TR A7175- Unrestricted

Report

The OPAL methodology for reliability analysis of power systems

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2012-07-03

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KEYWORDS:
Contingency analysis, reliability analysis, power systems, delivery point, reliability indices

VERSION
1

DATE
2012-07-03

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CLIENT(S)
Research Council of Norway, multiple clients

CLIENT'S REF.
Erland Staal Eggen

PROJECT NO.
12X683

NUMBER OF PAGES
101

ABSTRACT

This report describes the OPAL methodology for reliability analysis of power systems. Power system reliability assessment is a key part of security of supply analysis.

In reliability analysis the main objective is to determine the reliability of supply indices for the delivery points and power system under study, such as frequency and duration of interruptions, energy not supplied and interruption costs. OPAL is designed for these purposes, considering interruptions due to faults on power system components as well as protection system faults leading to missing or unwanted breaker operation, and time dependencies.

The OPAL methodology for reliability analysis is based on an analytical contingency enumeration approach. OPAL is in its current version primarily applicable for long-term planning purposes comprising the power generation and transmission system. Possible areas of application are value-based reliability planning, investment analysis, evaluation of reliability design criteria and as input to risk and vulnerability analysis on the identification of critical contingencies potentially leading to wide-area interruptions.

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REPORT NO.
TR A7175

ISBN
978-82-594-3506-4

CLASSIFICATION
Unrestricted

CLASSIFICATION THIS PAGE
Unrestricted

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1 Introduction

This report describes the OPAL methodology for reliability analysis of power systems. OPAL and the reliability assessment for delivery points in the power system constitute a corner stone in security of supply analysis in an integrated methodology combining network and market models. The report is a deliverable from the project "Integration of methods and tools for security of electricity supply analysis".

In reliability analysis, the main objective is to determine the reliability of supply indices for the delivery points under study, i.e. to estimate the frequency and duration of interruptions (or reduced supply), energy not supplied and the corresponding interruption costs such as cost of energy not supplied (CENS) e.g. according to the Norwegian regulation [16, 35]. OPAL is designed for these purposes, considering interruptions due to primary faults on power system components as well as protection system faults leading to missing or unwanted breaker operations.

Reliability methods for composite generation and transmission as well as distribution reliability analysis have been available for many decades, see e.g. [13, 14]. The methods have been under considerable development since then. There are two main approaches: simulation methods (Monte Carlo) and analytical methods. Both types of methods have strengths and weaknesses. Using Monte Carlo simulation the real system behaviour can, in general, be simulated and different operating strategies/policies can be included. Simulation is however usually a very time-consuming procedure compared to the analytical approaches which are computationally effective. On the other hand, analytical approaches suffer from problems by representing complex systems due to a variety of system behaviour, breaker- and operator-actions etc., and certain assumptions have to be made. Improvements have been made in both approaches over the years and there are also methods available combining the two, the so-called hybrid methods.

Although reliability methods have been available for a long time, the methods are not extensively used by the transmission system operators and network companies. Planning is still based on deterministic reliability criteria, while probabilistic approaches are used as a supplement to make relative comparisons between different operation schemes, different system alternatives, etc. [36]. The hesitation or reluctance to adopt such methods is often based on uncertainties associated with the calculated results, caused by limitations and inaccuracies in methods and reliability data. There has been a lot of activity and progress made within this field, shown by a comprehensive body of published papers [13, 14].

The OPAL methodology for reliability analysis is based on an analytical contingency enumeration approach. OPAL, in its current version, is primarily applicable for adequacy studies, i.e., long-term planning purposes for the composite power generation and transmission system, such as:

- Estimation of reliability of supply for delivery points in the transmission grid
- Estimation of interruption costs and value-based reliability planning
- Investment analysis
- Long-term operation planning
- Evaluation of reliability design criteria
- Evaluation of quality regulation design
- Input to risk and vulnerability analysis on the identification of critical contingencies potentially leading to wide-area interruptions (blackouts).

The methodology might be extended for short term operational security studies as outlined in [6].

The OPAL methodology is documented in detail in a requirement specification for software tool development [1] and it is implemented in a prototype tool in Matlab and Excel, documented in [26]. This report describes the models and methodology for reliability assessment. Comparisons are made with other analytical approaches, and examples and case studies are included demonstrating the OPAL methodology.

The report is structured as follows. Chapter 2 describes the integrated methodology for security of supply analysis, in which OPAL is a core methodology, and outlines the principles of the contingency enumeration approach. Chapter 3 gives an introduction to reliability analysis of power systems. It describes how delivery point interruptions are defined and the basic reliability models used in OPAL. The OPAL methodology is outlined in Chapter 4, in seven steps, through the use of an example network. OPAL gives the possibilities for including dependent faults in the protection system as well as time dependencies. How this is incorporated in the methodology is described in Chapter 5. Chapter 6 gives a comparison of OPAL and other analytical approaches, while Chapter 7 provides two case studies where OPAL is used. Conclusions and further work are finally discussed in Chapter 8. Three appendices are included, describing terms, definitions and symbols, basic reliability evaluation techniques, and give an example of using the accurate state space method compared to using minimal cut sets.

2 Integrated methodology for security of supply analysis

2.1 Overview of the integrated methodology

The OPAL methodology for reliability analysis of power systems and reliability assessment for delivery points in the power system is a core methodology in security of electricity supply analysis. Security of electricity supply (SoS) means the ability of an electricity system to supply final customers with electricity [3]. SoS is composed of *energy availability*, *power capacity* and *reliability*, with long term (system adequacy) and short term (security) perspectives [5]. This report considers the *adequacy* part of reliability, i.e. the capability of the system to supply the load in the steady state in which the power system may exist, considering normal operating conditions. The framework for security of supply analysis of power systems is depicted in Figure 1 and described in the following, based on [6, 7].

In a system where stochastic generation such as hydro and wind is a significant part, selection of operating states for contingency and reliability analysis becomes an important part of the framework. The approach integrates the power market analysis tools developed to handle stochastic generation and power market issues, with network models handling contingency and reliability analysis. This integrated approach provides a better information exchange and interaction between the different parts of the chain of analyses, thus improving the output generated by the different parts of the framework [6, 7]. Currently, the framework is being further developed in the project "Integration of methods and tools for security of supply analysis".

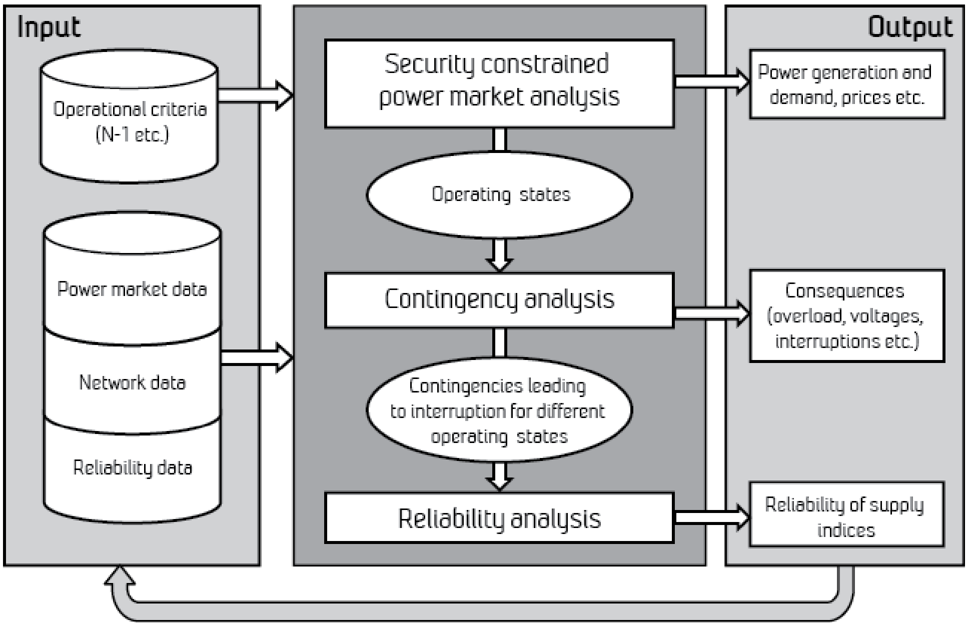


Figure 1 Framework of integrated methodology for security of supply analysis, from [7].

The integrated methodology enables a consistent analysis of societal impacts of energy or capacity shortage and interruptions, providing information about risk of high energy prices, risk of load curtailment and interruption costs for delivery points. These aspects are important input to the risk and vulnerability assessment of power systems as described in, e.g., [2, 8].

OPAL combines the two parts contingency analysis and reliability analysis in Figure 1 to determine the reliability of supply for delivery points. The security constrained power market analysis in the upper part of Figure 1 represents the part where generation and power market scenarios are combined to produce a set of operating states, taking into account network restrictions. An operating state[1] is defined as a system state valid for a period of time, characterized by load and generation composition including the electrical topological state (breaker positions etc.) and import/export to neighbouring areas [1]. The power market analysis thus provides important input to OPAL through information about the operating states of the system.

The combination of contingency analysis and reliability analysis in Figure 1 represents the contingency enumeration approach [1, 9, 10] for composite generation and transmission system reliability assessment. The security constrained power market analysis and the contingency enumeration approach are described in the following sections.

2.2 Security constrained power market analysis

The multi-area power market simulator EMPS is a software package for the optimization and simulation of hydro thermal power systems [2, 11]. It is the most commonly applied tool for power market analysis in the Nordic countries today. The EMPS tool calculates the value of stored water for different reservoir fillings, and simulates the optimal operation of the power system for a sequence of hydrological years. The corresponding equilibrium prices (market clearing or spot prices) affect demand, supply, transmission capacities and the use of water reservoirs. Transmission operational security constraints are included by subdividing the total system in a multi-area model where active power flow is controlled and kept within the predefined limits. These transmission capacities are constraints that by definition are exogenous to the market model, and ideally they should reflect thermal limits as well as stability limits for the system operation. In order to check if the set of generation and demand states generated by the power market model satisfies the transmission constraints, a DC power flow analysis is included. This represents an expansion and enhancement of EMPS, and is referred to as the EMPS with network constraints (EMPS-NC) [12].

By this approach, the power market model will generate operating states as input to the next step of the analysis, such that active power flow is physically modelled and kept within network constraints. Additionally, the method allows for the inclusion of deterministic reliability constraints (operational security criteria), such as N-1 as indicated in Figure 1.

2.3 Contingency enumeration approach for reliability assessment

Internationally, there are numerous methods and tools dealing with contingency and reliability (adequacy) analysis, see e.g. [13, 14]. Various alternative methods are described in the literature, and tools are developed, based on Monte Carlo simulations or analytical approaches. The essential task in either approach is to select different system states and assess their adequacy.

The electric power system is an extremely complex and comprehensive infrastructure. The number of system states increases exponentially by 2^n for a system of *n* components that typically are assumed to be in one of two possible states ("up" or "down"). For a real system, the number of system states will "explode" and it is a demanding task to assess the adequacy for all possible system states.

[1] The term "operating scenario" is sometimes used synonymously with operating state.

OPAL is based on the contingency enumeration approach. Instead of enumerating all possible system states, the OPAL approach brings the critical contingencies into focus. A contingency is an unplanned outage of one or more primary components in the power system due to failures, which may have technical, human or nature related causes. The critical contingencies are those potentially leading to delivery point interruptions. The basic structure of the contingency enumeration approach is depicted in Figure 2.

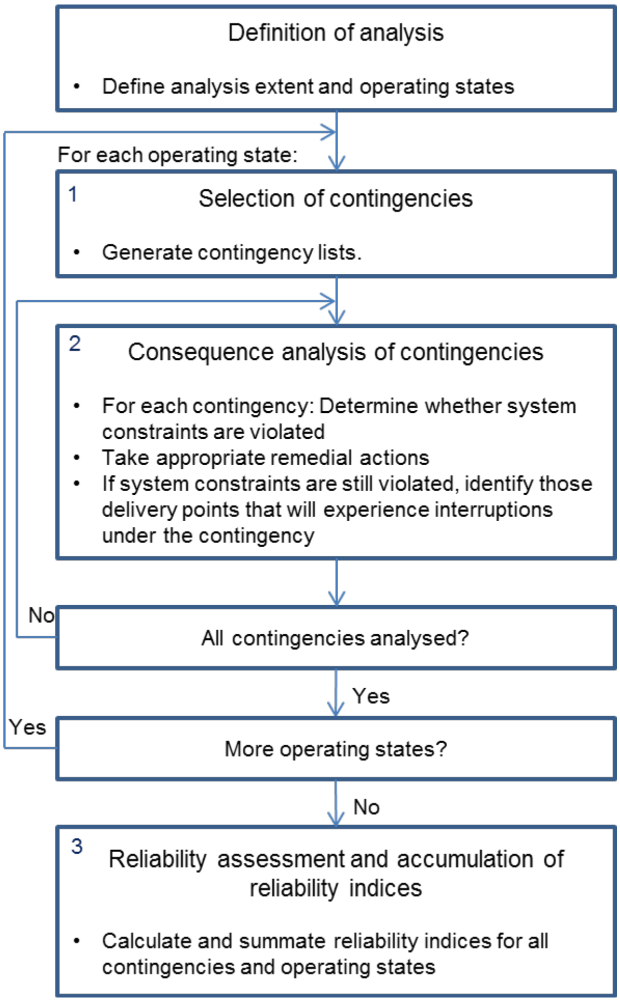


Figure 2 Basic structure of the contingency enumeration approach, based on [1, 10].

The contingency enumeration approach comprises three main steps (as indicated in Figure 2):

1 Selection of contingencies
2 Consequence analysis of contingencies
3 Reliability assessment and accumulation of reliability indices.

The first two steps constitute the contingency analysis where the major challenge is to identify those contingencies causing system problems, i.e. violating system constraints, for instance, related to voltage limits and thermal overload, and determine the consequences in terms of interruptions to delivery points. These results are combined with reliability models and data in the reliability assessment to calculate and accumulate the reliability indices.

As indicated in Figure 2, the analysis starts with defining its extent, i.e. defining which area/ part of the network and delivery points to be studied and the depth of contingencies (single, double, higher order combinations of outages, etc.) to be analysed. It should also be defined for which operating states (scenarios) the analysis should be performed.

As shown in Figure 2, the contingency analysis comprises selection and consequence analysis of contingencies. The aim is to identify the various contingencies that may lead to the interruption of a delivery point. The analysis starts with an operating state based on a network model with input data from a specific generation and load scenario for each node in the network. Often, the "worst case", i.e., the heavy (peak) load situation, is applied. The contingency analysis should ideally be carried out for a set of operating states regarded to be representative for a year.

In the first step of the contingency analysis, the objective is to reduce the number of contingencies for detailed analysis. The goal is to determine the set of contingencies that will cause violation of system operational constraints potentially leading to interruptions. Since the power system at the higher voltage levels is designed to withstand the loss of a single major component (N-1 criterion) it is usually necessary to select contingencies of higher order that potentially lead to interruptions. The total number of contingencies selected for the detailed studies may be based on some kind of cut-off criteria, e.g. according to the probability or frequency of the contingencies [10]. The number can be reduced by using screening or ranking techniques (see e.g. [1]). A typical analysis depth is to include all first and second order independent outages, and dependent outages such as common cause, station originated outages or other specified outages. While the probabilities of multiple independent outages can be very small, dependent outages may have significantly higher probabilities. As mentioned above it might be necessary to analyse higher order contingencies to reveal the high impact events. On the other hand, increasing the analysis depth means that a large number of contingencies need to be further analysed in detail, running the power flow analyses. These analyses can be very time consuming. The choice of analysis depth is thus a trade-off between the accuracy in identifying critical contingencies and the computational burden.

In the second step of the contingency analysis, the objective is to identify which delivery points in the power system under study will experience interruptions (or reduced supply). The selected contingencies are analysed to determine whether the contingency leads to violation of any system limits related to overloading of lines or transformers, too high or low voltages in some parts of the network or network separation. This analysis of electrical consequences is based on simulations of contingencies in the electricity system using physical power flow models (as described in e.g. [1, 10]).

If the system is outside its operating limits for a specific contingency, there might be possibilities for taking some automatic or manual corrective (remedial) actions to bring the system back within its limits. These include actions such as generation rescheduling, network reconfiguration, transformer tap adjustment, and automatic disconnection of specific generators or curtailable loads with low priority. If the corrective actions are not sufficient to bring the system back within its limits, load shedding is necessary, resulting in partial or total interruption for some delivery points. The computer programs used for contingency analysis try to mimic operation of the power system by representing remedial actions in the power flow analysis. In some programs, optimal power flow (OPF) is used to minimize the load shedding, say, based on the interruption cost which is an indicator of type and importance of different loads.

The consequence analysis of a given contingency under specified operating conditions yields information on those delivery points that will experience interruptions due to the contingency. The amount of load that can be served at each affected delivery point is determined. This is described in [1]. In OPAL, an interruption (partial or total) is defined as a situation when the total available capacity after the occurrence of a given contingency is unable to match the load at the delivery point:

$$P > SAC + LG \quad (1)$$

where P is the load at the delivery point, SAC is the System Available Capacity that can serve the load after the given contingency, and LG is the local generation at the delivery point, if available.

This principle is further outlined in the next chapter.

The main results from the contingency analysis in OPAL are the lists of delivery points that will be interrupted by the analysed contingencies as well as the corresponding system available capacities for the different operating states. These results are input to the reliability analysis.

The objective of the reliability analysis is to determine the reliability of supply for the system and delivery points under study. This is the third and final step of the contingency analysis approach (see Figure 2) comprised by the reliability of supply assessment for delivery points and accumulation of various electricity supply interruption measures. These measures are termed reliability indices.

The basic reliability indices comprise the frequency and duration of interruptions. In addition, there are various indices in use that describe the severity of the interruptions based on, for instance, interrupted power, energy not supplied and interruption costs. With the explicit application of interruption costs in system planning, total interruption costs are typically calculated using average specific costs (customer damage functions) for different customer groups, combined with interrupted power and energy not supplied to determine the reliability worth, see e.g. [16, 18, 33]. This process involves the combination of a reliability model, a load model and a cost model as illustrated in Figure 3. The OPAL methodology seeks to combine these three models.

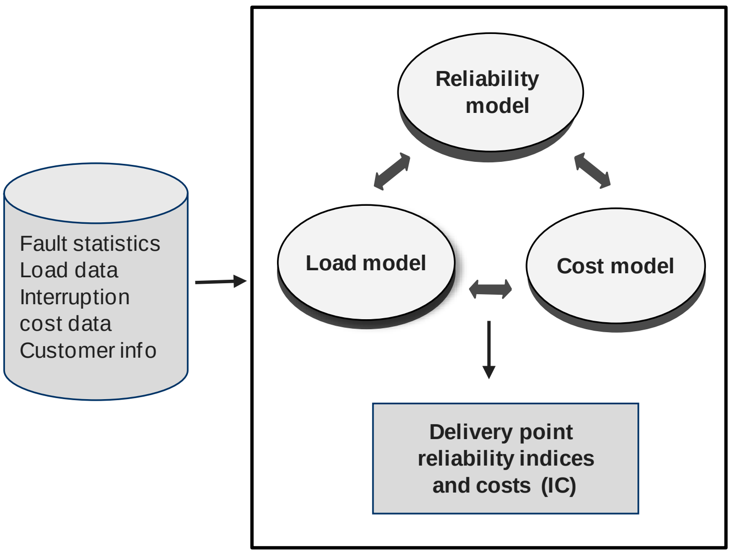


Figure 3 Assessment of delivery point reliability indices, based on [19].

The input data required for reliability and interruption cost assessment are:

- Topology
- Breaker positions and reserve connections
- Fault statistics
- Switching times
- Load data
- Customer data.

The main results from the analysis combining the three models are delivery point reliability indices (including interruption costs), as given in Table 1. Using analytical methods, these are usually expectation values, while Monte Carlo simulation methods can additionally provide probability distributions of the indices (see e.g. [1, 20, 21]).

Table 1 Main delivery point reliability indices (expected values).

Reliability of supply/interruption indices	Unit
Annual number of interruptions, λ	Number per year
Annual duration of interruptions, U	Hours per year
Average interruption duration, r	Hours per interruption
Annual interrupted power, P_{interr}	MW per year
Annual energy not supplied, ENS	MWh per year
Annual interruption cost, IC	NOK per year

The reliability analysis requires a reliability model as depicted in Figure 3, describing frequency and duration of electricity supply interruptions to delivery points. This reliability model is outlined in the next chapter.

3 Reliability analysis of power systems

The reliability analysis considers unplanned (or forced) outages caused by failures of the components in the system under study. These can be single failures or independent or dependent multiple failures. The component outages due to failures may cause electricity interruptions (partial or total) in one or more delivery points in the system. By delivery points is usually meant load points. A failure is the termination of the ability of an item to perform a required function. Failure is related to the power system components, while interruption is related to the delivery points according to the definitions in A.1. As stated in the previous chapter, the objective of reliability analysis is to determine the frequency and duration of electricity supply interruptions (and additional severity measures) for the delivery points and system under study. This chapter describes the principle of occurrence of interruptions, the reliability models, and the main reliability indices provided by the OPAL methodology.

3.1 Delivery point interruptions

Methods for reliability assessment in transmission or meshed systems are quite different from methods in use for distribution or radial systems [23]. The problem of assessing delivery point reliability is nevertheless quite general, even though the developed models and computer programs may be considerably different in type and complexity. This section addresses this generality and some basic principles, while the reliability model is described later in the chapter. The description in this section is based on (partial excerpt from) [19].

The occurrence of interruptions depends in general on the available capacity to supply the load P. The available capacity in the supply system is denoted System Available Capacity (SAC). The available power capacity (APC) at the supply terminals is the sum of SAC and the local generation (LG). According to Equation (1) interruption occurs when

$$P > SAC + LG$$

This equation represents the stationary situation after dynamic responses have faded away, and after possible actions to prevent interruptions or reduce the consequences have taken place. It is valid in the general case, except when LG represents reserve supply facilities which are connected after interruption has occurred. In such cases, interruptions occur when P > SAC, which is usually the case in radial systems. It should be noted here that this is a general and simplified description of the problem of assessing interruptions.

An example of a system available capacity (SAC)-curve is shown in Figure 4. By superimposing the SAC-curve on the curve for local generation (LG), the available power capacity (APC) profile is obtained (shown in Figure 4, APC = SAC + LG).

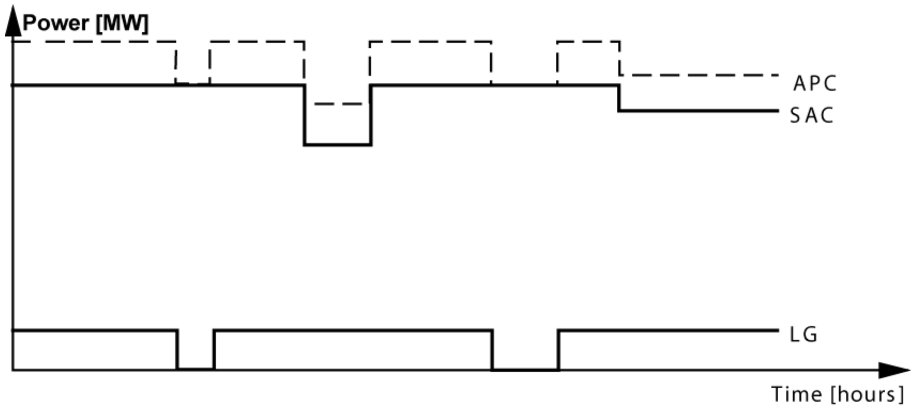


Figure 4 Available capacity models for a general delivery point, from [19].

The available capacity is determined by a combination of various factors related to generation and transmission resources and the loading conditions in the system:

- Installed capacity (generation)
- Ancillary services and other reserves (generation)
- Transmission capacity (transmission and distribution)
- Outages of generators, lines, cables, transformers, etc.
- Revisions and maintenance
- Local generation/reserve possibilities from underlying or neighbouring network
- Loading conditions.

The superimposition of the APC-curve on the hourly load curve gives the available margin. A negative margin implies that load has to be disconnected, yielding interrupted power and energy not supplied. The duration of the interruptions is given by the periods of negative margins. An example is shown in Figure 5.

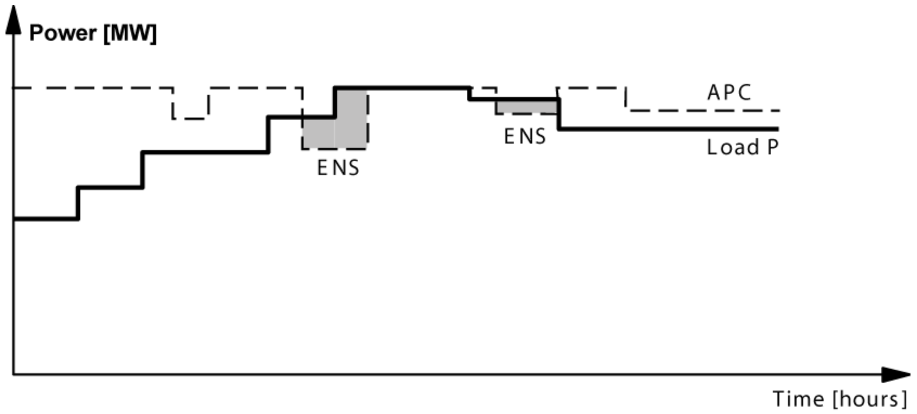


Figure 5 Superimposition of APC-curve on the hourly load curve for a general delivery point, from [19].

The procedure described here could be a general approach for the assessment of interruptions. APC is however obtained differently in meshed and radial systems.

The simplest way of assessing reliability is for radial distribution systems. Any component failure in the system will, with very few exceptions, cause interruptions to all the delivery points supplied by the same radial, and the total load is disconnected. It is therefore also quite simple to determine the power interrupted (P_{interr}) and the energy not supplied (ENS).

For composite systems and meshed distribution systems, the reliability assessment is more complicated. Only a few serious contingencies will lead to total interruptions. But any contingency can lead to a reduction in available power capacity to meet the load demand. Depending on the loading conditions and load demands during a contingency, some of the loads may be disconnected due to the violation of operating constraints. Corrective actions and preventive measures can be taken to prevent the disconnection of loads or reduce the volume of load curtailments. Examples of such measures are rescheduling of generation, alleviation of overload and load shedding. In radial distribution systems, a preventive measure often used is alternative supply from reserve connections. If disconnection of loads is necessary, the least critical load may (if possible) be disconnected first and so on.

