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Fractional Order on Modeling the Transmission of Devastative COVID-19 Infection: Efficacy of Vaccination

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Abstract

The second wave of COVID-19 is an unprecedented condition in India and began in mid February 2021. Individuals who were already suffering from other comorbidities were found with lung infection, and hence, the number of disease induced deaths were rising faster during the second wave in relation to the first wave. This paper has proposed a mathematical model with fractional order derivatives by correlating the model based number of infectives with the real number of infectives in India. For the system of fractional differential equations, a disease-free state has been computed and proved to be locally asymptotically stable with certain restrictions. The mathematical model has been numerically simulated using the predictor-corrector method to highlight the role played by fractional order in controlling the disease spread. Numerical simulations signify the fact that a vital role has been played by fractional order model over integer order model in determining the transmission of COVID-19. It can be visualized that the increment rate in the infectives is lower by taking into consideration the memory effect due to a previous exposure to COVID-19.

Keywords: COVID-19; Vaccination; Fractional Order; Stability; Sensitivity index

MSC 2020 No.: 92B05, 34C60, 34D20, 34D23

1. Introduction

India is the second-leading country in the entire world deteriorated with a massive surge of COVID-19 infectives and deaths. India has experienced devastating impacts during the second wave of COVID-19 disease. The new coronavirus known as SARS-Cov-2 has generated a contagious respiratory infection defined as COVID-19. The transmission of COVID-19 infection can occur either by direct or indirect contacts with actively infected individuals. The COVID-19 infection majorly affects those individuals who already have other immune-based comorbidities. The COVID-19 infection may differ between individuals. Many infected individuals with mild to moderate symptoms can be recovered without getting hospitalized for intensive medical care. In India, the second wave has become a catastrophe especially due to the insufficient availability of medical facilities to the infected individuals. During the second wave of COVID-19, the number of deaths were rising at a higher rate as some of the individuals who were already suffering from other comorbidities were experiencing the lack of medical treatment.

By the end of June 2021, more than 180 million infected individuals in the entire world were diagnosed with COVID-19 disease. Over 3.9 million COVID-19 infectives died in around 215 countries. In other words, it is estimated that out of per 1000 COVID-19 infected individuals more than 21 infectives have died so far. According to the report given by the World Health Organization, it is determined that the transmission of COVID-19 infection in India has occurred due to a cluster of cases. Due to the insufficient number of vaccination for the population, the government of India has decided to control the transmission of COVID-19 infection by working on non-pharmaceutical strategies such as restrictions on social gatherings and international traveling of individuals to and from India. In India, by the end of June 2021, more than 30 million individuals were infected with COVID-19 disease, out of which around 392 thousands infectives succumbed to death due to COVID-19 pandemic. As reported by WHO, out of total deaths due to COVID-19 infection, it is observed that more than 70% of infectives were suffering with other comorbidities.

In the current situation, vaccination to all individuals is considered to be the best strategy to control and reduce the transmission of COVID-19 infection. Therefore, different countries and worldwide scientists are collaborating and working together to introduce the best treatment procedure to reduce the COVID-19 infection. As reported by the World Health Organization, vaccination is not that efficient to eradicate the COVID-19 infection but reduces its infection level in a vaccinated individual. Therefore, the efficacy of vaccination depends on the non-pharmaceutical precautionary measures such as quarantine, social distancing, isolation of infectives and contact tracing.

The first and second wave of COVID-19 pandemic has majorly affected older-aged people, middle-aged people, and other adults especially those who were already infected with other immune-based comorbidities. This pandemic situation has become a major concern to both researchers and health care authorities. Various multidimensional mathematical models have been introduced to analyze and predict the solutions for infectious diseases (Tanvi and Aggarwal (2020a); Tanvi et al. (2020a); Tanvi and Aggarwal (2020b); Tanvi and Aggarwal (2021)). To analyze and control the transmission dynamics of COVID-19 infection, researchers have given different mathematical approaches (Zhang et al. (2020); Shahidul et al. (2020); Chen et al. (2020); Lau et al. (2020);

Colbourn (2020); Prem et al. (2020); Wilder-Smith and Freedman (2020); Tanvi et al. (2021b); Tanvi et al. (2020b); Tanvi et al. (2021a); Lin et al. (2020); Mandal et al. (2020); Ndairou et al. (2020); Ahmad et al. (2021); Khajanchi et al. (2021)). By considering the case study of New York and the entire US, a non-linear mathematical model has been investigated by Ngonghala et al. (2020) along with the incorporation of non-pharmaceutical interventions such as infection prevention and isolation to control of disease. A novel COVID-19 model has been proposed by Kucharski et al. (2020) to estimate the transmission dynamics of COVID-19 in Wuhan during December 2019 to February 2020 by taking into consideration four different databases from within and outside Wuhan.

Nowadays, fractional calculus has become an emerging field to investigate various complex phenomena, such as neural networks and signal processing (Podlubny (1998)). Being an extension of ordinary calculus, fractional calculus works as an important tool and provides more degree of freedom to model the transmission dynamics of infectious diseases. Recently many researchers have investigated fractional differential equation models to incorporate the nonlinear behavior and memory effect into the model. From a mathematical modeler point of view fractional calculus provides an advantage due to inclusion of memory effect in the model and to explore the hidden dynamics of an infectious disease. Fractional differential equations use the hereditary properties of infectives or prior knowledge of the disease due to some previous exposure to the disease and hence provides an advantage over ordinary differential equations as these properties have not been taken under consideration in ordinary differential equation models. In comparison to the classical integer order models, the study of infectious diseases becomes more realistic and highly predictable in case of fractional order models by incorporating memory effects and hereditary properties of the infection.

Bearing in consideration the advantage of fractional calculus in disease modeling, many researchers have introduced fractional order differential equation models to explore the dynamics of COVID-19 (Nazir et al. (2021); Alzaid and Alkahtani (2020)). Rajagopal et al. (2020) have introduced a SEIRD model with fractional order derivatives in Caputo sense to analyze the COVID-19 prevalence by incorporating memory effect into the model and found out the root mean-square value less then the classical one. Their results show that the fractional order model is more realistic due to the inclusion of memory effect. A fractional order model with different compartments under fractional order derivatives has been introduced by Ahmad et al. (2020). They have compared their model with the actual reported data against confirmed infectives and death cases for initial 67 days in Wuhan city. Baba and Nasidi (2021) proposed a nonlinear fractional order mathematical model to understand the transmissibility of COVID-19 in the human population. Numerical simulations have been performed to visualize the rich dynamics by varying the value of fractional order. A fractional order model for the case study of Wuhan China has been studied by Yadav and Verma (2021) and the comparative analysis has also been performed for classical model and fractional model along with the certified experimental data.

Keeping in view the aforementioned articles and the importance of including memory effect in disease modeling, we have introduced a Caputo fractional order mathematical model by incorporating the effect of vaccination in the transmission dynamics of COVID-19. Fractional order model is useful to capture the consequence of re-infection from COVID-19 or having a certain memory of COVID-19 on the transmission and control of the disease. Qualitative and quantitative analysis of the model has been done following Perko (1991), Strogatz (2014), and Podlubny (1998).

The content of this article is arranged in following manner. In the second section, a non-linear fractional order model has been developed in Caputo sense together with the justification of the well posedness of the model in a positively invariant region under consideration. The equilibrium points have been computed in the third section together with the analysis of the disease-free equilibrium point in the fourth section. The fifth section captures the effect of various parameters on the reproduction number with the help of sensitivity analysis. Numerical simulations have been performed to validate the current data of India with different values of the fractional order in the sixth section. In the last section, the proposed model results have been concluded in brief.

