $Y = \langle\{w_o, x_o, y_o\}, \{\phi\}, \{z_o\}\rangle$, then we have $2_{nc}\Psi = \{\phi_N, Y_N, W, X, Y\}$.

Define $h : (X, 2_{nc}\tau) \to (Y, 2_{nc}\Psi)$ as $h(l_o) = y_o$, $h(m_o) = y_o$, $h(n_o) = z_o$ and $h(o_o) = w_o$, then $2_{nc}\alpha Cts$ but not $2_{nc}Cts$, the set $h^{-1}(\langle\{w_o, y_o\}, \{\phi\}, \{x_o, z_o\}\rangle) = \langle\{l_o, m_o, o_o\}, \{\phi\}, \{n_o\}\rangle$ is a $2_{nc}\alpha os$ but not $2_{nc}os$.

Example 3.4.

Let $X = \{l_o, m_o, n_o, o_o\}$, ${}_{nc}\tau_1 = \{\phi_N, X_N, L, M, N\}$, ${}_{nc}\tau_2 = \{\phi_N, X_N\}$. $L = \langle\{n_o\}, \{\phi\}, \{l_o, m_o, o_o\}\rangle$, $M = \langle\{l_o, m_o\}, \{\phi\}, \{n_o, o_o\}\rangle$, $N = \langle\{l_o, m_o, n_o\}, \{\phi\}, \{o_o\}\rangle$, then we have $2_{nc}\tau = \{\phi_N, X_N, L, M, N\}$. Let $Y = \{w_o, x_o, y_o, z_o\}$, ${}_{nc}\Psi_1 = \{\phi_N, Y_N, W, X, Y\}$, ${}_{nc}\Psi_2 = \{\phi_N, Y_N\}$. $W = \langle\{x_o\}, \{\phi\}, \{w_o, y_o, z_o\}\rangle$, $X = \langle\{w_o, y_o\}, \{\phi\}, \{x_o, z_o\}\rangle$, $Y = \langle\{w_o, x_o, y_o\}, \{\phi\}, \{z_o\}\rangle$, then we have $2_{nc}\Psi = \{\phi_N, Y_N, W, X, Y\}$.

Define $h : (X, 2_{nc}\tau) \to (Y, 2_{nc}\Psi)$ as $h(l_o) = z_o$, $h(m_o) = y_o$, $h(n_o) = y_o$ and $h(o_o) = w_o$, then $2_{nc}\mathcal{S}Cts$ (respectively, $2_{nc}\gamma Cts$) but not $2_{nc}\alpha Cts$ (respectively, $2_{nc}\mathcal{P}Cts$), the set $h^{-1}(\langle\{w_o, y_o\}, \{\phi\}, \{x_o, z_o\}\rangle) = \langle\{m_o, n_o, o_o\}, \{\phi\}, \{l_o\}\rangle$ is a $2_{nc}\mathcal{S}os$ (respectively, $2_{nc}\gamma os$) but not $2_{nc}\alpha os$ (respectively, $2_{nc}\mathcal{P}os$).

Example 3.5.

In Example 3.4, Define $h : (X, 2_{nc}\tau) \to (Y, 2_{nc}\Psi)$ as $h(l_o) = w_o$, $h(m_o) = x_o$, $h(n_o) = z_o$ and $h(o_o) = y_o$, then $2_{nc}\mathcal{P}Cts$ (respectively, $2_{nc}\gamma Cts$) but not $2_{nc}\alpha Cts$ (respectively, $2_{nc}\mathcal{S}Cts$), the set $h^{-1}(\langle\{x_o\}, \{\phi\}, \{w_o, y_o, z_o\}\rangle) = \langle\{m_o\}, \{\phi\}, \{l_o, n_o, o_o\}\rangle$ is a $2_{nc}\mathcal{P}os$ (respectively, $2_{nc}\gamma os$)

but not $2_{nc}\alpha os$ (respectively, $2_{nc}\mathcal{S}os$).

Remark 3.1.

A map h satisfies the following statements are equivalent.

(i) h is $N_{nc}\gamma Cts$.

(ii) The inverse $h^{-1}(K_o)$ of each $N_{nc}os$ K_o in X_2 is $N_{nc}\gamma os$ in X_1.

Theorem 3.2.

A map h satisfies the statements are equivalent.

(i) $h(N_{nc}\gamma cl(K_o)) \subseteq N_{nc}cl(h(K_o))$, for all ncs K_o in X_1;

(ii) $N_{nc}\gamma cl(h^{-1}(M_o)) \subseteq h^{-1}(N_{nc}cl(M_o))$, for all ncs M_o in X_2.

Proof:

(i) Since $N_{nc}cl(h(K_o))$ is a $N_{nc}cs$ in X_2 and h is $N_{nc}\gamma Cts$, then $h^{-1}(N_{nc}cl(h(K_o)))$ is $N_{nc}\gamma c$ in X_1. Now, since $K_o \subseteq h^{-1}(N_{nc}cl(h(K_o)))$, $N_{nc}\gamma cl(K_o) \subseteq h^{-1}(N_{nc}cl(h(K_o)))$. Therefore, $h(N_{nc}\gamma cl(K_o)) \subseteq N_{nc}cl(h(K_o))$.

(ii) By replacing K_o with M_o in (i), we obtain $h(N_{nc}\gamma cl(h^{-1}(M_o))) \subseteq N_{nc}cl(h(h^{-1}(M_o))) \subseteq N_{nc}cl(M_o)$. Hence, $N_{nc}\gamma cl(h^{-1}(M_o)) \subseteq h^{-1}(N_{nc}cl(M_o))$. ∎

Remark 3.2.

If h is $N_{nc}\gamma Cts$, then:

(i) $h(N_{nc}\gamma cl(K_o))$ is not necessarily equal to $N_{nc}cl(h(K_o))$ where $K_o \subseteq X_1$.

(ii) $N_{nc}\gamma cl(h^{-1}(M_o))$ is not necessarily equal to $h^{-1}(N_{nc}cl(M_o))$ where $M_o \subseteq X_2$.

Example 3.6.

In Example 3.1, h is a $N_{nc}\gamma Cts$.

(i) Let $K_o = \langle\{l_o, m_o\}, \{\phi\}, \{n_o, o_o, p_o\}\rangle \subseteq X$. Then, $h(2_{nc}\gamma cl(K_o)) = h(2_{nc}\gamma cl(\langle\{l_o, m_o\}, \{\phi\}, \{n_o, o_o, p_o\}\rangle)) = h(\langle\{l_o, m_o\}, \{\phi\}, \{n_o, o_o, p_o\}\rangle) = \langle\{l_o, m_o\}, \{\phi\}, \{n_o, o_o, p_o\}\rangle$. But $2_{nc}cl(h(K_o)) = 2_{nc}cl(h(\langle\{l_o, m_o\}, \{\phi\}, \{n_o, o_o, p_o\}\rangle)) = 2_{nc}cl(\langle\{l_o, m_o\}, \{\phi\}, \{n_o, o_o, p_o\}\rangle) = \langle\{l_o, m_o, o_o, p_o\}, \{\phi\}, \{n_o\}\rangle$. Thus, $h(2_{nc}\gamma cl(K_o)) \neq 2_{nc}cl(h(K_o))$.