Since contingencies seldom lead to total interruptions of loads in transmission systems, it is relevant to look at the available power capacity in different time periods. For a bulk delivery point (local area), the situation is similar to the one illustrated in Figure 4 and Figure 5.

A similar capacity curve can, in principle, be obtained for a delivery point in a radial distribution system. In a radial system, however, for each component failure SAC will be zero, which means that the total load is disconnected. APC however, can be different from zero if there is any reserve supply available. In that case the power will be interrupted, but the amount of ENS will be reduced. Figure 6 shows an SAC-, APC- and a load curve for a delivery point fed from a radially-operated system.

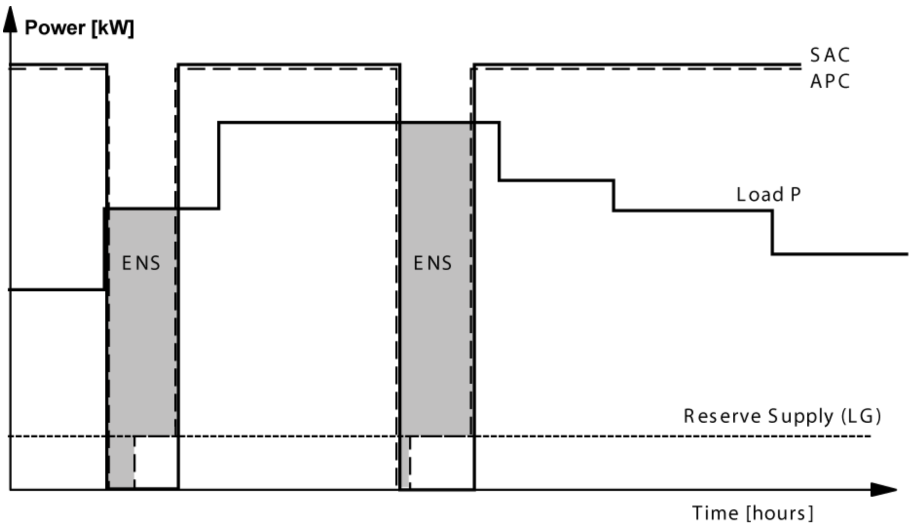


Figure 6 SAC and APC for a distribution delivery point, from [19].

Hence for reliability assessment in radial systems, it is not necessary to establish the SAC- and APC-curves to determine P_{interr} and ENS. Figure 6 is included to show the similarity with the assessment of reliability for a bulk delivery point.

For simplicity, SAC is held at a constant level in Figure 4 - Figure 6, except in periods of component failure.

SAC is determined by independent and dependent component failures and by system problems (violation of system constraints). The more meshed the system, the more complicated is the determination of SAC and the reliability level. If we consider failures on components only, the SAC profile can be obtained by combining the components' operating cycles, which are established from the stochastic failure/repair process for each component in the system. An example with two components is shown in Figure 7. The components are here represented by two states, either up or down.

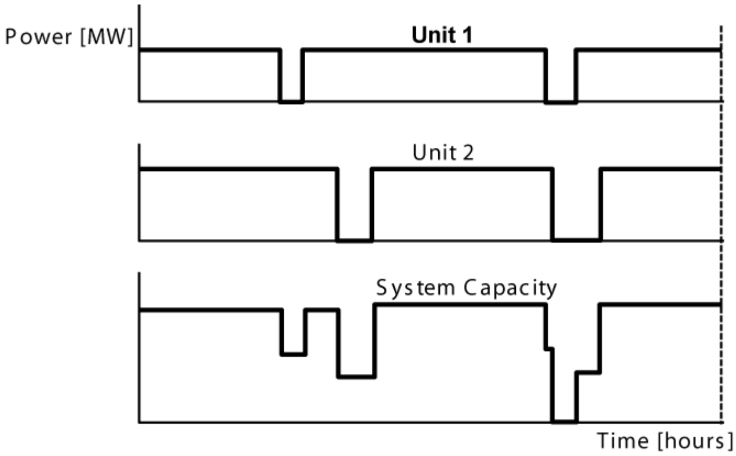


Figure 7 Example of System Available Capacity, from [19].

In large meshed systems, it is too demanding to analyse all possible contingencies. As mentioned, there are different techniques reported in the literature for screening and ranking the most important or the most severe contingencies (see Section 2.3 and [1]). Such contingencies are often outages of more than one component, since most systems are dimensioned to withstand outages of one major component according to the (N-1)-criterion.

An interruption occurs when the available capacity is unable to match the load. A negative margin (Figure 4 and Figure 5) implies that load has to be disconnected. The interrupted power P_{interr} is thus determined by:

$$P_{interr} = P - APC = P - SAC - LG \quad \textbf{(2)}$$

This equation might give an optimistic estimate of the interrupted power, since it assumes that it is possible to disconnect only the amount represented by the negative margin.

3.2 Reliability models for delivery points and components

The reliability model for a general delivery point which is utilized in the OPAL methodology is shown in Figure 8. The upper part of the figure gives a simplified picture of the power system supplying the delivery point while the lower part shows the reliability model as a minimal cut set[2] structure. The minimal cut set method is described in A.2.

[2] A cut set is a set of components which, when failed, causes system failure. A minimal cut set has no proper subset of components whose failure alone will cause system failure [38].

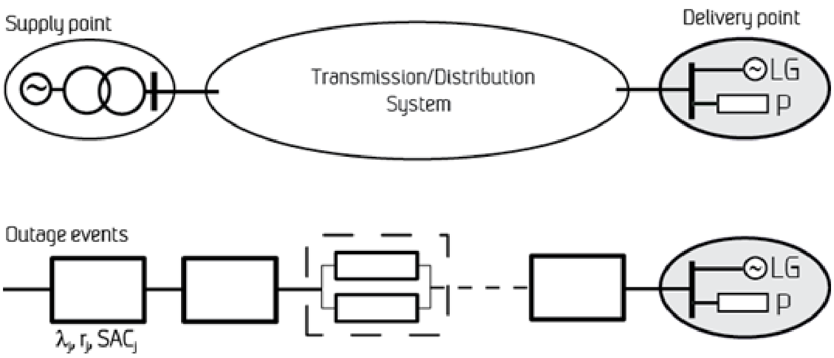


Figure 8 Reliability model for a general delivery point using minimal cut sets, based on [19].

This model takes the critical contingencies for a delivery point as a starting point. The critical contingencies are those found in the contingency analysis (in steps 1 and 2 described in Section 2.3) to cause interruptions in the delivery point. The annual reliability indices are thus found by a summation of contributions from different outage events as illustrated by the minimal cut set structure in Figure 8.

All combinations of contingencies (outage events) that will lead to delivery point interruptions according to Equation (1) can be viewed as the minimal cuts for a particular delivery point. The parallel components and dotted lines in Figure 8 illustrate a double contingency, i.e., simultaneous outage of two components. The minimal cuts may consist of various types of faults leading to component outages and other incidents in the system (such as violation of system constraints). Examples are:

- Single component faults
- Independent (multiple) overlapping faults
- Common mode faults
- Dependent (multiple) overlapping faults (protection and breaker faults)
- Overlapping faults in adverse weather.

Figure 9 gives examples of contributions from outage events in a minimal cut set structure.

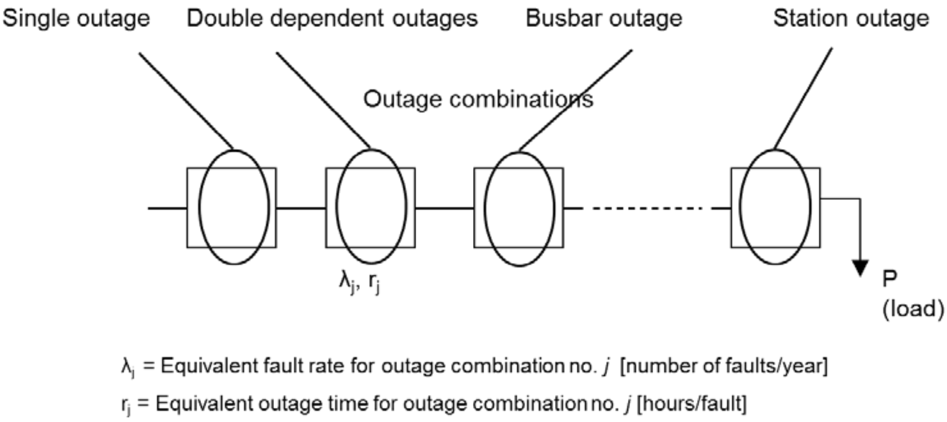


Figure 9 Examples of outage events (minimal cuts) contributing to interruptions in a delivery point.

A cut set may represent a single component failure or a multiple independent or dependent event as described above. Each minimal cut set (block) is represented by an equivalent fault rate (λ_j), outage time (r_j) and the available capacity (SAC_j) to supply the load (P) after the occurrence of contingency *j*. The equivalent fault rates (λ_j) and outage times (r_j) are determined from the fault rates and outage times of the individual components based on their failure and restoration processes.

The reliability model for the individual components is based on the two-state Markov model for function and failure/repair as shown in Figure 10:

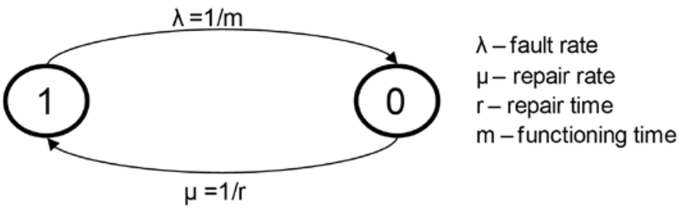


Figure 10 Two-state Markov model with failure and repair process.

The fault rate in Figure 10 is defined as:

$$\lambda = \frac{\text{number of faults on the component in a given time interval}}{\text{total time in operation}} \quad \textbf{(3)}$$

The components are assumed to be in one of the two possible states, i.e. either "up-state" (state 1) or "down-state" (state 0). The probabilities of these states are found based on the fault rate and repair rate of the component (see A.2 for details):

Down state:

$$P_0 = \frac{r}{m+r} = \frac{\lambda}{\lambda+\mu} \quad \textbf{(4)}$$

Up state:

$$P_1 = \frac{m}{m+r} = \frac{\mu}{\lambda+\mu} \quad \textbf{(5)}$$

Based on these basic probabilities and the relations between the fault and repair rates, and the repair and functioning times describing the state cycle of the component (see Figure 24 in A.2), the frequency of encountering a state and the state duration can be determined. This evaluation method is denoted the state space method. Frequency and duration techniques can be derived for series and parallel systems from the steady-state Markov probabilities as described in e.g. [22, 23], see A.2.

These basic techniques can be further utilized in approximate reliability evaluation such as the minimal cut set method. The components of a minimal cut set are in parallel since all of them must fail in order to cause system failure. Thus, the cut sets are in series as any minimal cut set can cause system failure.

Consider, for instance, the small example system shown in Figure 11. The system will fail if component 1 fails or if both components 2 and 3 fail. There are two minimal cut sets shown to the right in the figure. The first minimal cut set consists of component 1 and the second of the parallel of components 2 and 3.

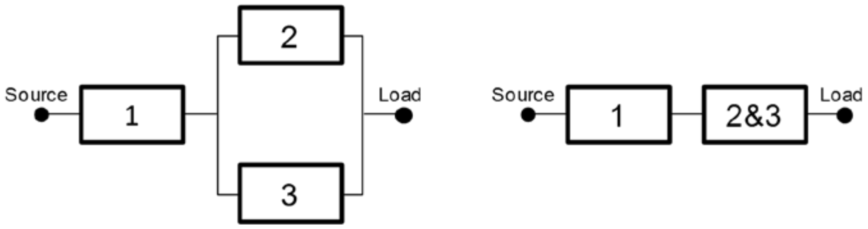


Figure 11 Example of series and parallel system and minimal cut sets.

The equivalent fault rate and repair time for the cut set containing the parallel components are found using the equations presented in A.2 for parallel systems. These are combined with the fault rate and repair time for the single component using the equations for series systems to give the overall system indices as shown in the following. The number of faults per year for the parallel:

$$\lambda_{2\&3} = \frac{\lambda_2 \lambda_3 (r_2 + r_3)}{8760 + \lambda_2 r_2 + \lambda_3 r_3} \qquad \text{[no of faults pr year]} \qquad \textbf{(6)}$$

The equivalent outage time for the parallel, in hours per fault:

$$r_{2\&3} = \frac{r_2 r_3}{r_2 + r_3} \qquad \text{[hours pr fault]} \qquad \textbf{(7)}$$

The overall indices for interruption to the load are found as follows (see A.2):

Expected number of interruptions per year:

$$\lambda_s = \sum_{i=1}^{n} \lambda_i = \lambda_1 + \lambda_{2\&3} \qquad \text{[no of interruptions pr year]} \qquad \textbf{(8)}$$

Expected annual interruption duration (hours per year), i.e. the unavailability U_s can be determined using the concepts of frequency and duration (see A.2):

$$U_s = f_s r_s \qquad \textbf{(9)}$$

where f_s is the frequency of entering the down state

For most practical purposes, the mean time to failure (MTTF =1/λ) and mean time between failures (MTBF =1/f) are almost numerically identical (if the repair time is small compared to the MTTF). The expected annual interruption duration may then be approximated to:

$$U_s \approx \lambda_s r_s = \sum_{i=1}^{n} \lambda_i r_i = \lambda_1 r_1 + \lambda_{2\&3} r_{2\&3}$$ [hours pr year] **(10)**

Average expected interruption duration (hours per interruption):

$$r_s = \frac{\sum_{i=1}^{n} \lambda_i r_i}{\lambda_s} = U_s / \lambda_s$$ [hours pr interruption] **(11)**

For a minimal cut set consisting of an independent overlapping outage event of two components, these are considered as two components in parallel as above while there are separate models for common cause and other dependent outage events [1, 23]. The equivalent fault rates and outage times are thus determined using the basic frequency and duration techniques as shown for the example above, see A.2 for further details.

3.3 Reliability indices

The basic frequency and duration techniques are now utilized to calculate the reliability indices for the general delivery point in Figure 8. In the following, we consider only those contingencies representing the minimal cut sets. The contingency (outage event) will give a certain level of SAC. The occurrence of this level is determined by the equivalent number of faults (i.e. the fault rate λ_j) of the cut *j*, while the duration is determined by the equivalent duration r_j. If we are able to determine the most important minimal cuts (outage events) and their corresponding SAC, we are able to assess the reliability of supply for the delivery point. The reliability indices for each delivery point are thus determined based on the series structure of the minimal cut sets and the indices are accumulated by the summation of contributions from the critical contingencies, using the principles for series systems as described above and outlined in [22].

The basic reliability indices for a delivery point are the expected frequency of interruptions and interruption duration. For a given operating state, the basic indices can be found as follows:

$$\lambda = \sum_{j=1}^{J} \lambda_j$$ [interruptions/year] **(12)**

$$U = \sum_{j=1}^{J} \lambda_j r_j$$ [hours/year] **(13)**

$$r = \frac{\sum_{j=1}^{J} \lambda_j r_j}{\sum_{j=1}^{J} \lambda_j}$$ [hours/interruption] **(14)**

where J = number of minimal cuts, λ = number of interruptions per year, U = annual interruption duration (unavailability) and r = average interruption duration.

In addition to the basic reliability indices represented by Equations (12), (13) and (14), it is possible to calculate the severity of the contingencies for the delivery point based on information about the load, i.e. expected consequences in terms of interrupted power, energy not supplied and interruption costs. This is shown in the following for each minimal cut *j* for a given operating state:

$$P_{\mathrm{interr},j} = P - SAC_j - LG \qquad \text{[MW/interruption]} \qquad \textbf{(15)}$$

$$ENS_j = r_j P_{\mathrm{interr},j} \qquad \text{[MWh/interruption]} \qquad \textbf{(16)}$$

$$IC_j = c(r_j)ENS_j = c(r_j)r_j P_{\mathrm{interr},j} \qquad \text{[NOK/interruption]} \qquad \textbf{(17)}$$

These indices for a given operating state are determined on an annual basis (per annum *a*) as follows:

$$P_{\mathrm{interr},j,a} = \lambda_j P_{\mathrm{interr},j} \qquad \text{[MW/year]} \qquad \textbf{(18)}$$

$$ENS_{j,a} = \lambda_j ENS_j = \lambda_j r_j P_{\mathrm{interr},j} \qquad \text{[MWh/year]} \qquad \textbf{(19)}$$

$$IC_{j,a} = c(r_j)\lambda_j ENS_j = \lambda_j r_j c(r_j) P_{\mathrm{interr},j} \qquad \text{[NOK/year]} \qquad \textbf{(20)}$$

where c_j is the specific cost of energy not supplied for the delivery point for the equivalent duration r_j. This specific cost can be found based on customer damage functions for different customer groups, see e.g. [1, 16, 17].

Equations ((18) – (20)) give the yearly contribution from the minimal cut *j* to the expected consequences in the delivery point. The total interrupted power, energy not supplied and interruption costs for the delivery point and for the given operating state, are found by the summation of the contributions from each cut as follows:

$$P_{\mathrm{interr},a} = \sum_{j=1}^{J} \lambda_j P_{\mathrm{interr},j} \qquad \text{[MW/year]} \qquad \textbf{(21)}$$

$$ENS_a = \sum_{j=1}^{J} \lambda_j ENS_j = \sum_{j=1}^{J} \lambda_j r_j P_{\mathrm{interr},j} \qquad \text{[MWh/year]} \qquad \textbf{(22)}$$

$$IC_a = \sum_{j=1}^{J} c(r_j)\lambda_j ENS_j = \sum_{j=1}^{J} \lambda_j r_j c(r_j) P_{\mathrm{interr},j} \qquad \text{[NOK/year]} \qquad \textbf{(23)}$$

Usually the reliability indices in Equations (12) – (14) and (15) – (20) are calculated on an annual basis for a given operating state. Often, the reliability assessment is performed for a single operating state, e.g. the heavy load situation only. In such a case, the reliability indices described by these equations are annualized, i.e. they are presented in units per year as if the operating state (e.g. the heavy load situation) lasts for the whole year. One should keep in mind that the heavy load situation is regarded as the worst case, but this situation only lasts for a small portion of the year. It is recommended to perform the contingency and reliability analyses for a set of operating states regarded to be representative for a year, as described in Chapter 2. When more than one operating state is used, the reliability indices for the different states are weighted together by their individual probabilities of occurrence, for instance, in terms of portions of the year, see Chapter 4 and [1].

The contingency enumeration approach combining contingency and reliability analyses can be used to derive annual indices as described above including all the critical contingencies for a set of operating states regarded to be representative for the year. The critical contingencies are defined here as those contingencies potentially leading to interruptions and thus constituting the minimal cut sets. As such, this methodology will be suitable for long term planning and operational planning of the electricity system. The approach can also be used in the assessment of consequences related to a specific contingency in a given operating state, i.e. for online operation as described in [6]. In online operation the operating state is provided by e.g. a state estimator and the SCADA system and regarded as known. Utilization of the contingency enumeration approach in online operation will however require short computation time, provision of reliability indices for a certain operating state, etc. Operational purposes are not further dealt with in this report.

3.4 Dependent faults and time dependencies

In practice, the reliability indices which are calculated based on the contingency enumeration approach as described in the previous section are influenced by a range of different factors [see e.g. 1, 27]: the reliability models and network solution (power flow) techniques used, the failure/interruption criteria (when do interruptions occur?), the load shedding philosophies, the analysis depth (contingency order) and how the corrective (or remedial) actions are represented.

In addition, there are various dependencies to be taken into account. In some parts of the electricity system, there are geographical dependencies, for instance two power lines on the same tower or in the same right-of-way, or cables in the same culvert. Such combinations of components or parts of the system are exposed to common cause faults. There are also functional dependencies related to the protection and control systems. It might happen that the required response from the protection upon a failure is missing and there are also possibilities for unwanted spontaneous tripping of protection or non-selectivity among protection devices in the fault clearance process. The protection system might have a significant influence on the reliability of supply, see e.g. [28, 29].

In addition, human factors may contribute to cascading events, e.g., situational unawareness or inadequate behaviour of operators. Human factors are usually not incorporated in the quantitative risk analysis methodology for power systems, and additional qualitative methods are necessary to analyse the impact. Unfortunate circumstances, such as generators or lines being out due to maintenance can be dealt with in defining the operating states for the analysis.

Furthermore, there are time dependencies to be considered. The load varies by time of year, day of week and time of day. So does the fault rate for some of the main components. For instance, overhead lines are exposed to weather and seasonal effects, while underground cables are exposed to digging, particularly on working days. Time dependencies might have a significant influence on the reliability indices [18, 19, 30].

Different fault types and dependencies mentioned in this section are modelled separately in the reliability analysis using more advanced models and methods. How protection system faults and time dependencies can be included in the analysis in the OPAL methodology is described in Chapter 5. The assumptions, included reliability aspects, representation of failure modes and solution techniques may differ between the available computer tools for contingency and reliability analysis. Thus, the results of two different tools based on the contingency enumeration approach will lead to differences in the results. The user of a tool for reliability assessment of power systems should be aware of these aspects when the results are evaluated.

4 The OPAL methodology for reliability analysis in power systems

This chapter describes the OPAL methodology for reliability analysis, based on [1]. The description is given in different steps in relation to an example network. OPAL is based on the contingency enumeration approach as described in Chapter 2. The contingency analysis results in a list of critical contingencies, i.e. the minimal cut sets, which are considered further in the reliability analysis for the accumulation of reliability indices as described in Chapter 3.

4.1 Overview of the methodology and example network

The reliability methodology consists of seven main steps [1]. The first four steps represent the contingency enumeration approach as shown in Figure 2 and described in Chapter 2. In addition, there are two steps for the inclusion of dependent faults related to the protection system and time-dependencies before the final accumulation of reliability indices:

1. Definition of analysis
2. Generation of contingency lists
3. Consequence analysis
4. Reliability assessment and calculation of reliability indices
5. Inclusion of protection system faults
6. Calculation of time-dependent variation and correlation between parameters
7. Accumulation of reliability indices.

The methodology is described in the following according to the steps above [1] except for the inclusion of protection system faults and time-dependencies in steps 5 and 6 (which is described in Chapter 5).

In the general case, the faults to be considered in the reliability analysis are faults in the primary equipment. The border between primary equipment and secondary equipment is defined to be the interface between the protection system and the circuit breaker (see A.1.1). Inclusion of faults in secondary equipment and circuit breakers is described in Chapter 5.

Example network

The different steps in the methodology are described through the use of an example network. The network is shown in Figure 12, with four bus bars (OPAL-8 – OPAL-11), two generators (G1, G2), two delivery points (L1, L2) and four overhead lines (1 – 4) between the bus bars. All lines have circuit breakers controlled with protection components at both ends (BE1A, BE1B, BE2A, BE2B, BE3A, BE3B, BE4A, BE4B). The overhead lines in the example network are equipped with distance protection. Data for the overhead lines are given in Table 2.

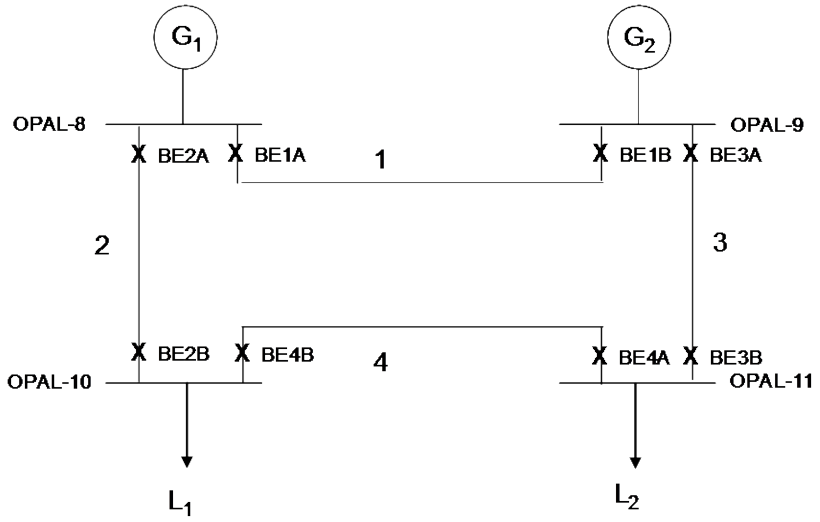


Figure 12 Example network, from [1].

The network is defined to be self-sufficient (no exchange to neighbouring areas), meaning that the operating states are given by load patterns for the delivery points L1 and L2. In a realistic network, exchange to the neighbouring areas has an impact on the definition of operating states, and must be taken into consideration in addition to the generation capacity constraints.

Table 2 Ratings and reliability data for the overhead lines in the example network, based on [1].

Line no	Rating [MW]	Fault rate [faults/year]	Outage time [hours/fault]
1	135	2	20
2	135	3	15
3	135	4	12
4	135	5	10

4.2 Definition of analysis

The first step in the analysis is to define the network, the analysis depth and the operating states. In addition, user-input is necessary regarding optional (manual) definition of additional contingencies as well as choosing whether the analysis should take into account time-dependent correlation or contribution from the protection system. It is required that a database exists with network representation for power flow and reliability analysis (technical data, load data, topology, interruption costs, etc.).

A set of operating states must be defined as basis for the reliability analysis of the defined network. The set of operating states should be a representation of a year in terms of different typical power flow states. It is recommended to perform the analysis at least for the heavy load and the light load situations [1].

Two distinct operating states are defined for the example network ("Heavy load" and "Light load"), and the network topology is assumed to be the same in both states, see Table 3.

Table 3 **Delivery point data, based on [1].**

Delivery point	Heavy load [MW]	Light load [MW]	Specific interruption cost [NOK/kWh]
1	100	60	66
2	75	30	13

The heavy load situation is defined to cover three months (December, January and February), while the light load situation covers nine months (March through November) for both delivery points [1].

Regarding the analysis extent for the example, an analysis depth is chosen including the 1st, 2nd and 3rd orders of independent outages. Only line outages are considered. No additional contingencies are defined. It is chosen to take into account the time-dependent correlation, as well as the protection and control equipment as outlined in Chapter 5. The specific interruption cost will be used to rank the delivery points in case load shedding is necessary.

4.3 Generation of contingency lists

The next step of OPAL is to generate a list of potentially critical contingencies for each operating state for the chosen depth of analysis, containing e.g. single and double independent outages within the area. The procedure follows the contingency enumeration approach as described in Chapter 2, and according to the analysis depth chosen in the previous step. For the example network, the chosen analysis depth is up to 3rd order line outages.