2. Model Formation

To analyze the transmission of COVID-19 infection, a seven dimensional mathematical model is introduced with mutually distinct classes. The system of differential equations is based on fractional order derivatives of order $\alpha \in (0,1)$ to justify the significance of memory effects. The memory effects of individuals involve the previous exposures from the COVID-19 infection and/or some other immune based comorbidities such as Human Immunodeficiency Virus (HIV), tuberculosis, diabetes and others. The total population is changing per day by the virtue of natural births represented by a constant recruitment rate Π and natural death of individuals represented by the parameter δ. The total population $N(t)$ is divided into seven distinct classes of population, $S_c(t)$: the class of individuals susceptibles to COVID-19 infection, $E_c(t)$: the class of individuals that are pre-symptomatically infected and are exposed to COVID-19 infection, $V_c(t)$: the class of susceptibles and exposed individuals who have attained COVID-19 vaccination. The total actively infected individuals from COVID-19 infection are categorized into three mutually distinct classes, $I_c(t)$: the class of isolated infectives that are either symptomatic or asymptomatic from COVID-19 infection, $U_c(t)$: the class of infected individuals in the need of intensive medical care but are unable to attain it due to lack of medical facilities and $H_c(t)$: the class of COVID-19 infected individuals who require intensive medical care and are hospitalized and $R_c(t)$: the class of recovered individuals from COVID-19 infection. Therefore, the total population $N(t)$ can be written as

$$N(t) = S_c(t) + E_c(t) + V_c(t) + I_c(t) + U_c(t) + H_c(t) + R_c(t).$$

Corresponding to the model, the transmission rate of COVID-19 infection is determined by the constant parameter β. Also, the force of infection (λ) is given as

$$\lambda = \frac{\beta}{N(t)}(\eta_1 I_c(t) + \eta_2 U_c(t) + H_c(t)).$$

The modification parameters $0 < \eta_1, \eta_2 < 1$ represent the infectiousness level of isolated and unattended infectives among the infected classes. With the force of infection λ, the susceptibles receive infection and become exposed to COVID-19 infection. At the constant rate ν_1 and ν_2, susceptibles and exposed individuals, respectively, attain vaccination against COVID-19 disease and move

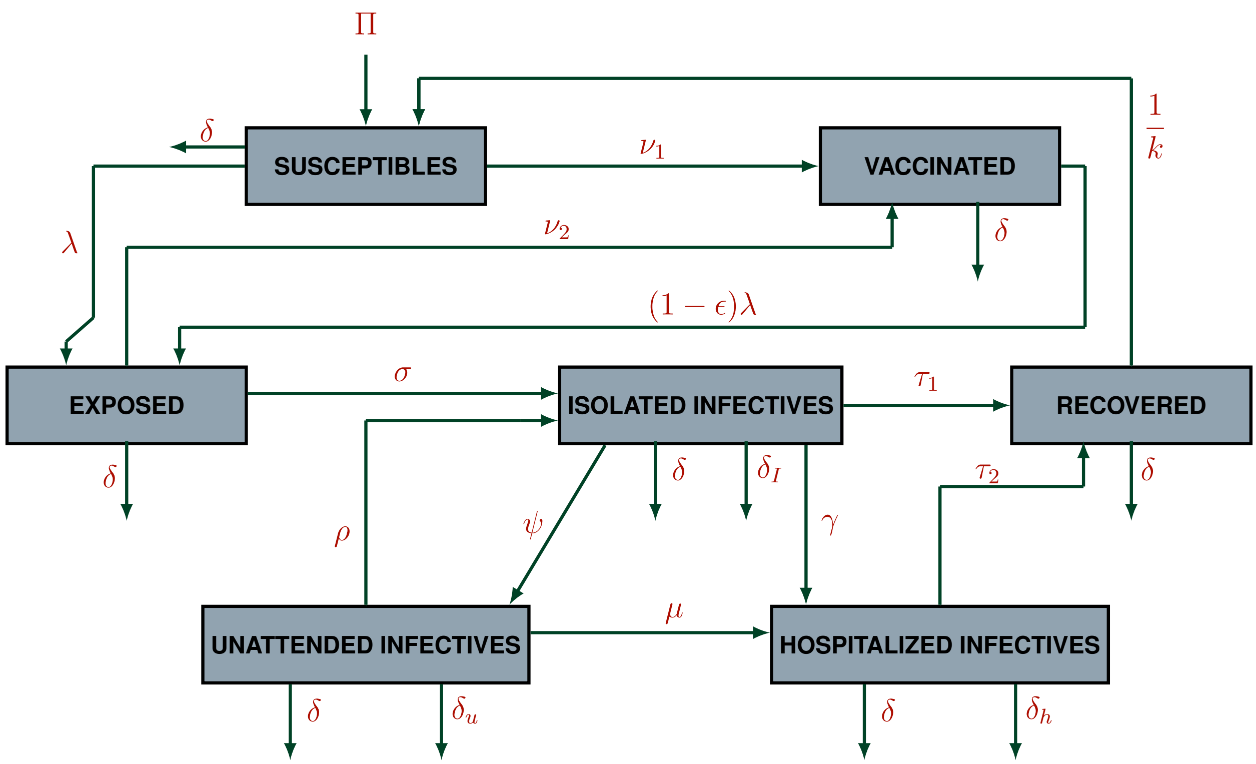


Figure 1. Schematic diagram illustrating the COVID-19 transmission

to class $V_c(t)$. After the incubation period denoted by $\frac{1}{\sigma}$, the exposed individuals start showing symptoms of COVID-19 infection and headway to the isolated class. Due to the incautiousness in precautionary measures, the vaccinated individuals get exposed with the disease at the rate $(1-\epsilon)\lambda$. The constant parameter ψ represents the rate at which isolated infectives remain unattended due to insufficient medical treatment facilities. However, the fraction ρ of unattended infectives have attained medical care during home isolation and enters into the class of isolated infectives again. By the constant rate γ, a fraction of isolated infectives attain medical treatment at hospitals and move to the class of hospitalized infectives. Also, μ is a fraction of unattended infectives that require intensive medical care and move into the hospitals at the μU_c. At the constant rate τ_1 and τ_2, the isolated infectives and hospitalized infectives, respectively, get recovered from COVID-19 infection and switch to the class of recovered individuals. As reported by the World Health Organization, individuals recovered from COVID-19 disease may not be immune for the long time, and therefore, again become susceptibles to COVID-19 infection.

Using the variables described above and model parameters in Table 1, the mathematical model is represented by the schematic diagram given in Figure 1 and the corresponding nonlinear system of

Table 1. The characterization of parameters

Parameter	Description
Π	constant natural birth rate
β	rate of transmission for COVID-19
ν_1	rate of vaccination for the susceptibles against COVID-19
ν_2	rate of vaccination for the exposed individuals against COVID-19
δ	natural death rate
η_1	modification parameter
δ_I	COVID-19 disease succumbed rate of isolated infectives
δ_u	COVID-19 disease succumbed rate of unattended infectives
δ_h	COVID-19 disease succumbed rate of hospitalized infectives
σ	rate at which exposed individuals become isolated infected
ϵ	efficacy of vaccination against COVID-19
ρ	fraction of unattended infectives becoming isolated infectives
ψ	rate at which isolated infectives become unattended
η_2	modification parameter
γ	rate at which isolated infectives attain medical care
τ_1	recovery rate of isolated infectives against COVID-19
τ_2	recovery rate of hospitalized infectives against COVID-19
k	period after which recovered individuals become susceptibles for COVID-19
μ	fraction of unattended infectives attaining medical care at hospitals

differential equations with fractional order derivative is given as

$$\begin{aligned}
\frac{d^\alpha S_c}{dt^\alpha} &= \Pi + \frac{1}{k}R_c - \lambda S_c - \nu_1 S_c - \delta S_c,\\
\frac{d^\alpha E_c}{dt^\alpha} &= \lambda S_c + (1-\epsilon)\lambda V_c - \nu_2 E_c - \sigma E_c - \delta E_c,\\
\frac{d^\alpha V_c}{dt^\alpha} &= \nu_1 S_c + \nu_2 E_c - (1-\epsilon)\lambda V_c - \delta V_c,\\
\frac{d^\alpha I_c}{dt^\alpha} &= \sigma E_c + \rho U_c - \psi I_c - \gamma I_c - \tau_1 I_c - (\delta+\delta_I)I_c,\\
\frac{d^\alpha U_c}{dt^\alpha} &= \psi I_c - \rho U_c - \mu U_c - (\delta+\delta_u)U_c,\\
\frac{d^\alpha H_c}{dt^\alpha} &= \mu U_c + \gamma I_c - \tau_2 H_c - (\delta+\delta_h)H_c,\\
\frac{d^\alpha R_c}{dt^\alpha} &= \tau_1 I_c + \tau_2 H_c - \frac{1}{k}R_c - \delta R_c,
\end{aligned} \tag{1}$$

with the initial conditions given as

$$\begin{aligned}
&S_c(0) = S_{c_0} \geqslant 0,\ E_c(0) = E_{c_0} \geqslant 0,\ V_c(0) = V_{c_0} \geqslant 0,\ I_c(0) = I_{c_0} \geqslant 0,\\
&U_c(0) = U_{c_0} \geqslant 0,\ H_c(0) = H_{c_0} \geqslant 0 \text{ and } R_c(0) = R_{c_0} \geqslant 0.
\end{aligned} \tag{2}$$

All the variables describing human population, that is, $S_c(t),\ E_c(t),\ V_c(t), I_c(t),\ U_c(t),\ H_c(t)$, and $R_c(t)$, must be positive for all time $t > 0$. Thus, for rest of the analysis the following biologically feasible region will be considered:

$$\Omega = \left\{ (S_c, E_c, V_c, I_c, U_c, H_c, R_c) \in \mathbb{R}^7_+ : N(t) \leqslant \frac{\Pi}{\delta} \right\}.$$

Theorem 2.1.

