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Sampling Plan for Lab Extracted Beverages Data with Multiple Dependent States Using the Odd Log-Logistic Distribution

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Abstract: In this paper, we propose a Multiple Dependent State Sampling (MDSS) plan for decision-making on lot acceptance/rejection based on properties of current and preceding lots sampled. The plan uses the Odd Log-Logistic Generalized Exponentiated (OLLGE) distribution to determine the median life of the product through a truncated-time life test. Optimal parameters such as sample size, acceptance/rejection numbers, and preceding lots are obtained using the operating characteristic curve (OC-curve). The plan's performance is compared to that of single sampling (SS) plans using Lab Extracted Beverages data sets on carbon dioxide pressure (MPa).

Keywords: Beverages; Carbon-dioxide pressure; Consumer's risk; Multiple dependent state; Odd log-logistic generalized exponentiated; Operating characteristic curve; Producer's Risk

1 Introduction

Products of high quality are the demands of most consumers and it has always placed higher pressure on producers also referred to manufacturing industry [1]. As a consequence, industries have frequently improved their production process efficiency to satisfy customers' thirst for high-quality products. In managing the quality of the product, producers must include continuous management of processes of manufacturing and at the end product produced should be tested for alignment of quality of measured characteristics [2].

Quality control charts are used for the management of the continuous process of production, while the risk of producers and consumers altogether are controlled through acceptance sampling plans (ASP) by the use of an operating characteristic curve [3]. The satisfaction of both risks (consumers and producers risks) simultaneously through the rejection and acceptance decision of a lot at the least inspection cost is the primary goal of sampling plans (SP) [4]. The sample for reaching an appropriate decision is randomly taken from the list of produced lots by the specific industry. Simultaneously consideration of both risks for the reliable decision of deposition of the taken lot for inspection at the optimal cost (minimum), thus is time and finance incurred to inspect all the products produced. The possibility minimized of incurring costs by the consumer of accepting the bad lot (consumer's risks β) and by the producer of rejecting a good lot (producer's risk δ). Quality expertise has set levels for quality which are linked to consumer's and producer's risks, and are referred to as Limiting Quality Level (LQL) and Acceptable Quality Level (AQL) respectively

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[5].

The nature and interest of researchers and authors have enabled a number of improvements specifically of average sampling plans (ASP). This paper is intended to focus on a time-truncated life test that has gained popularity and been confirmed to be more effective among techniques for acceptance sampling plans. Inspection of a lifetime of products and setting the limits of time, as the ultimate means in the provision of life assurance of the product has been the goal of the quality control team of all times [6].

The study on the reliability of manufacturing industries products, ASP while utilizing time-truncated life tests, for several distributions, are proposed for a number of sampling plans in the literature using lifetime tests (for quality characteristics) [7]. Extensive discussion on tightened group acceptance, and tightened normal sampling plan was done by Aslam [8], whereas, the product's quality, in this case, is considered to be equivalent to the product's percentile life score. The grouped sampling plan established by Aslam [9] while focusing on truncated life tests in the view of Inverse Gaussian (IG) distribution. The misspecification of the model parameters' impact is explored in the literature for group sampling plans. More work was revealed by Gui [10] on ASP focusing on truncated life tests. Extensive work has been worked by AL-Omari [11] for three parameter-kappa distributions for truncated life tests on ASP. Sudden death testing using Weibull distribution was proposed by Jun [12], for Double and single sampling plans for variable sampling inspection.

Optimal designing of Multiple Deferred State (MDS) Sampling Plans utilizing Weibull distribution lifetime for mean life was done by Balamurali [13]. Balamurali [13] utilized other distributions like Generalized Inverted Exponential (GIE) distribution in designing multiple deferred state sampling plans. So far as the literature we have gone through is concerned, there is no work done on Multiple Deferred State (MDS) Sampling Plans for Odd Log-Logistic Generalized Exponentiated (OLLGE) distribution, and hence this work intends to deal with it.

2 Odd Log-Logistic Generalized Exponentiated Distribution (OLLGED)

A baseline distribution in this case is a Generalized Exponentiated Distribution (GED), x is a random variable following GED with λ as scale and α as shape parameters and its cumulative (CDF) and probability density function (PDF) are specified as respectively:

$$F(x) = (1-e^{-\lambda x})^{\alpha} \tag{1}$$

$$f(x) = \lambda \alpha e^{-\lambda x}(1-e^{-\lambda x})^{\alpha-1} : x \geq 0, \alpha > 0, \lambda > 0 \tag{2}$$

Note: Substituting values of parameters equal to zero may result in other distributions as explained in other parts of this paper.

The researcher intends to use Odd Log-Logistic Generalized Exponential(OLLGE) distribution which may be referred to as best in modelling heavy-tailed data sets which are common in the field of lifetime data sets [14]. OLLGED originated from the exponential distribution (ED) with a total of three parameters simply the addition of two parameters (α and γ) to the original ED. The three parameters for the proposed distribution are γ and α as shape parameters and λ as scale parameters, which are introduced by procedures for generalization of the distribution [15]. lThe OLLGE distribution produces distributions with different shapes namely left and right-skewed, symmetrical and reversed-J, as parametric values of the distribution are changed. The flexibility of the OLLGE distribution enhances the possibility of it being applied in lifetime data sets and it can be stretched to quality control charts and acceptance sampling plans [16] and [17]. Using Equations (1) and (2), we can develop the OLLGED family with baseline distribution as GED and it is named as odd log-logistic generalized exponential distribution (OLLGED) see the generalization derived in [18]. The OLLGED's CDF and PDF are given in the following equations (3) and (4) respectively:

$$G(x) = \frac{(1-e^{-\lambda x})^{\alpha\gamma}}{(1-e^{-\lambda x})^{\alpha\gamma} + (1-(1-e^{-\lambda x})^{\alpha})^{\gamma}} \tag{3}$$

$$g(x) = \frac{\gamma\alpha\lambda e^{-\lambda x}(1-e^{-\lambda x})^{\alpha-1}(1-e^{-\lambda x})^{\alpha}(1-(1-e^{-\lambda x})^{\alpha})^{\gamma-1}}{((1-e^{-\lambda x})^{\alpha\gamma} + (1-(1-e^{-\lambda x})^{\alpha})^{\gamma})^{2}} : \lambda > 0, \gamma > 0, x \geq 0, \alpha > 0 \tag{4}$$