(ii) Let $M_o = \langle\{l_o\}, \{\phi\}, \{m_o, n_o, o_o, p_o\}\rangle \subseteq Y$. Then, $2_{nc}\gamma cl(h^{-1}(M_o)) \subseteq 2_{nc}\gamma cl(h^{-1}(\langle\{l_o\}, \{\phi\}, \{m_o, n_o, o_o, p_o\}\rangle)) = 2_{nc}\gamma cl(\langle\{l_o\}, \{\phi\}, \{m_o, n_o, o_o, p_o\}\rangle)$. But $h^{-1}(2_{nc}cl(M_o)) = h^{-1}(2_{nc}cl(\langle\{l_o\}, \{\phi\}, \{m_o, n_o, o_o, p_o\}\rangle)) = h^{-1}(Y) = X$. Thus, $2_{nc}\gamma cl(h^{-1}(M_o)) \neq h^{-1}(2_{nc}cl(M_o))$.

Theorem 3.3.

Let h be a function, then $h^{-1}(N_{nc}int(K_o)) \subseteq N_{nc}\gamma int(h^{-1}(K_o))$, for all ncs K_o in X_2.

Proof:

If h is $N_{nc}\gamma Cts$ and $K_o \subseteq X_2$. $N_{nc}int(K_o)$ is $N_{nc}o$ in X_2 and hence, $h^{-1}(N_{nc}int(K_o))$ is $N_{nc}\gamma o$ in X_1. Therefore, $N_{nc}\gamma int(h^{-1}(N_{nc}int(K_o))) = h^{-1}(N_{nc}int(K_o))$. Also, $N_{nc}int(K_o) \subseteq K_o$, implies that $h^{-1}(N_{nc}int(K_o)) \subseteq h^{-1}(K_o)$. Therefore, $N_{nc}\gamma int(h^{-1}(N_{nc}int(K_o))) \subseteq N_{nc}\gamma int(h^{-1}(K_o))$. That is, $h^{-1}(N_{nc}int(K_o)) \subseteq N_{nc}\gamma int(h^{-1}(K_o))$.

Conversely, let $h^{-1}(N_{nc}int(K_o)) \subseteq N_{nc}\gamma int(h^{-1}(K_o))$ for every subset K_o of X_2. If K_o is $N_{nc}o$ in X_2, then $N_{nc}int(K_o) = K_o$. By assumption, $h^{-1}(N_{nc}int(K_o)) \subseteq N_{nc}\gamma int(h^{-1}(K_o))$. Thus, $h^{-1}(K_o) \subseteq N_{nc}\gamma int(h^{-1}(K_o))$. But $N_{nc}\gamma int(h^{-1}(K_o)) \subseteq h^{-1}(K_o)$. Therefore, $N_{nc}\gamma int(h^{-1}(K_o)) = h^{-1}(K_o)$. That is, $h^{-1}(K_o)$ is $N_{nc}\gamma o$ in X_1, for every $N_{nc}os$ K_o in X_2. Therefore, h is $N_{nc}\gamma Cts$ on X_1. ■

Remark 3.3.

If h is $N_{nc}\gamma Cts$, then $N_{nc}\gamma int(h^{-1}(K_o))$ is not necessarily equal to $h^{-1}(N_{nc}int(K_o))$ where $K_o \subseteq X_2$.

Example 3.7.

In Example 3.1, h is a $2_{nc}\gamma Cts$. Let $K_o = \langle\{l_o, n_o\}, \{\phi\}, \{m_o, o_o, p_o\}\rangle \subseteq Y$. Then, $2_{nc}\gamma int(h^{-1}(K_o)) \subseteq 2_{nc}\gamma int(h^{-1}(\langle\{l_o, n_o\}, \{\phi\}, \{m_o, o_o, p_o\}\rangle)) = 2_{nc}\gamma int(\langle\{l_o, n_o\}, \{\phi\}, \{m_o, o_o, p_o\}\rangle) = \langle\{l_o, n_o\}, \{\phi\}, \{m_o, o_o, p_o\}\rangle$. But $h^{-1}(2_{nc}int(K_o)) = h^{-1}(2_{nc}int(\langle\{l_o, n_o\}, \{\phi\}, \{m_o, o_o, p_o\}\rangle)) = h^{-1}(\langle\{n_o\}, \{\phi\}, \{l_o, m_o, o_o, p_o\}\rangle) = \langle\{n_o\}, \{\phi\}, \{l_o, m_o, o_o, p_o\}\rangle$. Thus, $2_{nc}\gamma int(h^{-1}(K_o)) \neq h^{-1}(2_{nc}int(K_o))$.

Definition 3.2.

Let $K_o = \langle K_1, K_2, K_3\rangle$ a $N_{nc}s$ on X, then $pt = \langle pt_1, pt_2, pt_3\rangle$, $pt_1 \neq pt_2 \neq pt_3 \in X$ is called a N-neutrosophic crisp point (briefly, $N_{nc}P$).

A $N_{nc}P$, $pt = \langle pt_1, pt_2, pt_3\rangle$ belongs to a $N_{nc}s$ $K_o = \langle K_1, K_2, K_3\rangle$ of X, denoted by $pt \in K_o$, if it may be defined in two ways,

(i) $\{pt_1\} \subseteq K_1$, $\{pt_2\} \subseteq K_2$ & $\{pt_3\} \supseteq K_3$, or

(ii) $\{pt_1\} \subseteq K_1$, $\{pt_2\} \supseteq K_2$ & $\{pt_3\} \supseteq K_3$.

Theorem 3.4.

Let h be a map, then the following statements are equivalent.

(i) h is a $N_{nc}\gamma Cts$ function;

(ii) For every $N_{nc}P$ $pt_{(pt_1, pt_2, pt_3)} \in X_1$ and each ncs K_o of $h(pt_{(pt_1, pt_2, pt_3)})$, $\exists$ a $N_{nc}\gamma os$ M_o such that $pt_{(pt_1,pt_2,pt_3)} \in M_o \subseteq h^{-1}(K_o)$;

(iii) For every N_{nc} point $pt_{(pt_1,pt_2,pt_3)} \in X_1$ and each ncs K_o of $h(pt_{(pt_1,pt_2,pt_3)})$, $\exists$ a $N_{nc}\gamma os$ M_o such that $pt_{(pt_1,pt_2,pt_3)} \in M_o$ and $h(M_o) \subseteq K_o$.

