# STATISTICAL AVALANCHE ZONING[1]

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Abstract.--Runout for the extreme event on an avalanche path is calculated from confidence limits on a regression analysis of path topographic parameters. This results in redefinition of the traditional zoning problem by dividing it into an estimation of runout distance by an engineering decision on the choice of confidence limit to the estimate, and a dynamic problem with boundary conditions defined by the chosen limit.

## INTRODUCTION

The traditional method of avalanche zoning involves the joint solution of the avalanche runout and dynamics problems by selecting appropriate friction coefficients for an avalanche dynamics model. The physical problem involves a complex transition of states with many unknowns including: friction coefficients, constitutive laws and properties of flowing snow. These gaps in knowledge are significant; clearly the problem is far from a solution.

The avalanche zoning problem may be redefined by separation into two parts: (1) estimation of the runout distance for the extreme event on a path based on a regression equation involving topographic parameters and a selected confidence limit and (2) estimation of speeds along the incline between the start position and runout position. The latter position is defined by choice of a confidence limit based upon an engineering decision. In this paper part 1 of the problem is introduced in the simplest manner to illustrate the method; further details will be provided in a forthcoming paper.

## ANALYSIS OF TOPOGRAPHIC PARAMETERS

The data set used in the present analysis consists of estimates for 212 avalanche paths from the maritime climate regime of Western Norway. Extreme runout for time scales of at least 100 years was measured in the field and a number of other parameters for the paths were determined. The papers by Lied and Bakkehøi (1980) and Bakkehoi, Domaas and Lied (1983) have provided detailed descriptions of the data set, including methods and accuracy.

The parameters used in the present analysis consist of angles pictured in figure 1 and defined by the following equations:

$$\tan\alpha = \frac{1}{X_0}\int_0^{X_0}\left(\frac{dy}{dx}\right)dx = \frac{H_0}{X_0} \qquad (1)$$

$$\tan\beta = \frac{1}{X_\beta}\int_{\Delta X}^{X_0}\left(\frac{dy}{dx}\right)dx = \frac{H_\beta}{X_\beta} \qquad (2)$$

$$\tan\delta = \frac{1}{\Delta X}\int_0^{\Delta X}\left(\frac{dy}{dx}\right)dx = \frac{\Delta H}{\Delta X} \qquad (3)$$

A fourth parameter is the starting zone angle, which is defined by $\tan\theta$, the average slope in the first 100 m of the avalanche starting zone. For simplicity the origin of geometry is chosen at the extreme tip of the runout ($\alpha$ point) and the $\beta$ point ($\Delta H$, $\Delta X$) is chosen as that for which the slope angle first equals 10° proceeding downslope from the avalanche start position.

The $\beta$ point is chosen as a reference position from which runout is marked so that $\tan\delta$ is the average slope in the runout zone. Using the $\beta$ point as a zero reference means that runout can be regarding as taking positive, zero or negative values if the avalanche stop position is below, at or above the $\beta$ point, respectively. For a regression analysis approach, extreme runout is based on a prediction of the minimum value of $\alpha$, given values of the potential predictor variables ($\beta$, $\theta$, and $\delta$). Use of $\delta$ as a predictor variable is limited to cases where the runout zone is known to be at a constant angle, such as a flat valley floor. For the present data set, 131 paths have known $\delta$ angles.

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Correlation coefficients (R) were calculated for $\alpha$ with respect to $\beta$, $\theta$ (212 paths) and $\delta$ (131 paths). The results gave: 0.919, 0.388 and -0.111, respectively. This suggests that the best one parameter model is $\alpha = f(\beta)$ and this was confirmed by regression analysis.

An examination of residual plots for linear regression of $\alpha$ with $\beta$ showed that the predictive equation provides biased estimates. This suggests a transformation on the response variable $\alpha$. Power law regression gave a good unbiased relationship for 212 avalanche paths:

$$\hat{\alpha} = 0.730\beta^{1.06} \qquad (4)$$

with $R^2 = 0.861$ and $S = 0.0764$ , the latter quantity being the standard error. For comparison the linear regression gave $R^2 = 0.845$ and $S = 2.52°$.