For given shape parameters (α, γ), the only term the cdf depend on are $x\lambda$ and the q^{th} quantile, the (x_q) when its products lifetime follows the OLLGED it is given as:

$$x_q = -\frac{1}{\lambda}\ln(1-(1+(q^{-1}-1)^{\frac{1}{\gamma}})^{\frac{-1}{\alpha}}) \tag{5}$$

When quantile $q = 0.5$ is also called median:

$$x_{0.5} = -\frac{1}{\lambda}\ln(1-2^{\frac{-1}{\alpha}}) \tag{6}$$

The products' failure probability under experimental time x_0 for OLLGED is as follows:

$$p = \frac{(1-e^{-\lambda x_0})^{\alpha\gamma}}{(1-e^{-\lambda x_0})^{\alpha\gamma} + (1-(1-e^{-\lambda x_0})^{\alpha}))^{\gamma}} \tag{7}$$

The time of termination (x_0) is expressed as quantile for (specified truncated time) (x_q^0), which can be related to experimental time $x_0 = ax_q^0$, the quantile of the variable (x_q) can be used to express the scale parameter λ and its given as:

$$\lambda = -\frac{1}{x_q}\ln(1-(1-(1-q^{-1})^{\frac{1}{\gamma}})^{\frac{-1}{\alpha}}) \tag{8}$$

To simplify the computational formula, we can write as:

$$\lambda = \frac{\eta}{x_q} \tag{9}$$

$$\eta = -\ln(1-(1+(q^{-1}-1)^{\frac{1}{\gamma}})^{\frac{-1}{\alpha}}) \tag{10}$$

Now by letting $\frac{x_q}{x_q^0} = r$ which is referred to as quantile ratio (r) then, the probability of the product failing as displayed in Equation (7) can be written as:

$$p = \frac{(1-e^{-\frac{\eta a}{r}})^{\alpha\gamma}}{(1-e^{-\frac{\eta a}{r}})^{\alpha\gamma} + (1+(e^{-\frac{\eta a}{r}}-1)^{\alpha})^{\gamma}} \tag{11}$$

The expression for η can be obtained by making it the subject, in the Equation (11). For $r > 1$, the products' failure probability (p_1) is considered as simply AQL and for $r = 1$, the products' failure probability (p_2) is called simply LQL. In this article, the shape α and γ parameters are considered to be known for OLLGED. One of the ways aided to know the shape parameter is estimation done on the previous production process that the manufacturer maintains as history.

3 Multiple Dependent State Sampling Plan (MDSSP)

The MDSSP is among sampling plans that are conditional for a special purpose, the concept of MDSS plans was introduced by Wortham and Baker [19]. The MDSS plan is the modification of the sampling plan introduced by Dodge as MDS-1 ("c_2"and "c_1") [19] and [20] which was named the chain-sampling (CS) plan. The application of such a sampling plan (CS) is for the continuous submission of lots for serial inspection in the order of its production. The implementation of MDSSP reduces the sample size required for a decision, for the decision of current lot deposition is made on the base of results of drawn samples from both lots thus, current and successive lots [20]. MDSSP has been investigated by a number of scholars in a number of different situations. In keeping a record of some few in the matter of this article are like the discussion for given AQL and LQL, on the selection of MDSSP was done by Govindaraju and Subramani [**?**]. Also, the MDSSP was studied by [13], based on measured real data. The Bayesian methodology was used to investigate the MDSSP by [21]. [22], [23] and [24] give a wider detail on MDSSP and their applications in real-life data sets. Contemporary, the MDS sampling plans concept has been applicable in the design of control charts [25]. [25] Explored control charts specifically attribute, based on MDSS approaches in manufacturing processes' monitoring. Rao's work used the design of sampling plans for multiple-deferred state method [26]. The proposed MDSSP utilizes the products' median life on the bases of the life test for time-truncated, assuming the products' quality characteristics (lifetime of the products) follow OLLGED. The procedure for operating the MDSSP for OLLGED is shown in 3.1. The comparison of the recent sampling plans and the proposed sampling plans' performance unveils an outstanding sampling plan.

3.1 Designing of the MDSSP for OLLGED

The subsection describes, the operating procedure and designing methodology of the MDSSP for OLLGED.

3.1.1 Procedures for Operating MDSSP

The parameters are obtained, through the operating procedures as described next steps:

1. A random sample of n units is drawn from the current lot. For fixed time (x_0), all units "n" are simultaneously taken through a life test.

2. The number of all failed units before (x_0) is recorded, which is the fixed time or truncated period for testing the products, name it d.

3. The rejection or acceptance decision of the lot is based on the following if $d > c_2$, the current lot is rejected, if $d \leq c_1$, the lot is accepted, and the test is ended. If $c_1 < d \leq c_2$, the current lot is accepted, in the provision that previous m (successive m) lots, under the condition $d \leq c_1$. were accepted

4. The four parametric characteristics describing the proposed MDSSP are c_1, c_2, m, and n. Where: n: the size of the sample taken, m: the number of previous lots necessary for decision making, c_2: the maximum number of items failed for lots conditional acceptance, and c_1 is the maximum number of failed items for unconditional acceptance.