There exist a unique solution $X(t) = (S_c(t), E_c(t), V_c(t), I_c(t), U_c(t), H_c(t), R_c(t))$ for the model system (1), together with the initial conditions given by (2) in the positively invariant region Ω.

Proof:

Following Lin (2007), it can be easily verified that the solution $X(t) = (S_c(t), E_c(t), V_c(t), I_c(t), U_c(t), H_c(t), R_c(t))$ corresponding to the model system (1) exists uniquely. Further, it can be observed that

$$\begin{aligned}
D_t^\alpha S_{c|S_c=0} &= \Pi + \frac{1}{k} R_C > 0,\\
D_t^\alpha E_{c|E_c=0} &= \frac{\beta}{N}(\eta_1 I_c + \eta_2 U_c + H_c) S_c + (1-\epsilon)\frac{\beta}{N}(\eta_1 I_c + \eta_2 U_c + H_c) V_c \geqslant 0,\\
D_t^\alpha V_{c|V_c=0} &= \nu_1 S_c + \nu_2 E_c \geqslant 0,\\
D_t^\alpha I_{c|I_c=0} &= \sigma E_c + \rho U_c \geqslant 0,\\
D_t^\alpha U_{c|U_c=0} &= \psi I_c \geqslant 0,\\
D_t^\alpha H_{c|H_c=0} &= \mu U_c + \gamma I_c \geqslant 0,\\
D_t^\alpha R_{c|R_c=0} &= \tau_1 I_c + \tau_2 H_c \geqslant 0.
\end{aligned}$$

Thus, it follows that all the solution components are positive. Further, to show positive invariance of the region Ω, it is required to prove that every solution trajectory starting in Ω remains in Ω for all $t \geqslant 0$. Corresponding to the model system (1), the rate of change of total population can be written as

$$\begin{aligned}
D_t^\alpha N(t) &= \Pi - \delta_I I_c(t) - \delta_u U_c(t) - \delta_h H_c(t) - \delta N(t)\\
&\leqslant \Pi - \delta N(t).
\end{aligned}$$

After some algebraic calculations, we obtain

$$N(t) \leqslant N(0) E_\alpha(-\delta t^\alpha) + \frac{\Pi}{\delta}(1 - E_\alpha(-\delta t^\alpha)).$$

Thus, we have shown that $0 < N(t) \leqslant \frac{\Pi}{\delta}$, if $N(0) \leqslant \frac{\Pi}{\delta}$, for all $t \geqslant 0$. Therefore, all the solution components are bounded as the total population $N(t)$ is bounded between 0 and $\frac{\Pi}{\delta}$. Hence, the region Ω is positively invariant and the corresponding model system (1) is well-posed. ∎

3. Equilibrium Points

The disease free equilibrium point, $\mathcal{D}_{\mathcal{E}}$, for the model system (1) describes the state in which the population is free from COVID-19, and is given as

$$\mathcal{D}_{\mathcal{E}} = \left(\frac{\Pi}{\nu_1 + \delta}, 0, \frac{\nu_1 \Pi}{\delta(\nu_1 + \delta)}, 0, 0, 0, 0 \right).$$

For the model system (1), the basic reproduction number which is a threshold quantity counting the number of secondary infected cases generated by a single infected individual (Jones (2007)) can be computed using the next generation matrix approach (Driessche and Watmough (2020)). Correspondingly, the matrices F and V are determined as

$$F = \begin{bmatrix} 0 & \frac{\beta\eta_1(\delta+\nu_1(1-\epsilon))}{\delta+\nu_1} & \frac{\beta\eta_2(\delta+\nu_1(1-\epsilon))}{\delta+\nu_1} & \frac{\beta(\delta+\nu_1(1-\epsilon))}{\delta+\nu_1} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{3}$$

$$V = \begin{bmatrix} \nu_2 + \sigma + \delta & 0 & 0 & 0 \\ -\sigma & \psi + \gamma + \tau_1 + \delta + \delta_I & -\rho & 0 \\ 0 & -\psi & \rho + \mu + \delta + \delta_u & 0 \\ 0 & -\gamma & -\mu & \tau_2 + \delta + \delta_h \end{bmatrix}. \tag{4}$$

Using the spectral radius of FV^{-1}, the basic reproduction number is determined as

$$\mathcal{R}_0 = \frac{\beta\sigma\left(\delta + \nu_1(1-\epsilon)\right)\left\{(\gamma + \eta_1 B_4)B_3 + (\mu + \eta_2 B_4)\psi\right\}}{B_1 B_4(\delta + \nu_1)\left(B_2 B_3 - \psi\rho\right)}, \tag{5}$$

where

$$B_1 = \nu_2 + \sigma + \delta, B_2 = \psi + \gamma + \tau_1 + \delta + \delta_I, B_3 = \rho + \mu + \delta + \delta_u \text{ and } B_4 = \tau_2 + \delta + \delta_h.$$

The endemic equilibrium point, $\mathcal{E}_{\mathcal{E}}$, for the model system (1) is characterized by a steady state in which the disease will persist in the population and can be obtained by solving the simultaneous system of equations, given as

$$\begin{aligned} \Pi + \frac{1}{k}R_c - \lambda S_c - \nu_1 S_c - \delta S_c &= 0, \\ \lambda S_c + (1-\epsilon)\lambda V_c - \nu_2 E_c - \sigma E_c - \delta E_c &= 0, \\ \nu_1 S_c + \nu_2 E_c - (1-\epsilon)\lambda V_c - \delta V_c &= 0, \\ \sigma E_c + \rho U_c - \psi I_c - \gamma I_c - \tau_1 I_c - (\delta + \delta_I) I_c &= 0, \\ \psi I_c - \rho U_c - \mu U_c - (\delta + \delta_u) U_c &= 0, \\ \mu U_c + \gamma I_c - \tau_2 H_c - (\delta + \delta_h) H_c &= 0, \\ \tau_1 I_c + \tau_2 H_c - \frac{1}{k}R_c - \delta R_c &= 0. \end{aligned} \tag{6}$$

Here, the components of $\mathcal{E}_{\mathcal{E}} = (S_c^*, E_c^*, V_c^*, I_c^*, U_c^*, H_c^*, R_c^*)$ are computed as

$$\begin{aligned}
S_c^* &= \frac{1}{(\lambda+\nu_1+\delta)}\left(\Pi + \left(\frac{\tau_1 B_3 B_4 + \tau_2\left(\mu\psi+\gamma B_3\right)}{B_4\psi(1+\delta k)}\right)U_c^*\right),\\
E_c^* &= \frac{1}{\sigma}\left(\frac{B_2B_3}{\psi}-\rho\right)U_c^*,\\
V_c^* &= \frac{1}{((1-\epsilon)\lambda+\delta)}\left(\frac{\nu_1}{(\lambda+\nu_1+\delta)}\left(\Pi + \left(\frac{\tau_1 B_3 B_4 + \tau_2\left(\mu\psi+\gamma B_3\right)}{B_4\psi(1+\delta k)}\right)U_c^*\right)\right)\\
&\quad + \frac{1}{((1-\epsilon)\lambda+\delta)}\left(\frac{\nu_2}{\sigma}\left(\frac{B_2B_3}{\psi}-\rho\right)U_c^*\right),\\
I_c^* &= \frac{B_3}{\psi}U_c^*,\\
U_c^* &= \frac{\Pi\sigma\psi(1+\delta k)(\lambda+\nu_1C_1)(\delta+(1-\epsilon)\lambda)}{C_2-C_3},\\
H_c^* &= \frac{\mu\psi+\gamma B_3}{\psi B_4}U_c^*,\\
R_c^* &= \frac{k}{\psi(1+\delta k)}\left(\tau_1 B_3 + \frac{\tau_2}{B_4}\left(\mu\psi+\gamma B_3\right)\right)U_c^*,
\end{aligned} \tag{7}$$

where

$$\begin{aligned}
C_1 &= \frac{(1-\epsilon)\lambda}{(1-\epsilon)\lambda+\delta},\\
C_2 &= B_4(B_2B_3-\rho\psi)(1+\delta k)(\lambda+\nu_1+\delta)\left((\sigma+\delta)\left((1-\epsilon)\lambda+\delta\right)+\nu_2\delta\right),\\
C_3 &= \sigma\left(\tau_1B_3B_4+\tau_2(\mu\psi+\gamma B_3)\right)(\lambda+\nu_1C_1)(\delta+(1-\epsilon)\lambda).
\end{aligned} \tag{8}$$