Proof:

(i) $\Rightarrow$ (ii): If $pt_{(pt_1,pt_2,pt_3)}$ is a $N_{nc}P$ in X_1 and if K_o is a ncs of $h(pt_{(pt_1,pt_2,pt_3)})$, then there exists an $N_{nc}os$ W in X_2 such that $h(pt_{(pt_1,pt_2,pt_3)}) \in W \subset K_o$. Thus, h is a $N_{nc}\gamma Cts$, $M_o = h^{-1}(W)$ is a $N_{nc}\gamma os$ and $pt_{(pt_1,pt_2,pt_3)} \in h^{-1}(h(pt_{(pt_1,pt_2,pt_3)})) \subseteq h^{-1}(W) = M_o \subseteq h^{-1}(K_o)$. Thus, (ii) is a valid statement.

(ii) $\Rightarrow$ (iii): Let $pt_{(pt_1,pt_2,pt_3)}$ be a $N_{nc}P$ in X_1 and let K_o be a ncs of $h(pt_{(pt_1,pt_2,pt_3)})$. Then, there exists a $N_{nc}\gamma os$ M_o such that $pt_{(pt_1,pt_2,pt_3)} \in M_o \subseteq h^{-1}(K_o)$ by (ii). Thus, we have $pt_{(pt_1,pt_2,pt_3)} \in M_o$ and $h(M_o) \subseteq h(h^{-1}(K_o)) \subseteq K_o$. Hence, (iii) is valid.

(iii) $\Rightarrow$ (i): Let K_o be a $N_{nc}os$ in X_2 and let $pt_{(pt_1,pt_2,pt_3)} \in h^{-1}(K_o)$. Then, $h(pt_{(pt_1,pt_2,pt_3)}) \in h(h^{-1}(K_o)) \subset K_o$. Since K_o is a $N_{nc}os$, it follows that K_o is a ncs of $h(pt_{(pt_1,pt_2,pt_3)})$. Therefore, from (iii), there exists a $N_{nc}\gamma os$ M_o such that $pt_{(pt_1,pt_2,pt_3)}$ and $h(M_o) \subseteq K_o$. This implies that $pt_{(pt_1,pt_2,pt_3)} \in M_o \subseteq h^{-1}(h(M_o)) \subseteq h^{-1}(K_o)$. Therefore, we know that $h^{-1}(K_o)$ is a $N_{nc}\gamma os$ in X_1. Thus, h is a $N_{nc}\gamma Cts$ function. ■

Theorem 3.5.

Let h be a map, then the following statements are equivalent.

(i) h is a $N_{nc}\gamma Cts$ function;

(ii) For each $pt_{(pt_1,pt_2,pt_3)} \in X_1$ each $N_{nc}os$ $M_o \subset X_2$ containing $h(pt_{(pt_1,pt_2,pt_3)})$, there exists a $N_{nc}\gamma os$ $L \subset X_1$ containing $pt_{(pt_1,pt_2,pt_3)}$ such that $h(L) \subset M_o$.

Proof:

(i) $\Rightarrow$ (ii): Let M_o be any $N_{nc}os$ of X_2 containing $h(pt_{(pt_1,pt_2,pt_3)})$, then $h^{-1}(M_o) \in N_{nc}\gamma OS(X_1)$. The set $L = h^{-1}(M_o)$ which containing $pt_{(pt_1,pt_2,pt_3)}$, then $h(L) \subset M_o$.

(ii) $\Rightarrow$ (i): Let $M_o \subset X_2$ be $N_{nc}os$ and let $pt_{(pt_1,pt_2,pt_3)} \in h^{-1}(M_o)$. Then, $h(pt_{(pt_1,pt_2,pt_3)}) \in M_o$ and thus there exists $L_p \in N_{nc}\gamma OS(X_1)$ such that $pt_{(pt_1,pt_2,pt_3)} \in L_p$ and $h(W_p) \subset M_o$. Then, $pt_{(pt_1,pt_2,pt_3)} \in L_p \subset h^{-1}(M_o)$, and so $h^{-1}(M_o) = \bigcup_{p\in h^{-1}(M_o)} L_p$ but $\bigcup_{p\in h^{-1}(M_o)} L_p \in N_{nc}\gamma OS(X_1)$, then, $h^{-1}(M_o) \in N_{nc}\gamma OS(X_1)$. Therefore, h is $N_{nc}\gamma Cts$. ■

Theorem 3.6.

Let h be a map. Then, the following statements are equivalent.

(i) h is a $N_{nc}\gamma Cts$ function;

(ii) The inverse image of each $N_{nc}cs$ in X_2 is $N_{nc}\gamma cs$;

(iii) $N_{nc}int(N_{nc}cl(h^{-1}(K_o))) \cap N_{nc}cl(N_{nc}int(h^{-1}(K_o))) \subset h^{-1}(cl(K_o))$ for each $K_o \subset X_2$;

(iv) $h(N_{nc}int(cl(O))) \cap N_{nc}cl(N_{nc}int(O)) \subset N_{nc}cl(h(O))$ for each $O \subset X_1$.

Proof:

(i) $\Rightarrow$ (ii): Let $H \subset X_2$ be $N_{nc}cs$. Then, $X_2 \backslash H$ is $N_{nc}os$. $h^{-1}(X_2 \backslash H) \in N_{nc}\gamma OS(X_1)$, i.e., $X_1 \backslash h^{-1}(H) \in N_{nc}\gamma OS(X_1)$. Then $h^{-1}(H)$ is $N_{nc}\gamma cs$.

(ii) $\Rightarrow$ (iii): Let $K_o \subset X_2$, then $h^{-1}(N_{nc}cl(K_o))$ is $N_{nc}\gamma cs$ in X_1, i.e., $N_{nc}int(N_{nc}cl(h^{-1}(K_o))) \cap N_{nc}cl(N_{nc}int(h^{-1}(K_o))) \subset N_{nc}int(N_{nc}cl(h^{-1}(N_{nc}cl(K_o)))) \cap N_{nc}cl(N_{nc}int(h^{-1}(N_{nc} cl(K_o)))) \subset h^{-1}(N_{nc}cl(K_o))$.

(iii) $\Rightarrow$ (iv): Let $O \subset X_1$, $K_o = h(O)$ in (iii) then $N_{nc}int(N_{nc}cl(h^{-1}(h(O)))) \cap N_{nc}cl(N_{nc} int(h^{-1}(h(O)))) \subset h^{-1}(N_{nc}cl(h(O)))$. So that $N_{nc}int(N_{nc}cl(O)) \cap N_{nc}cl(N_{nc}int(O)) \subset h^{-1}(N_{nc}cl(h(O)))$. This gives $h(N_{nc}int(cl(O))) \cap N_{nc}cl(N_{nc}int(O)) \subset N_{nc}cl(h(O))$.