Another transformation explored was $\sqrt{\alpha}$. For this case the regression equation is:

$$\sqrt{\hat{\alpha}} = 0.0879\beta + 2.57 \qquad (5)$$

with $R^2 = 0.853$ and $S = 0.218$ . This equation removes some of the bias in estimates over a linear regression model but it is not as good in that respect as equation (4). Equation (5) is introduced because it appears more useful in zoning applications, as will be discussed below.

A number of multiple regression equations were derived in an attempt to improve the predictive schemes by addition of $\theta$ as a second variable. However, it was not found possible to improve the predictive scheme enough to warrant inclusion of $\theta$.

Addition of $\delta$ as a predictor variable does improve the predictive equations but this has very limited application and therefore $\delta$ is not introduced here, in favor of simplicity.

Another possibility for estimating runout when distances associated with the angles are known for the avalanche paths, is the calculation of horizontal reach ($\Delta X$) from the $\beta$ point. Using equations (1) to (3) (fig. 1) it is easily shown that:

$$\frac{\Delta X}{X_\beta} = \frac{\tan\beta - \tan\alpha}{\tan\alpha - \tan\delta} \qquad (6)$$

and

$$\frac{\Delta X}{H_\beta} = \frac{\left(1 - \frac{\tan\alpha}{\tan\beta}\right)}{(\tan\alpha - \tan\delta)} \qquad (7)$$

Calculations for 131 avalanche paths show that $\frac{\Delta X}{X_\beta}$ has a mean value of 0.171 and a standard deviation of 0.113. Similarly, $\frac{\Delta X}{H_\beta}$ has a mean of 0.276 and standard deviation of 0.197.

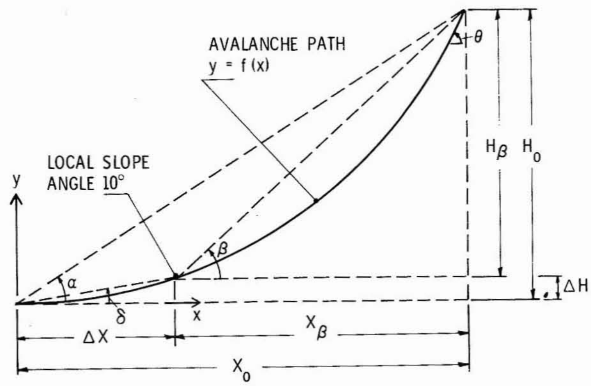


Figure 1.--Definitions of angles and length scales for an avalanche path.

Regression analyses showed that $\frac{\Delta X}{X_\beta}$ and $\frac{\Delta X}{H_\beta}$ are statistically independent of $\beta$, $\theta$ and $\delta$ to a good approximation ($R^2 \approx 0$).

## STATISTICAL AVALANCHE RUNOUT

Given the $\beta$ point as a reference, calculation of extreme runout depends on a prediction of the minimum value of $\alpha$ expected for a given model. The criterion used in this paper corresponds to the best fitting upper envelope on the distribution of $\beta$ (ordinate) versus $\alpha$ (abcissa, fig. 2). Upper envelopes were determined by calculating confidence limits for $\alpha = f(\beta)$ for linear regression, power law regression [equation (4)] and the square root transformation of equation (5).

The best fit to the upper limit on the $\beta - \alpha$ distribution throughout the ranges of $\beta$ and $\alpha$ was found from confidence limits of equation (5). By standard methods and using some approximations for the large number of data points, an estimate of the confidence limit for minimum value of $\alpha$ is given by:

$$\sqrt{\alpha_P} = 0.0879\beta + \left[2.57 - (0.218)t_{(1-\frac{P}{100})}\right] \qquad (8)$$

where $t_{(1-\frac{P}{100})}$ is a value of the t distribution for 210 degrees of freedom. Equation (8) states that P% of avalanches have $\alpha$ values greater than $\alpha_P$ for $50 \leq P < 100$. For example, for a 99% upper confidence limit, $t_{0.01}$ is 2.326 (taken from standard tables) and an expression for extreme runout for which 99% of avalanche paths would have greater values of $\alpha$ is obtained by substituting the value for $t_{0.01}$ into equation (8). Of course, choice of a value for P depends upon an engineering decision, which is determined by consideration of land prices and