4. Stability Analysis of the Disease-free Equilibrium

In this section, the stability of the disease-free equilibrium point is determined to be locally asymptotically stable in order to find the conditions under which small disturbances away from $\mathcal{D}_{\mathcal{E}}$ dissipate in time.

For the model system (1), the linearization matrix evaluated at $\mathcal{D}_{\mathcal{E}} = \left(\frac{\Pi}{\nu_1+\delta}, 0, \frac{\nu_1\Pi}{\delta(\nu_1+\delta)}, 0, 0, 0, 0\right)$ is computed as

$$J = \begin{bmatrix}
-(\nu_1+\delta) & 0 & 0 & -\frac{\beta\eta_1\delta}{\nu_1+\delta} & -\frac{\beta\eta_2\delta}{\nu_1+\delta} & -\frac{\beta\delta}{\nu_1+\delta} & \frac{1}{k}\\
0 & -B_1 & 0 & \frac{\beta\eta_1\delta}{\nu_1+\delta}+\frac{(1-\epsilon)\beta\eta_1\nu_1}{\nu_1+\delta} & \frac{\beta\eta_2\delta}{\nu_1+\delta}+\frac{(1-\epsilon)\beta\eta_2\nu_1}{\nu_1+\delta} & \frac{\beta\delta}{\nu_1+\delta}+\frac{(1-\epsilon)\beta\nu_1}{\nu_1+\delta} & 0\\
\nu_1 & \nu_2 & -\delta & -\frac{(1-\epsilon)\beta\eta_1\nu_1}{\nu_1+\delta} & -\frac{(1-\epsilon)\beta\eta_2\nu_1}{\nu_1+\delta} & -\frac{(1-\epsilon)\beta\nu_1}{\nu_1+\delta} & 0\\
0 & \sigma & 0 & -B_2 & \rho & 0 & 0\\
0 & 0 & 0 & \psi & B_3 & 0 & 0\\
0 & 0 & 0 & \gamma & \nu & -B_4 & 0\\
0 & 0 & 0 & \tau_1 & 0 & \tau_2 & -(\frac{1}{k}+\delta)
\end{bmatrix}.$$

The characteristic equation of the matrix $(\lambda^p I_7 - J)$ is computed as

$$(\lambda^p + \delta)(\lambda^p + \nu_1 + \delta)\left(\lambda^p + \frac{1}{k} + \delta\right)\left(\lambda^{4p} + P_3\lambda^{3p} + P_2\lambda^{2p} + P_1\lambda^p + P_0\right) = 0, \tag{9}$$

where the coefficients of the last factor are given as

$$\begin{aligned}
P_3 &= B_1 + B_2 + B_3 + B_4,\\
P_2 &= B_1B_2 + B_3B_4 + (B_1 + B_2)(B_3 + B_4) - \frac{\beta\sigma\eta(\delta + (1-\epsilon)\nu_1)}{\delta + \nu_1},\\
P_1 &= B_1B_2(B_3 + B_4) + B_3B_4(B_1 + B_2) - \rho\psi(B_1 + B_4) - \frac{\beta\sigma(\delta + (1-\epsilon)\nu_1)}{\delta + \nu_1}(\eta_2\psi + \gamma + \eta_2(B_3 + B_4)),\\
P_0 &= B_1B_2B_3B_4 - \rho\psi B_1B_4 - \frac{\beta\sigma(\delta + (1-\epsilon)\nu_1)}{\delta + \nu_1}(\gamma B_3 + \mu\psi + \eta_2\psi B_4 + \eta_1 B_3B_4).
\end{aligned} \tag{10}$$

The argument of each root of the first three factors is given as

$$\arg(\lambda_s) = \frac{\Pi}{p} + s\frac{2\Pi}{p} \geqslant \frac{\Pi}{p} > \frac{\Pi}{N} > \frac{\Pi}{2N},$$

for $s = 0, 1, 2, \ldots, (p-1)$.

Thus, all roots of the first three factors have arguments greater than $\frac{\Pi}{2N}$. Now, following the Routh-Hurwitz stability criterion for fractional derivatives (Ahmad et al. (2006)), it can be observed that all roots of the remaining quartic factor have argument greater than $\frac{\Pi}{2N}$, if the coefficients given by (10) satisfy the following conditions:

(i) $P_3 > 0$ and $P_0 > 0$,
(ii) $P_3P_2 > P_1$,
(iii) $P_1P_2P_3 > P_3^2P_0 + P_1^2$.

In the above discussion, we have proved the following theorem.

Theorem 4.1.

For the system of differential equations with fractional order derivative (1), the equilibrium point $\mathcal{D}_{\mathcal{E}}$ is locally asymptotically stable, provided the given conditions are satisfied:

(i) $P_3 > 0$ and $P_0 > 0$,
(ii) $P_3P_2 > P_1$,
(iii) $P_1P_2P_3 > P_3^2P_0 + P_1^2$,

where the coefficients P_3, P_2, P_1 and P_0 are given by equation (10).

5. Sensitivity Analysis

In this paper, the mathematical model illustrates the transmission of an epidemic disease known as COVID-19. Mathematically, the transmission of COVID-19 disease is described by the threshold

Table 2. Parameters value used in numerical simulations

Parameter	Value	Source	Parameter	Value	Source
Π	50000 day $^{-1}$	Assumed	β	2.47 day $^{-1}$	Data fitted
ν_1	0.015 day $^{-1}$	Estimated	ν_2	0.031 day $^{-1}$	Data fitted
δ	0.000039 day $^{-1}$	Estimated	δ_I	0.0000735708 day $^{-1}$	Estimated
δ_u	0.003323 day $^{-1}$	Estimated	δ_h	0.005536 day $^{-1}$	Estimated
σ	$1/4.68$ day $^{-1}$	Estimated	γ	0.007 day $^{-1}$	Estimated
ρ	0.32	Data fitted	ϵ	0.81	Estimated
k	$1/90$ day $^{-1}$	Estimated	μ	0.51	Data fitted
τ_1	0.972 day $^{-1}$	Estimated	τ_2	0.95 day $^{-1}$	Estimated
η_1	0.57	Data fitted	η_2	0.54	Data fitted

quantity $\mathcal{R}_0$. This section signifies the impact of aforementioned model parameters on the basic reproduction number $(\mathcal{R}_0)$ using the sensitivity analysis. This nonlinear mathematical model consists of the fractional derivatives, and therefore, we determine the impact of various parameters on $\mathcal{R}_0$ that illustrates the memory effects. We investigate the normalized forward sensitivity index of $\mathcal{R}_0$ by determining the partial derivative of $\mathcal{R}_0$ with respect to different parameters using the parameters value mentioned in Table 2.