The number of combinations **for each outage order** can be determined as follows:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \tag{24}$$

where

n = total number of lines

k = outage order

For example $\binom{4}{2}$ gives the number of possible combinations in pairs (2nd order outages) of 4 lines.

Since the network topology is the same for both operating states in the OPAL network, the outage lists are the same for both of them. The lines to be considered are: (1, 2, 3, 4).

For the contingency list covering 1st through 3rd order of a network consisting of 4 lines, this gives:

$$\binom{n}{k} = \binom{4}{1} + \binom{4}{2} + \binom{4}{3} = \frac{4!}{1!(4-1)!} + \frac{4!}{2!(4-2)!} + \frac{4!}{3!(4-3)!} = 4 + 6 + 4 = 14 \text{ possible combinations}$$

Table 4 shows the combinations to be analysed, i.e. the contingency list for both operating states for the OPAL network.

Table 4 Contingency list for the OPAL network for outage combinations of lines up to the third order, based on [1].

Contingency	Line(s) out
1	4
2	2,4
3	2,3,4
4	1,2,4
5	3,4
6	1,3,4
7	1,4
8	2
9	2,3
10	1,2,3
11	1,2
12	3
13	1,3
14	1

4.4 Consequence analysis

Consequence analysis at this stage means the identification of system problems by the use of power flow analysis, and the analysis of the consequences when it comes to interruption of loads. By consequences here, we only consider static conditions, referred to as adequacy in the literature [4], see A.1.

Consequently, a system problem is defined as *a power flow solution that does not satisfy the security constraints*[3] *for a given power network*. In the consequence analysis, potential system problems are identified by the use of full AC power flow analysis, analysing the consequences of each contingency primarily concerning the interruption of loads.

System problems can take different shapes and forms. In order to identify them, means are needed to recognize “symptoms” for the system problem definition above [1]. Such “symptoms” can be:
- Voltage violations at PQ buses
- Line/transformer thermal overloading

[3] Security constraints are given by the operational criteria or grid codes such as the Nordic Grid Code 2007 (Nordic Collection of Rules), https://www.entsoe.eu/index.php?id=62, and ENTSO-E Operation Handbook https://www.entsoe.eu/resources/publications/system-operations/operation-handbook/. Such grid codes typically comprise the N-1 criterion and operational limits for overload, voltage conditions and stability problems.

- Real power generation violations at PV-buses or swing-bus
- Reactive power generation violations at PV-buses or swing-bus
- Network separation/islanding
- Bus isolation
- Non-convergence of power flow.

We can differentiate between four possible stationary outcomes after the occurrence of a specific contingency:

- The system is within its limits set by the operational criteria. No corrective action or load shedding necessary (no interruption).
- The system is outside its operating limits. Corrective action is necessary to bring the system back within its limits. Load shedding is not necessary (no interruption).
- The system is outside its operating limits. Corrective action is not sufficient to bring the system back within its limits. Load shedding is necessary to bring the system back within its limits (interruption or reduced supply for some delivery points).
- System breakdown (blackout).

Corrective (remedial) actions to be considered might be, among others, as follows [1]:

- Generation rescheduling (active, reactive)
- Generation bus voltage adjustment
- Network reconfiguration (e.g. disconnection/reconnection of lines)
- Adjustment of capacitors/reactors
- Tap changer adjustment of transformers.

The required results from the consequence analysis of each contingency *j* under the specified operating conditions are:

- Will interruptions occur at some of the delivery points?
- Which delivery points will experience interruptions (or reduced supply)?
- For each affected delivery point: How much of the load in the delivery point can be served for a given contingency under a given operating state?

The results from the consequence analysis are expressed through the determination of $SAC_{j,n}$ which is the system available capacity for delivery point *n* due to contingency *j*.

Applied to the example network, consequence analysis is performed for each contingency in the two operating states, and the results are presented in Table 5. For contingencies resulting in the system being unable to supply both delivery points L1 and L2, load-shedding is performed. The load-shedding criterion here, is chosen as the specific interruption costs, meaning that shedding load at delivery point L2 (specific cost = 13 NOK/kWh) will be performed before shedding load at delivery point L1 (specific cost = 66 NOK/kWh).

For delivery point L1 (Table 5), only multiple contingencies lead to interruption, and total interruption occurs ($SAC_{j,n} = 0$) in all these cases. The same applies to both operating states.

For delivery point L2, both single and multiple contingencies lead to interruption in the heavy load state, while only multiple contingencies lead to interruption in the light load state. It can be observed that the SAC due to some contingencies during heavy load is greater than zero, meaning that it is possible to serve some (i.e. 35 MW) of the load in delivery point L2 during these contingencies.

Table 5 Consequence analysis for delivery points, based on [1].

Contingency	Line outage(s)	L1 Both states	L2 Heavy load	L2 Light load
		SAC [MW]	SAC [MW]	SAC [MW]
1	4	> 100	> 75	> 30
2	2,4	0	> 75	> 30
3	2,3,4	0	0	0
4	1,2,4	0	> 75	> 30
5	3,4	> 100	0	0
6	1,3,4	> 100	0	0
7	1,4	> 100	> 75	> 30
8	2	> 100	35	> 30
9	2,3	0	0	0
10	1,2,3	0	0	0
11	1,2	> 100	35	> 30
12	3	> 100	35	> 30
13	1,3	> 100	35	> 30
14	1	> 100	> 75	> 30

4.5 Reliability assessment

In the reliability assessment, the main task is to calculate the reliability indices for the delivery points. For this purpose, it is necessary to define the minimal cut sets and calculate the contribution to annual reliability indices for each delivery point and operating state.

Based on Table 5 the minimal cut sets for the example network are as shown in Figure 13 and Figure 14, for delivery point L1 and L2 respectively.

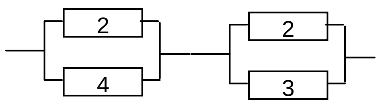


Figure 13 Minimal cuts for delivery point L1 (same minimal cuts for both operating states).

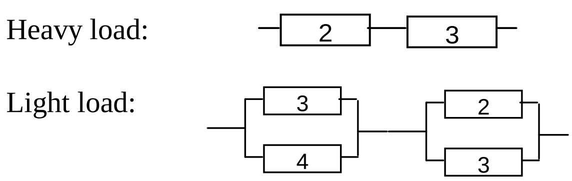


Figure 14 Minimal cuts for delivery point L2 in heavy and light load situations respectively.

The equivalent fault rate and outage times for the minimal cut sets are given in Table 6 and Table 7 together with corresponding specific interruption costs and interrupted power.

Table 6 Minimal cut sets for delivery point L1, from [1].

	Minimal cut	SAC	Equiv. fault rate [no/year]	Equiv. outage time (r) [hours/fault]	$c_{ref}(r_{av})$ [NOK/kWh]	P_{interr} [kW]
Heavy load						
	{2,4}	0	0,043	6	48,7	100
	{2,3}	0	0,037	6,7	47,0	100
Light load						
	{2,4}	0	0,043	6	48,7	60
	{2,3}	0	0,037	6,7	47,0	60

Table 7 Minimal cut sets for delivery point L2, from [1].

	Minimal cut	SAC	Equiv. fault rate [no/year]	Equiv. outage time (r) [hours/fault]	$c_{ref}(r_{av})$ [NOK/kWh]	P_{interr} [kW]
Heavy load						
	{2}	35	3	15	9,1	40
	{3}	35	4	12	9,2	40
Light load						
	{3,4}	0	0,050	5,455	10,2	30
	{2,3}	0	0,037	6,667	9,9	30

Results from all the minimal cuts in each operating state are then aggregated according to Equations (12) – (14) and (15) – (20) to determine the impact on the different delivery points in the system.

Calculations are shown in detail for delivery point L1 for the minimal cut {2,3}, operating state "Heavy load", and results are given for both delivery points in the next section. In the following **only one example is given for each equation.**

Interrupted power in delivery point (DP) L1 caused by cut {2,3} for the operating state (OS) "Heavy load":

$$P_{interr,DP,j,OS} = P_{DP,OS} - SAC_{DP,j,OS} - LG_{DP,OS} \quad \text{[kW pr interruption]}$$

$$P_{interr,L1,\{2,3\},Heavyload} = P_{L1,Heavyload} - SAC_{L1,\{2,3\},Heavyload} - LG_{L1,Heavyload} \quad \text{[kW pr interruption]}$$

$$= 100\,000 - 0 - 0 = 100\,000 \quad \text{[kW pr interruption]}$$

Energy not supplied in delivery point L1 caused by cut {2,3} for the operating state "Heavy load":

$$ENS_{DP,j,OS} = r_j \cdot P_{interr,DP,j,OS} \quad \text{[kWh pr interruption]}$$

$$ENS_{L1,\{2,3\},Heavyload} = r_{\{2,3\}} \cdot P_{interr,L1,\{2,3\},Heavyload} \quad \text{[kWh pr interruption]}$$

$$r_{\{2,3\}} = \frac{r_2 \cdot r_3}{r_2 + r_3} = \frac{15 \cdot 12}{15 + 12} = 6{,}6\overline{6}$$ [hours pr interruption]

And thus:

$$ENS_{L1,\{2,3\},Heavyload} = 6{,}6\overline{6} \cdot 100000 = 666667$$ [kWh pr interruption]

Interruption costs in delivery point L1 caused by cut {2,3} for the operating state "Heavy load":

$$IC_{DP,j,OS} = c_{DP}(r_j) \cdot ENS_{DP,j,OS}$$ [NOK pr interruption]

The specific cost $c_{DP}(r_j)$, as in [1]:

$$c_{L1}(6{,}67) = c_{L1,ref}(6{,}67) \cdot f_{L1,c,ENS} \approx 47 \cdot 1{,}086 = 51{,}0 \text{ NOK/kWh}$$

$$IC_{L1,\{2,3\},Heavyload} = c_{L1}(r_{\{2,3\}}) \cdot ENS_{L1,\{2,3\},Heavyload} = 35{,}7 \cdot 666667 = 34020017$$

[NOK pr interruption]

The number of interruptions in delivery point L1 caused by cut {2,3} for the operating state "Heavy load" (weighted by the probability of occurrence of the operating state):

$$\lambda_{DP,j,OS,a} = \lambda_j \cdot \frac{m_{OS}}{12}$$ [interruptions pr year]

Where *m* = number of months pr year covered by the operating state OS

Thus,

$$\lambda_{L1,\{2,3\},Heavyload,a} = \lambda_{\{2,3\}} \cdot \frac{m_{Heavyload}}{12}$$ [interruptions pr year]

Where $$\lambda_{\{2,3\}} \approx \frac{\lambda_2 \cdot \lambda_3 (r_2 + r_3)}{8760}$$ [interruptions pr year]

This gives:

$$\lambda_{L1,\{2,3\},Heavyload,a} = \lambda_{\{2,3\}} \cdot \frac{m_{Heavyload}}{12} \approx \frac{\lambda_2 \cdot \lambda_3 (r_2 + r_3)}{8760} \cdot \frac{m_{Heavyload}}{12}$$

$$= \frac{3 \cdot 4 \cdot (15 + 12)}{8760} \cdot \frac{3}{12} = 0{,}009$$ [interruptions pr year]

Annual interruption duration in delivery point L1 caused by cut {2,3} for the operating state "Heavy load":

$$U_{DP,j,OS,a} = \lambda_{DP,j,OS,a} \cdot r_j$$ [hours pr year]

$$U_{L1,\{2,3\},Heavyload,a} = \lambda_{L1,\{2,3\},Heavyload,a} \cdot r_{\{2,3\}}$$

$$= 0{,}009247 \cdot 6{,}6\overline{6} = 0{,}062$$ [hours pr year]

Annual interrupted power in delivery point L1 caused by cut {2,3} for the operating state "Heavy load":

$$P_{interr,DP,j,OS,a} = \lambda_{DP,j,OS,a} \cdot P_{interr,DP,j,OS} \qquad \text{[kW pr year]}$$

$$P_{interr,L1,\{2,3\},Heavyload,a} = \lambda_{L1,\{2,3\},Heavyload,a} \cdot P_{interr,L1,\{2,3\},Heavyload} \qquad \text{[kW pr year]}$$

$$= 0{,}009247 \cdot 100000 = 925$$

Annual energy not supplied in delivery point L1 caused by cut {2,3} for the operating state "Heavy load":

$$ENS_{DP,j,OS,a} = \lambda_{DP,j,OS,a} \cdot ENS_{DP,j,OS} \qquad \text{[kWh pr year]}$$

$$ENS_{L1,\{2,3\},Heavyload,a} = \lambda_{L1,\{2,3\},Heavyload,a} \cdot ENS_{L1,\{2,3\},Heavyload} \qquad \text{[kWh pr year]}$$

$$= 0{,}009247 \cdot 666667 = 6165 \qquad \text{[kWh pr year]}$$

Annual customer interruption costs in delivery point L1 caused by cut {2,3} for the operating state "Heavy load":

$$IC_{DP,j,OS,a} = c_{DP}(r_j) \cdot ENS_{DP,j,OS,a} \qquad \text{[NOK pr year]}$$

$$IC_{L1,\{2,3\},Heavyload,a} = c_{L1}(r_{\{2,3\}}) \cdot ENS_{L1,\{2,3\},Heavyload,a} = 51{,}0 \cdot 6164 \qquad \text{[NOK pr year]}$$

$$= 314\ 600 \qquad \text{[NOK pr year]}$$

All the reliability indices are calculated for each delivery point (i.e. accumulated based on minimal cut sets, and weighted for all operating states), and an excerpt is given below for delivery point L1, taken from [1]. Remark: the deviation between the above calculated interruption cost and the results in Table 8 are due to rounding off of values when calculating the average specific cost [1, Appendix F].

Table 8 Reliability indices for L1, contributions from minimal cut sets and operating states, from [1].

Operating state	Light load		Heavy load		$\sum_{j,OS}$
Minimal cut	{2,4}	{2,3}	{2,4}	{2,3}	
[interruptions pr year]	0,032	0,028	0,011	0,009	$\lambda_{L1,a}$ = 0,080
[hours pr year]	0,193	0,185	0,064	0,062	$U_{L1,a}$ = 0,503
[hours pr interruption]	6,0	6,7	6,0	6,7	r_{L1} = 6,3
[kW pr year]	1 926	1 664	1 070	925	$P_{interr,L1,a}$ = 5586
[kWh pr year]	11 558	11 096	6 421	6 165	$ENS_{L1,a}$ = 35240
[NOK pr year]	593 563	550 156	329 757	305 642	$IC_{L1,a}$ = 1 779 118

4.6 Accumulation of reliability indices

In the previous sections, it has been shown how to calculate the contribution to the reliability indices for a particular delivery point from each minimal cut and operating state. In the final step of OPAL and the contingency enumeration approach as described in Chapter 2, the objective is to determine the aggregate reliability indices by accumulation of the contributions from all contingencies leading to interruptions, for all operating states. In addition to the total aggregate results pr delivery point and for the system as a whole, it can be of interest to determine the total contributions in the system from each minimal cut for each operating state. The procedure for the accumulation can be described as follows [1]:

- Calculate indices pr delivery point
  - Sum up values for each delivery point DP (summarized over all minimal cuts and operating states)
  - Average values for each delivery point DP
- Calculate indices pr minimal cut
  - Sum up values for contribution from each minimal cut *j* (summarized over all delivery points and operating states where the minimal cut occurs)
- Calculate indices pr operating state
  - Sum up values for contribution from each operating state OS (summarized over all delivery points and minimal cuts)
- Calculate system indices
  - Sum up values for the system (summarized over all delivery points and operating states)
  - Average values for the system.

The following sections describe the indices that can be calculated.

4.6.1 Indices pr delivery point

Annual or annualized values for each delivery point DP are found by summing up contributions (summarized over all minimal cuts *j* and operating states *OS)*:

Number (frequency) of interruptions in delivery point *DP*:

$$\lambda_{DP,a} = \sum_{j,OS} \lambda_{DP,j,OS,a} \qquad \text{[interruptions pr year]} \qquad \textbf{(25)}$$

Interruption duration in delivery point *DP*:

$$U_{DP,a} = \sum_{j,OS} U_{DP,j,OS,a} \qquad \text{[hours pr year]} \qquad \textbf{(26)}$$

Interrupted power in delivery point *DP*:

$$P_{interr,DP,a} = \sum_{j,OS} P_{interr,DP,j,OS,a} \qquad \text{[kW pr year]} \qquad \textbf{(27)}$$

Energy not supplied in delivery point *DP*:

$$ENS_{DP,a} = \sum_{j,OS} ENS_{DP,j,OS,a} \qquad [\text{kWh pr year}] \tag{28}$$

Interruption costs in delivery point *DP*:

$$IC_{DP,a} = \sum_{j,OS} IC_{DP,j,OS,a} \qquad [\text{NOK pr year}] \tag{29}$$

The annual reliability indices for L1 and L2 in the OPAL network are summarized in Table 9 and Table 10 respectively.

Table 9 Annual reliability indices for L1, from [1].

		Delivery point L1					
		Cut {2,4} light	Cut {2,3} light	Cut {2,4} heavy	Cut {2,3} heavy	sum	Unit
No. of interr. pr year	Lambda	0,032	0,028	0,011	0,009	0,080	No/year
Annual interr. duration	U	0,193	0,185	0,064	0,062	0,503	Hours/year
Average interr. duration	r	6,0	6,7	6,0	6,7	6,3	Hours/interruption
Annual power interrupted	P_{interr}	1,9	1,7	1,1	0,9	5,6	MW/year
Annual ENS	ENS	11,6	11,1	6,4	6,2	35,2	MWh/year
Annual interr. cost	IC	594	550	330	306	1 779	1000 NOK/year

Table 10 Annual reliability indices for L2, from [1].

		Delivery point L2					
		Cut {3,4} light	Cut {2,3} light	Cut {2} heavy	Cut {3} heavy	sum	Unit
No. of interr. pr year	Lambda	0,038	0,028	0,750	1,0	1,8	No/year
Annual interr. duration	U	0,205	0,185	11,3	12,0	23,6	Hours/year
Average interr. duration	r	5,5	6,7	15,0	12,0	13,0	Hours/interruption
Annual power interrupted	P_{interr}	1,1	0,8	30,0	40,0	72,0	MW/year
Annual ENS	ENS	6,2	5,5	450	480	942	MWh/year
Annual interr. cost	IC	107	93	6 971	7 565	14737	1000 NOK/year

Average values pr interruption for each delivery point DP are found as follows:

Average interruption duration in delivery point *DP*:

$$r_{DP} = \frac{U_{DP,a}}{\lambda_{DP,a}} \qquad \text{[hours pr interruption]} \qquad \textbf{(30)}$$

Average interrupted power in delivery point DP:

$$P_{interr,DP} = \frac{P_{interr,DP,a}}{\lambda_{DP,a}} \qquad \text{[kW pr interruption]} \qquad \textbf{(31)}$$

Average energy not supplied in delivery point *DP*:

$$ENS_{DP} = \frac{ENS_{DP,a}}{\lambda_{DP,a}} \qquad \text{[kWh pr interruption]} \qquad \textbf{(32)}$$

Average interruption costs in delivery point *DP*:

$$IC_{DP} = \frac{IC_{DP,a}}{\lambda_{DP,a}} \qquad \text{[NOK pr interruption]} \qquad \textbf{(33)}$$

The average values pr interruption for L1 and L2 are summarized in Table 11.

Table 11 Average values pr interruption for L1 and L2.

	L1	L2	Unit
Average interruption duration	6,3	13,0	Hours/interruption
Average interrupted power	70	40	MW/interruption
Average energy not supplied	440	523,3	MWh/ interruption
Average interruption costs	22237,5	8187,2	1000 NOK/ interruption

4.6.2 Indices pr minimal cut

Sum up values for the contributions from each minimal cut *j* (summarized over all delivery points DP and operating states OS):

Interrupted power in the system due to cut j:

$$P_{interr,S,j,a} = \sum_{DP,OS} P_{interr,DP,j,OS,a} \qquad \text{[kW pr year]} \qquad \textbf{(34)}$$

Energy not supplied in the system due to cut j:

$$ENS_{S,j,a} = \sum_{DP,OS} ENS_{DP,j,OS,a} \qquad \text{[kWh pr year]} \tag{35}$$

Interruption costs in the system due to cut j:

$$IC_{S,j,a} = \sum_{DP,OS} IC_{DP,j,OS,a} \qquad \text{[NOK pr year]} \tag{36}$$

The contributions from each minimal cut are summarized in Table 12.

Table 12 Contributions to the reliability indices pr minimal cut, summarized over all delivery points and operating states for the example network.

	Cut {2}	Cut {3}	Cut {2,3}	Cut {2,4}	Cut {3,4}	Sum	Unit
Annual power interrupted	30,0	40,0	3,4	3,0	1,1	77,5	MW/year
Annual ENS	450	480	22,8	18,0	6,2	977,0	MWh/year
Annual interr. costs	6971	7565	949	924	107	16516	1000 NOK/year

The deviation in the total sum over the year compared to the sum of values in Table 9 and Table 10, are due to rounding off in the calculations.

4.6.3 Indices pr operating state

Sum up values for the contributions from each operating state OS (summarized over all delivery points DP and minimal cuts j):

Interrupted power in the system due to interruptions occurring in operating state OS:

$$P_{interr,S,OS,a} = \sum_{DP,j} P_{interr,DP,j,OS,a} \qquad \text{[kW pr year]} \tag{37}$$

Energy not supplied in the system due to interruptions occurring in operating state *OS*:

$$ENS_{S,OS,a} = \sum_{DP,j} ENS_{DP,j,OS,a} \qquad \text{[kWh pr year]} \tag{38}$$

Interruption costs in the system due to interruptions occurring in operating state *OS*:

$$IC_{S,OS,a} = \sum_{DP,j} IC_{DP,j,OS,a} \qquad \text{[NOK pr year]} \tag{39}$$

The indices pr operating state are summarized for the example network in Table 13.

Table 13 Indices pr operating state for the example network.

	Light load	Heavy load	Sum	Unit
Annual interrupted power	5,5	72	77,5	MW/year
Annual ENS	34,4	942,6	977,0	MWh/year
Annual interr. costs	1344	15172	16516	1000 NOK/year

The deviation in the total sum over the year compared to the sum of values in Table 9 and Table 10, are due to rounding off in the calculations.

4.6.4 Indices for the system as a whole

Various system indices are defined for composite generation and transmission reliability as described in e.g. [10, 23]. Here, we only consider the aggregate system indices based on the load point or delivery point indices as described above.

Annual or annualized aggregate system indices (summarized for all operating states):

Total interrupted power:

$$P_{interr,S,a} = \sum_{OS} P_{interr,S,OS,a} \qquad \text{[kW pr year]} \tag{40}$$

Total energy not supplied:

$$ENS_{S,a} = \sum_{OS} ENS_{S,OS,a} \qquad \text{[kWh pr year]} \tag{41}$$

Total interruption costs:

$$IC_{S,a} = \sum_{OS} IC_{S,OS,a} \qquad \text{[NOK pr year]} \tag{42}$$

The aggregate system indices for the example network are shown in Table 14:

Table 14 Aggregate annual system indices for the example network.

Annual interrupted power	77,5	MW/year
Annual ENS	977,0	MWh/year
Annual interr. costs	16516	1000 NOK/year

Average system indices (annual or annualized), where N = number of delivery points DP:

Average number (frequency) of interruptions pr delivery point:

$$\lambda_{S,DP} = \frac{1}{N}\sum_{N}\lambda_{DP,a} \qquad \text{[interruptions pr DP pr year]} \qquad \textbf{(43)}$$

Average interruption duration pr delivery point:

$$U_{S,DP} = \frac{1}{N}\sum_{N}U_{DP,a} \qquad \text{[hours pr DP pr year]} \qquad \textbf{(44)}$$

Average duration pr interruption pr delivery point:

$$r_{S,DP} = \frac{U_{S,DP}}{\lambda_{S,DP}} \qquad \text{[hours pr interruption]} \qquad \textbf{(45)}$$

Average interrupted power pr delivery point:

$$P_{interr,S,DP} = \frac{1}{N}\sum_{N}P_{interr,DP,a} \qquad \text{[kW pr DP pr year]} \qquad \textbf{(46)}$$

Average energy not supplied pr delivery point:

$$ENS_{S,DP} = \frac{1}{N}\sum_{N}ENS_{DP,a} \qquad \text{[kWh pr DP pr year]} \qquad \textbf{(47)}$$

Average interruption costs pr delivery point:

$$IC_{S,DP} = \frac{1}{N}\sum_{N}IC_{DP,a} \qquad \text{[NOK pr DP pr year]} \qquad \textbf{(48)}$$

The average system indices for the example network are shown in Table 15.

Table 15 Average system indices for the example network, pr delivery point (DP) pr year.

Average number of interruptions	0,94	No. of interruptions pr DP/year
Average annual interruption duration	12,05	Hours pr DP/year
Average duration pr interruption	12,82	Hours pr DP/interruption
Average interrupted power	38,75	MW pr DP/year
Average ENS	488,50	MWh pr DP/year
Average interruption cost	8258	1000 NOK pr DP/year

5 Inclusion of protection system faults and time dependencies

5.1 Protection system faults

The description given in this section is based on [1, Appendix E][4].

A cut set may represent a single-component fault or a multiple event: Independent or dependent overlapping faults, fault-bunching effects, etc. In the following sections, the calculation of fault rates including protection system faults for both independent and dependent components are illustrated by the use of the example network in Figure 12.

Including faults in the protection system, the fault types to be considered are as follows:

Fault type 1: A fault occurs on the transmission line *i*, upon which the line's protection system, or backup protection system, clears the fault.

Fault type 2: The transmission line *i* is fault-free, but because of faulty operation of the line's protection system or circuit breaker, an unwanted spontaneous tripping of the circuit breaker(s) occurs. This results in isolation of the healthy line *i*.

Fault type 3: A fault occurs on the neighbouring transmission line, but because of the faulty operation of the neighbouring line's protection system, its corresponding circuit breaker(s) fails to act. This results in a missing operation of the circuit breaker(s), because of which the faulted neighbouring line cannot be isolated by its own circuit breaker(s). In such a case, the protection system of line *i* acts as backup to isolate the faulted neighbouring line. This **also** results in isolation of the healthy line *i*.

Fault type 4: A fault occurs on the neighbouring transmission line, upon which the neighbouring line's protection system clears the fault correctly. However, because of faulty operation of the protection system of line *i* (which is also designed as backup for a neighbouring line), an unwanted non-selective tripping of the circuit breaker(s) of line *i* occurs. This results in isolation of the healthy line *i*.

