A note on the two-country Cournot model with an exchange rate linkage

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Abstract

In Aoki (2007), I claimed that, under a Cournot oligopoly model with international linkage, a long-run equilibrium exchange rate deviates, into the same direction but in a larger extent, from the purchasing power parity (PPP) in factor prices than the PPP in product prices does, with the extent of asymmetry in the degree of competitiveness, consumer's preference and market volume between countries, or with the extent of asymmetry in firms' marginal costs between countries. In this note, I show the detailed computational process for deriving equilibrium prices and quantities, and the equilibrium exchange rate under the settings of the model.

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1. Basic model

As explained in Aoki (2007), we set the following model: There exist two countries (country 1 and 2) and one traded product. There are n identical firms (denoted by x) in country 1, and m identical firms (denoted by y) in country 2. Firm x has a constant marginal cost c_x and produces quantity x_i for country i (i=1,2). Similarly, firm y has a constant marginal cost c_y and produces quantity y_i for country i (i=1,2). Country i has an inverse demand function $p_i=\alpha_i-\beta_i q_i$, where $q_i=nx_i+my_i$ is the quantity supplied in country i, and p_i is the *real* price of the product in country i. n and m represent the degree of competitiveness in country 1 and 2, respectively. The larger α_i implies greater consumer's preference for the product, and the less β_i implies larger (inherent) market volume. The current *real* exchange rate of country 2 in terms of country 1's *real* price level is denoted by e.

Under the above assumptions, further settings are as follows.

1. Each firm behaves as a Cournot oligopolist, and is allowed to set a differentiated price in each country's market for each good it produces.
2. Each firm takes a current exchange rate as given.
3. Each firm exits from the market, if the current Cournot equilibrium makes no profit.
4. A long-run equilibrium exchange rate is determined so that the account of each country's currency is zero, that is, the demand and supply of the currency meet.
5. Every price including exchange rate and cost is represented in *real* terms, not *nominal*.
6. We relax the assumption of the law of PPP (Purchasing Power Parity) in product prices, and allow firm's price differentiation across countries. (That is, we assume that there do not occur reversible

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and instantaneous flows of products across countries.)

7. The factors in each country are not traded between countries, and the factor price (represented by the real marginal cost) of each firm is exogenously given.
8. We assume the linear demand for each market, allowing to set diversified consumer's preferences (*diversified intercepts*), diversified market volumes (*diversified slopes*) across markets, or diversified degree of competitiveness (*diversified number of firms*) between countries, or to set diversified marginal costs between firms.

In addition, we consider the following two cases:

Case A: Each firm has its production line only in its mother country (country 1 for firm x and country 2 for firm y).

Case B: Each firm produces the product in each country where the market exists. (That is, firm x produces the quantity x_i and firm y produces the quantity y_i in country i.)

The long-run equilibrium free exchange rate is determined, so that the demand and supply of the currency meet. Then, in case A and B, the following equations hold:

In case A,

$$ep_2\, nx_2 = p_1\, my_1, \qquad (1)^{1}$$

and in case B,

$$e(p_2 - c_x)nx_2 = (p_1 - c_y)my_1. \qquad (2)$$

Also we define the *PPP ratio in factor prices* (in terms of country 1) as $\tilde{c} \equiv c_x/c_y$, and the *PPP ratio in product prices as* $\tilde{p} \equiv p_1/p_2$. Then the law of PPP can be interpreted as the following relation:

$$\tilde{p} = e = \tilde{c} \qquad (3)$$

Note that, in equilibrium, generally we may have $\tilde{p} \neq \tilde{c}$ and $e \neq \tilde{c}$.

2. Derivation and results

Case A

In country i (i=1,2), the quantity supplied is $q_i = nx_i + my_i$, and the price of the product is p_i. Taking the *current* exchange rate of country 2 in terms of country 1's price level, e, as given, we denote the Cournot equilibrium prices and quantities (supplied in and produced for country i (i=1,2)), by p_i^*, q_i^*, x_i^* and y_i^*.

In country 1, in equilibrium, firm x takes other firms' equilibrium quantities as given, and solves the following problem in the country:

$$x_1^* = \arg\max_{x_1}\ (p_1 - c_x)x_1, \qquad (4)$$

where $p_1 = \alpha_1 - \beta_1\, q_1$ and $q_1 = x_1 + (n-1)x_1^* + my_1^*$.

The F.O.C. of problem (4) and setting an equilibrium condition, $x_1 = x_1^*$, derive

$$(\alpha_1 - c_x) = \beta_1 \{(n+1)nx_1^* + my_1^*\}. \qquad (5)$$

Similarly, firm y, in country 1, takes other firms' equilibrium quantities as given, and solves the following problem in the country:

$$y_1^* = \arg\max_{y_1}\ (p_1 - ec_y)y_1, \qquad (6)$$

where $p_1 = \alpha_1 - \beta_1\, q_1$ and $q_1 = y_1 + nx_1^* + (m-1)y_1^*$.

1 From equation (1) we have $\frac{de}{e} = d(\frac{p_1}{p_2})/(\frac{p_1}{p_2}) + d(\frac{my_1}{nx_2})/(\frac{my_1}{nx_2})$.