Now, we will determine the sensitivity index of $\mathcal{R}_0$ with respect to the parameters value given in Table 2. For the constant parameter ϵ, the sensitivity index signifies the immense impact of efficacy of vaccination on $\mathcal{R}_0$, given by Chitnis et al. (2008) as

$$\begin{aligned}\Upsilon_{\epsilon}^{R_0} &:= \frac{\partial \mathcal{R}_0}{\partial \epsilon}\frac{\epsilon}{\mathcal{R}_0}\\ &= -4.20561.\end{aligned}$$

For the basic reproduction number $\mathcal{R}_0$, the sensitivity indices corresponding to the remaining parameters are mentioned in Table 3. Following observations can be made from the sensitivity analysis:

- $\Upsilon_{\epsilon}^{\mathcal{R}_0} = -4.20561$: This value indicates the high impact of the efficacy of vaccination on $\mathcal{R}_0$. It is estimated that $\mathcal{R}_0$ decreases with a huge difference of 42% if ϵ increases by 10%. From this, it is observed that the efficacy of vaccination provides a significant impact into the model. Thus, the transmission of COVID-19 infection can be majorly reduced by increasing the efficacy of vaccination among the vaccinated individuals.
- $\Upsilon_{\beta}^{\mathcal{R}_0} = 1$: For the transmission rate β, the sensitivity index is estimated to be 1 which signifies that β and $\mathcal{R}_0$ are directly proportional. Therefore, the reproduction of new infection can be controlled if transmission of infection can be reduced by providing vaccination with higher rate and keeping the infectives under isolation.
- $\Upsilon_{\rho}^{\mathcal{R}_0} = -0.202759$: From this value of sensitivity index for ρ, is it observed that the reproduction number decreases by approximately 2% if ρ increases by 10%. It indicates that to control

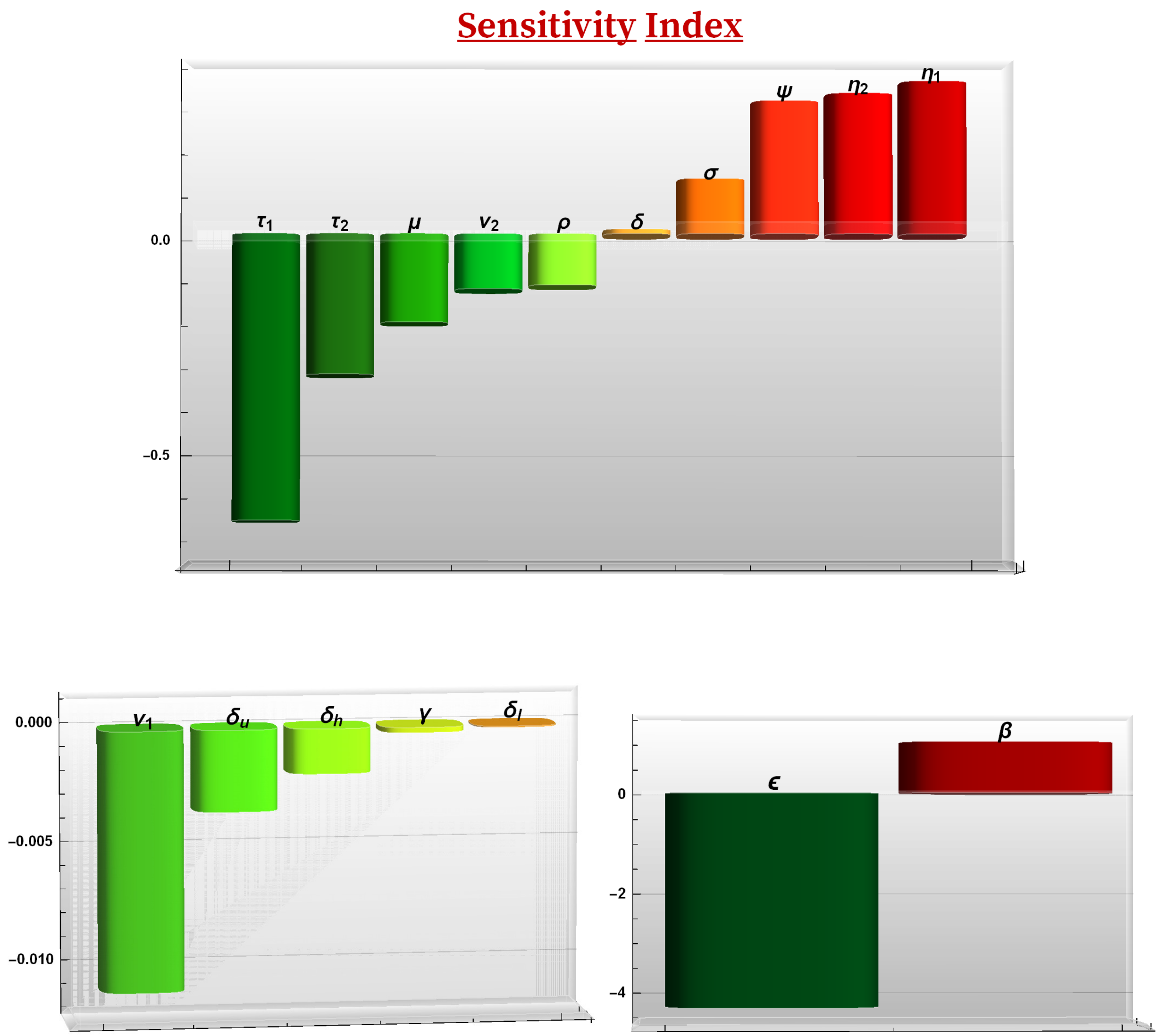


Figure 2. Graph depicting the impact level of parameters on $\mathcal{R}_0$

the transmission of COVID-19 infection, the unattended infectives must obtain the medical treatment as early as possible.

- $\Upsilon_{\eta_1}^{\mathcal{R}_0} = +0.351649$ and $\Upsilon_{\eta_2}^{\mathcal{R}_0} = +0.323802$: For the fractions η_1 and η_2, the sensitivity indices indicates that the reproduction of new infectives must be controlled by keeping all the infectives under isolation and strictly follow the precautionary measures.

From the above observations, it is concluded that the basic reproduction number $\mathcal{R}_0$ is highly affected with variation in the efficacy of vaccination. From the sensitivity index, it is concluded that the transmission of COVID-19 infection can be reduced if the vaccination is provided with its optimal level and the efficacy of vaccination is increased by strictly obeying the precautionary measures against COVID-19 disease. Also, the medical care must be provided with precedence to those who need intensive medical care.

Table 3. Sensitivity indices of $\mathcal{R}_0$ to the parameters

Parameter	Sensitivity index $(\mathcal{R}_0)$	Parameter	Sensitivity index $(\mathcal{R}_0)$
τ_1	-0.657608	γ	-0.000217
τ_2	-0.322656	δ_I	-0.000049
μ	-0.202759	δ	+0.010667
ν_2	-0.126678	σ	+0.126838
ρ	-0.117597	ψ	+0.306252
ν_1	-0.010906	η_2	+0.323802
δ_u	-0.003406	η_1	+0.351649
δ_h	-0.001880	β	+1.0

6. Numerical Simulations

For the COVID-19 model system (1) discussed above, the numerical simulation has been done in this section using the predictor-corrector method which is an algorithm designed to solve the system of fraction differential equations in MATLAB software. The parameters value prescribed in Table 2 have been referred to numerically solve the system of equations. Considering the time duration from April 20 - June 10, 2021 for the study period, the initial conditions for the variables defined in the model system (1) are chosen as $S_c(0) = 1300000000$, $E_c(0) = 10000000$, $V_c(0) = 127129113$, $I_c(0) = 1617428$, $U_c(0) = 323486$, $H_c(0) = 215657$ and $R_c(0) = 13276039$. The baseline parameters value have been estimated using the data provided on the World Health Organization (2020b) and World Health Organization (2020c). MoHFW (2020) and the literature published for the case study of India (Ngonghala et al. (2020); Mandal et al. (2020)). On the basis of daily new births in India, which ranges from $45,000-70,000$ per day, we have assumed the constant recruitment rate Π to be 50,000 for India. In India, for an individual the average life expectancy is estimated as 69.3 years which assisted us in calculating the natural death rate δ to be $\frac{1}{69.3}\times\frac{1}{365}$ day^{-1} = 3.9×10^{-5} day^{-1}. Further, the per day active cases for the month of April - May 2021 were recorded as 3244040 (Worldometer (2020)), whereas the per day deaths were reported to be 3592 (Worldometer (2020)). It has been reported that more than 75% of the total infectives recover on their own during home isolation and do not require any specific medical assistance for their treatment, due to which out of total active cases, we have estimated 2433030 to be isolated active cases per day. However, out of the remaining 25% active cases approximately 10% cases are daily hospitalized which are estimated as 324404 and the remaining are still considered to be unattended. On the other hand, approximately 50% of the total COVID-19 induced death cases occur in hospitals due to comorbidities along with the COVID-19 infection. Directed by this, approximately 1796 deaths are estimated in hospitals per day. Thus, the estimated cases fatality rate of hospitalized infectives in India is computed as $\delta_h = \frac{1796}{324404} = 0.005536$ per day. However, the case fatality rate for the isolated infectives is estimated at $\delta_I = \frac{179}{2433030} = 0.0000735708$ per day by considering the fact that the number of individuals succumbed to COVID-19 in home isolation without any specific requirement of medical assistance is less than 5%.