Taking the protection system into account, one has to generate lists of contingencies including neighbouring components, i.e. components connected to the same bus bar. The reason for this is that a protection system fault usually leads to outage of neighbouring components (fault type 3 and 4 as defined above).

The analysis for the example network is chosen to cover all the 1st, 2nd and 3rd order outages. Thus, neighbouring components are already included as part of the 2nd order outage combinations.
Power lines 2, 3 and 4 are present in the minimal cut sets while line 1 is not. For all the power lines present in the minimal cut sets an overview of how different fault types contribute to the failures must be established, e.g. as shown in Table 16. The table is established as follows, using power line 2 as an example:

Starting with power line 2, fault type 1 is fault in power line 2 where protection and breakers (V2A, V2B, BE2A, BE2B or backup protection) clear the fault correctly.

[4] This methodology is currently being revised and generalised, and changes may occur as a result of this.

Fault type 2 is unwanted spontaneous tripping of circuit breakers at either end of power line 2 due to protection fault.

Fault type 3 is fault in a power line combined with fault in associated breaker or protection leading to missing tripping of breaker. This must be considered for all neighbouring units of power line 2, i.e. power lines 1 and 4, ref. Figure 12. Neighbouring units are defined as units connected to the same bus bar. Interpretation of the table is that fault in power line 1 and missing operation of BE1A will lead to tripping of backup protection and outage of power line 2. Similarly, fault in power line 4 and missing operation of BE4B will lead to tripping of backup protection and outage of power line 2.

Fault type 4 is fault in a power line combined with fault in protection resulting in unwanted non-selective tripping of breaker. This must be considered for all neighbouring units of power line 2, i.e. power lines 1 and 4, ref. Figure 12. Interpretation of the table is that fault in power line 1 and unwanted operation of BE2B results in outage of power line 2. Similarly fault in power line 4 and unwanted operation of BE2A results in outage of power line 2.

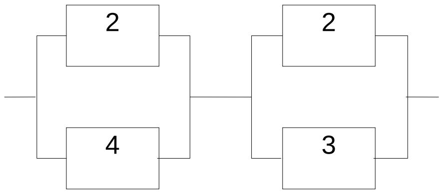
As regards the probability of tripping of backup protection ($p_{missing}$) and probability of lack of selectivity ($p_{unwanted}$), they are only considered for the *closest* neighbouring unit, i.e. all power lines connected to the same bus bar (at the same voltage level). In this example, faults in generators and delivery point are also neglected.

Table 16 Combinations of faults leading to outages of power lines 2, 3 and 4 in the example network.

Power line outage due to	Fault type	Primary fault	Secondary fault	Comment
2	1	Power line 2		Protection and breakers clear the fault correctly
	2	BE2A BE2B V2A V2B		Unwanted spontaneous tripping of circuit breaker
	3	Power line 1 Power line 4	Missing BE1A Missing BE4B	Missing tripping of breaker
	4	Power line 1 Power line 4	Unwanted BE2B Unwanted BE2A	Unwanted non-selective tripping of breaker
3	1	Power line 3		Protection and breakers clear the fault correctly
	2	BE3A BE3B V3A V3B		Unwanted spontaneous tripping of circuit breaker
	3	Power line 1 Power line 4	Missing BE1B Missing BE4A	Missing tripping of breaker
	4	Power line 1 Power line 4	Unwanted BE3B Unwanted BE3A	Unwanted non-selective tripping of breaker
4	1	Power line 4		Protection and breakers clear the fault correctly
	2	BE4A BE4B V4A V4B		Unwanted spontaneous tripping of circuit breaker
	3	Power line 2 Power line 3	Missing BE2B Missing BE3B	Missing tripping of breaker
	4	Power line 2 Power line 3	Unwanted BE4A Unwanted BE4B	Unwanted non-selective tripping of breaker

The following minimal cuts are deduced for delivery points L1 and L2, as described in Chapter 4:

Delivery point L1:

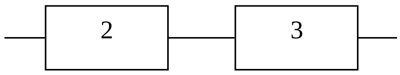


Heavy load and light load:

{2, 4} Power line 2 and power line 4
{2, 3} Power line 2 and power line 3

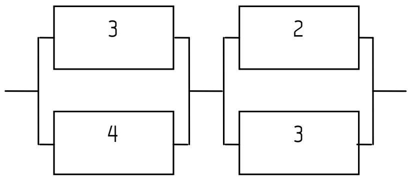
Delivery point L2:

Heavy load:



{2} Power line 2
{3} Power line 3

Light load:



{3, 4} Power line 4 and power line 4
{2, 3} Power line 2 and power line 3

There are three different types of minimal cut sets present in this example:

Second order cut sets – neighbouring units: {2, 4}, {3, 4}
Second order cut sets – not neighbouring units: {2, 3}
First order cut sets: {2}, {3}

Based on Table 16 and the minimal cuts, calculation of fault rates and outage times may be carried out. This is shown in the following.

5.1.1 Independent components

The minimal cut {2, 3}, meaning a simultaneous outage of lines 2 and 3, is used to demonstrate calculation of equivalent fault rates λ_{pp} for the case of independent components.

$$\lambda_{pp} = \frac{\lambda_{\{2\}}\lambda_{\{3\}}(r_{\{2\}} + r_{\{3\}})}{1 + \lambda_{\{2\}}r_{\{2\}} + \lambda_{\{3\}}r_{\{3\}}} \approx \lambda_{\{2\}}\lambda_{\{3\}}(r_{\{2\}} + r_{\{3\}})$$
$$\text{when} \quad \lambda_{\{i\}}r_{\{i\}} \ll 1 \tag{49}$$

$\lambda_{\{2\}}$ and $\lambda_{\{3\}}$ are fault rates for the lines 2 and 3 respectively, while $r_{\{2\}}$ and $r_{\{3\}}$ are the respective outage times. With the units [no. of faults/year] for fault rate and [hours pr fault] for the outage time this gives:

$$\lambda_{pp} \approx \lambda_{\{2\}}\lambda_{\{3\}}(r_{\{2\}} + r_{\{3\}})/8760 \quad \text{faults/year} \tag{50}$$

Fault rates for overhead lines 2 and 3 including contributions from breakers and protection:

$$\lambda_{\{2\}} = \lambda_{f1,2} + \lambda_{f2,2} + \lambda_{f3,2} + \lambda_{f4,2}$$
$$\lambda_{\{3\}} = \lambda_{f1,3} + \lambda_{f2,3} + \lambda_{f3,3} + \lambda_{f4,3} \qquad \textbf{(51)}$$

The fault rate subscript $f_{1,2}$ refers to fault type 1, overhead line 2, $f_{2,2}$ refers to fault type 2, line 2 and so on (as defined above). The contributions from fault types 1 – 4[5]:

$$\begin{aligned}
\lambda_{f1,2} &= \lambda_2 \\
\lambda_{f2,2} &= \lambda_{\text{unwanted BE2A}} + \lambda_{\text{unwanted BE2B}} \\
\lambda_{f3,2} &= \lambda_1 \cdot p_{\text{missing BE1A}} + \lambda_4 \cdot p_{\text{missing BE4B}} \\
\lambda_{f4,2} &= \lambda_1 \cdot p_{\text{unwanted BE2B}} + \lambda_4 \cdot p_{\text{unwanted BE2A}}
\end{aligned} \qquad \textbf{(52)}$$

$$\begin{aligned}
\lambda_{f1,3} &= \lambda_3 \\
\lambda_{f2,3} &= \lambda_{\text{unwanted BE3A}} + \lambda_{\text{unwanted BE3B}} \\
\lambda_{f3,3} &= \lambda_1 \cdot p_{\text{missing BE1B}} + \lambda_4 \cdot p_{\text{missing BE4A}} \\
\lambda_{f4,3} &= \lambda_1 \cdot p_{\text{unwanted BE1B}} + \lambda_4 \cdot p_{\text{unwanted BE3A}}
\end{aligned} \qquad \textbf{(53)}$$

The subscript *unwanted BE2A* refers to unwanted operation of breaker BE2A (or its protection equipment), while subscript *missing BE1A* refers to missing operation of breaker BE1A (or its protection equipment), see Figure 12.

A model of the cut set {2, 3} with contribution from all the four fault types is shown in Figure 15:

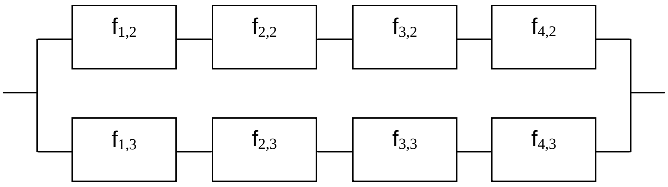


Figure 15 Cut set {2, 3} represented by the fault types 1 – 4.

A state diagram for the minimal cut {2, 3} is shown in Figure 16.

[5] Subscript "missing" means "missing operation of protection equipment after a fault on primary equipment" and subscript "unwanted" means "unwanted operation of protection equipment, i.e. spontaneous or non-selective tripping of circuit breaker(s)".

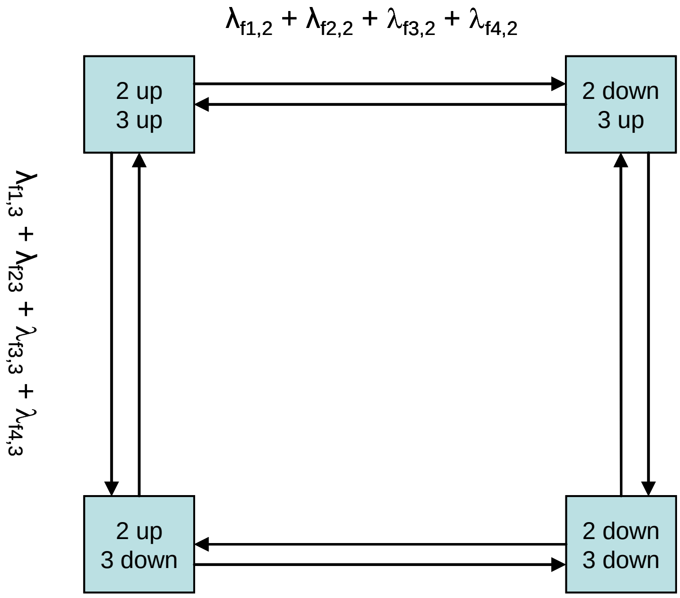


Figure 16 State diagram for minimal cut {2, 3} (independent components).

5.1.2 Dependent components

The minimal cut {2, 4} is used to demonstrate the calculation of equivalent fault rates in case of neighbouring components.

$$\lambda_{pp} = \frac{\lambda_{\{2\}}\lambda_{\{4\}}(r_{\{2\}} + r_{\{4\}})}{1 + \lambda_{\{2\}}r_{\{2\}} + \lambda_{\{4\}}r_{\{4\}}} + \lambda_A \approx \lambda_{\{2\}}\lambda_{\{4\}}(r_{\{2\}} + r_{\{4\}}) + \lambda_A \quad \text{when} \quad \lambda_{\{i\}}r_{\{i\}} << 1 \tag{54}$$

$\lambda_{\{2\}}$, $\lambda_{\{4\}}$ are fault rates for overhead lines 2 and 4, respectively, while $r_{\{2\}}$ and $r_{\{4\}}$ are the respective outage times. λ_A represents dependent faults, i.e. outage of overhead line 2 due to fault in overhead line 4 together with fault in protection or breaker, or outage of overhead line 4 due to fault in overhead line 2 together with fault in protection or breaker. With the units [no. of faults/year] for fault rate and [hours pr fault] for the outage time this gives:

$$\lambda_{pp} \approx \lambda_{\{2\}}\lambda_{\{4\}}(r_{\{2\}} + r_{\{4\}})/8760 + \lambda_A \quad \text{faults/year} \tag{55}$$

Dependent faults may be described by the sum of the terms where fault in overhead line 4 causes outage of overhead line 2, and vice versa:

$$\lambda_A = \lambda_4 \cdot (p_{\text{missing BE4B}} + p_{\text{unwanted BE2A}}) + \lambda_2 \cdot (p_{\text{missing BE2B}} + p_{\text{unwanted BE4A}}) \tag{56}$$

The term $\lambda_4 \cdot (p_{missing\ BE4B} + p_{unwanted\ BE2A})$ represents faults in overhead line 4 where missing operation of breaker BE4B or unwanted non-selective operation of breaker BE2A leads to simultaneous outage of overhead line 2 due to correct operation of back-up protection.

The term $\lambda_2 \cdot (p_{missing\ BE2B} + p_{unwanted\ BE4A})$ represents faults in overhead line 2 where missing operation of breaker BE2B or unwanted non-selective operation of breaker BE4A leads to simultaneous outage of overhead line 4 due to correct operation of back-up protection.

Remaining independent parts of fault rates for overhead lines 2 and 4, including contributions from breakers and protection:

$$\begin{aligned} \lambda_{\{2\}} &= \lambda^*_{f1,2} + \lambda_{f2,2} + \lambda^*_{f3,2} + \lambda^*_{f4,2} \\ \lambda_{\{4\}} &= \lambda^*_{f1,4} + \lambda_{f2,4} + \lambda^*_{f3,4} + \lambda^*_{f4,4} \end{aligned} \tag{57}$$

The fault rates for fault types 1, 3 and 4 are renamed with an asterisk and modified to contain only the independent parts. These fault rates are calculated as follows:

$$\begin{aligned} \lambda^*_{f1,2} &= \lambda_2 \cdot (1 - p_{missing\ BE2B} - p_{unwanted\ BE4A}) \\ \lambda_{f2,2} &= \lambda_{unwanted\ BE2A} + \lambda_{unwanted\ BE2B} \\ \lambda^*_{f3,2} &= \lambda_1 \cdot p_{missing\ BE1A} \\ \lambda^*_{f4,2} &= \lambda_1 \cdot p_{unwanted\ BE2B} \\ \\ \lambda^*_{f1,4} &= \lambda_4 \cdot (1 - p_{missing\ BE4B} - p_{unwanted\ BE2A}) \\ \lambda_{f2,4} &= \lambda_{unwanted\ BE4A} + \lambda_{unwanted\ BE4B} \\ \lambda^*_{f3,4} &= \lambda_3 \cdot p_{missing\ BE3B} \\ \lambda^*_{f4,4} &= \lambda_3 \cdot p_{unwanted\ BE4B} \end{aligned} \tag{58}$$

A model of the cut set {2, 4} with contribution from all the four fault types is shown in Figure 17.

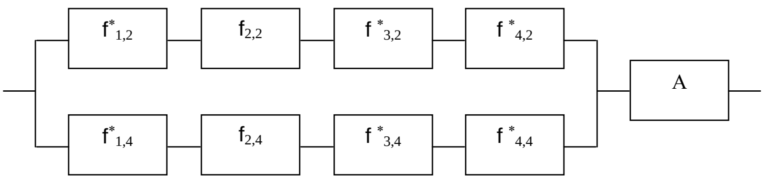


Figure 17 Cut set {2, 4} with fault types 1 – 4 (neighbouring components).

The dependent faults are represented by the cut *A* (or serial element), meaning that each dependent fault will lead to interruption for the delivery point (L1). A simplified state diagram for the minimal cut {2, 4} is shown in Figure 18.

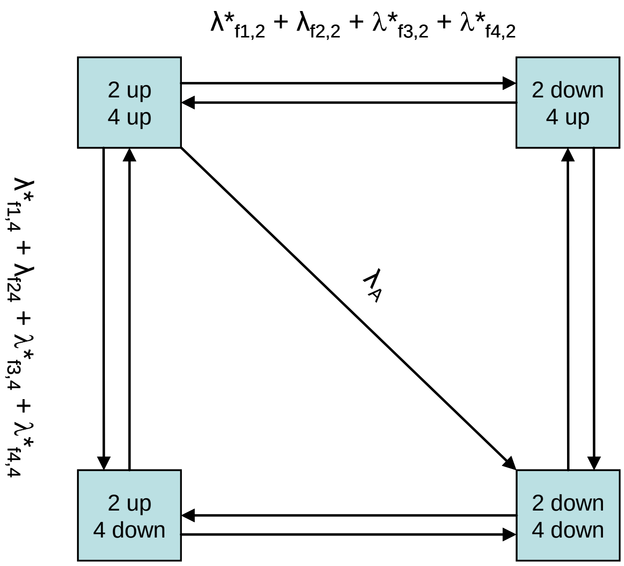


Figure 18 State diagram for minimal cut {2, 4} (neighbouring components).

5.1.3 Applied to the example network

Calculations are shown in detail for delivery point L1 for the minimal cut {2,3}, operating state "Heavy load", and results are given for both delivery points in [1]. In the following, **only one example is given for each equation.**

Table 17 Input data, fault rates and outage times.

Component	Fault rate λ [pr year]	Outage time r [hours]
Power line 1	2	20
Power line 2	3	15
Power line 3	4	12
Power line 4	5	10
Circuit breaker and protection (BE) *)	0,025	2

*) Fault rate for unwanted spontaneous tripping of circuit breaker (fault type 2, see below)

Table 18 Input data, conditional probabilities.

Component	$p_{missing}$*	$p_{unwanted}$**	Restoration time*** [hours]
Protection and breaker	0,0205	0,007	0,5

*) Conditional probability of missing tripping of circuit breaker (fault type 3, see below)
**) Conditional probability of unwanted tripping of circuit breaker (fault type 4, see below)
***) Time to restore supply after missing or unwanted tripping of breaker

All breaker and protection units are assumed to be identical.

The contribution from protection and control to delivery point L1 for the minimal cut {2,3} can now be taken into account by calculating the "new" fault rate and interruption duration for the minimal cut {2,3}:

$$\lambda_{\{2,3\}} = 0{,}040 \text{ interruptions pr year}$$
$$r_{\{2,3\}} = 6{,}3 \quad \text{hours pr interruption}$$

Thus, the following indices are obtained:

Interrupted power in delivery point (DP) L1 caused by cut {2,3} for operating state (OS) "Heavy load":

$$P_{\text{interr,DP,j,OS}} = P_{\text{DP,OS}} - SAC_{\text{DP,j,OS}} - LG_{\text{DP,OS}} \qquad \text{[kW pr interruption]}$$
$$P_{\text{interr,L1,\{2,3\},Heavyload}} = P_{\text{L1,Heavyload}} - SAC_{\text{L1,\{2,3\},Heavyload}} - LG_{\text{L1,Heavyload}} \qquad \text{[kW pr interruption]}$$
$$= \mathit{100\,000 - 0 - 0 = 100\,000} \qquad \text{[kW pr interruption]}$$

Energy not supplied in delivery point L1 caused by cut {2,3} for operating state "Heavy load":

$$ENS_{\text{DP,j,OS}} = r_j \cdot P_{\text{interr,DP,j,OS}} \qquad \text{[kWh pr interruption]}$$
$$ENS_{\text{L1,\{2,3\},Heavyload}} = r_{\{2,3\}} \cdot P_{\text{interr,L1,\{2,3\},Heavyload}} \qquad \text{[kWh pr interruption]}$$
$$ENS_{\text{L1,\{2,3\},Heavyload}} = 6{,}3 \cdot 100000 = 630000 \qquad \text{[kWh pr interruption]}$$

Interruption costs in delivery point L1 caused by cut {2,3} for operating state "Heavy load":

$$IC_{\text{DP,j,OS}} = c_{\text{DP}}(r_j) \cdot ENS_{\text{DP,j,OS}} \qquad \text{[NOK pr interruption]}$$

The specific interruption cost is taken from [1, app. F]:

$$c_{\text{L1}}(5{,}8) = c_{\text{L1,ref}}(5{,}8) \cdot f_{\text{L1,c,ENS}} \approx 49{,}3 \cdot 1{,}086 = 53{,}5 \text{ NOK/kWh, thus}$$
$$IC_{\text{L1,\{2,3\},Heavyload}} = c_{\text{L1}}(r_{\{2,3\}}) \cdot ENS_{\text{L1,\{2,3\},Heavyload}}$$
$$= 53{,}5 \cdot 630000 = 33\,642\,000 \qquad \text{[NOK pr interruption]}$$

Number of interruptions in delivery point L1 caused by cut {2,3} for the operating state "Heavy load" (weighted with its probability of occurrence):

$$\lambda_{\text{DP,j,OS,a}} = \lambda_j \cdot \frac{m_{\text{OS}}}{12} \qquad \text{[interruptions pr year]}$$
$$\lambda_{\text{L1,\{2,3\},Heavyload,a}} = \lambda_{\{2,3\}} \cdot \frac{m_{\text{Heavyload}}}{12}$$
$$= 0{,}040 \cdot \frac{3}{12} = 0{,}01 \qquad \text{[interruptions pr year]}$$

Annual interruption duration in delivery point L1 caused by cut {2,3} for the operating state “Heavy load”:

$$U_{DP,j,OS,a} = \lambda_{DP,j,OS,a} \cdot r_j$$ [hours pr year]

$$U_{L1,\{2,3\},Heavyload,a} = \lambda_{L1,\{2,3\},Heavyload,a} \cdot r_{\{2,3\}}$$
$$= 0{,}01 \cdot 6{,}3 = 0{,}063$$ [hours pr year]

Annual interrupted power in delivery point L1 caused by cut {2,3} for the operating state “Heavy load”:

$$P_{interr,DP,j,OS,a} = \lambda_{DP,j,OS,a} \cdot P_{interr,DP,j,OS}$$ [kW pr year]

$$P_{interr,L1,\{2,3\},Heavyload,a} = \lambda_{L1,\{2,3\},Heavyload,a} \cdot P_{interr,L1,\{2,3\},Heavyload}$$
$$= 0{,}01 \cdot 100000 = 1000$$ [kW pr year]

Annual energy not supplied in delivery point L1 caused by cut {2,3} for the operating state “Heavy load”:

$$ENS_{DP,j,OS,a} = \lambda_{DP,j,OS,a} \cdot ENS_{DP,j,OS}$$ [kWh pr year]

$$ENS_{L1,\{2,3\},Heavyload,a} = \lambda_{L1,\{2,3\},Heavyload,a} \cdot ENS_{L1,\{2,3\},Heavyload}$$ [kWh pr year]

$$= 0{,}01 \cdot 630000 = 6300$$ [kWh pr year]

Annual customer interruption costs in delivery point L1 caused by cut {2,3} for the operating state “Heavy load”:

$$IC_{DP,j,OS,a} = c_{DP}(r_j) \cdot ENS_{DP,j,OS,a}$$ [NOK pr year]

$$IC_{L1,\{2,3\},Heavyload,a} = c_{L1}(r_{\{2,3\}}) \cdot ENS_{L1,\{2,3\},Heavyload,a} = 53{,}5 \cdot 6300$$ [NOK pr year]

$$= 337\ 050$$ [NOK pr year]

All reliability indices are calculated, and an excerpt is given in Table 19 below for delivery point L1.

Table 19 Reliability indices for L1 including protection system faults, from [1].

Operating state	Light load		Heavy load		$\sum_{j,OS}$
Minimal cut	{2,4}	{2,3}	{2,4}	{2,3}	
[interruptions pr year]	0,196	0,030	0,065	$\lambda_{L1,\{2,3\},Heavyload,a}$ = 0,01	0,302
[hours pr year]	0,266	0,186	0,089	$U_{L1,\{2,3\},Heavyload,a}$ = 0,062	0,603
[hours pr interruption]	1,4	6,3	1,4	$r_{\{2,3\}}$ = 6,3	2,0
[kW pr year]	11 790	1 788	6 550	$P_{interr,L1,\{2,3\},Heavyload,a}$ = 993	21 121
[kWh pr year]	15 948	11 190	8 860	$ENS_{L1,\{2,3\},Heavyload,a}$ = 6217	42 215
[NOK pr year]	818 999	554 814	454 999	$IC_{L1,\{2,3\},Heavyload,a}$ = 308 230	2 137 043

Deviation between the table and the above calcuations in the total result is due to the approximate calculation of contribution from fault types.
Table 20 shows the contribution from different fault types to the reliability indices for L1.

Table 20 Contribution from fault types to the reliability indices for L1 including protection system faults, from [1].

For delivery point L1	Fault type				$\sum_{\text{Fault types } 1-4}$
	1	2	3	4	
[interruptions pr year]	0,080	0,001	0,165	0,056	$\lambda_{L1,a}$ = 0,302
[hours pr year]	0,503	0,002	0,073	0,025	0,603
[hours pr interruption]	6,3	1,7	0,442	0,442	2,0
[kW pr year]	5 586	87	11 516	3 932	21 120
[kWh pr year]	35 240	149	5 090	1 740	42 218
[NOK pr year]	1 779 118	7 495	261 283	89 301	2 137 196

The results of the reliability assessment including the protection and control system are given in Table 21 for L1 and in Table 22 for L2, and a comparison is given with the results in Table 9 and Table 10. The tables show that protection and control contribute significantly to the reliability indices for L1 since these are dominated by higher order outages. The contribution to the reliability of supply for L2 is less than 20 %. The main reason for this is that interruptions to L2 are dominated by single order outages.

Table 21 Reliability indices for L1 including faults in protection and control, from [1].

		Delivery point L1					% deviation from results without considering faults in protection and control
		Cut {2,4} light	Cut {2,3} light	Cut {2,4} heavy	Cut {2,3} heavy	sum	
No. of interr. pr year	Lambda	0,196	0,030	0,065	0,010	0,302	278,1
Annual interr. duration [hours pr year]	U	0,266	0,186	0,089	0,062	0,603	19,8
Average interr. duration [hours pr interruption]	r	1,4	6,3	1,4	6,3	2,0	-68,3
Annual power interrupted [MW pr year]	P_{interr}	11,8	1,8	6,5	1,0	21,1	278,1
Annual ENS [MWh pr year]	ENS	15,9	11,2	8,9	6,2	42,2	19,8
Annual interr. cost [1000 NOK pr year]	IC	819	555	455	308	2 137	20,1

Table 22 Reliability indices for L2 including faults in protection and control, from [1].

		Delivery point L2				sum	% deviation from results without considering faults in protection and control
		Cut {3,4} light	Cut {2,3} light	Cut {2} heavy	Cut {3} heavy		
No. of interr. pr year	Lambda	0,222	0,030	0,811	1,1	2,1	17,0
Annual interr. duration [hours pr year]	U	0,288	0,186	11,3	12,0	23,8	0,8
Average interr. duration [hours pr interruption]	r	1,3	6,3	13,9	11,4	11,2	-13,8
Annual power interrupted [MW pr year]	P_{interr}	6,7	0,9	32,4	42,4	82,4	14,5
Annual ENS [MWh pr year]	ENS	8,6	5,6	452	482	948	0,7
Annual interr. cost [1000 NOK pr year]	IC	150	94	7 002	7 596	14 842	0,7

5.2 Time-dependent variation and correlation

5.2.1 Time-varying parameters

The description given in this section is mainly based on [19], also included in [1, Appendix C].