The F.O.C. of problem (6) and setting an equilibrium condition, $y_1 = y_1^*$, derive

$$(\alpha_1 - ec_y) = \beta_1 \{nx_1^* + (m+1)y_1^*\}. \tag{7}$$

Then, from (5) and (7), the equilibrium quantity produced by each firm for country 1 is calculated as a function of parameters explaining the market structure, α_1, β_1, n, m, c_x, c_y, and a current exchange rate, e, as follows.

$$x_1^* = \frac{1}{\beta_1(m+n+1)} \{(m+1)(\alpha_1 - c_x) - m(\alpha_1 - ec_y)\} \tag{8}$$

$$y_1^* = \frac{1}{\beta_1(m+n+1)} \{-n(\alpha_1 - c_x) + (n+1)(\alpha_1 - ec_y)\} \tag{9}$$

Then the equilibrium quantity supplied for country 1 is

$$q_1^* = nx_1^* + my_1^* = \frac{1}{\beta_1(m+n+1)} \{n(\alpha_1 - c_x) + m(\alpha_1 - ec_y)\}, \tag{10}$$

and the equilibrium market price is

$$p_1^* = \alpha_1 - \frac{1}{(m+n+1)} \{n(\alpha_1 - c_x) + m(\alpha_1 - ec_y)\}. \tag{11}$$

Quite similarly, we obtain in country 2,

$$x_2^* = \frac{1}{\beta_2(m+n+1)} \{(m+1)(\alpha_2 - \frac{c_x}{e}) - m(\alpha_2 - c_y)\}, \tag{12}$$

$$y_2^* = \frac{1}{\beta_2(m+n+1)} \{-n(\alpha_2 - \frac{c_x}{e}) + (n+1)(\alpha_2 - c_y)\}, \tag{13}$$

$$q_2^* = nx_2^* + my_2^* = \frac{1}{\beta_2(m+n+1)} \{n(\alpha_2 - \frac{c_x}{e}) + m(\alpha_2 - c_y)\}, \tag{14}$$

$$p_2^* = \alpha_2 - \frac{1}{(m+n+1)} \{n(\alpha_2 - \frac{c_x}{e}) + m(\alpha_2 - c_y)\}. \tag{15}$$

By substituting (9), (11), (12), (15) into y_1, p_1, x_2, p_2 in (1), respectively, the equilibrium exchange rate, e^*, is derived as e which satisfies the following simplified equation.

$$e\beta_1 [\alpha_2 - \frac{1}{(m+n+1)} \{n(\alpha_2 - \frac{c_x}{e}) + m(\alpha_2 - c_y)\}]\{(m+1)(\alpha_2 - \frac{c_x}{e}) - m(\alpha_2 - c_y)\}$$

$$= \beta_2 [\alpha_1 - \frac{1}{(m+n+1)} \{n(\alpha_1 - c_x) + m(\alpha_1 - ec_y)\}]\{-n(\alpha_1 - c_x) + (n+1)(\alpha_1 - ec_y)\} \tag{16}$$

<u>Case B</u>

Quite similarly as in case A, we obtain, in place of (8) ~ (15),

$$x_1^* = \frac{1}{\beta_1(m+n+1)} \{(m+1)(\alpha_1 - c_x) - m(\alpha_1 - c_y)\}, \tag{8'}$$

$$y_1^* = \frac{1}{\beta_1(m+n+1)} \{-n(\alpha_1 - c_x) + (n+1)(\alpha_1 - c_y)\}, \tag{9'}$$

$$q_1^* = nx_1^* + my_1^* = \frac{1}{\beta_1(m+n+1)} \{n(\alpha_1 - c_x) + m(\alpha_1 - c_y)\}, \tag{10'}$$

$$p_1^* = \alpha_1 - \frac{1}{(m+n+1)} \{n(\alpha_1 - c_x) + m(\alpha_1 - c_y)\}. \tag{11'}$$

$$x_2{}^*=\frac{1}{\beta_2(m+n+1)}\ \{(m+1)(\alpha_2-c_x)-m(\alpha_2-c_y)\}, \tag{12'}$$

$$y_2{}^*=\frac{1}{\beta_2(m+n+1)}\ \{-n(\alpha_2-c_x)+(n+1)(\alpha_2-c_y)\}, \tag{13'}$$

$$q_2{}^*=nx_2{}^*+my_2{}^*=\frac{1}{\beta_2(m+n+1)}\ \{n(\alpha_2-c_x)+m(\alpha_2-c_y)\}, \tag{14'}$$

$$p_2{}^*=\alpha_2-\frac{1}{(m+n+1)}\ \{n(\alpha_2-c_x)+m(\alpha_2-c_y)\}. \tag{15'}$$

By substituting (9'), (11'), (12'), (15') into y_1, p_1, x_2, p_2 in (2), respectively, the equilibrium exchange rate, e^*, is derived as e which satisfies the following simplified equation.

$$e\beta_1\ \{(m+1)(\alpha_2-c_x)-m(\alpha_2-c_y)\}^2=\beta_2\ \{-n(\alpha_1-c_x)+(n+1)(\alpha_1-c_y)\}^2 \tag{16'}$$

The results in Aoki (2007) can be derived, in (16) and (16') by treating e as a function of moving parameters and solving the equation.

3. Final remarks

In this note, a long-run (sustainable) equilibrium exchange rate was strictly derived, according to the determination mechanism as in Aoki (2007) under an international Cournot oligopoly setting, relaxing one significant assumption of the law of single price for one good. These results prove to show how the larger deviation in equilibrium exchange rate arises, compared with the PPP in product prices, with some asymmetric elements in the international economic system.

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