(iv) $\Rightarrow$ (i): Let $M_o \subset X_2$ be $N_{nc}os$. $L = X_2 \backslash M_o$ and $O = h^{-1}(L)$, then $h(N_{nc}int(N_{nc}cl(h^{-1}(L)))) \cap N_{nc}cl(N_{nc}int(h^{-1}(L))) \subset N_{nc}cl(h(h^{-1}(L))) \subset N_{nc}cl(L) = L$, i.e., $h^{-1}(L)$ is $N_{nc}\gamma cs$ in X_1, so h is $N_{nc}\gamma Cts$. ■

4. Weakly N-neutrosophic Crisp γ Continuous

Definition 4.1.

A map h is said to be a weakly N-neutrosophic crisp continuous (briefly, $WeN_{nc}Cts$) function if for each $pt_{(pt_1,pt_2,pt_3)} \in X_1$ and each $N_{nc}os$ M_o of X_2 containing $h(pt)$, there exists an $N_{nc}os$ of K_o containing $pt_{(pt_1,pt_2,pt_3)}$ such that $h(K_o) \subset N_{nc}cl(M_o)$.

Definition 4.2.

A map h is said to be a weakly N-neutrosophic crisp γ continuous (briefly, $WeN_{nc}\gamma Cts$) function if for each $pt_{(pt_1,pt_2,pt_3)} \in X_1$ and each $N_{nc}os$ M_o of X_2 containing $h(pt)$, there exists $K_o \in N_{nc}\gamma OS(X_1)$ such that $h(K_o) \subset N_{nc}cl(M_o)$.

Remark 4.1.

Every $WeN_{nc}Cts$ function is $WeN_{nc}\gamma Cts$ function. Also, $WeN_{nc}\gamma Cts$ function is implied by $N_{nc}\gamma Cts$. The following examples show that the reverse implications are not true in general.

Example 4.1.

Let $X = \{l_o, m_o, n_o, o_o\} = Y$, ${}_{nc}\tau_1 = \{\phi_N, X_N, L, M, N\}$, ${}_{nc}\tau_2 = \{\phi_N, X_N\}$. $L = \langle \{n_o\}, \{\phi\}, \{l_o, m_o, o_o\} \rangle$, $M = \langle \{l_o, m_o\}, \{\phi\}, \{n_o, o_o\} \rangle$, $N = \langle \{l_o, m_o, n_o\}, \{\phi\}, \{o_o\} \rangle$. Then, we have $2_{nc}\tau = \{\phi_N, X_N, L, M, N\}$. Let ${}_{nc}\Psi_1 = \{\phi_N, Y_N, O = \langle \{l_o, m_o\}, \{\phi\}, \{n_o, o_o\} \rangle \}$, ${}_{nc}\Psi_2 = \{\phi_N, Y_N, P = \langle \{n_o, o_o\}, \{\phi\}, \{l_o, m_o\} \rangle \}$. Then, we have $2_{nc}\Psi = \{\phi_N, Y_N, O, P\}$.

Define $h : (X, 2_{nc}\tau) \to (Y, 2_{nc}\Psi)$ as identity function, then h is $We2_{nc}\gamma Cts$ but not $We2_{nc}Cts$ and $2_{nc}\gamma Cts$.

Definition 4.3.

Let $(X_1, N_{nc}\Gamma)$ be $N_{nc}ts$, $pt \in N_{nc}P$ in X_1, a $N_{nc}s$ $K_o = \langle K_1, K_2, K_3\rangle \in \Gamma$ is called N-neutrosophic crisp neighborhood (briefly, $N_{nc}nbhd$) of p in $(X_1, N_{nc}\Gamma)$ if $pt \in K_o$, if there is a $N_{nc}os$, $M_o = \langle M_1, M_2, M_3\rangle$ containing pt such that $M_o \subseteq K_o$.

Lemma 4.1.

Let h be a map, then the following statements are equivalent.

(i) h is $WeN_{nc}\gamma Cts$ at $pt_{(pt_1,pt_2,pt_3)} \in X_1$;

(ii) $pt \in N_{nc}cl(N_{nc}int(h^{-1}(N_{nc}cl(M_o)))) \cup N_{nc}int(N_{nc}cl(h^{-1}(N_{nc}cl(M_o))))$ for each $N_{nc}nbhd$ M_o of $h(pt)$;

(iii) $h^{-1}(M_o) \subset N_{nc}\gamma int(h^{-1}(N_{nc}cl(M_o)))$ for every $N_{nc}os$ M_o of X_2.

Proof:

(i) $\Rightarrow$ (ii): Let M_o be any $N_{nc}nbhd$ of $h(pt)$. Since h is $WeN_{nc}\gamma Cts$ at pt, there exist $L \in N_{nc}\gamma OS(X_1)$ such that $h(L) \subset N_{nc}cl(M_o)$. Then, $L \subset h^{-1}(N_{nc}cl(M_o))$. Since L is $N_{nc}\gamma o$, $pt \in L \subset N_{nc}cl(N_{nc}int(L)) \cup N_{nc}int(N_{nc}cl(L)) \subset N_{nc}cl(N_{nc}int(h^{-1}N_{nc}cl(M_o))) \cup N_{nc}int(N_{nc}cl(h^{-1}(N_{nc}cl(M_o))))$.

(ii) $\Rightarrow$ (iii): Let $pt \in h^{-1}(M_o)$ so $h(pt) \in M_o$. Then, $pt \in h^{-1}(N_{nc}cl(M_o))$ and since $pt \in N_{nc}cl(N_{nc}int(h^{-1}(N_{nc}cl(M_o)))) \cup N_{nc}int(N_{nc}cl(h^{-1}(N_{nc}cl(M_o))))$. We have $pt \in h^{-1}(N_{nc}cl(M_o)) \cap [N_{nc}cl(N_{nc}int(h^{-1}(N_{nc}cl(M_o)))) \cup N_{nc}int(N_{nc}cl(h^{-1}(N_{nc}cl(M_o))))] = N_{nc}\gamma int(h^{-1}(N_{nc}cl(M_o)))$. Hence, $h^{-1}(M_o) \subset N_{nc}\gamma int(h^{-1}(N_{nc}cl(M_o)))$.

(iii) $\Rightarrow$ (i): Let M_o be any $N_{nc}nbhd$ of $h(pt)$. Then, $pt \in h^{-1}(M_o) \subset N_{nc}\gamma int(h^{-1}(N_{nc}cl(M_o)))$. The set $L = N_{nc}\gamma int(h^{-1}(N_{nc}cl(M_o)))$ then $L \in N_{nc}\gamma OS(X_1)$ and $h(L) \subset N_{nc}cl(M_o)$. This shows that h is $WeN_{nc}\gamma Cts$ at $pt \in X_1$. ■

Theorem 4.1.

Let h be a map, then the following statements are equivalent.