It is well known that the load has cyclic variations on a daily, weekly and yearly basis. Similarly, the available capacity (SAC_j) to supply the load after the occurrence of outage event *j* will depend on the total loading of the network and the actual topology (i.e. the operating conditions), when the contingency occurs. Also, the fault rate and outage time vary with the time of fault occurrence.

The chronology may be handled by time-sequential simulation methods such as those described in e.g. [20]. Ideally, the 8760 hourly loads during a year should be taken into account. This would, however, be a very time consuming approach, and requires a large amount of data.

The approach described in the following is based on patterns found from the fault statistics. The observed time-dependent fault rate pattern in this approach is the aggregated result of all causes contributing to faults of a particular component, representing weather related, technical or human related faults [18, 19]. A model which is able to generate this pattern, analytically or by simulation, will therefore produce outage events according to what is expected to happen in the real system.

Available statistics describing component fault rates can be grouped into three typical time periods: 24 hours, 7 days and 24 months. Let the conditional probability of having faults in a certain month *m* be:

$$q_{\lambda m} = \frac{\lambda_m}{\lambda_{av}} \quad \textbf{(59)}$$

where
λ_m = the number of faults occurring in month m
λ_{av}= the average annual fault rate for a particular component

Assume analogous profiles for variations over the week, $q_{\lambda d} = \lambda_d/\lambda_{av}$ and for the daily variation $q_{\lambda h} = \lambda_h/\lambda_{av}$. The probability of fault in a certain hour *h*, on a certain day *d* and in a certain month *m* is determined from the following equation, assuming independence between the three factors:

$$q_{\lambda(h,d,m)} = q_{\lambda_h} q_{\lambda_d} q_{\lambda_m} = \frac{\lambda_h}{\lambda_{av}} \frac{\lambda_d}{\lambda_{av}} \frac{\lambda_m}{\lambda_{av}} \qquad \textbf{(60)}$$

This equation can now be used to predict and time-tag the component faults by month, weekday and hour, which is the key to picking up right load, interruption duration and interruption costs and to calculate reliability indices for the delivery points.

The time-dependent fault rate for a component (or cut set) can be written as:

$$\lambda_{h,d,m} = \frac{\lambda_h}{\lambda_{av}} \frac{\lambda_d}{\lambda_{av}} \frac{\lambda_m}{\lambda_{av}} \lambda_{av} \qquad \textbf{(61)}$$

where
λ_{av} = the annual average number of faults for the component

The relative expected number of faults (conditional probability) occurring in hour *h*, independent of weekday and month:

$$q_h = \frac{\lambda_h}{\lambda_{av}}, \text{ under the restriction } \sum_{h=1}^{24} \frac{\lambda_h}{\lambda_{av}} = 1.0 \qquad \textbf{(62)}$$

The relative expected number of faults occurring in day *d*, independent of month:

$$q_d = \frac{\lambda_d}{\lambda_{av}}, \text{ under the restriction } \sum_{d=1}^{7} \frac{\lambda_d}{\lambda_{av}} = 1.0 \qquad \textbf{(63)}$$

The relative expected number of faults (conditional probability) occurring in month *m*:

$$q_m = \frac{\lambda_m}{\lambda_{av}}, \text{ under the restriction } \sum_{m=1}^{12} \frac{\lambda_m}{\lambda_{av}} = 1.0 \qquad \textbf{(64)}$$

Equation (64) gives the number of faults occurring in a certain time period (*h*,*d*,*m*) of the year, in the long run.

Time-dependent variations of outage time *r*, load *P*, and specific interruption cost *c* can be described by similar profiles and factors. The annual average load (expectation value) can also be found using the relative load factors as follows:

$$E(P_{h,d,m}) = E(\frac{P_h}{P_{av}}) \cdot E(\frac{P_d}{P_{av}}) \cdot E(\frac{P_m}{P_{av}}) \cdot P_{av} = 1 \cdot 1 \cdot 1 \cdot P_{av} = P_{av} \quad \textbf{(65)}$$

where
P_{av} = annual average load = annual electricity consumption/8760 [kWh/h]

Equation (65) for the average load assumes independent load profiles on a daily, weekly and monthly basis. This may be a questionable assumption. The daily load profile may be different on working days versus weekends/holidays, giving two different daily load curves and relative profiles. Such a representation is described in [18, 19].

Time-dependent specific interruption cost $c_{h,d,m}$ can also be represented by relative profiles, usually referred to the specific cost at reference time, c_{ref}, or a customer damage function based on surveys on customer interruption costs:

$$c_{h,d,m} = \frac{c_h}{c_{ref}} \cdot \frac{c_d}{c_{ref}} \cdot \frac{c_m}{c_{ref}} \cdot c_{ref} \quad \textbf{(66)}$$

Information about customer interruption costs is usually collected and estimated on the basis of customer surveys. The surveys provide information about the costs at a reference time. The reference time is usually in the heavy (maximum) load situation, typically on a weekday in January during working hours, but different for the different customer categories. In addition, the surveys often give information about the deviation in the cost from the reference time if the interruption occurs at another time of day, on another weekday or in another month. The specific interruption cost may be represented as discrete values in, say, NOK/kWh energy not supplied, or as continuous cost functions in, say, NOK/kW power interrupted. These are normalized costs, representing average specific costs for different customer categories.

The relative factors for the absolute interruption cost are assumed independent similar to the factors for the load. However, to be able to take the correlation between cost and load into account, relative factors for the specific cost can also be established as follows:

$$c_{h,d,m} = f_{c,h} \cdot f_{c,d} \cdot f_{c,m} \cdot \frac{P_{ref} \cdot c_{ref}}{P_{av}} \quad \textbf{(67)}$$

where
$f_{c,h}$ = relative daily variation in specific interruption cost, independent of weekday and month
$= (C_h/C_{ref})/(P_h/P_{av})$
$f_{c,d}$ = relative weekly variation in specific interruption cost, independent of month
$= (C_d/C_{ref})/(P_d/P_{av})$
$f_{c,m}$ = relative yearly variation in specific interruption cost
$= (C_m/C_{ref})/(P_m/P_{av})$

Thus, the relative factors for the specific cost handle the correlation between the cost and load. As can be seen from Equation (67) the relative factors should be further multiplied with the factor P_{ref}/P_{av}, giving the specific cost in time period (*h*,*d*,*m*) as a function of c_{ref}.

5.2.2 Calculation of reliability indices including time-dependent correlation

The inclusion of time-dependent correlation can be made by the following procedure to be executed for each minimal cut and each operating state.

1. Determine the time of occurrence (Loop through the months (m=1,..,12), weekdays (d=1,..,7) and hours (h=1,..24).

2. Determine the number of faults in hour *h*, on weekday *d* and in month *m*:

   Fault rate for each minimal cut *j* in hour *h*, weekday *d* and month *m*

$$\lambda_{j,h,d,m} = q_{j,h} \cdot q_{j,d} \cdot q_{j,m} \cdot \lambda_{j,av} \qquad \textbf{(68)}$$

3. Determine the expected load $P_{h,d,m}$:

   Load in each delivery point DP in hour *h*, weekday *d* and month *m*

$$P_{DP,h,d,m} = \frac{P_{DP,h}}{P_{DP,av}} \cdot \frac{P_{DP,d}}{P_{DP,av}} \cdot \frac{P_{DP,m}}{P_{DP,av}} \cdot P_{DP,av} \qquad \text{[kW]} \qquad \textbf{(69)}$$

4. Does an interruption occur: $P_{h,d,m}$ > SAC + LG ?

   If so, determine the power interrupted: $\Delta P_{h,d,m} = P_{h,d,m}$ - SAC - LG

5. Determine the expected duration $r_{j\ h,d,m}$ of the interruption:

   Outage time for each minimal cut *j* in hour *h*, weekday *d* and month *m*

$$r_{j,h,d,m} = \frac{r_{j,h}}{r_{j,av}} \frac{r_{j,d}}{r_{j,av}} \frac{r_{j,m}}{r_{j,av}} r_{j,av} \qquad \text{[hours pr outage]} \qquad \textbf{(70)}$$

6. Determine the expected interruption cost $c_{\ h,d,m}(r_{j\ h,d,m})$:

   Specific interruption costs in each delivery point DP in hour *h*, weekday *d* and month *m*

$$c_{DP,h,d,m}(r_{j,h,d,m}) = f_{c,DP,h} \cdot f_{c,DP,d} \cdot f_{c,DP,m} \cdot \frac{P_{DP,ref}}{P_{DP,av}} \cdot c_{DP,ref}(r_{j,h,d,m}) \qquad \text{[NOK pr kWh]} \qquad \textbf{(71)}$$

7. Calculate the contribution to the reliability indices from the time period (h,d,m).

$$
\begin{aligned}
&(\Delta\lambda)_j = \lambda_{jh,d,m} \\
&(\Delta U)_j = \lambda_{jh,d,m} r_{jh,d,m} \\
&\Delta P_{interr,j} = \lambda_{jh,d,m} \Delta P_{h,d,m} \\
&\Delta ENS_j = \lambda_{jh,d,m} r_{jh,d,m} \Delta P_{h,d,m} \\
&\Delta IC_j = \Delta ENS_j c_{h,d,m}(r_{jh,d,m})
\end{aligned} \qquad \textbf{(72)}
$$

8. Calculate the contributions to the annual indices for the delivery point from the minimal cut *j*, by the summation of contributions from each time period, 24*7*12 = 2016 in all (only IC_j is shown):

$$IC_j = \sum_{h=1}^{24}\sum_{d=1}^{7}\sum_{m=1}^{12} \Delta IC_{jh,d,m} \qquad \textbf{(73)}$$

9. Sum up the contributions to the delivery point from all minimal cuts (only IC is shown):

$$IC = \sum_{j=1}^{J} IC_j \qquad \textbf{(74)}$$

where *J* is the total number of minimal cuts affecting the delivery point.

The results of the reliability assessment including time dependent correlation are given for L1 in Table 23 and for L2 in Table 24. The results show that in comparison to the indices presented in Chapter 4 (Table 9 and Table 10) without considering protection and control and time-dependencies, L1 is hardly influenced by the time dependencies, while the indices for L2 are significantly higher when time dependencies are included.

Table 23 Reliability indices for L1 incorporating time-dependent correlation, from [1].

		Delivery point L1					
		Cut {2,4} light	**Cut {2,3} light**	**Cut {2,4} heavy**	**Cut {2,3} heavy**	**sum**	**% deviation from results without considering time dependencies**
No. of interr. pr year	Lambda	0,024	0,020	0,019	0,017	0,080	0,0
Annual interr. duration [hours pr year]	U	0,132	0,127	0,091	0,088	0,439	-12,8
Average interr. duration [hours pr interruption]	r	5,6	6,2	4,8	5,3	5,5	-12,8
Annual power interrupted [MW pr year]	P_{interr}	1,5	1,3	2,0	1,7	6,4	15,0
Annual ENS [MWh pr year]	ENS	8,1	7,8	9,4	9,0	34,3	-2,7
Annual interr. cost [1000 NOK pr year]	IC	461	429	495	459	1 844	3,6

% deviation from reliability indices without considering time dependencies, Table 9

Table 24 Reliability indices for L2 incorporating time-dependent correlation, from [1].

		Delivery point L2				sum	% deviation from results without considering time dependencies
		Cut {3,4} light	Cut {2,3} light	Cut {2} heavy	Cut {3} heavy		
No. of interr. pr year	Lambda	0,028	0,020	1,3	1,8	3,2	75,6
Annual interr. duration [hours pr year]	U	0,141	0,127	16,0	17,1	33,4	41,2
Average interr. duration [hours pr interruption]	r	5,1	6,2	11,9	9,5	10,5	-19,6
Annual power interrupted [MW pr year]	P_{interr}	0,858	0,632	57,1	76,2	135	87,3
Annual ENS [MWh pr year]	ENS	4,3	3,9	669	714	1 391	47,7
Annual interr. cost [1000 NOK pr year]	IC	80	70	9 776	10 644	20 570	39,6

% deviation from reliability indices without considering time dependencies, Table 10

5.3 Reliability indices including protection system faults and time dependencies

Finally, the representation of time-dependent variation described above makes it possible to calculate the annual expectation values of the reliability indices taking into account the chronology and time-dependent correlation between the variables. This can be performed both analytically and by a Monte Carlo simulation [21]. In the latter approach, the stochastic variations in the input variables can also be taken into account.

If protection system faults are considered, this is handled separately including various fault types in the minimal cut sets, both for independent and dependent components. The detailed calculations of reliability indices including protection system faults and/or time dependent correlation are described in the requirement specification [1].

Reliability indices including protection system faults and time dependencies are summarized in Table 25 and
Table 26 for the two delivery points in the example network.

Table 25 shows that when calculating the reliability indices for delivery point L1, protection system faults are important to be included in the analysis. This is because there is no single contingency forming a cut for L1 (see Table 6), and hence the dependent double contingencies originating from protection system faults play an important role. However, including time dependencies has a relatively small influence on the results for L1.

Table 25 Reliability indices for delivery point L1.

Index	Without protection faults and time dependencies	With protection faults	With time dependencies
λ (no./year)	0,080	0,302	0,080
U (duration/year)	0,503	0,603	0,439
ENS (MWh/year)	35,2	42,2	34,3
IC (1000 NOK/year)	1779	2137	1844

Table 26 Reliability Indices for Delivery Point L2.

Index	Without protection faults and time dependencies	With protection faults	With time dependencies
λ (no./year)	1,8	2,1	3,2
U (duration/year)	23,6	23,8	33,4
ENS (MWh/year)	942	948	1391
IC (1000 NOK/year)	14737	14842	20570

On the contrary, including time dependencies is important for the results of delivery point L2, while the inclusion of protection equipment faults changes the indices to a lesser degree, see
Table 26. The reason for this is that two single contingencies are minimal cuts for L2 in the heavy load situation (see Table 7), and these cuts dominate the indices due to their relatively large fault rates.

6 Comparison of OPAL and other analytical approaches

As mentioned, the reliability indices which are calculated based on the contingency enumeration approach, are in practice influenced by a range of different factors: the reliability models and network solution (power flow) techniques used, the failure criteria (when do interruptions occur?), the load shedding philosophies, the analysis depth (contingency order), and how the corrective (or remedial) actions are represented, see e.g. [1, 27]. This is illustrated later in this chapter in a comparison of different tools for the analytical reliability assessment.

First in this chapter, it is described how the reliability of supply can be accurately calculated using the state space method on the OPAL example network. Then, a comparison is made with the approximate evaluation technique based on minimal cut sets.

6.1 Approximate evaluation using minimal cut sets versus the state space method

The minimal cut set method is in itself an approximate reliability evaluation method compared with the accurate analytical state space method (see A.2). The assumptions and analysis depth etc., also lead to approximations influencing the results.

The state space method is briefly outlined in A.2. The accurate reliability calculation is based on the steady-state probabilities in the Markov model and the derived frequency and duration techniques. Using the state space method, the whole state space is investigated in calculating the exact probabilities and frequencies of interruptions, while the minimal cut set method represents an approximate reliability evaluation. The minimal cut set approach only takes the failure modes represented by the minimal cuts into account, while the other system states are unknown and thus not considered.

An example is given in A.3 of using the state space method for the OPAL network. The results are summarized in Table 27. For comparison purposes, Table 28 gives the results using the minimal cut set method described in Chapter 4.

Table 27 Reliability indices for the OPAL network using the state space method.

	Light load		Heavy load		Combined, for the year	
	L1	L2	L1	L2	L1	L2
Probability of interruption	5,67E-05	5,86E-05	5,67E-05	0,0105	5,67E-05	0,00267
Interruption frequency [no./year]	0,0786	0,0859	0,0786	6,935	0,0786	1,798
Annual interruption duration [hours/year] = Probability * 8760	0,4967	0,5135	0,4967	92,01	0,4967	23,39
Mean duration [hours/interruption]	6,32	5,98	6,32	13,27	6,32	13,01
Interrupted power [MW/year]	4,72	2,58	7,86	278,18	5,50	71,48
ENS [MWh/year]	29,80	15,41	49,67	3691	34,77	934,3

Table 28 Reliability indices for the OPAL network using the minimal cut set method.

	Light load		Heavy load		Combined, for the year	
	L1	L2	L1	L2	L1	L2
Interruption frequency (λ) [no./year]	0,0789	0,0863	0,0789	7	0,0789	1,815
Annual interruption duration [hours/year]	0,498	0,515	0,498	93	0,498	23,64
Mean duration [hours/interruption]	6,31	5,97	6,31	13,29	6,31	13,02
Interrupted power [MW/year]	4,74	2,59	7,89	280	5,53	71,94
ENS [MWh/year]	29,88	15,45	49,81	3720	34,87	941,6

The tables show that the cut set method gives slightly higher frequencies, annual interruption duration, interrupted power and energy not supplied. The approximate method tends to overestimate the unreliability. However, the differences are small and negligible for most practical purposes. The approximate method is faster and simpler given that the minimal cut sets are established.

In this OPAL example where the generators are considered 100 % reliable with "infinite" capacity, line no. 1 has no influence on the reliability. Lines 2 and 3 are the most important lines. The single outages dominate the reliability of supply to L2 in the heavy load situation, while line no. 4 is involved in one of the minimal cuts in the light load situation.

Sometimes, it so happens that multiple outages can improve the situation for some delivery points, while for others the situation will worsen. Which delivery points are possible to serve in the different operating states for a specific contingency depends on the topology, available capacity and load shedding procedures (prioritization etc.). This problem is illustrated by the OPAL network, varying from the operating state "light load" to "heavy load" (see A.3). For L1, the failure modes (minimal cuts) and SAC are unchanged for the two operating states since this delivery point is prioritized in case of load shedding. However, for L2 the situation changes as described in the following.

In the light load situation, there is no single outage causing interruption for L2, while single outage of line 2 or line 3 causes a reduced SAC and partial interruption in the heavy load situation. These single outages were therefore identified as minimal cuts for L2 in the operating state "heavy load". While single outage of line 2 gives SAC equal to 35 MW, double outage of lines 2 and 4 gives SAC > 75 MW. The reason for this is that when both lines 2 and 4 are on outage, it is not possible to supply L1, and the whole load in L2 can be supplied instead. This state will be handled in the state space method as a success state, i.e. no interruption, while in the minimal cut set method this double outage will not be regarded as a cut, and as such, gives no contribution to interruptions in the load point.

When lines 2 and 3, or lines 3 and 4 are on outage, SAC for L2 changes from 35 to 0 MW. These two double outages actually constitute minimal cut sets for L2 in addition to the single outages of line 2 and 3. These changes can be considered in the reliability assessment including additional higher order minimal cut sets. The single outages cause partial interruption with SAC equal to 35 MW, and the double outages cause total interruption with SAC equal to 0 MW. Using approximate evaluation techniques including the additional minimal cut sets, we get the results shown in Table 29:

Table 29 Reliability indices for the OPAL network using the minimal cut set method including the additional higher order minimal cut sets. Results highlighted for L2 in the heavy load situation.

	Light load		Heavy load		Combined, for the year	
	L1	L2	L1	L2	L1	L2
Interruption frequency (λ) [no./year]	0,0789	0,0863	0,0789	7,09	0,0789	1,84
Annual interruption duration [hours/year]	0,498	0,515	0,498	93,51	0,498	23,76
Mean duration [hours/interruption]	6,31	5,97	6,31	13,20	6,31	12,94
Interrupted power [MW/year]	4,74	2,59	7,89	286,47	5,53	73,56
Energy not supplied (ENS) [MWh/year]	29,88	15,45	49,81	3780,44	34,87	956,70

The difference compared with Table 28 and the results given in Chapter 4 is less than or about 1 % except for interrupted power which increases by about 2 % including the higher order minimal cut sets.

Taking the two double outages in the minimal cut sets into account slightly increases the difference compared to the state space method in Table 27. Again, we see that the approximate method overestimates the unreliability in terms of frequency of interruptions and interrupted power, as well as energy not supplied. Compared with the state space method, the interrupted power and energy not supplied is 2,9 % and 2,4 % higher respectively. In this calculation we have not considered that the single outages of lines 2 and 3, and the double outages of lines 2 and 3, and 3 and 4, respectively, are not mutually exclusive. The indices in Table 29 can therefore be regarded as an uppr bound for the reliability indices. See 0 for a description of approximations regarding the use of minimal cut sets.

6.2 Comparisons of different reliability analysis tools

The assumptions about how different reliability aspects are included and modelled, the representation of failure modes (e.g. dependencies) and network solution techniques may differ between the available computer tools for contingency and reliability analysis. Thus, the results using two different tools based on the contingency enumeration approach will lead to differences in the results.

A comparison of OPAL is made with three different analytical tools for reliability analysis of power systems. These tools are MECORE developed by the University of Saskatchewan [31, 32], PSS®SINCAL owned by Siemens where the reliability module is based on the former Zuber tool developed by German universities and the Forschungsgemeinschaft für Elektrische Anlagen und Stromwirtschaft (FGH) e. V., and PowerFactory developed by DIgSILENT [46]. A common feature shared by these tools is that they represent an analytical contingency enumeration approach, although they differ in the way this approach is implemented and the types of reliability models used.

In these comparisons, the OPAL prototype tool in Matlab [26] is used, which has been developed according to the requirement specification in [1].

Table 30 gives a comparison of the methodologies, basic assumptions, network solution techniques, corrective measures, load shedding procedure etc., insofar as the references give information about the various aspects.

Table 30 Comparison of methodology used in MECORE, OPAL prototype tool, PSS®SINCAL and PowerFactory.

Topic	MECORE (Univ. of Saskatchewan)	OPAL (SINTEF Energy Research)	Sincal (Siemens)	PowerFactory (DIgSILENT)
Methodology	Analytical contingency enumeration approach	Analytical contingency enumeration approach	Semi-Markov process (Analytical contingency enumeration approach)	State enumeration algorithm and Weibull-Markov modelling
λ and r	For lines and generators – permanent faults	Permanent faults	Permanent faults and temporary for single independent failures	Permanent faults for lines and transformers
Interruption (failure criteria)	Interruption, partial or total (*probably*)	Interruption, partial or total	Interruption, partial or total	Interruption, partial or total
Load shedding	Depends on the interruption cost	Depends on the interruption cost	Identifies/defines different areas where load shedding is needed, performed proportionally. ”Optimistic”: proportionate. ”Pessimistic”: total load shed	Depends on the interruption cost or manually set
Degree of supply if interruption	Probably the same as in OPAL	Load is reduced until the system is within its limits	”Optimistic”: proportionate. Tries to load available components up to 100 % when possible	Load is reduced until the system is within its limits
Over-/undervoltage	Load shedding if over-/undervoltage	Load shedding if over-/undervoltage. These limits may be disregarded	Undervoltage: May choose to disregard. Overvoltage is not considered	Load shedding if over-/undervoltage. These limits may be disregarded
Prioritization of generators	Determined by production cost	Determined by production cost	Disconnected generators can be prioritized by an index high – low	Determined by production cost or prioritization list
Reliability of swing generator			100 % reliable, is not affected by failures. On reduced supply, the swing generator will produce the deviation within its limits	Reliability evaluation of generators seems not to be included
Automatic corrective measures		Optimal power flow[6]	Redispatching of production if swing generator is overloaded	Several possibilities, incl. generator redispatch and optimal power flow.
Other corrective measures			Reconnections of breakers, user defined	
Overload factor		Not yet implemented for lines and transformers	Each component can be allocated an overload factor, can be time limited	Lines can be allocated a time-limited overload factor
Islanding	Possible	Possible	Possible	Possible

[6] The OPAL prototype tool in Matlab as described in [26] is based on using the optimal power flow.

6.3 A small example using a reliability test system

As an example of using the contingency enumeration approach results for the well-known Roy Billinton Test System RBTS [24, 25] are included. The RBTS is shown in Figure 19. The RBTS system which is developed for educational purposes is frequently used to examine new techniques and methods. It is a six-bus power system consisting of 9 transmission overhead lines and 11 generators. The annual peak load is 185 MW and the voltage level is 230 kV. The individual loads are given in Figure 19, while line lengths and reliability data are given in Table 31.

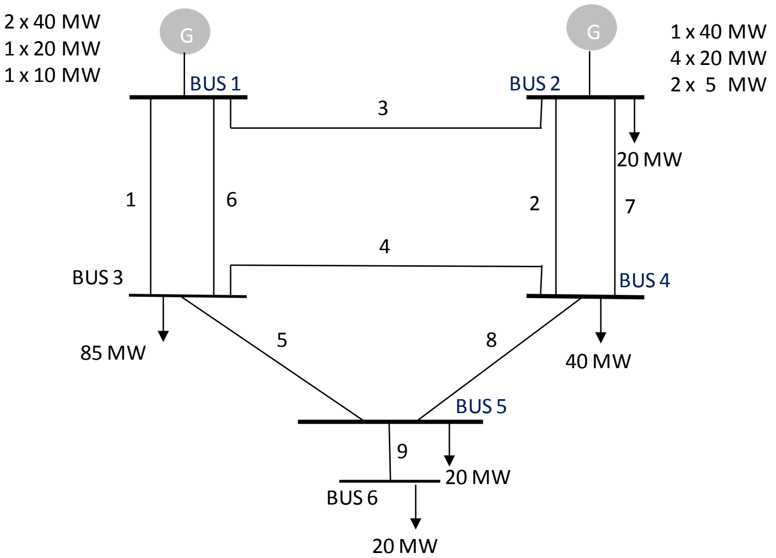


Figure 19 Roy Billinton Test System (RBTS) based on [24].

Table 31 Transmission line length and reliability data, based on [24].

Line	From bus* no	To bus no	Length km	Permanent fault rate (pr year)	Outage time (hrs)
1	1	3	75	1,5	10
2	2	4	250	5,0	10
3	1	2	200	4,0	10
4	3	4	50	1,0	10
5	3	5	50	1,0	10
6	1	3	75	1,5	10
7	2	4	250	5,0	10
8	4	5	50	1,0	10
9	5	6	50	1,0	10

* BUS = Delivery point

The contingency and reliability analyses are performed using the OPAL methodology implemented in a prototype tool [26]. This tool performs reliability analysis for calculation of reliability indices for the delivery points and for the system as a whole using MATPOWER [44] for power flow analysis in the consequence assessment of contingencies. The tool utilizes optimal power flow to decide the amount of disconnected load. The contingencies (or outage combinations) selected for further power flow analyses include line outages up to third order and generator outages up to fourth order as well as third order combinations of line and generator outages. Common cause or other dependencies are not included in the analysis. The results are shown in Table 32 for delivery point (bus) no 3 and 6. The reliability indices for the other delivery points are negligible compared to these two delivery points. Note that the indices are annualized as the analysis is performed for the peak load situation only.