   Note: The attributes MDSS plan is the general case of a Single sampling plan (SSP), and it also converges to SSP when either $m \to \infty$, c_1=c_2=c.

5. The MDSS plan's operating characteristic function for OLLGED of the time truncated life test as given in Equation (12)

$$P_a(p) = P(c_1 < d \leq c_2)(P(d \leq c_1))^m + P(d \geq c_1) \quad (12)$$

6. The probability of acceptance of the lot at probability p, while considering it following binomial distribution is shown in Equation (13).

$$P_a(p) = [\sum_{d=c_1+1}^{c_2} \binom{n}{d} p^d (1-p)^{n-d}][\sum_{d=0}^{c_1} \binom{n}{d} p^d (1-p)^{n-d}]^m + [\sum_{d=0}^{c_1} \binom{n}{d} (1-p)^{n-d} p^d] \quad (13)$$

3.2 Design of The Methodology

Sampling plans have various optimization goals, for example, the minimization of the average sample number (ASN) is the focus of any sampling plan. Among all sampling plans, the plan which gives the minimum ASN is most preferable. With the minimum ASN, the corresponding inspection cost in terms of monetary and time are all reduced. The attempt to have minimal ASN for the MDSS plan for OLLGE distribution is the focus of this article considering truncated life tests. The optimal parameters are obtained by optimization problem resulting in the minimal value of ASN for the proposed design as shown in Equations (14), (15), and (16):

Minimize ASN(p) = n (14)

subjected to

$$P_a(p_1) \geq 1 - \delta \quad (15)$$

$$P_a(p_2) \leq \beta \tag{16}$$

Where the failure probability at the risk of the producer is p_1 and that at the risk of the consumer is p_2. x_q/x_q^0 is the true lifetime quantile ratio which expresses the level of quality. The producers have their product quality enhanced through the utilization of the concept of the lifetime quantile ratio. The lot's acceptance probabilities at LQL and AQL probabilities of acceptance of the lot at AQL and LQL under an MDSS plan are respectively obtained as shown in Equation (17) and (18):

$$P_a(p_2) = \sum_{d=0}^{c_1} \binom{n}{d} p_2^d (1-p_2)^{n-d} + (\sum_{d=c_1+1}^{c_2} \binom{n}{d} p_2^d (1-p_2)^{n-d})(\sum_{d=0}^{c_1} \binom{n}{d} p_2^d (1-p_2)^{n-d})^m \tag{17}$$

$$P_a(p_1) = \sum_{d=0}^{c_1} \binom{n}{d} p_1^d (1-p_1)^{n-d} + (\sum_{d=c_1+1}^{c_2} \binom{n}{d} p_1^d (1-p_1)^{n-d})(\sum_{d=0}^{c_1} \binom{n}{d} p_1^d (1-p_1)^{n-d})^m \tag{18}$$

Where:

$$p_1 = \frac{(1-e^{-\frac{\eta a}{r}})^{\alpha\gamma}}{(1-e^{-\frac{\eta a}{r}})^{\alpha\gamma} + (1-(1-e^{-\frac{\eta a}{r}})^{\alpha})^{\gamma}} \tag{19}$$

$$p_2 = \frac{(1-e^{-\eta a})^{\alpha\gamma}}{(1-e^{-\eta a)^{\alpha\gamma}} + (1-(1-e^{-\eta a})^{\alpha})^{\gamma}} \tag{20}$$

The acceptance of a good quality product is always the wish of the producer.The computational was done, using R package, an open-source developed by Ihaka and Gentleman [27]. Therefore, $x_q/x_q^0 = 2,4,6,8,10$ are considered as the quantile ratio at the risk of the producers. Also, the wish of the consumers is to reject all poor-quality level products received. Therefore, at the risk of the consumer, the consideration here is at $x_q/x_q^0 = 1$ as the mean ratio. The optimal parametric values are given in Tables 1, 2, 3, and 4, while assuming the risk of the producer at $\delta = 0.05$ and risk of the consumer at β= 0.25, 0.10, 0.05, and 0.01, and at 50^{th} percentile, at known shape parameters (γ,α)=$(2,2)$,$(1.5,2)$,$(1.5,1.5)$ and $(2,1.5)$ of proposed MDSS plan for OLLGE distribution under life test for truncated time.

(i) When fixing a combination of other parametric values, it is noted that while increasing the termination ratio "0.5-0.1" in his range, the needed size of the sample "n" decreases.

(ii) The observed phenomena are observed when the risk of consumer decreases as consequence the size of the sample "n" increases, this happens when fixing the combination of parametric values of the proposed MDSS plan of OLLGE distribution.

(iii) Moreover, the size of the sample "n" was found to be influenced by the shape parametric value of the proposed MDSS plan of OLLGE distribution.

Table 1: The Design Estimate of Parameters for MDSSP when $\alpha = 1$ and $\gamma = 1$