(i) h is $WeN_{nc}\gamma Cts$;

(ii) $N_{nc}\gamma cl(h^{-1}(N_{nc}int(N_{nc}cl(K_o)))) \subset h^{-1}(N_{nc}cl(K_o))$ for every subset K_o of X_2;

(iii) $N_{nc}\gamma cl(h^{-1}(N_{nc}int(F))) \subset h^{-1}(F)$ for every $N_{nc}rcs$ F of X_2;

(iv) $N_{nc}\gamma cl(h^{-1}(M_o)) \subset h^{-1}(N_{nc}cl(M_o))$ for every $N_{nc}os$ M_o of X_2;

(v) $h^{-1}(M_o) \subset N_{nc}\gamma int(h^{-1}(N_{nc}cl(M_o)))$ for every $N_{nc}os$ M_o of X_2;

(vi) $h^{-1}(M_o) \subset N_{nc}cl(N_{nc}int(h^{-1}(N_{nc}cl(M_o)))) \cup N_{nc}int(N_{nc}cl(h^{-1}(N_{nc}cl(M_o))))$ for every $N_{nc}os$ M_o of X_2.

Proof:

(i) $\Rightarrow$ (ii): Let K_o be any subset of X_2. Assume that $pt \in X_1 \backslash h^{-1}(N_{nc}cl(K_o))$. Then, $h(pt) \in X_2 \backslash N_{nc}cl(K_o)$ and there exists a $N_{nc}os$ M_o containing $h(pt)$ such that $M_o \cap K_o = \varnothing$; hence $N_{nc}cl(M_o) \cap N_{nc}int(N_{nc}cl(K_o)) = \varnothing$. Since h is $WeN_{nc}\gamma Cts$, there exists $L \in N_{nc}\gamma OS(X_1)$ such that $h(L) \subset N_{nc}cl(M_o)$. Therefore, we have $L \cap h^{-1}(N_{nc}int(N_{nc}cl(K_o))) = \varnothing$. Hence, $pt \in X_1 \backslash N_{nc}\gamma cl(h^{-1}(N_{nc}int(N_{nc}cl(K_o))))$. Thus, we obtain $N_{nc}\gamma cl(h^{-1}(N_{nc}int(N_{nc}cl(K_o)))) \subset h^{-1}(N_{nc}cl(K_o))$.

(ii) $\Rightarrow$ (iii): Let F be any $N_{nc}rcs$ of X_2. Then, we have $N_{nc}\gamma cl(h^{-1}(N_{nc}int(F))) \subset N_{nc}\gamma cl(h^{-1}(N_{nc}int(N_{nc}cl(N_{nc}int(F))))) \subset h^{-1}(N_{nc}cl(N_{nc}int(F))) = h^{-1}(F)$.

(iii) $\Rightarrow$ (iv): For any $N_{nc}os$ M_o of X_2, $N_{nc}cl(M_o)$ is $N_{nc}rcs$ in X_2 and we have $N_{nc}\gamma cl(h^{-1}(M_o)) \subset N_{nc}\gamma cl(h^{-1}(N_{nc}int(N_{nc}cl(M_o)))) \subset h^{-1}(N_{nc}cl(M_o))$.

(iv) $\Rightarrow$ (v): Let M_o be any $N_{nc}os$ of X_2. Then, $X_2 \backslash N_{nc}cl(M_o)$ is $N_{nc}os$ in X_2 and using $N_{nc}\gamma cl(X_1 \backslash A) = X_1 \backslash N_{nc}\gamma int(A)$, we have $X_1 \backslash N_{nc}\gamma int(h^{-1}(N_{nc}cl(M_o))) = N_{nc}\gamma cl(h^{-1}(X_2 \backslash N_{nc}cl(M_o))) \subset h^{-1}(N_{nc}cl(X_2 \backslash N_{nc}cl(M_o))) \subset X_1 \backslash h^{-1}(M_o)$. Therefore, we obtain $h^{-1}(M_o) \subset N_{nc}\gamma int(h^{-1}(N_{nc}cl(M_o)))$.

(v) $\Rightarrow$ (vi): Let M_o be any $N_{nc}os$ of X_2. By using $N_{nc}\gamma int(A) = A \cap (N_{nc}int(N_{nc}cl(A)) \cup N_{nc}cl(N_{nc}int(A)))$, we have $h^{-1}(M_o) \subset N_{nc}\gamma int(h^{-1}(N_{nc}cl(M_o))) \subset N_{nc}cl(N_{nc}int(h^{-1}(N_{nc}cl(M_o)))) \cup N_{nc}int(N_{nc}cl(h^{-1}(N_{nc}cl(M_o))))$.

(vi) $\Rightarrow$ (i): Let pt be any $N_{nc}P$ of X_1 and M_o any $N_{nc}os$ of X_2 containing $h(pt)$. Then, $pt \in h^{-1}(M_o) \subset N_{nc}cl(N_{nc}int(h^{-1}(N_{nc}cl(M_o)))) \cup N_{nc}int(N_{nc}cl(h^{-1}(N_{nc}cl(M_o))))$. It follows from Lemma 4.1, h is $WeN_{nc}\gamma Cts$. ■

Theorem 4.2.

Let h be a map, then the following statements are equivalent.

(i) h is $WeN_{nc}\gamma Cts$;

(ii) $N_{nc}\gamma cl(h^{-1}(N_{nc}int(N_{nc}cl(M_o)))) \subset h^{-1}(N_{nc}cl(M_o))$ for each $M_o \in N_{nc}\gamma OS(X_2)$;

(iii) $N_{nc}\gamma cl(h^{-1}(M_o)) \subset h^{-1}(N_{nc}cl(M_o))$ for each $M_o \in N_{nc}\mathcal{P}OS(X_2)$;

(iv) $h^{-1}(M_o) \subset N_{nc}\gamma int(h^{-1}(N_{nc}cl(M_o)))$ for each $M_o \in N_{nc}\mathcal{P}OS(X_2)$;

(v) $N_{nc}\gamma cl(h^{-1}(N_{nc}int(N_{nc}cl(M_o)))) \subset h^{-1}(N_{nc}cl(M_o))$ for each $M_o \in N_{nc}\mathcal{S}OS(X_2)$.

Proof:

(i) $\Rightarrow$ (ii): Let M_o be any $N_{nc}\gamma os$ of X_2. Then, $N_{nc}cl(M_o)$ is $N_{nc}rcs$. Since h is $WeN_{nc}\gamma Cts$, from Theorem 4.1 we have $N_{nc}\gamma cl(h^{-1}(N_{nc}int(N_{nc}cl(M_o)))) \subset h^{-1}(N_{nc}cl(M_o))$.

(ii) $\Rightarrow$ (iii): Clear, since $N_{nc}\mathcal{P}OS(X_2) \subset N_{nc}\gamma OS(X_2)$ and $M_o \subset N_{nc}int(N_{nc}cl(M_o))$.