Table 32 Reliability indices (annualized) for delivery point 3 and 6 in the RBTS system.

Reliability index		Delivery point (bus) no 3	Delivery point (bus) no 6
No of interruptions pr year	λ	6,9	1,0
Annual interruption duration (hours/year)	U	123,4	10,0
Average interruption duration (hours/interruption)	r	17,9	10,0
Interrupted power (MW/year)	P_{interr}	50,6	20,1
Energy not supplied (MWh/year)	ENS	1183,6	200,3
Interruption cost (1000 USD/year)	IC	5545,6	1088,4

Delivery point (DP) 3 is expected to experience almost 7 interruptions pr year. This number is influenced by outage of either line 1 or line 6 causing overload on the other line due to low voltage, as well as combinations of outages of lines or generators. The expected number of interruptions for DP 6 is about 1 pr year dominated by the single outage of line no 9. The interruption costs for DP 3 sum up to about 5,5 million USD pr year. This DP serves large industrial users and some commercial services, while DP 6 serves an industrial farm.

The tools MECORE, OPAL, Sincal and PowerFactory are tested by analysing the RBTS test network shown in Figure 19. In this analysis, the assumptions, analysis depth, power flow techniques etc., are selected for the purpose of making comparisons possible. The definition of the analysis is given in Table 33 while the results are given in Table 34 for delivery points (bus) 3 and 6.

Table 33 Definition of analysis of the RBTS test network.

Topic	MECORE (Univ. of Saskatchewan)	OPAL (SINTEF Energy Research)	Sincal (Siemens)	PowerFactory (DIgSILENT)
Outage combinations	Third order line outages, fourth order generator outages and third order combinations	Third order line outages, fourth order generator outages and third order combinations	Third order line and generator outages	Second order line and generator outages
Common mode	No	No	No	With and without
Power flow	DC	DC and AC	AC	DC and AC
Tolerance for overload and voltage	0	0	No tolerance for overload Undervoltage not considered	No tolerance for overload , voltage within +/- 3%
Islanding	Yes	Yes	Yes	Yes
Load shedding	Different interruption cost for all delivery points	Same interruption costs as MECORE	Optimistic	Prioritization
Production cost	Different for all generators	Same costs as in MECORE	Not possible to define	Not considered in the comparison
Load curve	Load constant over the year	Load constant over the year	Load constant over the year	Load constant over the year

Table 34 Results for the RBTS test network, for delivery point (bus) no 3 and 6.

Reliability index	Bus no.	MECORE (DC)	OPAL (DC)	OPAL (AC)	Sincal (AC)	PF (DC and AC[7])	PF (AC and CM[8])
λ [no/yr]	3	4,08	3,69	6,89	3,86	3,33	3,35
	6	1,37	1,003	1,004	5,14	1	1,02
U [h/yr]	3		84,6	123,35	36,52	90,61	90,72
	6		10,01	10,03	48,69	10,01	10,06
r [h/interr]	3		22,93	17,9	9,47	27,25	27,08
	6		9,99	9,99	9,47	9,97	9,86
P_{interr} [MW/yr]	3	48,16	36,81	50,58	49,87		
	6	24,01	20,05	20,05	37,58		
ENS [MWh/yr]	3	849,64	827,14	1183,56	468,73	909,39	915,41
	6	216,11	200,24	200,27	354,49	200,19	201,19

A comparison of the results in Table 34 shows that the expected number of interruptions (λ) differs between the tools, and so do the other indices. MECORE gives the highest numerical values comparing the results using DC power flow. However, the table shows the importance of using full AC power flow. In OPAL, this yields 87 % higher number of interruptions, 46 % higher annual interruption time, 38 % higher interrupted power and 43 % higher energy not supplied than using DC (in OPAL). Sincal gives slightly higher number

[7] The impedances are not fully represented in this analysis, thus the voltage problems will not appear and DC and AC analysis give the same results
[8] CM = Common Mode faults are included

of interruptions than OPAL (DC) for delivery point no. 3 but five times higher for delivery point no. 6. In the RBTS network there apparently are voltage problems which cannot be discovered using DC power flow. However, the differences in results between OPAL (AC) and Sincal are quite large despite the common basis of AC power flow. Another observation from the table is that PowerFactory and OPAL give fairly close results (up to 10 % deviation). The difference between MECORE and OPAL is a bit higher for the number of interruptions, about 30 % for the interrupted power, while for the energy not supplied the difference is only about 3 %.

Even if the basic premises for the analysis are chosen as equal as possible, the comparison shows that different tools give different results. The main reasons for this, among others, are related to the differences in load shedding procedures, handling of overload and voltage problems, and other corrective measures.

7 Case studies

This chapter presents two case studies in which the OPAL methodology is used. The first case study considers the reliability of supply for two delivery points in the Norwegian transmission grid. In the second study the reliability of supply is analysed in a simple four area test system.

7.1 Reliability of supply for delivery points in the transmission grid

In this case study, as described in [6, 7], the reliability of supply was analysed for two different delivery points in the 420 kV Norwegian transmission grid, both supplying major industrial sites. One point is situated close to the centre of the transmission grid, and the other at the end of a line with single sided supply.

The studied delivery points L1 (centrally located) and L2 (with single sided supply) are situated in the middle of Norway, represented by area “10” in the power market model shown in Figure 20. This figure only shows the areas and the connections between areas included in the market model, which gives the operating states used as input to the contingency and reliability analyses. For this purpose, detailed information about the network and components is needed. This is restricted information and the physical lines within the different areas are not shown.

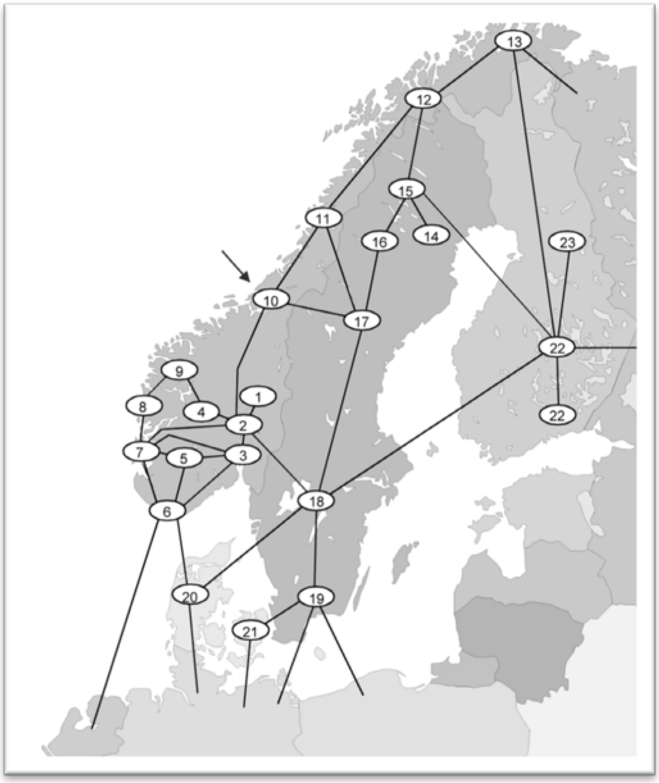


Figure 20 Power market model and division of areas.

The Norwegian electricity system is dominated by hydro power generation (about 95 %). Total annual electricity consumption in Norway is about 125 TWh, the maximum electricity generation about 140 TWh and the maximum load approximately 24 000 MW.

Since the electricity generation and loads vary during the year, the contingency analysis should ideally be carried out for a set of operating states representative for a year. For simplicity, the system in the middle of Norway has been analysed using three different operating states from the market model to represent the year; these were the weeks 4, 16 and 30. Week 4 represents a heavy load situation, week 16 also represents a heavy load situation (but less than that of week 4) and hydro reservoirs running out of water, while week 30 represents light load. Power flow and market models for the year 2010 have been used as a basis with constant loads of 650 MW in L1 and 220 MW in L2.

The first analysis step was to initialize the three operating states through interaction between the market model and the power flow model in a security constrained power market analysis. In the estimation of transmission capacities to the neighbouring areas, the system constraints related to voltages and loading of lines must be taken into account. In this case, the voltage level turned out to be the limiting factor. The market model was updated with the transmission capacities in the different operating states.

A total of 330 single and 46 double contingencies were analysed. For each of the 376 contingencies, the system consequences were found. This consisted of deciding whether or not the contingency would lead to interruption of electricity supply for the studied delivery points. Voltages and loads were checked, and it was revealed whether or not the system was within its defined operating limits. The consequence analysis led to lists of contingencies causing interruptions for delivery points L1 and L2 in the different operating states represented by weeks 4, 16 and 30. The interruptions were due to network separation, overload or voltage deviations. No blackout-situations (i.e. interruption for the entire system) were revealed for the chosen operating states.

The last step was to calculate the reliability of supply indices. A number of 4 – 6 minimal cuts were identified both for L1 and L2, depending on the operating state. Only first and second order cuts (i.e. single and double outages) were taken into account. Inputs to the reliability analysis included information about the protection configuration, fault statistics and specific interruption costs. The main reliability input data are listed in Table 35 og Table 36. For each delivery point and operating state, interruption frequency and duration was calculated by summing up the contributions from the different minimal cut sets. The indices were then weighted according to the probability of the different operating states to obtain the annual indices. The results are listed in Table 37.

Table 35 Reliability analysis input data, component failures (source Statnett).

Component	Voltage [kV]	Failure rate [no./year]	MTTF* [years]	Average outage time** [hours]
Overhead line (1 km)	420	0,0104	96,2	2,2
Bus bar		0,0118	84,7	0,25
Power transformer		0,0645	15,5	360
Circuit breaker		0,0035	285,7	0,25
Distance protection		0,0543	18,4	0,25
Overhead line (1 km)	300	0,0088	113,6	5,9
Bus bar		0,0146	68,5	0,25
Power transformer		0,0283	35,3	430
Circuit breaker		0,0035	256,4	0,25
Distance protection		0,0549	18,2	0,25
Overhead line (1 km)	132	0,0118	84,7	6.9
Bus bar		0,0044	227,3	0,25
Power transformer		0,0106	94,3	245
Circuit breaker		0,0019	526,3	0,25
Distance protection		0,0231	43,3	0,25

*) "Mean Time To Failure"
**) Expected reconnection time is used for bus bars, circuit breaker and protection

Table 36 Reliability analysis input data, dependent protection failures (source Statnett).

Component	Voltage [kV]	Missing operation p(missing)*	Unwanted operation p(unwanted)**
Distance protection	420/300 kV	0,0009	0,01412
Circuit breaker	420/300 kV		-

*) Probability of missing operation of circuit breaker
**) Probability of unwanted operation of circuit breaker

Table 37 Reliability of supply indices for the 420 kV delivery points.

	λ [No. of interruptions pr year]	U [Annual interruption duration (hrs/yr)]	ENS [Energy not supplied (MWh/yr)]	IC [Cost of energy not supplied (€/yr)], (approx.)
L1 (650 MW)	0,03	0,007	4,78	8 000
L2 (220 MW)	1,36	2,37	521	4,30 million

This example shows that the expected number of interruptions is much higher for delivery point L2 compared with those of L1. This is as expected since L2 has single sided supply and the single outages are decisive for the reliability of supply indices. The expected mean time between interruptions is more than 30 years for L1, while L2 will experience an interruption more than once pr year on average, resulting in an expected interruption cost of about 4.3 million euro pr year. For L1, only double outages contribute to the unreliability. These are dependent outages, mainly arising from the dependencies related to the protection system of missing and unwanted operation of the breakers.

The reliability indices neglecting any dependencies related to the protection system are shown in Table 38. A comparison between the tables shows that protection system dependencies only have a relatively small influence on L2, while the reliability of supply would have been practically 100 % for L1 if protection had not been taken into consideration since only second or higher order independent outages contribute to the unreliability.

Table 38 Reliability of supply indices neglecting protection system dependencies.

	λ [No. of interruptions pr year]	U [Annual interruption duration (hrs/yr)]	ENS [Energy not supplied (MWh/yr)]	IC [Cost of energy not supplied (€/yr)], (approx.)
L1 (650 MW)	0	0	0	0
L2 (220 MW)	1,34	2,36	520	4,29 million

7.2 Reliability of supply in four area test system

This section describes the reliability of supply in a simple test system with 25 bus bars and four areas. The description is taken from [36]. This test system as depicted in Figure 21 is established with corresponding market and network models.

The areas "1", "2" and "3" are connected by 130 kV AC lines. There are two lines between areas "2" and "3". Between areas "3" and "4", there is a HVDC line. Internal lines are 66 kV AC. The corresponding power market model has 50 hydrological series (years) with a resolution of 52 weeks and four price sections pr week. The main input data for the power market model is listed in Table 39.

Rated power is specified for all generation units. Demand is represented as both "firm power" and "net purchase/ sale", with a distribution based on fixed ratios. Actual production and demand for the different units and bus bars varies with the different operating states, and is a result from the power market simulations. "Typical" electrical parameters are used for lines and power transformers, while reliability data are based on the Norwegian fault statistics.

Table 39 Input data for power market model, from [36].

		Area 1	Area 2	Area 3	Area 4
Hydro	Capacity [MW]	610,25	533,29	819,6	
Thermal	Capacity [MW]	5,12			230
Wind	Capacity [MW]				119,05
Pump	Capacity [MW]			19,9	
Load	Demand [MW]	342	250	380	480

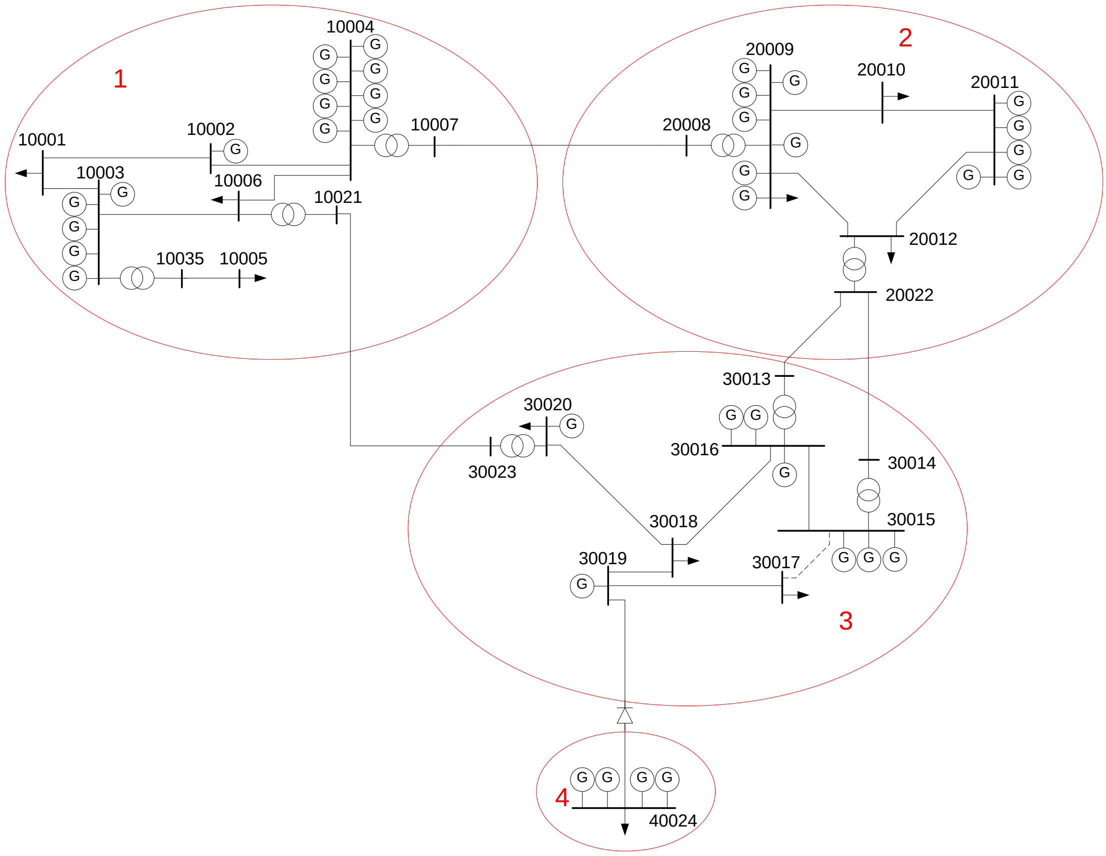


Figure 21 Four area test system, from [36].

In this example, the test system was analysed using the OPAL methodology as described earlier. The security constrained power market analysis was carried out using EMPS with network constraints [12], resulting in a total of 10400 different operating states (50 hydrological series * 52 weeks * 4 price sections pr week).

The contingency analyses were limited to single outages of power lines and transformers. The consequence analyses were carried out as AC power flow simulations in PSS^{TM}E. To get a more realistic picture of how the contingencies in the network are handled, rule based subsequent corrective actions reducing demand and generation were modelled. In this case only overload problems were identified, and all single outages leading to overload for each operating state were further analysed to decide the consequence. No problems related to islanding or separations from the network were identified (except for area 4); this is as expected since only first order outages were analysed.

Figure 22 presents an overview of energy not supplied (ENS) for each of the studied delivery points and for each of the hydrological series (years). It is obvious that there is a large variation both in the total ENS for the system and for each of the delivery points. The minimum total ENS is 712 MWh/year, while the maximum total ENS is 8549 MWh/year. ENS in the year closest to the average is 2766 MWh/year.

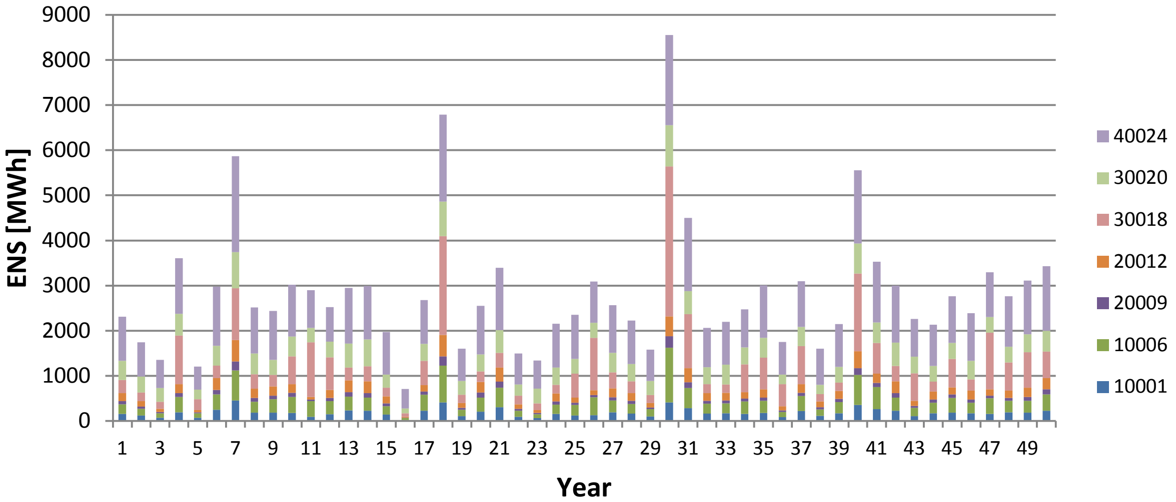


Figure 22 Energy not supplied pr year and delivery point, from [36].

Further details for the different delivery points are presented in the following. Table 40 shows the results for the year with the lowest total ENS for the system (year 16), Table 41 shows the results for the year with the highest total ENS for the system (year 30), and Table 42 shows results from the year with ENS closest to the average for all 50 years (year 45).

Table 40 Average reliability of supply indices for the test network, in the year with minimum total ENS, from [36].

Delivery point	λ [no/ year]	U [h/year]	r [hours/ interr.]	P_{interr} [MW]	ENS [MWh/ year]	IC [kNOK/ year]	ENS [%]	Demand [MWh/ year]
10001	0,0091	0,41	45,0	0,57	24,53	671,2	1,84E-03	1331434
10006	0,0111	0,65	58,0	0,82	43,05	1163,3	2,59E-03	1664322
20009	0,0074	0,25	33,4	0,20	7,90	240,7	1,31E-03	604438
20012	0,0021	0,12	54,1	0,40	17,65	533,8	1,12E-03	1571366
30018	0,0096	1,00	103,1	0,95	78,75	2170,0	4,45E-03	1768510
30020	0,0274	2,07	75,8	1,79	109,31	3051,0	6,97E-03	1568301
40024	0,1922	13,94	72,6	5,81	431,01	8806,2	10,1E-03	4279609
Sum					**712,20**	**16636,3**		**12787981**

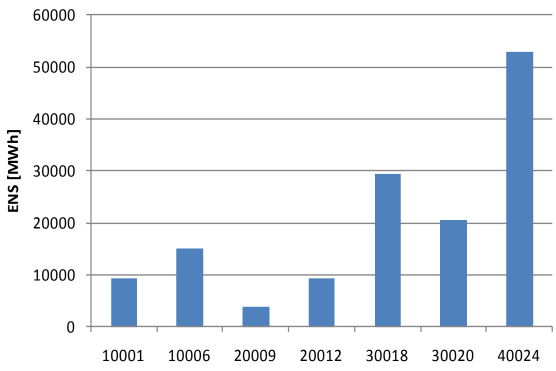
Table 41 Average reliability of supply indices for the test network, in the year with maximum total ENS, from [36].

Delivery point	λ [no./year]	U [h/year]	r [hours/ interr.]	P_{interr} [MW]	ENS [MWh/year]	IC [kNOK/year]	ENS [%]	Demand [MWh/ year]
10001	0,0407	3,64	89,5	4,56	416,39	10950,4	3,13E-02	1331633
10006	0,1572	12,23	77,8	13,98	1207,13	31807,1	7,25E-02	1664571
20009	0,0370	3,33	90,0	2,74	252,60	7349,4	4,17E-02	605890
20012	0,0201	1,90	94,8	4,70	438,74	12760,4	2,79E-02	1575139
30018	0,2475	18,62	75,2	44,62	3326,24	91887,8	18,79E-02	1769890
30020	0,1122	9,40	83,8	10,56	911,73	25046,7	5,81E-02	1569525
40024	0,2090	15,58	74,5	21,54	1996,26	40454,9	4,66E-02	4279609
Sum					**8549,09**	**220256,8**		**12796258**

Table 42 Average reliability of supply indices for the test network, in the year with total ENS closest to the average for the 50 years, from [36].

Delivery point	λ [no./ year]	U [h/year]	r [hours/ interr.]	P_{interr} [MW]	ENS [MWh/ year]	IC [kNOK/year]	ENS [%]	Demand [MWh/ year]
10001	0,0204	1,70	83,5	2,09	189,14	4990,2	1,42E-02	1330361
10006	0,0352	2,93	83,1	3,51	319,74	8433,6	1,92E-02	1662981
20009	0,0141	1,30	92,3	0,82	80,16	2342,6	1,33E-02	604990
20012	0,0093	1,02	109,2	1,57	159,68	4660,2	1,02E-02	1572800
30018	0,0825	6,55	79,5	7,75	624,51	17219,8	3,53E-02	1768924
30020	0,0525	4,81	91,6	3,93	357,54	9815,8	2,28E-02	1568668
40024	0,1935	14,24	73,6	11,03	1035,54	20976,5	2,42E-02	4279609
Sum					**2766,31**	**68438,7**		**12788334**

By inspecting the results from the analysis, it can be seen that two of the delivery points, 30018 and 40024, on average have worse reliability of supply in terms of number of interruptions, annual interruption duration and percentage of energy not supplied. This is also supported by Figure 23 showing to the left the total energy not supplied pr delivery point for the 50 year period. For delivery point 40024, all indices are high on average. This might be expected since it is an import area with single sided connection to the rest of the system. For delivery point 30018, the indices vary a lot from year to year, with several years of high values influencing the average, as can be seen to the right in Figure 23.



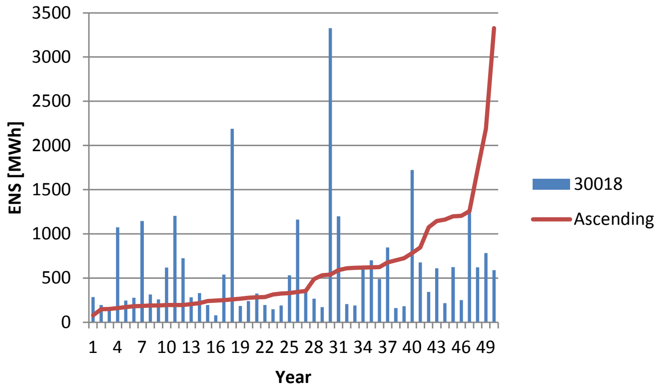


Figure 23 Total energy not supplied pr delivery point for the 50 year period (left) and energy not supplied for delivery point 30018 pr year and in ascending order (right), from [36].

Since delivery point 40024 is a "specialty" with its HVDC connection, it was decided to look at possible measures to improve the reliability for delivery point 30018. The total IC for 30018 over 50 years is about 800 MNOK, an average of 16 MNOK pr year, indicating a certain investment potential. The investment analysis is described in [36].

8 Conclusions and further work

This report has described the OPAL methodology for reliability analysis of power systems. Power system reliability assessment is a key part of security of supply analysis.

In reliability analysis the main objective is to determine the reliability of supply indices for the delivery points and system under study, such as frequency and duration of interruptions, energy not supplied and interruption costs. OPAL is designed for these purposes, considering interruptions due to faults on power system components as well as protection system faults leading to missing or unwanted breaker operation, and time dependencies.

The OPAL methodology for reliability analysis is based on an analytical contingency enumeration approach. OPAL is in its current version primarily applicable for long-term planning purposes comprising the power generation and transmission system. Possible areas of application are value-based reliability planning, investment analysis, evaluation of reliability design criteria and as input to risk and vulnerability analysis on the identification of critical contingencies potentially leading to wide-area interruptions.

The OPAL methodology is previously documented in detail in a requirement specification for software tool development and it is implemented in a prototype tool in Matlab. This report describes the models and methodology for reliability assessment. Both the methods and the tool are being further developed in the project "Integration of methods and tools for security of electricity supply analysis". In particular, protection system reliability and the influence on reliability of supply are currently under development, and the methodology described in Chapter 5 might be changed and enhanced as a result of this.