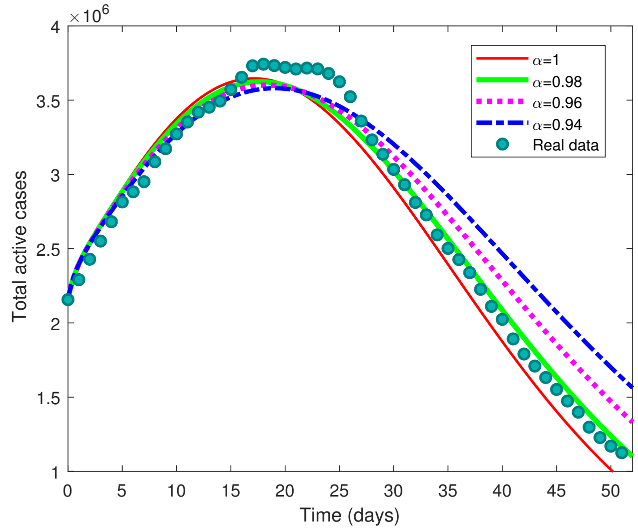


Figure 3. Graph validating the model by comparing the estimated number of infectives with the real data of India with different values of the order of derivative for the time interval April 20, 2021 - June 10, 2021.

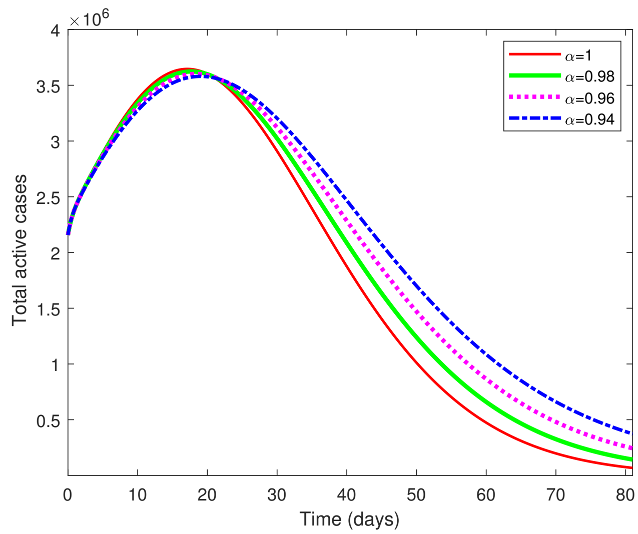
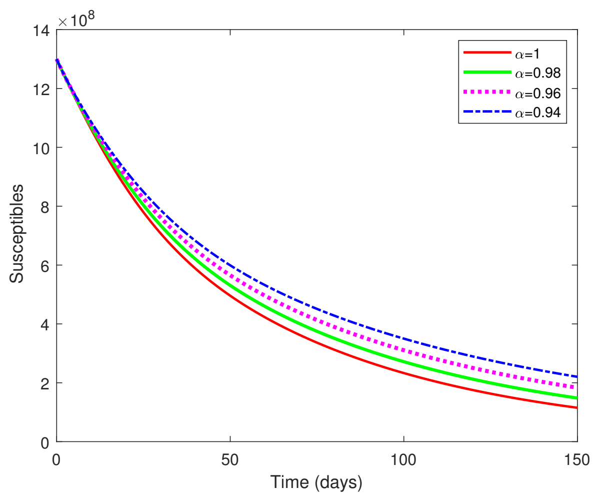


Figure 4. Graph illustrating the predicted cases of India with different values of the order of derivative for the time interval April 20, 2021 - July 10, 2021.

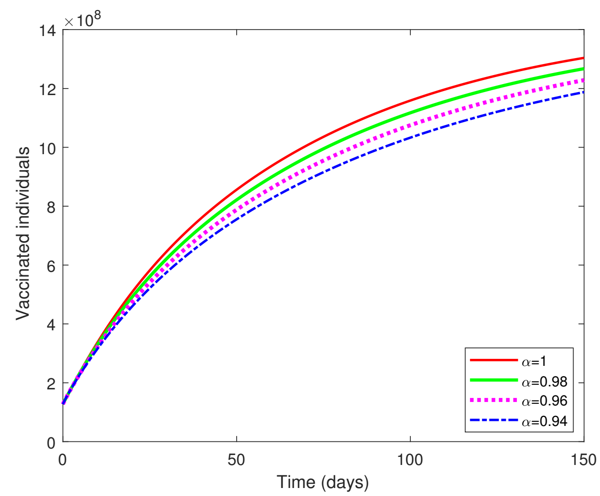
Consequently, the COVID-19 death rate of unattended infectives is computed as $\delta_u = \frac{1617}{486606} = 0.00332$ per day. As reported by the World Health Organization (2020a), an individual exposed to a COVID-19 infected individual will spend approximately $4 - 6$ days in incubation period after acquiring the infection. Thus, in accordance with the real data of India and the fact produced by WHO, the progression rate of exposed individuals to the isolation is estimated at $\sigma = \frac{1}{4.68}$ per day. It is known that isolated and unattended infectives have lesser chance of spreading infection in comparison to the hospitalized infectives due to the fact that they are isolated at home and do not come in contact with health care workers. Thus, the relative infectiousness of isolated and unattended infectives is less than that of hospitalized infectives, which in turn made us to assume $\eta_1 = 0.57$ and $\eta_2 = 0.54$.

The vaccination rate for susceptibles computed by the ratio of no. of individuals vaccinated per day to the target population has been estimated at $\nu_1 = 0.015$. As per the report by World Health Organization, recovery rate for COVID-19 has reached to approximately 95% by the end of May 2021, corresponding to which we have estimated $\tau_1 = 0.972$ and $\tau_2 = 0.95$. We assume that $\gamma = 0.07$, due to the fact that the progression of infectives to hospitals is less than 10%. Furthermore, after getting recovered from COVID-19 an individual is supposed to acquire immunity for approximately 3 months and therefore k is estimated at 90 days. In accordance with the real data of India for active cases and the proposed model, certain parameters value have been fitted and hence chosen as $\rho = 0.32$, $\nu_2 = 0.031$, $\mu = 0.51$.

Using the MATLAB software, the estimated data is validated for the numerical simulation of the model. Therefore for the data validation, the real data of India is correlated with the predicted number of infectives for integer and fractional values of α. From Figure 3, the demolition of COVID-19 infection in India can be observed for the time period April 20 - June $10, 2021$ for the different values of order α of differential equations. It can be visualized from the graph that infectives increase until May $16, 2021$ and then start decreasing. It can also be observed from Figure 3 that the trajectory corresponding to the total infectives predicted by the model is in close approximation with the real data of India for the fractional order $\alpha = 0.98$ from beginning until the end of the prediction period. This may happen due to the fact that by April 2021 more than 15.38 million individuals had already been recovered from COVID-19 in the first wave. These individuals have certain memories of the disease for how to adopt practices in order to prevent themselves from reinfection with COVID-19 or which practices to be taken if again become infected with COVID-19. This justifies the importance of inclusion of memory effect in the model. It can be observed from Figure 4 that following trajectory corresponding to $\alpha = 0.98$, the number of infectives reaches approximately 0.2×10^6 by July $10, 2021$.



(a)



(b)

Figure 5. Graphs depicting the stability of the equilibrium point $\mathcal{D}_{\mathcal{E}} = (2.81865 \times 10^6, 0, 1.27923 \times 10^9, 0, 0, 0, 0)$ with different values of the order of derivative.