β		0.5					0.7					1.0				
	r	n	c_1	c_2	m	$P(p_1)$	n	c_1	c_2	m	$P(p_1)$	n	c_1	c_2	m	$P(p_1)$
	2	32	7	17	3	0.9590	24	7	17	3	0.9583	18	7	11	3	0.9575
	4	10	1	3	1	0.9551	10	2	3	1	0.9723	7	2	3	2	0.9735
0.25	6	9	1	8	2	0.9855	7	1	2	1	0.9788	5	1	2	1	0.9810
	8	9	1	8	2	0.9945	4	0	2	1	0.9530	5	1	2	1	0.9919
	10	5	0	1	1	0.9656	3	0	2	3	0.9520	3	0	2	1	0.9645
	2	52	10	14	1	0.9510	41	11	21	2	0.9556	29	10	14	1	0.9511
	4	17	2	12	2	0.9514	13	2	4	1	0.9587	9	2	8	2	0.9524
0.10	6	12	1	4	2	0.9617	9	1	2	1	0.9538	7	1	3	1	0.9671
	8	12	1	4	2	0.9846	9	1	2	1	0.9792	7	1	3	1	0.9868
	10	12	1	4	2	0.9927	9	1	2	1	0.9891	7	1	3	1	0.9938
	2	68	13	17	1	0.9514	51	13	18	1	0.9565	40	14	17	1	0.9513
	4	24	3	5	2	0.9579	18	3	13	2	0.9625	13	3	12	2	0.9639
0.05	6	20	2	4	1	0.9864	11	1	3	1	0.9504	11	2	5	1	0.9875
	8	15	1	2	1	0.9623	11	1	3	1	0.9796	8	1	3	1	0.9788
	10	15	1	2	1	0.9797	11	1	3	1	0.9902	8	1	3	1	0.9898
0.01	2	103	19	25	1	0.9528	77	19	26	1	0.9536	57	19	25	1	0.9527
	4	37	4	9	1	0.9635	27	4	9	1	0.9639	20	4	10	1	0.9589
	6	26	2	5	1	0.9664	19	2	5	1	0.9657	14	2	6	1	0.9634
	8	21	1	5	1	0.9501	15	1	4	1	0.9507	14	2	6	1	0.9891
	10	20	1	2	1	0.9534	15	1	4	1	0.9750	11	1	5	1	0.9727

Table 2: The Design Estimate of Parameters for MDSSP when $\alpha = 2$ and $\gamma = 2$

β		0.5					0.7					1.0				
	r	n	c_1	c_2	m	$P(p_1)$	n	c_1	c_2	m	$P(p_1)$	n	c_1	c_2	m	$P(p_1)$
	2	21	0	10	3	0.9667	7	0	6	2	0.9705	3	0	2	1	0.9651
	4	21	0	10	3	0.9998	7	0	6	2	0.9998	3	0	2	1	0.9997
0.25	6	21	0	10	3	1.0000	7	0	6	2	1.0000	3	0	2	1	1.0000
	8	21	0	10	3	1.0000	7	0	6	2	1.0000	3	0	2	1	1.0000
	10	21	0	10	3	1.0000	7	0	6	2	1.0000	3	0	2	1	1.0000
	2	39	0	2	1	0.9600	13	0	3	1	0.9514	7	1	3	1	0.9940
	4	34	0	1	2	0.9995	11	0	10	2	0.9995	4	0	1	1	0.9993
0.10	6	34	0	1	2	1.0000	11	0	10	2	1.0000	4	0	1	1	1.0000
	8	34	0	1	2	1.0000	11	0	10	2	1.0000	4	0	1	1	1.0000
	10	34	0	1	2	1.0000	11	0	10	2	1.0000	4	0	1	1	1.0000
	2	70	1	4	2	0.9932	22	1	3	2	0.9913	8	1	3	1	0.9901
	4	44	0	2	2	0.9993	14	0	10	2	0.9991	5	0	2	1	0.9992
0.05	6	44	0	2	2	1.0000	14	0	10	2	1.0000	5	0	2	1	1.0000
	8	44	0	2	2	1.0000	14	0	10	2	1.0000	5	0	2	1	1.0000
	10	44	0	2	2	1.0000	14	0	10	2	1.0000	5	0	2	1	1.0000
	2	98	1	2	1	0.9727	31	1	3	1	0.9840	11	1	5	1	0.9735
	4	68	0	1	1	0.9988	21	0	1	1	0.9986	7	0	2	1	0.9985
0.01	6	68	0	1	1	0.9999	21	0	1	1	0.9999	7	0	2	1	0.9999
	8	68	0	1	1	1.0000	21	0	1	1	1.0000	7	0	2	1	1.0000
	10	68	0	1	1	1.0000	21	0	1	1	1.0000	7	0	2	1	1.0000

Table 3: The Design Estimate of Parameters for MDSSP when $\alpha = 1.5$ and $\gamma = 1.5$

β		0.5					0.7					1				
	r	n	c_1	c_2	m	$P(p_1)$	n	c_1	c_2	m	$P(p_1)$	n	c_1	c_2	m	$P(p_1)$
	2	17	1	11	3	0.9540	9	1	5	2	0.9604	7	2	3	2	0.9754
	4	9	0	2	2	0.9896	5	0	1	1	0.9902	3	0	2	1	0.9882
0.25	6	9	0	2	2	0.9981	5	0	1	1	0.9982	3	0	2	1	0.9978
	8	9	0	2	2	0.9995	5	0	1	1	0.9995	3	0	2	1	0.9994
	10	9	0	2	2	0.9998	5	0	1	1	0.9998	3	0	2	1	0.9998
	2	33	2	7	2	0.9702	17	2	12	2	0.9659	9	2	8	2	0.9561
	4	14	0	3	2	0.9765	7	0	6	2	0.9746	4	0	1	1	0.9729
0.10	6	14	0	3	2	0.9956	7	0	6	2	0.9952	4	0	1	1	0.9949
	8	14	0	3	2	0.9987	7	0	6	2	0.9986	4	0	1	1	0.9985
	10	14	0	3	2	0.9995	7	0	6	2	0.9995	4	0	1	1	0.9994
	2	40	2	4	1	0.9507	21	2	5	1	0.9518	13	3	12	2	0.9674
	4	18	0	10	2	0.9630	9	0	8	2	0.9601	5	0	2	1	0.9692
0.05	6	18	0	10	2	0.9929	9	0	8	2	0.9922	5	0	2	1	0.9942
	8	18	0	10	2	0.9979	9	0	8	2	0.9977	5	0	2	1	0.9983
	10	18	0	10	2	0.9992	9	0	8	2	0.9991	5	0	2	1	0.9993
	2	62	3	6	1	0.9515	36	4	10	2	0.9656	19	4	14	2	0.9508
	4	28	0	2	1	0.9543	14	0	2	1	0.9508	11	1	5	1	0.9962
0.01	6	28	0	2	1	0.9913	14	0	2	1	0.9905	7	0	2	1	0.9888
	8	28	0	2	1	0.9975	14	0	2	1	0.9972	7	0	2	1	0.9967
	10	28	0	2	1	0.9990	14	0	2	1	0.9989	7	0	2	1	0.9987