(iii) $\Rightarrow$ (iv): This is similar to the proof of the implication (iv) $\Rightarrow$ (v) in Theorem 4.1.

(iv) $\Rightarrow$ (i): This follows from Theorem 4.1, since every $N_{nc}os$ is $N_{nc}\mathcal{P}os$. ■

Lemma 4.2.

If h is $WeN_{nc}\gamma Cts$ and g is $N_{nc}Cts$, then the composition $g \circ h : (X_1, N_{nc}\Gamma) \to (X_3, N_{nc}\Phi)$ is $WeN_{nc}\gamma Cts$.

Proof:

Let $pt \in X_1$ and K_o be a $N_{nc}os$ of X_3 containing $g(h(pt))$. Then, $g^{-1}(K_o)$ is a $N_{nc}os$ of X_2 containing $h(pt)$ and there exists $M_o \in N_{nc}\gamma OS(X_1)$ such that $h(M_o) \subset N_{nc}cl(g^{-1}(K_o))$. Since g is $N_{nc}Cts$, we obtain $(g \circ h)(M_o) \subset g(N_{nc}cl(g^{-1}(K_o))) \subset g(g^{-1}(N_{nc}cl(K_o))) \subset N_{nc}cl(K_o)$. ■

5. N-neutrosophic Crisp γ-contra Continuous Function

Definition 5.1.

A map h is said to be a N-neutrosophic crisp γ contra continuous (briefly, $N_{nc}\gamma CCts$) function if the inverse image of each $N_{nc}os$ K_o in $(X_2, N_{nc}\Psi)$ is a $N_{nc}\gamma cs$ set in $(X_1, N_{nc}\Gamma)$.

Example 5.1.

Let $X = \{l_o, m_o, n_o, o_o\} = Y$, ${}_{nc}\tau_1 = \{\phi_N, X_N, L, M, N\}$, ${}_{nc}\tau_2 = \{\phi_N, X_N\}$. $L = \langle\{l_o\}, \{\phi\}, \{m_o, n_o, o_o\}\rangle$, $M = \langle\{m_o, n_o\}, \{\phi\}, \{l_o, o_o\}\rangle$, $N = \langle\{l_o, m_o, n_o\}, \{\phi\}, \{o_o\}\rangle$. Then, we have $2_{nc}\tau = \{\phi_N, X_N, L, M, N\}$. ${}_{nc}\Psi_1 = \{\phi_N, Y_N, O, P\}$, ${}_{nc}\Psi_2 = \{\phi_N, Y_N\}$. $O = \langle\{l_o, o_o\}, \{\phi\}, \{m_o, n_o\}\rangle$, $P = \langle\{m_o, n_o\}, \{\phi\}, \{l_o, o_o\}\rangle$, then we have $2_{nc}\Psi = \{\phi_N, Y_N, O, P\}$. Let $h : (X, 2_{nc}\tau) \to (Y, 2_{nc}\Psi)$ be an identity function. Then, h is a $2_{nc}\gamma CCts$.

Theorem 5.1.

Let h be a map, then the following statements are equivalent.

(i) h is a $N_{nc}\gamma CCts$ function;

(ii) $h^{-1}(M_o)$ is a $N_{nc}\gamma cs$ set in X_1, for each $N_{nc}os$ M_o in X_2.

Proof:

(i) $\Rightarrow$ (ii): Let h be any $N_{nc}\gamma CCts$ function and let M_o be any $N_{nc}os$ in X_2. Then, $\overline{M_o}$ is a $N_{nc}cs$ in X_2. Based on these assumptions, $h^{-1}(\overline{M_o})$ is a $N_{nc}\gamma os$ in X_1. Hence, $h^{-1}(M_o)$ is a $N_{nc}\gamma cs$ in

X_1.

(ii) $\Rightarrow$ (i): Same way. ■

Theorem 5.2.

Let h be a bijective mapping. Then, h is $N_{nc}\gamma CCts$, if $N_{nc}cl(h(K_o)) \subseteq h(N_{nc}\gamma int(K_o))$, for each ncs K_o in X_1.

Proof:

Let M_o be any $N_{nc}cs$ in X_1. Then, $N_{nc}cl(M_o) = M_o$ and h is onto, by assumption, which shows that $h(N_{nc}\gamma int(h^{-1}(M_o))) \supseteq N_{nc}cl(h(h^{-1}(M_o))) = N_{nc}cl(M_o) = M_o$. Hence, $h^{-1}(h(N_{nc}\gamma int(h^{-1}(M_o)))) \supseteq h^{-1}(M_o)$. Since h is an into mapping, we have $N_{nc}\gamma int(h^{-1}(M_o)) = h^{-1}(h(N_{nc}\gamma int(h^{-1}(M_o)))) \supseteq h^{-1}(M_o)$. Therefore, $N_{nc}\gamma int(h^{-1}(M_o)) = h^{-1}(M_o)$, so $h^{-1}(M_o)$ is a $N_{nc}\gamma os$ in X_1. Hence, h is a $N_{nc}\gamma CCts$ mapping. ■

Theorem 5.3.

Let h be a map, then the following statements are equivalent.

(i) h is a $N_{nc}\gamma CCts$ map;

(ii) For each $N_{nc}P$ $pt_{(pt_1,pt_2,pt_3)} \in X_1$ and $N_{nc}cs$ M_o containing $h(pt_{(pt_1,pt_2,pt_3)})$, there exists a $N_{nc}\gamma os$ K_o in X_1 containing $pt_{(pt_1,pt_2,pt_3)}$ such that $M_o \subseteq h^{-1}(K_o)$;

(iii) For each $N_{nc}P$ $pt_{(pt_1,pt_2,pt_3)} \in X_1$ and $N_{nc}cs$ M_o containing $h(pt_{(pt_1,pt_2,pt_3)})$, there exists a $N_{nc}\gamma os$ K_o in X_1 containing $pt_{(pt_1,pt_2,pt_3)}$ such that $h(K_o) \subseteq M_o$.

Proof:

(i) $\Rightarrow$ (ii): Let h be a $N_{nc}\gamma CCts$ mapping, let M_o be any $N_{nc}cs$ in X_2 and let $pt_{(pt_1,pt_2,pt_3)}$ be a $N_{nc}P$ in X_1 and such that $h(pt_{(pt_1,pt_2,pt_3)}) \in M_o$. Then, $pt_{(pt_1,pt_2,pt_3)} \in h^{-1}(M_o) = N_{nc}\gamma int(h^{-1}(M_o))$. Let $K_o = N_{nc}\gamma int(h^{-1}(M_o))$. Then, K_o is a $N_{nc}\gamma os$ and $K_o = N_{nc}\gamma int(h^{-1}(M_o)) \subseteq h^{-1}(M_o)$.

(ii) $\Rightarrow$ (iii): The results follow from evident relations $h(K_o) \subseteq h(h^{-1}(M_o)) \subseteq M_o$.