There are needs for further development of the methodology to include corrective measures in the consequence analysis, restoration times and procedures, different station configurations and busbar faults, extraordinary events, wind power and distributed generation etc., and to extend the OPAL methodology for applications in operational planning and online operation.

In particular, the project currently deals with the following topics of development:

- Modelling and implementation of corrective actions
- Definition of contingency lists including use of screening techniques
- Testing and comparisons of methods for handling extraordinary (low probability, high impact) events
- Verification, further development and generalisation of implemented line protection models.
- Modelling of transformer protection, station configurations, and protection reliability input data.

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Appendices

A.1 Terms, definitions and list of abbreviations

A.1.1 Terms and definitions

Component
A *component* is a device which performs a major operating function and which is regarded as an entity for purposes of recording and analysing data on outage occurrences. Note 1: Some examples of components are line sections, transformers, AC/DC converters, series capacitors or reactors, shunt capacitors or reactors, circuit breakers, line protection systems, and bus sections [37].

A *component* is a piece of electrical or mechanical equipment, a line or circuit, or a section of a line or circuit, or a group of items that is viewed as an entity for the purposes of reliability evaluation [38].

Contingency (outage event)
A *contingency* is an unplanned *outage* of one or more primary equipment components, i.e. one or more primary components are in the outage state [1], [37].

Curtailment
Curtailment is planned reduction of demand other than through market prices. Curtailment can be realized in several ways. A distinction can be made between physical curtailment by rotating disconnection or quota allocation [2].

Delivery point
A *Delivery point* is a point in the network where electrical energy is exchanged [1, 15].

Energy Not Supplied (ENS)
Energy Not Supplied (*ENS*) is the estimated amount of energy that would have been supplied to the end-user if the supply *fault*[9] did not occur [1].

Failure, fault
A *failure* is the termination of the ability of an item to perform a required function. After failure, the item has a *fault* [39].

Failure is an event, as distinguished from *fault*, which is a state.

Fault is the state of an item characterized by inability to perform a required function. A fault is often the result of a failure of the item itself, but may exist without prior failure [39].

The term fault will mainly be used throughout this report, meaning either failure or fault.

[9] ENS is the consequence of contingencies, i.e., unplanned outages, which are due to failure events. After failure the item has a fault.

Fault rate
The *fault rate* is the number of faults of a continuously required function (of a component), pr unit of time exposed to such faults = number of faults of a particular type pr unit exposure time. The fault rate is usually expressed in faults pr year[10]. (Fault rate = Failure rate in the ref.) [37, 38].

Interruption
An *interruption* is a condition characterized by missing or reduced supply of electric energy to one or more end users [1, 15]. A supply interruption is a condition in which the voltage at the supply terminals is lower than 5 % of the reference voltage [40].

Interruption cost
Interruption cost is the cost of interruptions to customers/end-users, usually measured through customer surveys based on methods for contingent valuation or other. The interruption cost typically expresses the customer's willingness to pay to avoid an interruption of certain duration, or the direct cost of a similar interruption [16, 17]. Interruption costs are expressed in units of monetary values for a period of time (NOK pr interruption, pr year). *Specific interruption costs* are normalised interruption costs (NOK/kW interrupted power) for different customer groups (typically residential, industry, commercial, public services, etc.), given as a function of the interruption duration, and sometimes also as a function of the time of interruption. Specific interruption costs are often called customer damage functions [see e.g. 16, 17].

Minimal cut set
A *minimal cut set* is a set of components that, if removed from the system, results in loss of continuity to the delivery point being investigated and does not contain as a subset any set of components that is itself a cut-set of the system. In the present context, the components in a minimal cut set are just those components whose overlapping outage results in an interruption according to the interruption definition adopted [1], [38].

Neighbouring units
Primary equipment components connected to the same busbar [1].

Operating state
An *operating state* is a system state valid for a period of time, characterized by load and generation composition including the electrical topological state (breaker positions etc) and import/export to neighbouring areas [1]. The term operating scenario is sometimes used with a similar meaning.

Outage
An *outage* is the state of a component or system when it is not available to properly perform its intended function due to some event[11] directly associated with that component or system [6], [38].

Outage state
An *outage state* is when the component or unit is not in the in-service state; i.e., it is partially or fully isolated from the system [1], [37].

Outage time
Outage time is the accumulated time in which one or more components or units are in the outage state during the reporting period [37]. In OPAL outage time represents the *repair time* for a single component or the equivalent outage time for a minimal cut set. Sometimes, it may also represent *restoration time*.

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[10] For adequacy studies and long term planning purposes, the fault rate is usually measured as an average over several years
[11] Outages and contingencies are in this report related to *failure* events

Primary equipment[12]
Units and functions for generation, transmission and distribution of electrical energy. Note 1: This includes lines, cables, transformers, generators, capacitors, breakers etc. [42].

Protection equipment
Equipment incorporating one or more protection relays and, if necessary, logic elements intended to perform one or more specified protection functions. Note: Protection equipment is part of a *protection system* [43].

Protection and control equipment
Secondary equipment, i.e.: Units and functions necessary for monitoring, protection and control of *primary equipment* [1].

Protection system
An arrangement of one or more *protection equipments*, and other devices intended to perform one or more specified protection functions. Note 1: A protection system includes one or more protection equipments, instrument transformer(s), wiring, tripping circuit(s), auxiliary supply(s), and, where provided, communication system(s). Depending upon the principle(s) of the protection system, it may include one end or all ends of the protected section and, possibly, automatic reclosing equipment. Note 2: The circuit breaker(s) are excluded [43].

Reliability of a power system
Reliability of a power system is the degree to which the performance of the elements of that system results in power being delivered to consumers within accepted standards and in the amount desired. The degree of reliability may be measured by the frequency, duration and magnitude of adverse effects on the consumer service. Reliability can be divided into *power system security* and *power system adequacy* [4]:

- **Power system security**
  Power system security is the ability of the power system to withstand sudden disturbances such as short circuits or un-anticipated loss of system components. Security refers to the degree of risk in its ability to survive imminent disturbances (contingencies) without the interruption of customer service. It relates to the robustness of the system in a context of imminent disturbances, and depends on the power system operating condition before the disturbance and the contingent probability of disturbances [4].

- **Power system adequacy**
  Power system adequacy is the ability of the system to supply the aggregate electric power and energy requirements of the customers at all times, taking into account scheduled and unscheduled outages of the system components [4].

Repair time
Repair time (often denoted mean time to repair (MTTR)) is the mean time to repair or replace a failed component. The most common unit for repair time is hours. Administrative delay is not included [15]. It differs between the various references depending on whether the time for necessary preparations such as logistics, transport, spare part acquisitions and crew mobilization, is included or not [15], [38].

[12] The interface between primary and secondary equipment is defined between the circuit breaker (tripping device included) and the protection system [1]

Restoration time
Restoration time is the time for restoration of supply after a contingency leading to interruption or reduced supply has occurred. Restoration time may consist of time for repair, coupling, reconnection etc., [1].

Risk
Risk is the effect of uncertainty on objectives. It can also be defined as the combination of probability of an event and its consequence [41].

Secondary equipment
Units and functions necessary for monitoring, protection and control of *primary equipment*. Note: This includes settings, measuring transformers, communication, field units (including protection and control functions), power supply etc. [42].

Security of electricity supply
Security of electricity supply means the ability of an electricity system to supply final customers with electricity [3]. It can be divided into long-term and short-term security of supply. Long-term security of supply can be split into the following aspects: access to primary fuels, generation adequacy, network adequacy and market adequacy [5]. Short-term security of supply means the operational reliability (i.e., *power system security*) of the system as a whole and its assets, including the ability to overcome short-term failures of individual components of the system [5].

System available capacity (SAC)
System available capacity (*SAC*) is the available capacity to supply a certain load after the occurrence of a specific contingency, based on [1].

A.1.2 List of symbols and abbreviations

λ	Fault rate for components, in no. of faults pr year, or frequency of interruptions for delivery points, in no. of interruptions pr year
r	Repair/outage time for components, in hours pr fault, or average interruption duration for delivery points, in hours pr interruption
U	Annual interruption duration for delivery points, in hours pr year
ENS	Energy not supplied, in MWh or kWh
P_{interr}	Interrupted power, in MW or kW
IC	Interruption cost, in NOK
$c_{DP}(r)$	Specific interruption cost as a function of duration r, in NOK/kW or NOK/kWh
APC	Available power capacity, in MW or kW
SAC	System available capacity, in MW or kW
LG	Local generation, in MW or kW
DP	Delivery point
OS	Operating state
P	Load, in MW or kW
m	Functioning time
MTTF	Mean time to failure (= m)
MTTR	Mean time to repair (= r)
MTBF	Mean time between failures (m+r)
P_X	Probability of state 'X'
f_X	Frequency of encountering a state

A.2 Basic reliability evaluation techniques

This appendix gives a brief introduction to the basic reliability evaluation techniques based on the Markov model and frequency and duration techniques, as well as the approximate evaluation techniques derived for series and parallel systems. The description given here is mainly based on [22, 23].

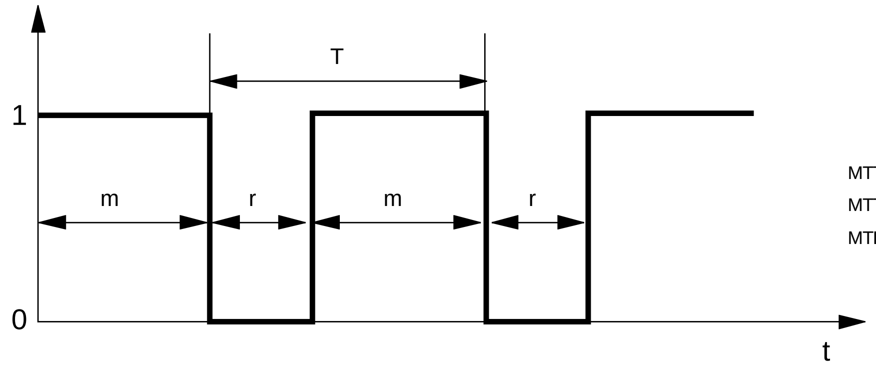
A.2.1 Markov model, state space method and frequency and duration techniques

The Markov model is named after the Russian mathematician Andrey Markov. There are both discrete and continuous Markov (stochastic) processes, where the continuous Markov processes is the most interesting for reliability evaluation of power systems. The continuous process is discrete in space and continuous in time. The main properties are listed below:

- Stationary Markov processes:
  - The conditional probability of failure is constant in a fixed time interval → state residence time is exponentially distributed
- Markov property:
  - The conditional probability distribution of future states of the process depend only upon the present state
  - The process has no memory
- Markov models can be used for systems with constant fault rates and repair rates

The state space method based on the Markov model and frequency and duration techniques provide precise modelling and evaluation of reliability of supply (provided that the assumptions are acceptable). The frequency and duration techniques are based on the steady-state constant probabilities based on a continuous Markov model.

In the basic Markov model a single component can reside in either “up-state” (state 1) or “down-state” (state 0). Figure 24 shows the state cycle of the component and defines the mean time to failure (MTTF) and repair (MTTR), as well as mean time between failures (MTBF).



MTTF = mean time to failure = m = $1/\lambda$

MTTR = mean time to repair = $1/\mu$

MTBF = mean time between failures = T = m + r = 1/f

Figure 24 State cycle of a component that can reside in one of two states.

The probabilities of the component being in state 0 (down) or 1 (up) and the relations between the mean time to failure and repair, as well as mean time between failures is given below:

$$P_0 = \frac{r}{m+r} = \frac{r}{T} = \frac{1}{\mu T} = \frac{f}{\mu}$$

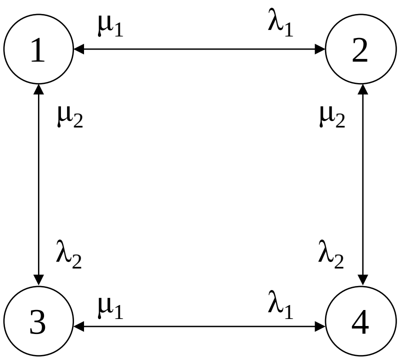
$$P_1 = \frac{m}{m+r} = \frac{m}{T} = \frac{1}{\lambda T} = \frac{f}{\lambda} \qquad \textbf{(75)}$$

$$f = P_0\mu = P_1\lambda$$

From these relations, the frequency of encountering a state can be derived as follows:

Frequency of encountering the up state:
= $P_1\lambda$ = (Probability of being in the state) X (rate of departure from the state) OR
= $P_0\mu$ = (Probability of NOT being in the state) X (rate of entry into the state)

The state probabilities and transition rates of a system composed by two components are shown in Figure 25:



State 1: Both component in up state:

$$P_1 = \frac{\mu_1}{\lambda_1 + \mu_1}\frac{\mu_2}{\lambda_2 + \mu_2}$$

State 2: Comp. 1 down, comp. 2 up:

$$P_2 = \frac{\lambda_1}{\lambda_1 + \mu_1}\frac{\mu_2}{\lambda_2 + \mu_2}$$

State 3: Comp. 1 up, comp. 2 down:

$$P_3 = \frac{\mu_1}{\lambda_1 + \mu_1}\frac{\lambda_2}{\lambda_2 + \mu_2}$$

State 4: Both components in down state:

$$P_4 = \frac{\lambda_1}{\lambda_1 + \mu_1}\frac{\lambda_2}{\lambda_2 + \mu_2}$$

Figure 25 State space (Markov) model for two components and four states.

The state residence times can be determined based on the departure rates:

$$m_1 = \frac{1}{\lambda_1 + \lambda_2}$$

$$m_2 = \frac{1}{\lambda_2 + \mu_1}$$

$$m_3 = \frac{1}{\lambda_1 + \mu_2} \qquad \textbf{(76)}$$

$$m_4 = \frac{1}{\mu_1 + \mu_2}$$

Consider the two components to be in series. In such a case, it is only state 1 where both components are up which is considered to be a system up state (or success state), see Figure 26, and the probability of the system being in up-state is found from the probability of state 1.

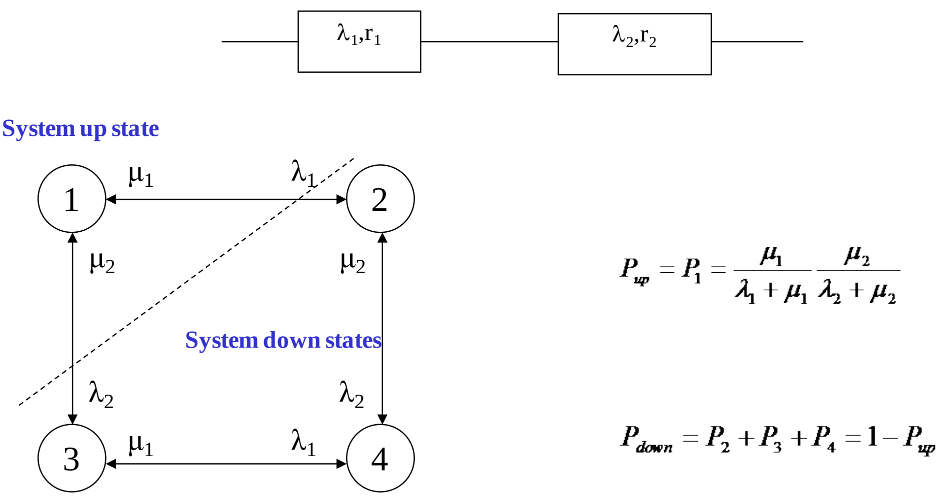


Figure 26 Markov model applied to series structure.

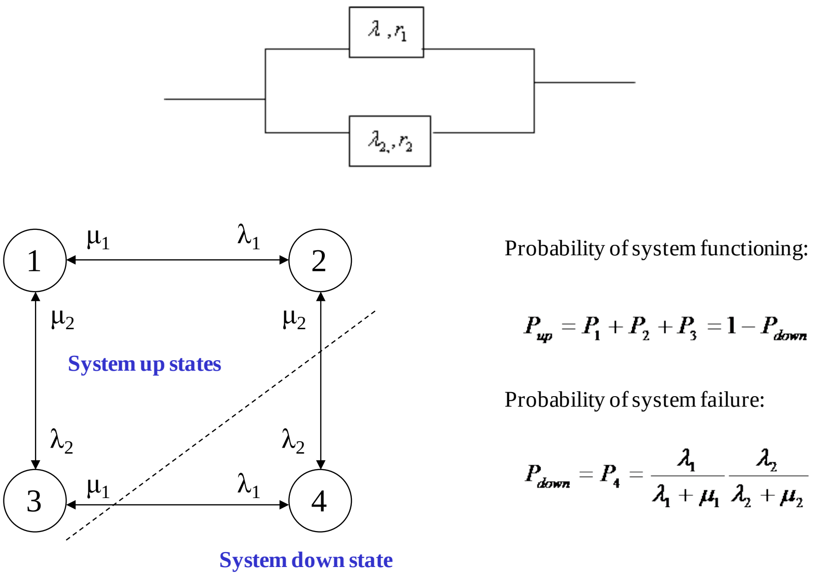


Figure 27 Markov model applied to parallel structure.

If we instead consider the two components to be in parallel, as shown in Figure 27, both components have to be in down-state (i.e. state 4) for the system to be in down-state (or failure state). In such a case, the probability of the system being in down-state is found from the probability of state 4, while the probability of the system being in up-state equals 1 minus this probability (P_4).

Frequency and duration of interruptions ("system failure")

From the basic models presented above and the relationships between probabilities, transition rates and frequencies, we can determine the frequency and duration of interruptions ("system failure") in the simple series and parallel systems presented in the figures above:

For the **series system** in Figure 26 the interruption frequency equals the frequency of departure from up state (1) which again equals the frequency of arrivals to the cumulated down states (2, 3, 4):

$$f_1 = P_1(\lambda_1 + \lambda_2) = P_2\mu_1 + P_3\mu_2 \quad \textbf{(77)}$$

Number of departures:

$$f_1 = P_1(\lambda_1 + \lambda_2) \quad \textbf{(78)}$$

Number of arrivals:

$$f_1 = P_2\mu_1 + P_3\mu_2 \quad \textbf{(79)}$$

$$f_{up} = f_1 = P_1(\lambda_1 + \lambda_2) = \frac{\mu_1\mu_2(\lambda_1 + \lambda_2)}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} \quad \textbf{(80)}$$

The frequency of interruptions can also be determined from the frequencies to the down-states minus the transitions between the down-states. This means that the contributions to the interruption (or system failure) frequency only include the transitions between the system success- and failure-states.

$$\begin{aligned} f_{down} &= f_2 + f_3 + f_4 - (\text{frequency of encounters between 2 and 3,} \\ &\quad \text{between 3 and 4, between 2 and 4)} \\ &= P_2(\mu_1 + \lambda_2) + P_3(\lambda_1 + \mu_2) + P_4(\mu_1 + \mu_2) - P_3\lambda_1 - P_4\mu_1 - P_2\lambda_2 - P_4\mu_2 \\ &= P_2\mu_1 + P_3\mu_2 \\ &= f_1 \end{aligned} \quad \textbf{(81)}$$

Similarly for the **parallel system**:

The interruption frequency equals the frequency of departure from down state (4) to the cumulated up states which again equals the arrivals to the system down state:

$$f_4 = P_4(\mu_1 + \mu_2) = P_3\lambda_1 + P_2\lambda_2 \quad \textbf{(82)}$$

Number of departures:

$$f_4 = P_4(\mu_1 + \mu_2) \quad \textbf{(83)}$$

Number of arrivals:

$$f_4 = P_3\lambda_1 + P_2\lambda_2 \quad \textbf{(84)}$$

$$f_{down} = f_4 = \frac{\lambda_1\lambda_2(\mu_1 + \mu_2)}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} \quad \textbf{(85)}$$

The frequency of interruptions can be again determined from the frequencies to the down-state minus the transitions between the up-states, i.e. the contributions to the interruption (or system failure) frequency only includes the transitions between the system success- and failure-states.

$$\begin{aligned} f_{up} &= f_1 + f_2 + f_3 - (\text{frequency of encounters between 1 and 2, and 3)} \\ &= P_1(\lambda_1 + \lambda_2) + P_2(\lambda_2 + \mu_1) + P_3(\lambda_1 + \mu_2) - P_1\lambda_1 - P_2\mu_1 - P_1\lambda_2 - P_3\mu_2 \\ &= P_3\lambda_1 + P_2\lambda_2 \\ &= f_4 \end{aligned} \quad \textbf{(86)}$$

Duration of interruption:

The mean interruption duration (or duration of “system failure”) is found by the division of the probability of interruption (system down-state) and the interruption frequency (frequency of entering system down-state):

$$r = \frac{P_{down}}{f_{down}} \quad \textbf{(87)}$$

Using the previously calculated probabilities and frequencies, we get the following expressions for the mean duration in series and parallel systems, respectively:

Series system:

$$r = \frac{P_2 + P_3 + P_4}{P_1(\lambda_1 + \lambda_2)}$$

$$r = \frac{\lambda_1\mu_2 + \mu_1\lambda_2 + \lambda_1\lambda_2}{\mu_1\mu_2(\lambda_1 + \lambda_2)} \quad \textbf{(88)}$$

Substituting the repair rates (μ) with the reciprocal of the average outage (repair) times (r):

$$r = \frac{\lambda_1\mu_2 + \mu_1\lambda_2 + \lambda_1\lambda_2}{\mu_1\mu_2(\lambda_1 + \lambda_2)}$$

$$= \frac{\lambda_1\frac{1}{r_2} + \frac{1}{r_1}\lambda_2 + \lambda_1\lambda_2}{\frac{1}{r_1}\frac{1}{r_2}(\lambda_1 + \lambda_2)} = \frac{\lambda_1 r_1 + \lambda_2 r_2 + \lambda_1\lambda_2 r_1 r_2}{\lambda_1 + \lambda_2} \quad \textbf{(89)}$$

Parallel system:

$$r = \frac{P_4}{P_4(\mu_1 + \mu_2)} = \frac{1}{\mu_1 + \mu_2} \quad \textbf{(90)}$$

Substituting the repair rates (μ) with the reciprocal of the average outage (repair) times (r):

$$r = \frac{1}{\mu_1 + \mu_2} = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{r_1 r_2}{r_1 + r_2} \quad \textbf{(91)}$$

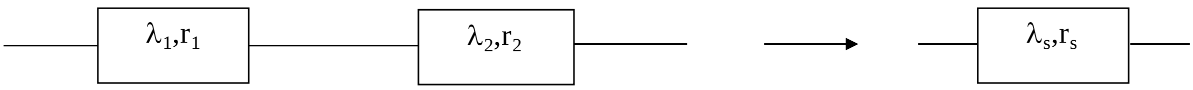
A.2.2 Approximate reliability evaluation

For many engineering systems, MTTF ≈ MTBF, i.e. the fault rate λ is ≈ the failure frequency f.

Furthermore, in many practical systems the product ($\lambda_i r_i$) is very small and therefore $\lambda_1 r_1 \lambda_2 r_2 \ll \lambda_1 r_1$ and $\lambda_2 r_2$. In addition, $\lambda_1 r_1$ and $\lambda_2 r_2$ are often much less than unity.

These approximations are utilized in the equations presented above for series and parallel systems in approximate evaluation techniques. This is shown in the following [22].