Table 4: The Design Estimate of Parameters for MDSSP when $\alpha = 1.5$ and $\gamma = 2.5$

β		0.5					0.7					1.0				
	r	n	c_1	c_2	m	$P(p_1)$	n	c_1	c_2	m	$P(p_1)$	n	c_1	c_2	m	$P(p_1)$
	2	24	0	10	4	0.9643	7	0	1	3	0.9680	3	0	2	1	0.9740
	4	24	0	10	4	0.9997	7	0	1	3	0.9998	3	0	2	1	0.9998
0.25	6	24	0	10	4	1.0000	7	0	1	3	1.0000	3	0	2	1	1.0000
	8	24	0	10	4	1.0000	7	0	1	3	1.0000	3	0	2	1	1.0000
	10	24	0	10	4	1.0000	7	0	1	3	1.0000	3	0	2	1	1.0000
	2	43	0	1	1	0.9560	13	0	1	1	0.9532	7	1	3	1	0.9966
	4	40	0	1	2	0.9995	12	0	10	2	0.9996	4	0	1	1	0.9996
0.10	6	40	0	1	2	1.0000	12	0	10	2	1.0000	4	0	1	1	1.0000
	8	40	0	1	2	1.0000	12	0	10	2	1.0000	4	0	1	1	1.0000
	10	40	0	1	2	1.0000	12	0	10	2	1.0000	4	0	1	1	1.0000
	2	82	1	2	2	0.9891	24	1	3	2	0.9943	8	1	3	1	0.9943
	4	52	0	4	2	0.9993	15	0	10	2	0.9994	5	0	2	1	0.9995
0.05	6	52	0	4	2	1.0000	15	0	10	2	1.0000	5	0	2	1	1.0000
	8	52	0	4	2	1.0000	15	0	10	2	1.0000	5	0	2	1	1.0000
	10	52	0	4	2	1.0000	15	0	10	2	1.0000	5	0	2	1	1.0000
	2	115	1	2	1	0.9773	34	1	3	1	0.9891	11	1	5	1	0.9841
	4	79	0	4	2	0.9985	23	0	1	1	0.9989	7	0	2	1	0.9990
0.01	6	79	0	4	2	0.9999	23	0	1	1	0.9999	7	0	2	1	1.0000
	8	79	0	4	2	1.0000	23	0	1	1	1.0000	7	0	2	1	1.0000
	10	79	0	4	2	1.0000	23	0	1	1	1.0000	7	0	2	1	1.0000

4 Application of Proposed Sampling Plan for Lab Extracted Beverages Products' Data

The real data set on carbon dioxide pressure (MPa) was collected in the laboratory from randomly selected locally (Tanzania) produced beverages. Used beverage products were collected for the period of six months (February 2021 to July 2021) to allow the collection of various batches of beverage products in the market. The carbon dioxide pressure (MPa) was measured by the portable carbon dioxide analyzer which measures in the range of 0.00 to 0.6 (MPa) at the average atmospheric temperature of 24.97^0C. A sample of 156 carbonated beverage products of locally produced beverages was used. The collected data set on the carbon dioxide pressure were used here to test the OLLGED distribution's goodness of fit. The transformation of the original data set on the value of carbon dioxide pressure was done by adding 0.128 value, to enable fitting well of the data set to the proposed probability distribution OLLGED and the final used data set are as follows:

0.298, 0.318, 0.343, 0.388, 0.393, 0.358, 0.318, 0.338, 0.338, 0.338, 0.343, 0.310, 0.398, 0.328, 0.395, 0.328, 0.388, 0.338, 0.388, 0.288, 0.315, 0.377, 0.388, 0.399, 0.395, 0.398, 0.388, 0.328, 0.308, 0.358, 0.318, 0.370, 0.208, 0.448, 0.418, 0.473, 0.458, 0.508, 0.418, 0.503, 0.449, 0.448, 0.378, 0.388, 0.428, 0.458, 0.397, 0.428, 0.428, 0.408, 0.468, 0.448, 0.452, 0.393, 0.448, 0.418, 0.428, 0.468, 0.428, 0.402, 0.418, 0.478, 0.318, 0.408, 0.448, 0.498, 0.390, 0.458, 0.438, 0.448, 0.449, 0.458, 0.488, 0.478, 0.456, 0.448, 0.468, 0.398, 0.328, 0.473, 0.455, 0.473, 0.458, 0.418, 0.448, 0.478, 0.367, 0.448, 0.378, 0.448, 0.437, 0.458, 0.388, 0.448, 0.372, 0.298, 0.198, 0.317, 0.180, 0.368, 0.218, 0.428, 0.398, 0.178, 0.208, 0.428, 0.473, 0.328, 0.228, 0.138, 0.398, 0.488, 0.468, 0.537, 0.368, 0.488, 0.398, 0.378, 0.218, 0.538, 0.376, 0.528, 0.458, 0.318, 0.418, 0.448, 0.460, 0.228, 0.208, 0.227, 0.358, 0.391, 0.518, 0.458, 0.388, 0.518, 0.388, 0.353, 0.397, 0.478, 0.457, 0.398, 0.358, 0.428, 0.368, 0.428, 0.538, 0.348, 0.218, 0.352, 0.408, 0.348, 0.388, 0.543, 0.388, 0.368, 0.368.