(iii) $\Rightarrow$ (i): Let M_o be any $N_{nc}cs$ in X_2 and let $pt_{(pt_1,pt_2,pt_3)}$ be a $N_{nc}P$ in X_1 such that $pt_{(pt_1,pt_2,pt_3)} \in h^{-1}(M_o)$. Then, $h(pt_{(pt_1,pt_2,pt_3)}) \in M_o$. According to the assumption, there exists a $N_{nc}\gamma os$ K_o in X_1 such that $pt_{(pt_1,pt_2,pt_3)} \in K_o$ and $h(K_o) \subseteq M_o$. Hence, $pt_{(pt_1,pt_2,pt_3)} \in K_o \subseteq h^{-1}(h(K_o)) \subseteq h^{-1}(M_o)$. Therefore, $pt_{(pt_1,pt_2,pt_3)} \in K_o = N_{nc}\gamma int(K_o) \subseteq N_{nc}\gamma int(h^{-1}(M_o))$. Since $pt_{(pt_1,pt_2,pt_3)}$ is an arbitrary $N_{nc}P$ and $h^{-1}(M_o)$ is the union of all $N_{nc}P$'s in $h^{-1}(M_o)$, we obtain that $h^{-1}(M_o) \subseteq N_{nc}\gamma int(h^{-1}(M_o))$. Thus, h is a $N_{nc}\gamma CCts$ mapping. ■

6. Application: Neutrosophic Score Function

In this section, we present a neutrosophic score function based on methodical approach for decision-making problem with neutrosophic information.

Definition 6.1.

Let $\mathcal{N}_i = \langle \mu_i, \sigma_i, \gamma_i \rangle$, $i = 1, 2, 3, \cdots, k$ be neutrosophic sets, then the neutrosophic score function (shortly, $NSF(\mathcal{N}_i)$) is defined by $NSF(\mathcal{N}_i) = \frac{1}{3(k+2)} \left[\sum_{i=1}^{k+2} [2 + \mu_i - \sigma_i - \gamma_i] \right]$.

The following essential steps are proposed the precise way to deal with select the proper attributes and alternative in the decision-making situation.

Step 1: Consider the universe of discourse (set of objects) $t_1, t_2, \cdots, t_m$, the set of alternatives $\tau_1, \tau_2, \cdots, \tau_n$, the set of decision attributes $\nu_1, \nu_2, \cdots, \nu_p$.

Step 2: Construct a neutrosophic matrix of alternative verses objects and object verses decision attributes.

Step 3: Construct the neutrosophic topologies τ_i^* and ν_i^*.

Step 4: Find the score values by Definition 6.1 of each of the entries of the NTS.

Step 5: Arrange neutrosophic score values for the alternatives $\tau_1 \leq \tau_2 \leq \cdots \leq \tau_n$ and the attributes $\nu_1 \leq \nu_2 \leq \cdots \leq \nu_p$. Choose the attribute ν_p for the alternative τ_1 and ν_{p-1} for the alternative τ_2 etc. If $n < p$, then ignore ν_k, where $k = 1, 2, \cdots, n - p$.

Example 6.1.

Medical diagnosis has increased volume of data accessible to doctors from new medical innovations and includes vulnerabilities. In medical diagnosis, very difficult task is the way toward classifying different set of symptoms under a single name of an illness. In this part, we exemplify a medical diagnosis problem for effectiveness and applicability of above proposed approach.

Step 1: Consider the following tables giving informations when consulted physicians about four patients, Jhon, Bob, Raja and Thaya and symptoms is a universe of discourse are Temperature (Temp), Headache (Ha), Stomach pain (Spa), Cough (co) and Chest pain (Cpa). We need to find the patient and to find the disease such as Viral fever (Vf), Malaria (Ma), Stomach problem (Sp), Typhoid (Ty) and Chest problem (Cp) of the patient. The data in Table 1 and Table 2 are explained by the membership, the indeterminacy and the non-membership functions of the patients and diseases respectively.

Step 2: Construct a neutrosophic matrix of alternative verses objects and object verses decision attributes.

Step 3: Construct the neutrosophic topologies τ_i^* and ν_i^*.

Table 1. Neutrosophic values for patients

SymptomsPatients	Jhon	Bob	Raja	Thaya
Temperature	(0.8,0.1,0.1)	(0.0,0.2,0.8)	(0.8,0.1,0.1)	(0.6,0.3,0.1)
Headache	(0.6,0.3,0.1)	(0.4,0.2,0.4)	(0.8,0.1,0.1)	(0.5,0.1,0.4)
Stomach pain	(0.2,0.0,0.8)	(0.6,0.3,0.1)	(0.0,0.4,0.6)	(0.3,0.3,0.4)
Cough	(0.6,0.3,0.1)	(0.1,0.2,0.7)	(0.2,0.1,0.7)	(0.7,0.1,0.2)
Chest pain	(0.1,0.3,0.6)	(0.1,0.1,0.8)	(0.0,0.5,0.5)	(0.3,0.3,0.4)

Table 2. Neutrosophic values for disease

DiseaseSymptoms	Temperature	Headache	Stomach pain	Cough	Chest pain
Viral fever	(0.4,0.6,0.0)	(0.3,0.2,0.5)	(0.1,0.2,0.7)	(0.4,0.3,0.3)	(0.1,0.2,0.7)
Malaria	(0.7,0.3,0.0)	(0.2,0.2,0.6)	(0.0,0.1,0.9)	(0.7,0.3,0.0)	(0.1,0.1,0.8)
Stomach problem	(0.1,0.2,0.7)	(0.2,0.4,0.4)	(0.8,0.2,0.0)	(0.2,0.1,0.7)	(0.2,0.1,0.7)
Typhoid	(0.3,0.4,0.3)	(0.6,0.3,0.1)	(0.2,0.1,0.7)	(0.2,0.2,0.6)	(0.1,0.0,0.9)
Chest problem	(0.1,0.1,0.8)	(0.0,0.2,0.8)	(0.2,0.0,0.8)	(0.2,0.0,0.8)	(0.8,0.1,0.1)

Neutrosophic topologies for patients are τ_i^*:

(i) $\tau_1^* = \{(0,0,1),(1,1,0),(0.8,0.1,0.1),(0.6,0.3,0.1),(0.2,0.0,0.8),(0.1,0.3,0.6),(0.8,0.3,0.1),(0.2,0.3,0.6),(0.6,0.1,0.1),(0.1,0.1,0.6),(0.1,0.0,0.8)\}$.

(ii) $\tau_2^* = \{(0,0,1),(1,1,0),(0.0,0.2,0.8),(0.4,0.2,0.4),(0.6,0.3,0.1),(0.1,0.2,0.7),(0.1,0.1,0.8),(0.1,0.2,0.8),(0.0,0.1,0.8)\}$.