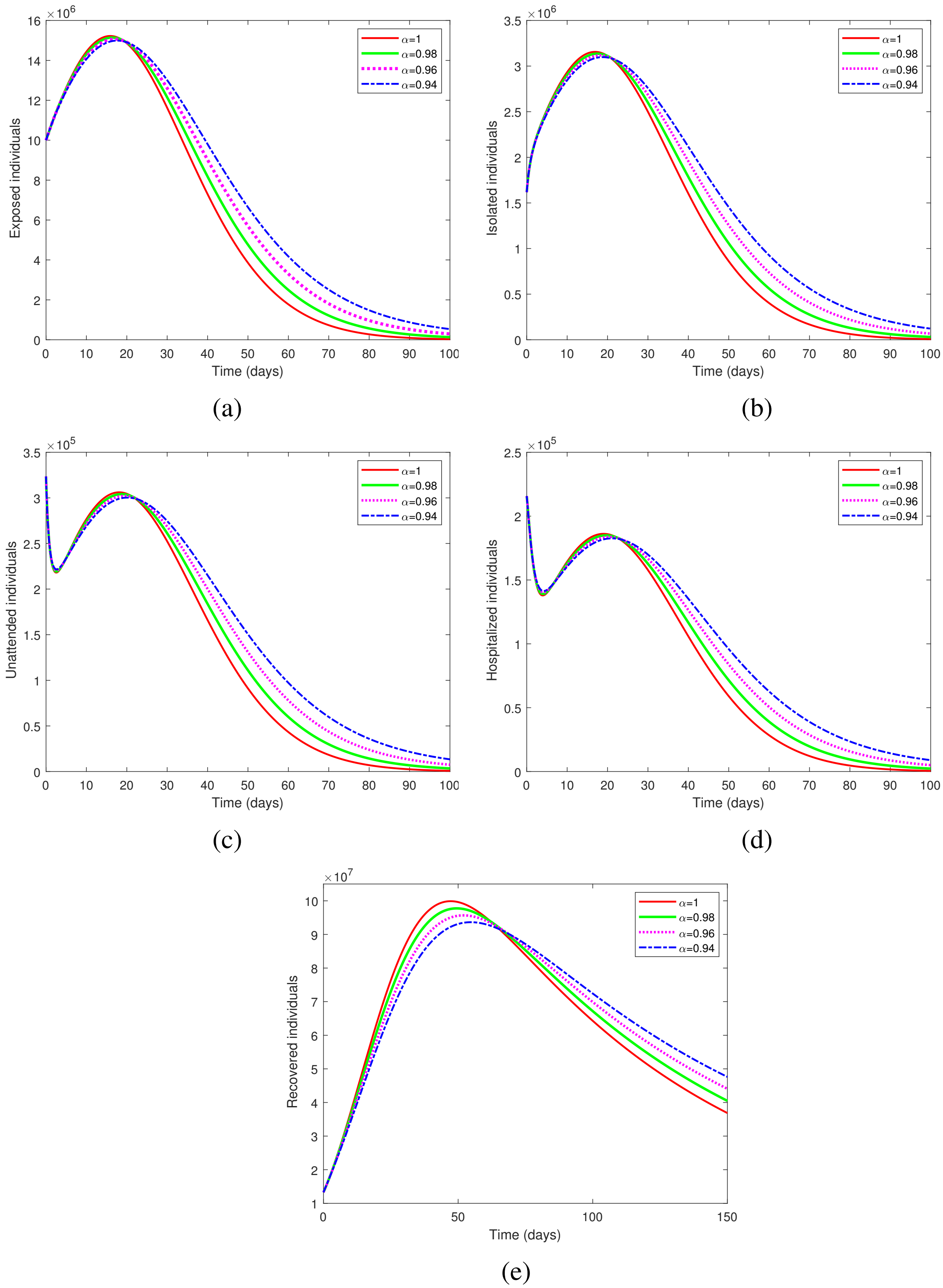


Figure 6. Graphs depicting the stability of the equilibrium point $\mathcal{D}_{\mathcal{E}} = (2.81865 \times 10^6, 0, 1.27923 \times 10^9, 0, 0, 0, 0)$ with different values of the order of derivative.

With the chosen parameters value, we have obtained the basic reproduction number to be less than unity, consequently the disease-free equilibrium point is obtained as $\mathcal{D}_{\mathcal{E}} = (2.81865 \times$

$10^6, 0, 1.27923 \times 10^9, 0, 0, 0, 0)$. In our model, Figure 5 and 6 show the trajectories corresponding to COVID-19 for four different values of the order of derivatives (α), to represent the role that memory plays in the treatment and recovery from COVID-19 infection. The local asymptotic stability of the disease-free equilibrium point $\mathcal{D}_{\mathcal{E}}$ can also be visualized in Figure 5 and 6 in which all the solution trajectories are converging towards their respective equilibrium components.

In Figure 5(a), we can visualize the effect of fractional order on the population susceptible to COVID-19. As the order of derivative decreases from $\alpha = 1$ to $\alpha = 0.94$, memory effect increases, which in turn decreases the decrement rate of susceptibles as can be seen in Figure 5(a). The key reason behind the decrease in decrement rate of susceptibles could be the lower rate of transfer of susceptibles to infection class as memory effect increases. Thus, Figure 5(a) shows the crucial role played by the inclusion of memory effect (characterized by the fractional order α) in reducing the infection rate and increasing the susceptibles. It can also be observed from Figure 5(a), the number of susceptibles approach at a faster rate to the corresponding component value 2.81865×10^6 of equilibrium as the order of differential equations α raises from 0.94 to 1. This justifies higher convergence rate for a higher value of α. The effect of fractional order on the vaccinated population can be seen in Figure 5(b). It can be concluded that the requirement of increment in vaccinated individuals is more for integer order model as compared to the case when fractional order model is considered. This may happen due to the fact that fractional order includes the memory of individuals who have a previous exposure to the disease and hence are taking more precautionary measures to keep themselves away from the infected individuals.

Also, the immune system of individuals having a previous exposure to the disease may also have the memory of being attacked by the same virus before and hence produces antibodies in the early stage of infection or already have active antibodies in their system which in turn reduces the emergency of getting vaccinated. Keeping this in mind, the health care authorities are vaccinating susceptibles on priority rather than those who have recovered from the disease and have prescribed a time duration of three months from the date of infection for those who have previous exposure to the disease. However, for integer order models where memory effect is not considered, there is an emergent need of vaccination and high increment in the vaccination rate is required as can be seen from Figure 5(b), which is quite infeasible for any community. Thus, fractional order is more feasible as far as the real world scenario is considered. The trajectory corresponding to the vaccinated population can be seen to be converging towards the equilibrium component 1.27923×10^9 in approximately 5 months with a higher convergence rate for integer order as compared to fractional order.

Figure 6(a) and (b) show the prevalence of infection in exposed and isolated individuals, respectively. It can be observed that the nunber of exposed and isolated individuals increases for the initial 25 days and reaches its peak value until May 16, 2021. The growth in the number of infectives is slow for small fractional order, as smaller fractional order indicates more memory effect and infected individuals with previous exposure will be cautious with following precautionary measures that reduces the accretion rate of infectives. However, it can also be observed that after approximately 25 days infectives start decreasing with a slower decay rate for lower fractional order which is a result of higher vaccination rate for higher order.

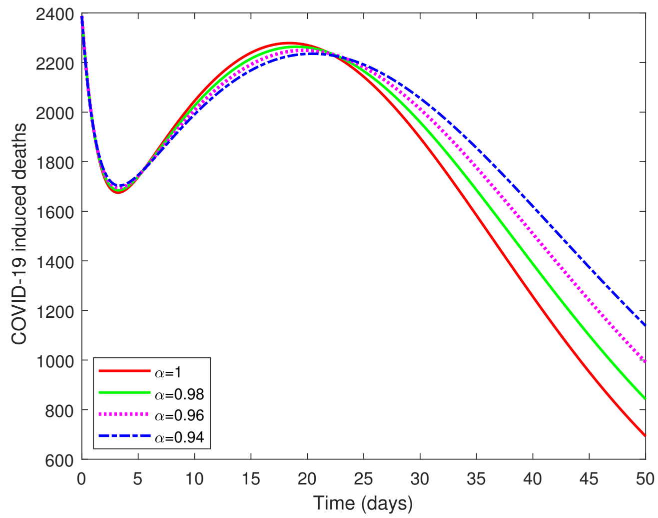


Figure 7. Graphs illustrating the predicted death cases for the model system (1) with different values of the order of derivative.