Figure 1 displays the theoretical and empirical Q-Q and CDFs plots to highlight the goodness of fit of OLLGED for real data on the values of carbon dioxide pressure (MPa). Hence, from Figure 1 the OLLGED yields a good fit for the applied data set used for application. Moreover, the estimated parameters of OLLGED using the maximum likelihood approach are $\hat{\alpha}$ = 0.8557, $\hat{\gamma}$ = 6.8189, and $\hat{\lambda}$ =1.4792. The Kolmogorov-Smirnov test = 0.0879 and p-value = 0.1793. This shows that the carbon dioxide pressure data set is reasonably well fitted to OLLGED.

Table 5: The Goodness of test and Maxmumlikehood estimation of Parameters

Model	**-2Likehood**	**AIC**	**BIC**	**W***	**A***
OLLGED (λ,α,γ)	334.793	328.7930	319.6434	0.2270	2.6253
				KS	**P-Value**
	1.4792 (1.1743)	0.8557 (0.6154)	6.8189 (3.2112)	0.0879	0.1793

For instance, the industrialist would prefer the use of developed MDSS plans in the implementation of the product's median life percentile, when the product lifetime quality characteristics follow OLLGED with the known shape parametric values of γ = 6.8178 and α = 0.8558. The industrialist suggests that given the median carbon dioxide pressure from beverage products in the market 0.18 whereas the industrialist expected that the median carbon dioxide pressure from beverage products is 0.36 MPa. The risk of the consumer is 0.05 when the actual median carbon dioxide pressure from the beverage product is 0.18 MPa and the risk of the producer is 0.10 when the actual median carbon dioxide pressure from the beverage product is 0.36 MPa. The constrained optimal parameters as selected in Table (6) are listed as; $n = 44$, $c_1 = 0$, $c_2 = 1$, and $m = 2$ with values of $\gamma = 6.8178$ and $\alpha = 0.8558$, $x_q^0 = 0.18$, $\delta = 0.05$, $\beta = 0.10$, $r = 2$ at $a = 0.7$. The MDSS plans are illustrated by considering a sample of 44 randomly selected beverage products from which the quality characteristic measured is the carbon dioxide pressure is 0.18 MPa. If the carbon dioxide pressure before 0.18 MPa is zero number of items then, the batch (lot) of the product will be accepted; and the batch (lot) of the product will be rejected if it is greater than 1 number of items. There will be indecision of the batch (lot) of the product is deferred until the 2 preceding the batch (lot) of the product will be tested in case of the number of items 0 and 1. In this real example, there are2 cases before the carbon dioxide pressure before 0.18 MPa and after. Hence, reject the present batch (lot) of the product. Thus industrialists could suggest to the government or public that the median carbon dioxide pressure (MPa) from beverage products is at an unacceptable level.