For a series system (two components in series is substituted by an equivalent component):



$$P_{up} = \frac{\mu_s}{\lambda_s + \mu_s} \tag{92}$$

Probability of up-state for the two component series system:

$$P_{up} = \frac{\mu_1 \mu_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} = \frac{\mu_s}{\lambda_s + \mu_s} \tag{93}$$

Transition rate from the system up-state:

$$\lambda_s = \lambda_1 + \lambda_2$$

$$r_s = \frac{1}{\mu_s} = \frac{\lambda_1 r_1 + \lambda_2 r_2 + \lambda_1 \lambda_2 r_1 r_2}{\lambda_1 + \lambda_2} = \frac{\lambda_1 r_1 + \lambda_2 r_2 + \lambda_1 \lambda_2 r_1 r_2}{\lambda_s} \tag{94}$$

when the product ($\lambda_i r_i$) is very small, then $\lambda_1 \lambda_2 r_1 r_2 \ll \lambda_1 r_1$ and $\lambda_2 r_2$, this equation reduces to:

$$r_s = \frac{\lambda_1 r_1 + \lambda_2 r_2}{\lambda_s} \tag{95}$$

For a general n-component series system, the failure rate and average outage duration may be deduced as:

$$\lambda_s = \sum_{i=1}^{n} \lambda_i \tag{96}$$

$$r_s = \frac{\sum_{i=1}^{n} \lambda_i r_i}{\lambda_s} \qquad \textbf{(97)}$$

The probability of the system being in down state, i.e. the unavailability U_s can be determined using the concepts of frequency and duration:

$$P_{down} = \frac{f_s}{\mu_s} = f_s r_s \qquad \textbf{(98)}$$

$$U_s = f_s r_s \qquad \textbf{(99)}$$

where f_s is the frequency of entering the down state

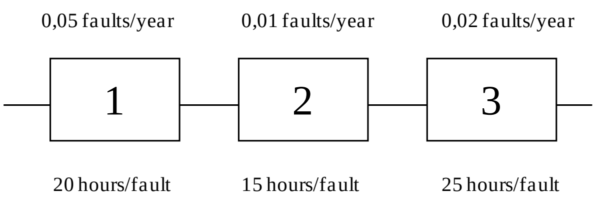
MTTF and MTBF are conceptually different, but as mentioned above in many cases MTTF (=1/λ) and MTBF (=1/f) are numerically almost identical. The unavailability may then be approximated to:

$$U_s \approx \lambda_s r_s = \sum_{i=1}^{n} \lambda_i r_i \qquad \textbf{(100)}$$

Summary: The set of equations for a series system is:

$$\boxed{\lambda_s = \sum_{i=1}^{n} \lambda_i \qquad U_s = \lambda_s r_s = \sum_{i=1}^{n} \lambda_i r_i \qquad r_s = \frac{\sum_{i=1}^{n} \lambda_i r_i}{\lambda_s} = U_s / \lambda_s} \qquad \textbf{(101)}$$

Example:

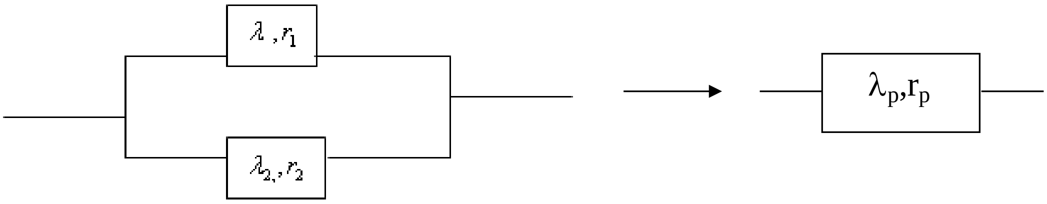


λ_s=0,05 + 0,01 + 0,02 = 0,08 faults/year

U_s=0,05 x 20 + 0,01 x 15 + 0,02 x 25 = 1,65 hours/year

r_s= 1,65/0,08 = 20,6 hours/fault

For a parallel system (two components in parallel are substituted by an equivalent component):



$$P_{down} = \frac{\lambda_1 \lambda_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} \quad \textbf{(102)}$$

Probability of down-state for the two component parallel system:

$$\frac{\lambda_1 \lambda_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} = \frac{\lambda_p}{\lambda_p + \mu_p} \quad \textbf{(103)}$$

Transition rate from the system down-state:

$$\mu_p = \mu_1 + \mu_2$$

$$r_p = \frac{1}{\mu_p} = \frac{1}{\mu_1 + \mu_2} = \frac{r_1 r_2}{r_1 + r_2} \quad \textbf{(104)}$$

$$\lambda_p = \frac{\lambda_1 \lambda_2 (r_1 + r_2)}{1 + \lambda_1 r_1 + \lambda_2 r_2}$$

Pr year:

$$\lambda_p = \frac{\lambda_1 \lambda_2 (r_1 + r_2)}{8760 + \lambda_1 r_1 + \lambda_2 r_2} \quad \textbf{(105)}$$

when $\lambda_1 r_1$ and $\lambda_2 r_2$ are much less than unity this equation reduces to:

$$\lambda_p \approx \lambda_1 \lambda_2 (r_1 + r_2) \quad \textbf{(106)}$$

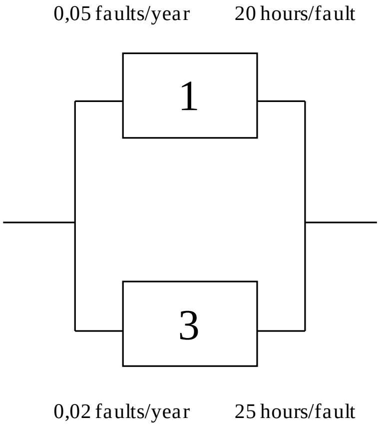
Pr year:

$$\lambda_p \approx \lambda_1 \lambda_2 (r_1 + r_2) / 8760$$

$$\textbf{(107)}$$

$$U_p = f_p r_p \approx \lambda_p r_p = \lambda_1 \lambda_2 r_1 r_2$$

Example:



λ_p= 0,05 x 0,02 (20 + 25)/8760 = 5,14 x 10^{-6} faults/year

r_p=(20 x 25)/(20 + 25) = 11,1 hours/fault

U_p= $\lambda_p r_p$ = 5,71 x 10^{-5} hours/year

Minimal cut sets method

The minimal cut sets-method enables a reliability network [22] of series and parallel systems.

It can be recalled that a cut set is a set of components which, when failed, causes system failure, while minimal cut set has no propr subset of components whose failure alone will cause system failure. The components of a minimal cut set are in parallel since all of them must fail in order to cause system failure. Thus, the cut sets are in series as any minimal cut set can cause system failure. For instance for the small example system shown in Figure 28 there are two minimal cut sets shown to the right in the figure. The first minimal cut set consists of component 1 and the second of the parallel of components 2 and 3 since both must fail for the system to fail.

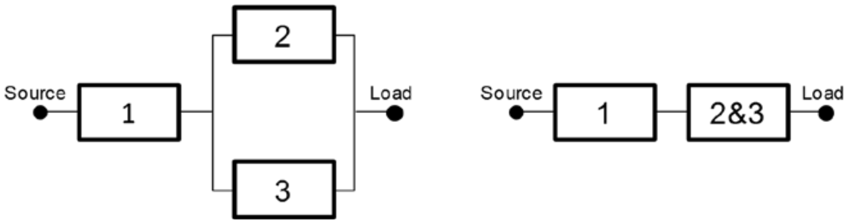


Figure 28 Example of series and parallel system and minimal cut sets.

The equivalent fault rate and repair time for the cut set containing the parallel components are found using the equations presented above for parallel systems, and these are combined with the fault rate and repair time for the single component using the equations for series systems to give the overall system indices.

Example of series and parallel system:

Consider the small example of a transmission system consisting of two lines in parallel and one transformer given in Figure 29 together with the reliability data (fault rates and repair times).

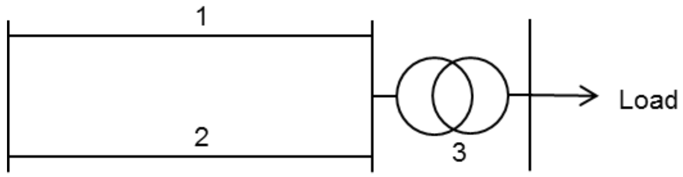


Figure 29 Example transmission system.

$\lambda_1 = \lambda_2 = 3$ faults/year
$r_1 = r_2 = 8$ hours
$\lambda_3 = 0{,}1$ fault/year
$r_3 = 100$ hours

$$\lambda_{12} = \lambda_1\lambda_2(r_1 + r_2) = 3{,}3\left(\frac{16}{8760}\right) \text{faults / year} = 0{,}0164 \text{ faults / year}$$

$$r_{12} = \frac{r_1 r_2}{r_1 + r_2} = \frac{8 \cdot 8}{8 + 8} \text{hours} = 4 \text{ hours / fault}$$

$$\lambda_S = \lambda_{123} = \lambda_{12} + \lambda_3 = 0{,}1164 \text{ faults / year}$$

$$r_S = r_{123} = \frac{0{,}1 \cdot 100 + 0{,}0164 \cdot 4}{0{,}1164} \text{hours} = 86{,}47 \text{hours / fault}$$

$$U_S = \lambda_S r_S = 10{,}06 \text{ hours / year}$$

As shown above, once the minimal cut sets have been obtained, the reliability indices can be determined by the application of the formulas using the approximate techniques. Just summing up the contributions from each minimal cut set to obtain the frequency of interruption (system failure) might overestimate the frequency and as such, yields an uppr bound for the frequency. This is shown in the following, based on the description in [38, 45]:

The probability of system failure for *m* minimal cut sets are given by

$$\begin{aligned} P_f = P\{\overline{C_1} \cup \overline{C_2} \cup \overline{C_3} \ldots \cup \overline{C_m}\} \\ = P\{\overline{C_1}\} + \cdots + P\{\overline{C_m}\} - (P\{\overline{C_1} \cap \overline{C_2}\} + \cdots \\ + P\{\overline{C_i} \cap \overline{C_j}\}_{i \neq j}) \quad \ldots (-1)^{m-1} P\{\overline{C_1} \cap \overline{C_2} \cap \ldots \overline{C_m}\} \end{aligned} \quad \textbf{(108)}$$

Where
C_i = Minimal cut set i
$\overline{C}_i$ = Failure of all components in C_i

As indicated in the equation, adding the first order terms only, will give an uppr bound. Subtracting the second order terms, gives a lower bound. Successive additions of odd and even order terms give increasingly closer uppr and lower bounds [45].

If two minimal cut sets C_i and C_k are mutually exclusive, i.e., there could be no transition between the two cut sets, then the frequency contribution due to C_i and C_k will be the sum of their frequencies. In practice, some of the minimal cut sets have overlapping subsets. Then, the minimal cut sets are not mutually exclusive, and the probability and frequency will be overestimated if only the first order terms in the above equation are taken into account. The frequency can be found similarly using the accurate calculations based on the relations described in A.2.1 between the probability and frequency of system failure (interruption) [38, 45]:

The frequency of encountering the subset S_i of the state space, representing the minimal cut C_i, is

$f_f = P\{\overline{C}_i\}\overline{\mu}_i$

Where

$\overline{\mu}_i$ = Sum of μ_j over all *j* components in C_i

In general for *m* cut sets:

$$f_f = (P\{\overline{C_1}\}\overline{\mu_1} + P\{\overline{C_2}\}\overline{\mu_2} + \cdots + P\{\overline{C_m}\}\overline{\mu_m}) - (P\{\overline{C_1} \cap \overline{C_2}\}\overline{\mu_{1+2}} + P\{\overline{C_1} \cap \overline{C_3}\}\overline{\mu_{1+3}} + \cdots + P\{\overline{C_\imath} \cap \overline{C_\jmath}\}_{i \neq j}\overline{\mu_{\imath+\jmath}}) \quad \ldots (-1)^{m-1}P\{\overline{C_1} \cap \overline{C_2} \cap \ldots \overline{C_m}\}\overline{\mu_{1+2+\cdots+m}} \qquad \textbf{(109)}$$

The frequency of encountering higher order contingencies becomes increasingly smaller. Therefore, uppr and lower bounds for the frequency of system failure are:

$$f_U = \sum_i P\{\overline{C}_i\}\overline{\mu}_i$$
$$f_L = \sum_i P\{\overline{C}_i\}\overline{\mu}_i - \sum_{i<k} P\{\overline{C_\imath} \cap \overline{C_k}\}\overline{\mu_{i+k}} \qquad \textbf{(110)}$$

See [38, 45] for more details.

A.3 Example using the state space method

The minimal cut set method is in itself an approximate reliability evaluation method compared with the accurate analytical state space method which is briefly outlined in A.2. The accurate reliability calculation is based on the steady-state probabilities in the Markov model and the derived frequency and duration techniques.

The probability of component 'i' being in up (operating) state according to Equation (5):

$$P_{iu} = \frac{m_i}{m_i + r_i} = \frac{\mu_i}{\lambda_i + \mu_i}$$

The probability of component 'i' being in down (failed) state according to Equation (4):

$$P_{id} = \frac{r_i}{m_i + r_i} = \frac{\lambda_i}{\lambda_i + \mu_i}$$

In the following, we consider line faults only. The probabilities for the four lines in the OPAL network are calculated on the basis of the reliability data given in Table 2, and presented in Table 43 together with MTTF and MTBF.

Table 43 Component probabilities of being in operating or failed states, OPAL network.

Component (Line) probabilities	U = Up state P_{iu} =p	D = Down state P_{id} =q	sum	MTTF (hrs)	MTBF (hrs)
1	0,995454545	0,004545455	1	4380	4400
2	0,994889267	0,005110733	1	2920	2935
3	0,994550409	0,005449591	1	2190	2202
4	0,994324631	0,005675369	1	1752	1762

If there is insufficient power to supply the load, it is assumed that the load is interrupted (at least partially). In the following, the total state space is analysed, i.e., $2^4 = 16$ states. Table 44 shows the result for the delivery points in the heavy load and light load situations respectively.

Table 44 Consequences for the delivery points of the different system states. U = up state, D = down state. S = Success state, F = Failed state.

System state	Line outages		Component			Case 1 a)	Light load			Case 1 b)	Heavy load		
		1	2	3	4	L1 = 60 MW	SAC	L2 = 30 MW	SAC	L1 = 100 MW	SAC	L2 = 75 MW	SAC
1	None	U	U	U	U	S	> 60	S	> 30	S	> 100	S	> 75
2	1	D	U	U	U	S	> 60	S	> 30	S	> 100	S	> 75
3	2	U	D	U	U	S	> 60	S	> 30	S	> 100	F	35
4	3	U	U	D	U	S	> 60	S	> 30	S	> 100	F	35
5	4	U	U	U	D	S	> 60	S	> 30	S	> 100	S	> 75
6	1, 2	D	D	U	U	S	> 60	S	> 30	S	> 100	F	35
7	1, 3	D	U	D	U	S	> 60	S	> 30	S	> 100	F	35
8	1, 4	D	U	U	D	S	> 60	S	> 30	S	> 100	S	> 75
9	3, 4	U	U	D	D	S	> 60	F	0	S	> 100	F	0
10	2, 3	U	D	D	U	F	0	F	0	F	0	F	0
11	2, 4	U	D	U	D	F	0	S	> 30	F	0	S	> 75
12	1, 2, 3	D	D	D	U	F	0	F	0	F	0	F	0
13	1, 2, 4	D	D	U	D	F	0	S	> 30	F	0	S	> 75
14	1, 3, 4	D	U	D	D	S	> 60	F	0	S	> 100	F	0
15	2, 3, 4	U	D	D	D	F	0	F	0	F	0	F	0
16	1, 2, 3, 4	D	D	D	D	F	0	F	0	F	0	F	0

The probability of the states in the above table is found by combining the component probabilities in Table 43 as shown in A.2:

Table 45 System state probabilities.

P1	0,97937988
P2	0,004472054
P3	0,005031061
P4	0,005366465
P5	0,005590068
P6	2,29729E-05
P7	2,45044E-05
P8	2,55254E-05
P9	3,06305E-05
P10	2,75675E-05
P11	2,87161E-05
P12	1,25879E-07
P13	1,31124E-07
P14	1,39865E-07
P15	1,57349E-07
P16	7,18486E-10
sum	**1**

Table 44 shows for delivery point L1 in the light load situation the following success (no interruption) and failed (interruption) states:

Success states: {1, 2, 3, 4, 5, 6, 7, 8, 9, 14}

Failed states: {10, 11, 12, 13, 15, 16} corresponding to the following outage combinations:

{2,3}, {2,4},{1,2,3},{1,2,4},{2,3,4},{1,2,3,4}

The probability of interruption to the load L1 is determined on the basis of the state probabilities for the failed states:

$$P_f = P_{10} + P_{11} + P_{12} + P_{13} + P_{15} + P_{16} = 5{,}66986\text{E-}05$$

The frequency of interruption to the load L1 is found from the transitions between the success states and failed states:

$$f_f = P_{10}(\mu_2 + \mu_3) + P_{11}(\mu_2 + \mu_4) + P_{12}(\mu_2 + \mu_3) + P_{13}(\mu_2 + \mu_4) + P_{15}\mu_2 + P_{16}\mu_2 = 0{,}078598305$$

interruptions/year

Mean (average) interruption duration:

$$r_f = P_f / f_f = 6{,}319220154 \text{ hrs/interruption}$$

For delivery point L2, the success (no interruption) and failed (interruption) states in the light load situation are as follows:

Success states: {1, 2, 3, 4, 5, 6, 7, 8, 11, 13}

Failed states: {9, 10, 12, 14, 15, 16} corresponding to the following outage combinations:

{3,4}, {2,3},{1,2,3},{1,3,4},{2,3,4},{1,2,3,4}

The probability of interruption to the load L2:

$$P_f = P_9 + P_{10} + P_{12} + P_{14} + P_{15} + P_{16} = 5{,}86218\text{E-}05$$

Frequency of interruption to the load L2:

$$f_f = P_9(\mu_3 + \mu_4) + P_{10}(\mu_2 + \mu_3) + P_{12}(\mu_2 + \mu_3) + P_{14}(\mu_3 + \mu_4) + P_{15}\mu_3 + P_{16}\mu_3 = 0{,}085921653$$

interruptions/year

Mean (average) interruption duration: $r_f = P_f / f_f = 5{,}976686304$ hrs/interruption

Similar calculations are performed for the heavy load situation. As stated in Chapter 4, the heavy load situation is defined to cover three months while the light load situation covers nine months for both delivery points. Thus, there are two operating states with the following probabilities:

$p_1 = 9/12 = 0{,}75$
$p_2 = 3/12 = 0{,}25$

The probability, frequency and mean duration of interruptions for the whole year are found by weighting the indices for the light load and heavy load situation respectively with their probabilities, except for the mean interruption duration *r* which is found as the probability divided by the frequency. The results using the state space method for the probability, frequency and mean duration of interruptions are summarized in Table 46 later in the section.

For the purpose of calculating the consequences of interruptions in terms of interrupted power and energy not supplied, we need information about the system available capacity to supply the load for each failed state, i.e. for each contingency leading to interruption. See Table 44. For example SAC > 60 MW means that the capacity to supply the load is at least 60 MW (the load demand).

In the light load situation, SAC is equal to zero for all the failed states for both delivery points, while it is at least equal to the demand for all the success states. For delivery point L2, there are four new failed states in the heavy load situation compared to that of light load, for which SAC is reduced to 35 MW.

Interrupted power pr year and energy not supplied pr year are calculated for each of the failed states, based on the interrupted power and the frequency for the failed state, and summed up. This is shown in the following.

Light load:

Since SAC = 0 for all the failed states for both L1 and L2 in this case, interrupted power is found as the difference between the load and SAC multiplied by the total frequency of interruptions. Energy not supplied is the product of interrupted power and mean interruption duration.

Delivery point L1:

$P_{interr} = (P - SAC) \cdot f = 60 \cdot 0{,}078598305 = 4{,}715898271$ MW/year

$ENS = P_{interr} \cdot r_f = 29{,}8007994$ MWh/year

Similarly for L2:

$P_{interr} = (P - SAC) \cdot f = 30 \cdot 0{,}085921653 = 2{,}577649605$ MW/year

$ENS = P_{interr} \cdot r_f = 15{,}4058$ MWh/year

Heavy load:

Delivery point L1:

Since SAC = 0 for all the failed states for L1 in this case as well, interrupted power and energy not supplied is given by:

$P_{interr} = (P - SAC) \cdot f = 100 \cdot 0{,}078598305 = 7{,}85983045$ MW/year

$ENS = P_{interr} \cdot r_f = 49{,}667999$ MWh/year

Delivery point L2**:**

Since SAC was equal to zero for L1 and L2 in the light load situation, we calculate interrupted power directly based on the difference between the load and SAC multiplied with the total interruption frequency. In the heavy load case, SAC is different for the different failed states for L2, see Table 44. In order to calculate the interrupted power we now have to consider SAC individually for each of the states included in the frequency of interruptions.

The frequency is obtained as described above for the heavy load situation:

$$f_f = P_3(\mu_2 + \lambda_4) + P_4\mu_3 + P_6(\mu_2 + \lambda_4) + P_7\mu_3 + P_9\mu_3 + P_{14}\mu_3 + P_{15}\mu_3 + P_{16}\mu_3 = 6{,}93481147$$

interruptions/year

This gives the interrupted power:

$$\begin{aligned} P_{\text{int err}} &= (75-35)P_3(\mu_2 + \lambda_4) + (75-35)P_4\mu_3 + (75-35)P_6(\mu_2 + \lambda_4) + (75-35)P_7\mu_3 \\ &+ 75(P_9\mu_3 + P_{14}\mu_3 + P_{15}\mu_3 + P_{16}\mu_3) = 278{,}182681 \ [\text{MW/year}] \end{aligned}$$

ENS = $P_{interr} \cdot r_f$ = 3690,955 MWh/year

Combined for the whole year:

Interrupted power and energy not supplied from the two cases are weighted with the probabilities of the operating states (9/12 and 3/12, respectively).

Delivery point L1**:**

P_{interr} = 5,501881316 MW/year

ENS = 34,7675993 MWh/year

Delivery point L2**:**

P_{interr} = 71,47890734 MW/year

ENS = 934,2931035 MWh/year

The results using the state space method are given in Table 46 and the results using the minimal cut set method described in Chapter 4 are given in Table 47.

Table 46 Reliability indices for the OPAL network using the state space method.

	Light load		Heavy load		Combined, for the year	
	L1	L2	L1	L2	L1	L2
Probability of interruption	5,67E-05	5,86E-05	5,67E-05	0,0105	5,67E-05	0,00267
Interruption frequency [no./year]	0,0786	0,0859	0,0786	6,935	0,0786	1,798
Annual interruption duration [hours/year] = Probability * 8760	0,4967	0,5135	0,4967	92,01	0,4967	23,39
Mean duration [hours/interruption]	6,32	5,98	6,32	13,27	6,32	13,01
Interrupted power [MW/year]	4,72	2,58	7,86	278,18	5,50	71,48
ENS [MWh/year]	29,80	15,41	49,67	3691	34,77	934,3

Table 47 Reliability indices for the OPAL network using the minimal cut set method.

	Light load		Heavy load		Combined, for the year	
	L1	L2	L1	L2	L1	L2
Interruption frequency (λ) [no./year]	0,0789	0,0863	0,0789	7	0,0789	1,815
Annual interruption duration [hours/year]	0,498	0,515	0,498	93	0,498	23,64
Mean duration [hours/interruption]	6,31	5,97	6,31	13,29	6,31	13,02
Interrupted power [MW/year]	4,74	2,59	7,89	280	5,53	71,94
ENS [MWh/year]	29,88	15,45	49,81	3720	34,87	941,6

Using the state space method, the whole state space is investigated in calculating the exact probabilities and frequencies of interruptions, while the minimal cut set method represents an approximate reliability evaluation. The minimal cut set approach only takes the failure modes represented by the minimal cuts into account, while the other system states are unknown and thus not considered.

The tables show that the cut set method gives slightly higher frequencies, annual interruption duration, interrupted power and energy not supplied. The approximate method tends to overestimate the unreliability. However, the differences are small and negligible for most practical purposes. The approximate method is faster and simpler given that the minimal cut sets are established.

In this OPAL example where the generators are considered 100 % reliable with "infinite" capacity, line no. 1 has no influence on the reliability. Lines 2 and 3 are the most important lines. The single outages dominate the reliability of supply to L2 in the heavy load situation, while line no. 4 is involved in one of the minimal cuts in the light load situation.

Sometimes, it so happens that multiple outages can improve the situation for some delivery points, while for others the situation will worsen. Which delivery points are possible to serve in the different operating states for a specific contingency depends on the topology, available capacity and load shedding procedures (prioritization etc.). This problem is illustrated by the OPAL network, varying from the operating state “light load” to “heavy load”. For L1, the failure modes (minimal cuts) and SAC are unchanged for the two operating states since this delivery point is prioritized in case of load shedding. However, for L2 the situation changes as described in the following.

In the light load situation, there is no single outage causing interruption for L2, while single outage of line 2 or line 3 causes a reduced SAC and partial interruption in the heavy load situation. These single outages were therefore identified as minimal cuts for L2 in the operating state “heavy load”. Looking at the table of states and corresponding system available capacity (SAC) for this case, a change can be noticed in SAC for L2 for the double outages of line 2 and 3, line 2 and 4, and line 3 and 4, respectively, compared to the single outages of line 2 and 3. This is highlighted in Table 48 below.

Table 48 Consequences for the delivery points of the different system states. Highlighting changes in SAC for L2 for different operating states.

System state	Line outages		Component			Case 1 a)				Case 1 b)			
		1	2	3	4	L1 = 60 MW	SAC	L2 = 30 MW	SAC	L1 = 100 MW	SAC	L2 = 75 MW	SAC
1	None	U	U	U	U	S	> 60	S	> 30	S	> 100	S	> 75
2	1	D	U	U	U	S	> 60	S	> 30	S	> 100	S	> 75
3	2	U	D	U	U	S	> 60	S	> 30	S	> 100	F	35
4	3	U	U	D	U	S	> 60	S	> 30	S	> 100	F	35
5	4	U	U	U	D	S	> 60	S	> 30	S	> 100	S	> 75
6	1, 2	D	D	U	U	S	> 60	S	> 30	S	> 100	F	35
7	1, 3	D	U	D	U	S	> 60	S	> 30	S	> 100	F	35
8	1, 4	D	U	U	D	S	> 60	S	> 30	S	> 100	S	> 75
9	3, 4	U	U	D	D	S	> 60	F	0	S	> 100	F	0
10	2, 3	U	D	D	U	F	0	F	0	F	0	F	0
11	2, 4	U	D	U	D	F	0	S	> 30	F	0	S	> 75
12	1, 2, 3	D	D	D	U	F	0	F	0	F	0	F	0
13	1, 2, 4	D	D	U	D	F	0	S	> 30	F	0	S	> 75
14	1, 3, 4	D	U	D	D	S	> 60	F	0	S	> 100	F	0
15	2, 3, 4	U	D	D	D	F	0	F	0	F	0	F	0
16	1, 2, 3, 4	D	D	D	D	F	0	F	0	F	0	F	0

While single outage of line 2 gives SAC equal to 35 MW, double outage of lines 2 and 4 gives SAC > 75 MW. The reason for this is that when both lines 2 and 4 are on outage, it is not possible to supply L1, and the whole load in L2 can be supplied instead. This state will be handled in the state space method as a success state, i.e. no interruption, while in the minimal cut set method this double outage will not be regarded as a cut, and as such, gives no contribution to interruptions in the load point.

When lines 2 and 3, or lines 3 and 4 are on outage, SAC for L2 changes from 35 to 0 MW. These two double outages actually constitute minimal cut sets for L2 in addition to the single outages of line 2 and 3. These changes can be considered in the reliability assessment including additional higher order minimal cut sets. While the single outages cause partial interruption with SAC equal to 35 MW, the double outages cause total interruption with SAC equal to 0 MW. Using approximate evaluation techniques including the additional minimal cut sets, we get the results shown in Table 49:

Table 49 Reliability indices for the OPAL network using the minimal cut set method including the additional higher order minimal cut sets. Results highlighted for L2 in the heavy load situation.

	Light load		Heavy load		Combined, for the year	
	L1	L2	L1	L2	L1	L2
Interruption frequency (λ) [no./year]	0,0789	0,0863	0,0789	7,09	0,0789	1,84
Annual interruption duration [hours/year]	0,498	0,515	0,498	93,51	0,498	23,76
Mean duration [hours/interruption]	6,31	5,97	6,31	13,20	6,31	12,94
Interrupted power [MW/year]	4,74	2,59	7,89	286,47	5,53	73,56
Energy not supplied (ENS) [MWh/year]	29,88	15,45	49,81	3780,44	34,87	956,70

The difference compared with Table 47 and the results given in Chapter 4 is less than or about 1 % except for interrupted power which increases by about 2 % including the higher order minimal cut sets.

Taking the two double outages in the minimal cut sets into account slightly increases the difference compared to the state space method in Table 46. Again, we see that the approximate method overestimates the unreliability in terms of frequency of interruptions and interrupted power, as well as energy not supplied. Compared with the state space method, the interrupted power and energy not supplied is 2,9 % and 2,4 % higher respectively. In this calculation we have not considered that the single outages of lines 2 and 3, and the double outages of lines 2 and 3, and 3 and 4, respectively, are not mutually exclusive. The indices in Table 49 can therefore be regarded as an uppr bound for the reliability indices. See 0 for a description of approximations regarding the use of minimal cut sets.

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