The similar behavior of solution trajectories corresponding to unattended infectives and hospitalized infectives can be seen in Figure 6(c) and (d) for different values of fractional order. It can also be seen in Figure 6(e) that for higher order of derivative, infection rate is more and hence the number of recovered individuals also increase and further converge towards the equilibrium component zero as infectives decrease in the long run.

From Figure 7, it can be observed that the death cases increase for initial few days as the COVID-19 cases rise with a lower increment rate for fractional order and then start decreasing as infectives decrease with a lower decrement rate for higher value of order of differential equations. This justify the fact that if the memory effect is taken into consideration the health care facilities are made available to the infectives so, the death cases increases slowly. Thus, the fractional order derivatives plays a vital role in deduction of the number of infectives and makes a mathematical model realistic.

The COVID-19 vaccines were introduced worldwide several months ago. The graph in Figure 8 illustrates the effect of vaccination rate ν_1 on COVID-19 infectives by varying the value of ν_1 from 0.01 to 0.0175 for $\alpha = 0.98$. It can be visualized from Figure 8 that the number of infectives decreases with a higher rate for $\nu_1 = 0.15$ and the rate of increment is much lesser in this case as compared to the cases when vaccination rate is lesser. Therefore, the spread of infection can be effectively controlled if the vaccination rate can be increased by encouraging the susceptibles to take vaccination along with increasing the vaccination production and vaccination centers.

As per the World Health Organization, it is difficult to exactly determine the efficacy of a vaccination, whether it is Covaxin or Covishield, yet both the vaccines are proved to be very effective in controlling the disease. However, no vaccine is 100% effective in controlling the spread of the disease and a small fraction of individuals acquire COVID-19 infection even after getting vaccination with either first dose or both doses.

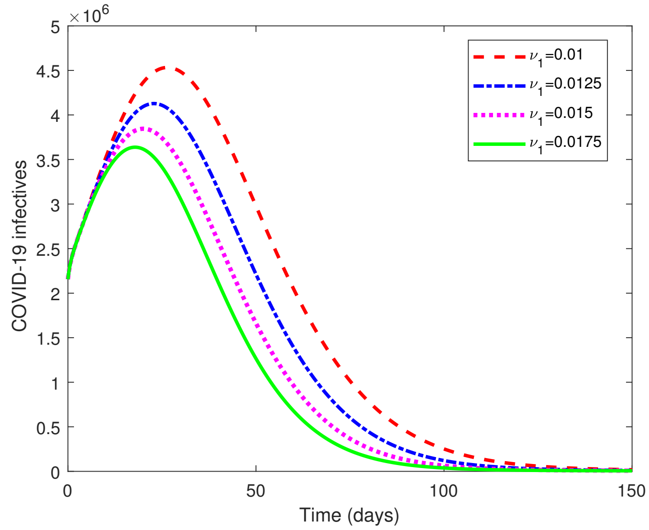


Figure 8. Graph illustrating the effect of vaccination rate on COVID-19 infectives by varying the value of ν_1 from 0.01 to 0.0175.

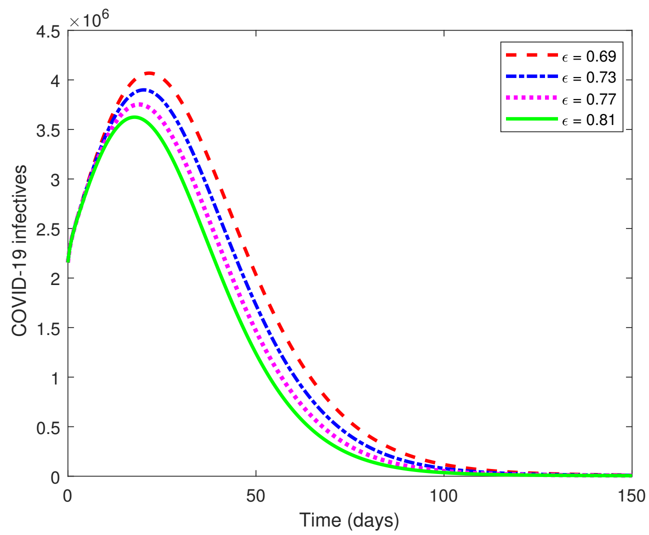


Figure 9. Graph illustrating the changes in COVID-19 infectives with efficacy of vaccination (ϵ) ranges from 0.69 to 0.81.

Certain other factors such as an individual's health condition, previous exposure to COVID-19, infection with some other comorbidities and an individual's age may leave an impact on the efficacy of vaccination. On the other hand, vaccines are effective and the development of antibodies may be hampered due to a suppressed immune system. Keeping this in view, health care authorities advise all the vaccinated individuals to follow all the precautionary measures such as mask wearing, breathing etiquette, social distancing, avoiding large gathering and the consumption of alcohol for at least 45 days in order to develop a strong and healthy immune system. All these practices have an immense impact on the efficacy of vaccination.

Keeping all the practices in mind, we have portrayed the number of active infectives in Figure

9 by varying the value of efficacy of vaccination, that is, ϵ from 0.69 to 0.81 to visualize the vital role played by the efficacy of vaccination in reducing the infection prevalence. From Figure 9, it can be visualized that infectives start decreasing more rapidly as the efficacy of vaccination increases. Thus, it can be concluded that even after getting vaccinated against COVID-19 infection, the population must follow all the precautionary measures such as infection prevention, social distancing and avoiding alcohol consumption for at least 45 days after the vaccination in order to increase the efficacy of vaccination.

The behavior of solution trajectories, corresponding to the COVID-19 model for different values of the order of derivative, indicates the novelty of this article over other papers on COVID-19.

7. Discussion

In this paper, we have analyzed a COVID-19 model by incorporating vaccination for both susceptibles and exposed individuals. The basic reproduction number has been computed along with the determination of two equilibrium points, namely, the equilibrium point with disease free and the equilibrium point with existence of disease. We have also estimated the sensitivity index for the threshold quantity $\mathcal{R}_0$ that signifies the impact of distinct parameters. It has been observed from sensitivity analysis that the transmission rate of the disease and efficacy of vaccination have an immense impact on the reproduction number and can play a crucial role in reducing the spread of the disease. Thus, proper precautionary measures such as wearing masks, body sanitization, social distancing and isolation of infectives must be followed to reduce the transmission rate and efficacy of vaccination against infection which in turn reduces the disease prevalence.

The model is based on the Caputo fractional order derivative and is numerically simulated for α ranging from 0.94 to 1. Numerical results show the crucial role played by fractional order derivatives in the reduction of infection prevalence. It has been visualized that the real data of India is in close approximation to predicted model trajectory for the fractional order $\alpha = 0.98$, which consequently shows that if the memory effect is taken into consideration along with strict implementation of precautionary measures and proper management of vaccination rate, the infected population decreases to approximately 0.2×10^6 in the beginning of July 2021. It has been also observed that for higher order of derivative infection rate is higher which in turn increases the number of recovered individuals. The graphs corresponding to the solutions trajectories also predict that India can be made free from COVID-19 in next five months if vaccination is given properly and strict precautionary measures such as infection prevention, home isolation, quarantine and work from home are followed.

Also, the convergence rate towards the equilibrium point is less for a fractional order model which justifies the actual solutions to the real problems. Numerical results also signify the fact that if the vaccination rate and the efficacy of vaccination is increased by strictly following all the precautionary measures and avoiding practices which reduce human immunity such as consuming alcohol, the COVID-19 infectives can be decreased.

8. Conclusion

In this paper, a fractional order model has been formed with an aim to explain the transmission dynamics of population by taking the memory effect into consideration. It has been observed that the fractional order α can portray the role of precautionary measures in decreasing the COVID-19 infection, as a lower increment rate can be observed in the number of infectives by reducing the value of α and increasing the memory effect. It must be noticed that for smaller fractional order growth as well as decay rate is slower. The results presented in this article can be fruitful for the COVID-19 outbreak and may be utilized for making defensive strategies against the infection prevalence.

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