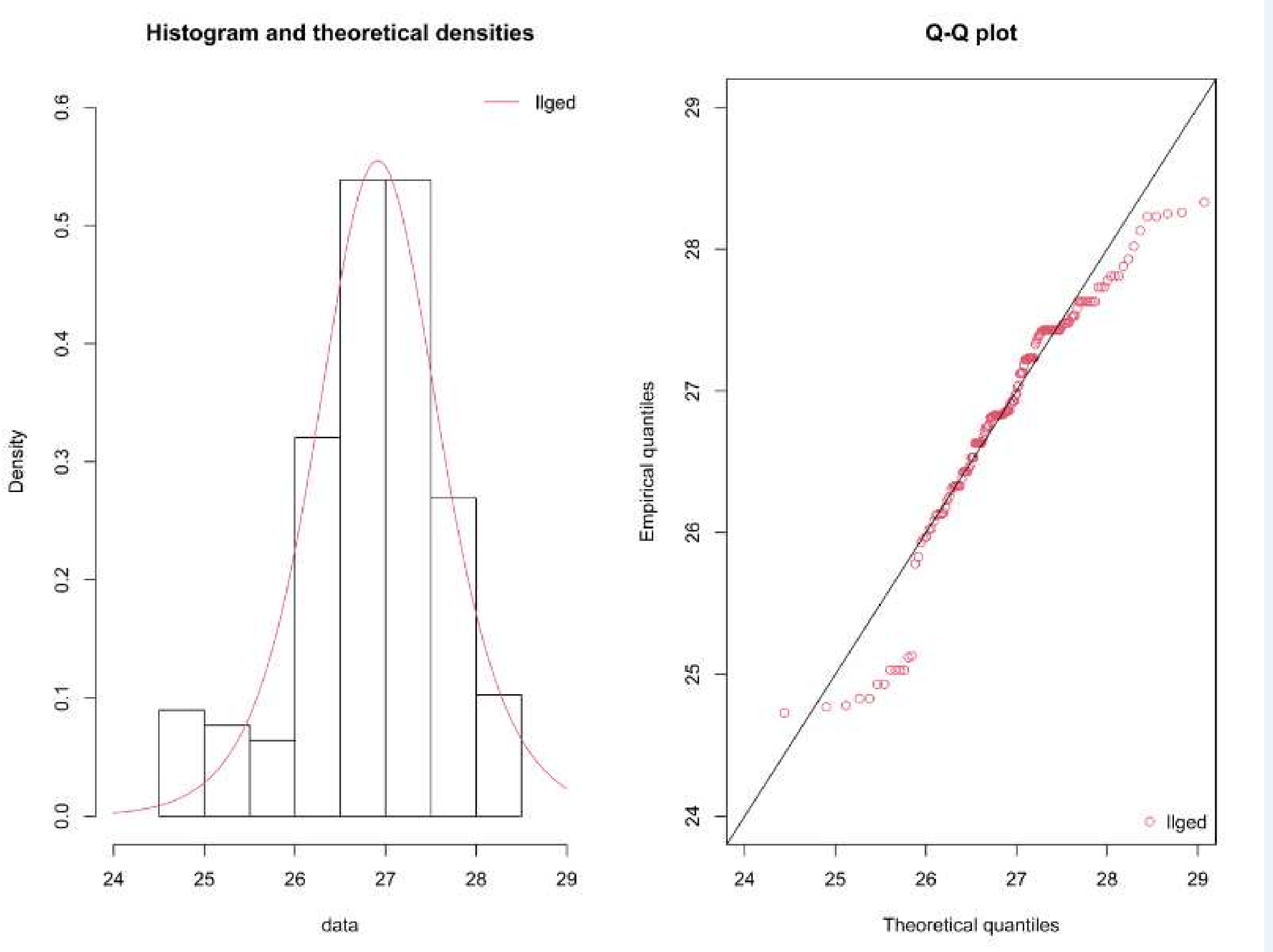

Fig. 1: CDFs and Q-Q Plots for Carbon Dioxide Pressure in *MPa*

Table 6: The Design Estimate of Parameters for MDSSP when $\alpha = 0.8558$ and $\gamma = 6.8175$

β	r	a=0.5					a=0.7					a=1.0				
		n	C_1	C_2	m	$Pa(p_1)$	n	C_1	C_2	m	$Pa(p_1)$	n	C_1	C_2	m	$Pa(p_1)$
	2	321	0	10	5	0.9994	27	0	10	3	0.9997	3	0	2	1	0.9998
	4	321	0	10	5	1.0000	27	0	10	3	1.0000	3	0	2	1	1.0000
0.25	6	321	0	10	5	1.0000	27	0	10	3	1.0000	3	0	2	1	1.0000
	8	321	0	10	5	1.0000	27	0	10	3	1.0000	3	0	2	1	1.0000
	10	321	0	10	5	1.0000	27	0	10	3	1.0000	3	0	2	1	1.0000
	2	532	0	10	4	0.9986	44	0	1	2	0.9994	4	0	1	1	0.9996
	4	532	0	10	4	1.0000	44	0	1	2	1.0000	4	0	1	1	1.0000
0.10	6	532	0	10	4	1.0000	44	0	1	2	1.0000	4	0	1	1	1.0000
	8	532	0	10	4	1.0000	44	0	1	2	1.0000	4	0	1	1	1.0000
	10	532	0	10	4	1.0000	44	0	1	2	1.0000	4	0	1	1	1.0000
	2	692	0	2	3	0.9982	57	0	3	2	0.9992	5	0	2	1	0.9995
	4	692	0	2	3	1.0000	57	0	3	2	1.0000	5	0	2	1	1.0000
0.05	6	692	0	2	3	1.0000	57	0	3	2	1.0000	5	0	2	1	1.0000
	8	692	0	2	3	1.0000	57	0	3	2	1.0000	5	0	2	1	1.0000
	10	692	0	2	3	1.0000	57	0	3	2	1.0000	5	0	2	1	1.0000
	2	1064	0	2	2	0.9972	57	0	3	2	0.9992	7	0	2	1	0.9991
	4	1064	0	2	2	1.0000	57	0	3	2	1.0000	7	0	2	1	1.0000
0.01	6	1064	0	2	2	1.0000	57	0	3	2	1.0000	7	0	2	1	1.0000
	8	1064	0	2	2	1.0000	57	0	3	2	1.0000	7	0	2	1	1.0000
	10	1064	0	2	2	1.0000	57	0	3	2	1.0000	7	0	2	1	1.0000

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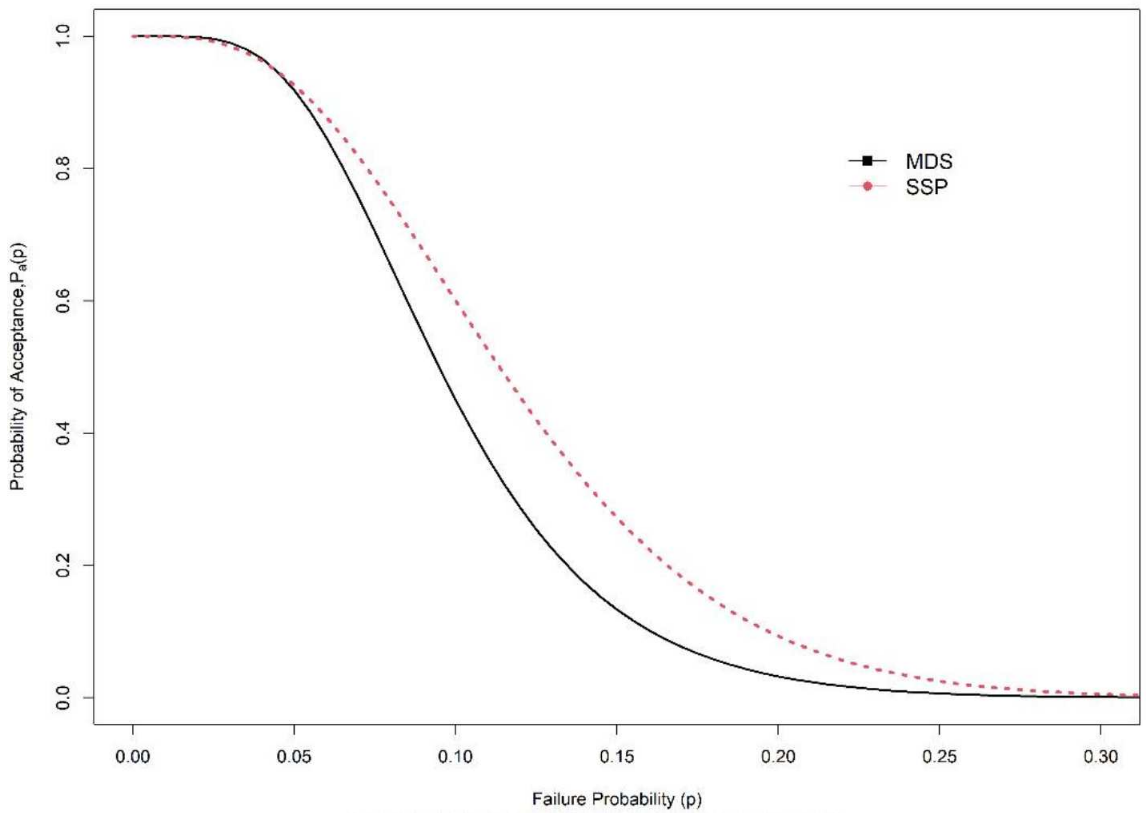
4.1 Comparative Study