(iii) $\tau_3^* = \{(0,0,1),(1,1,0),(0.8,0.1,0.1),(0.0,0.4,0.6),(0.2,0.1,0.7),(0.0,0.5,0.5),(0.8,0.4,0.1),(0.8,0.5,0.1),(0.2,0.4,0.6),(0.2,0.5,0.5),(0.0,0.1,0.6),(0.0,0.1,0.5),(0.0,0.1,0.7)\}$.

(iv) $\tau_4^* = \{(0,0,1),(1,1,0),(0.6,0.3,0.1),(0.5,0.1,0.4),(0.3,0.3,0.4),(0.7,0.1,0.2),(0.7,0.3,0.1),(0.5,0.3,0.4),(0.7,0.3,0.2),(0.6,0.1,0.2),(0.3,0.1,0.4)\}$.

Neutrosophic topologies for diseases are ν_i^*:

(i) $\nu_1^* = \{(0,0,1),(1,1,0),(0.4,0.6,0.0),(0.3,0.2,0.5),(0.1,0.2,0.7),(0.4,0.3,0.3)\}$.

(ii) $\nu_2^* = \{(0,0,1),(1,1,0),(0.7,0.3,0.0),(0.2,0.2,0.6),(0.0,0.1,0.9),(0.1,0.1,0.8)\}$.

(iii) $\nu_3^* = \{(0,0,1),(1,1,0),(0.1,0.2,0.7),(0.2,0.4,0.4),(0.8,0.2,0.0),(0.2,0.1,0.7),(0.2,0.2,0.7),(0.8,0.4,0.0),(0.1,0.1,0.7),(0.2,0.2,0.4)\}$.

(iv) $\nu_4^* = \{(0,0,1),(1,1,0),(0.3,0.4,0.3),(0.6,0.3,0.1),(0.2,0.1,0.7),(0.2,0.2,0.6),(0.1,0.0,0.9),(0.6,0.4,0.1),(0.3,0.3,0.3)\}$.

(v) $\nu_5^* = \{(0,0,1),(1,1,0),(0.1,0.1,0.8),(0.0,0.2,0.8),(0.2,0.0,0.8),(0.8,0.1,0.1),(0.1,0.2,$

$0.8), (0.2, 0.1, 0.8), (0.2, 0.2, 0.8), (0.8, 0.2, 0.1), (0.0, 0.1, 0.8), (0.1, 0.0, 0.8), (0.0, 0.0, 0.8), (0.0, 0.1, 0.8)\}$.

Step 4: Find the neutrosophic score values.

Neutrosophic score values for the patients are

$NSF(\tau_1) = 0.5818$, $NSF(\tau_2) = 0.4666$, $NSF(\tau_3) = 0.5076$, $NSF(\tau_4) = 0.6545$.

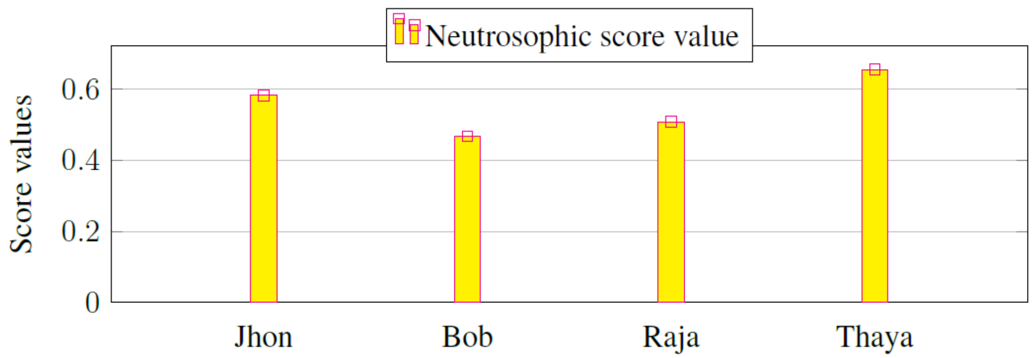

Figure 2. Neutrosophic score values for patients

Neutrosophic score values for the diseases are

$NSF(\nu_1) = 0.5222$, $NSF(\nu_2) = 0.5$, $NSF(\nu_3) = 0.54$, $NSF(\nu_4) = 0.5407$, $NSF(\nu_5) = 0.4761$.

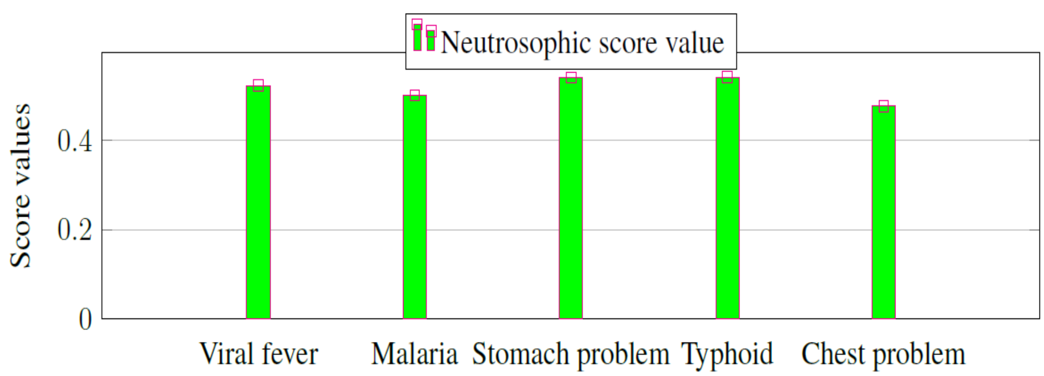

Figure 3. Neutrosophic score values for diseases

Step 5: Arrange neutrosophic score values for the alternatives $\tau_1, \tau_2, \tau_3, \tau_4$ and the attributes $\nu_1, \nu_2, \nu_3, \nu_4, \nu_5$ in run-up order. We consider the sequences below $\tau_2 \leq \tau_3 \leq \tau_1 \leq \tau_4$ and $\nu_5 \leq \nu_2 \leq \nu_1 \leq \nu_3 \leq \nu_4$. Thus, the patient Jhon suffers from Viral fever, the patient Bob suffers from Stomach problem, the patient Raja suffers from Typhoid, and the patient Thaya suffers from Malaria.

7. Conclusion

In this article, we have examined some new types of N-neutrosophic crisp γ-continuous, weakly N-neutrosophic crisp γ-continuous, N-neutrosophic crisp γ-contra continuous maps in N-neutrosophic crisp topological spaces and we analysed the difference between these maps. This can be improved to N-neutrosophic crisp γ-irresolute function, N-neutrosophic crisp γ-homeomorphism functions of N-neutrosophic crisp topological spaces are the further research areas can be covered in future tasks. Finally, we applied a neutrosophic score function in a medical diagnosis problem along with graphical representation.

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