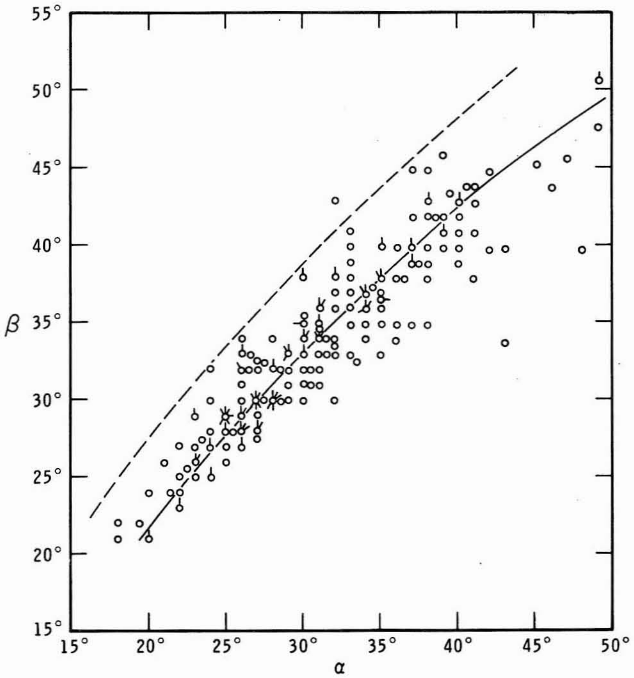


Figure 2.--Plot of β versus α for 212 avalanche paths from Western Norway.

—— Regression line for square root transformation on Equation (5).

---- Prediction for extreme runout ($\alpha_{99}$) for the regression line shown.

Multiple plotted points at the same location on the graph are denoted with extra bars attached to the circle, e.g. ꓤ represents 3 data points.

margin of safety desired, coupled to knowledge of local climate records and avalanche return periods. In many instances a 90% confidence limit may be adequate for estimating the extreme runout distance.

Another possibility for estimation of extreme runout distance consists of extrapolation to the upper limits of $\frac{\Delta X}{X_\beta}$ or $\frac{\Delta X}{H_\beta}$ given mean values and standard deviations. Since these quantities are statistically independent of the predictor values, the assumption that they are Gaussian variables suggests the model:

$$\left(\frac{\Delta X}{H_\beta}\right)_P = 0.276 + 0.197\left[t_{(1-\frac{P}{100})}\right] \quad (9)$$

and a similar expression for $\left(\frac{\Delta X}{X_\beta}\right)_P$ may be derived.

For the present data set, there are two disadvantages to equation (9) as a predictive equation: (1) calculations with actual examples show that the accuracy is not as good throughout the ranges of β and α as compared to equation (8); (2) values of $H_\beta$ and $X_\beta$ were not measured for the present data set and it is unknown whether ΔX is proportional to $H_\beta$ or $X_\beta$ as a model, as equation (9) would imply. In spite of these disadvantages, a rough estimate of runout can be given once a value of $t_{(1-\frac{P}{100})}$ is determined by an engineering decision.

## DISCUSSION

Prediction of extreme avalanche runout distance has been presented for two types of models expressed by equations (8) and (9). Either of these may be used to prepare a statistical map of confidence limits for calculation of runout. Choice of a limit depends upon an engineering decision, and this places the zoning problem in the same language that other problems concerned with risk and safety are phrased in modern practice.

Two advantages of the approach presented are: (1) it eliminates the necessity for solving avalanche dynamics equations to determine runout using an arbitrary choice of friction coefficients, as is usually done; (2) the dynamics problem is reduced to prediction of speeds along the incline between the start position and stop position, once a given confidence limit is chosen (i.e. a set of boundary conditions for the dynamics problem is defined by choice of a confidence limit).

Many unanswered questions need to be investigated with regard to the present approach to runout. For example, field experience strongly indicates that starting zone size should have an influence. Also, the effect of climate regime needs to be quantified.

The one parameter model is useful because extreme avalanches reach slope angles near 10°. A solution of the dynamics problem requires an understanding of its dependence on the relevant length scales and the effect of parameters such as avalanche mass; this may emerge from a solution of the runout problem. Until these questions are answered and until a good physical model for flowing snow is developed, solution of the runout and dynamics problems together, as is common in practice, amounts to nothing more than a curve fitting exercise by adjustment of friction coefficients.

## REFERENCES

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