The comparison is made for single and MDS sampling plan when quality characteristics follow OLLGED, the OC curve shows the plan's efficiency. The probabilities of rejecting of a lot that is bad and accepting the lot that is good are displayed in the curve to reveal their differences. The proposed MDSS plan efficiency over the SSP is revealed in Table (6), and the distribution of quality characteristics is assumed to follow OLLGED. When quantile ratio is considered $r = 2, 4, 6, 8, 10$ for each risk of the consumer $\beta = 0.25, 0.10, 0.05, 0.01$ at producer's risk of $\delta = 0.05$. Basically, the comparison is on the probability of acceptance $P_a(p_1)$ and sample size n. The acceptance sample size for the existing SSP is greater than that of the proposed MDSS plan for a number of set parameter values as shown in Table (6). For $r = 2$, the plan parameters for MDSS plan are $n = 17$, $c_1 = 1$, $c_2 = 11$, $m = 3$, in another hand SS plan the design parametric values are $n = 32$, $c_1 = 3$ and the corresponding designs probability of acceptances for MDSS and SSP are given as 0.9540 and 0.9690 respectively. The size of the sample for MDSS plan is smaller in comparison to SSP. The acceptance sample size decreases for both sampling plans as the quantile ratio increases. The MDSS plan with plan parametric values of $n = 17$, $c_1 = 1$, $c_2 = 11$, $m = 3$ and SSP with plan parametric values of $n = 32$, $c_1 = 3$, are compared by OC curve as displayed in Figure (2). MDSS plan is notably to be more efficient than SSP when the sample size is considered.



2.eps

Fig. 2: Operating Characteristic Curves of MDS Sampling Plan and SS Plan Schemes

Table 7: The Optimal Parametric Values Comparison for the Proposed MDSS plan and SS Plan for OLLGED with $\alpha = 1.5$ and $\gamma = 1.5$

β	r	$a = 0.5$								$a = 1.0$							
		MDSSP					SSP			MDSSP					SSP		
		n	c_1	c_2	m	$P_a(p_1)$	n	c	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$	n	c	$P_a(p_1)$
	2	17	1	11	3	0.954	32	3	0.969	7	2	3	2	0.9754	12	4	0.9719
	4	9	0	2	2	0.9896	17	1	0.9911	3	0	2	1	0.9882	5	1	0.9869
0.25	6	9	0	2	2	0.9981	9	0	0.9693	3	0	2	1	0.9978	3	0	0.9688
	8	9	0	2	2	0.9995	9	0	0.9836	3	0	2	1	0.9994	3	0	0.9832
	10	9	0	2	2	0.9998	9	0	0.99	3	0	2	1	0.9998	3	0	0.9897
	2	33	2	7	2	0.9702	50	4	0.9608	9	2	8	2	0.9561	17	5	0.9619
	4	14	0	3	2	0.9765	24	1	0.9827	4	0	1	1	0.9729	7	1	0.9738
0.10	6	14	0	3	2	0.9956	14	0	0.9526	4	0	1	1	0.9949	7	1	0.9951
	8	14	0	3	2	0.9987	14	0	0.9746	4	0	1	1	0.9985	4	0	0.9668
	10	14	0	3	2	0.9995	14	0	0.9844	4	0	1	1	0.9994	4	0	0.9795
	2	40	2	4	1	0.9507	65	5	0.9647	13	3	12	2	0.9674	18	5	0.9502
	4	18	0	10	2	0.963	29	1	0.9753	5	0	2	1	0.9692	8	1	0.966
0.05	6	18	0	10	2	0.9929	29	1	0.9954	5	0	2	1	0.9942	8	1	0.9935
	8	18	0	10	2	0.9979	18	0	0.9675	5	0	2	1	0.9983	5	0	0.9586
	10	18	0	10	2	0.9992	18	0	0.98	5	0	2	1	0.9993	5	0	0.9744
	2	62	3	6	1	0.9515	98	7	0.9684	19	4	14	2	0.9508	27	7	0.9511
	4	28	0	2	1	0.9543	40	1	0.9553	11	1	5	1	0.9962	14	2	0.9858
0.01	6	28	0	2	1	0.9913	40	1	0.9914	7	0	2	1	0.9888	11	1	0.9876
	8	28	0	2	1	0.9975	40	1	0.9975	7	0	2	1	0.9967	11	1	0.9963
	10	28	0	2	1	0.999	28	0	0.9691	7	0	2	1	0.9987	7	0	0.9644

5 Conclusions

The MDSS plan was developed in the article as presented considering the assumption that the quality characteristics lifetime of the beverages products follows an OLLGE distribution while under truncated lifetime tests. The optimal parametric values of the sampling plan proposed were obtained by satisfying the respective risk of consumer (β) and risk of producer (δ) simultaneously. the practical application was provided in extensive tables. A comparative study of an MDSS plan proposed and with the SS plan was done using OC curves. It is concluded that the MDSS plan proposed is more effective than the SS plans to secure the producer and consumer with less cost of the inspection. The suggested plan is illustrated with the real data set on carbon dioxide pressure (MPa) produced in beverages product.

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Availability of data and material
The used data sets are given in the manuscript.

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