Transferring Avalanches Between Paths

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ABSTRACT. Estimates of exceedance probabilities of runout lengths of avalanches for a specific path can rarely be based on measured avalanches in that slope alone, if they are to become statistically reliable. Thus one has, directly or indirectly, to include information of known runout lengths in other paths, i.e. to transfer runout lengths between paths. An attempt is made to classify such transfer methods, including both topographical methods that only make use of information on the shape of the path, such as methods based on runout ratios and α/β-models, as well as physical methods which also make use of physical models, simulating the avalanche as it runs down the path. By introducing a specific standard slope all avalanches in a given dataset can be transferred to that slope. The length of the transferred avalanche in the standard slope then becomes a slope-independent measure of its length. Using an Icelandic dataset of 196 avalanches we demonstrate how estimates of exceedance probabilities of runout lengths may vary with the choice of transfer method and how the order of the slope-independent lengths of the avalanches in the dataset will vary. The implication for avalanche risk assessment is briefly discussed.

INTRODUCTION

An integral part of snow avalanche risk assessment is to estimate the probability that an avalanche that sets out in a given path will run beyond a specified location. Such an estimate may be based on

a) climatological records that allow the estimation of e.g. the probability that the snow accumulated in a given number of days in the starting zone will have exceeded a certain depth when the avalanche sets out. With the help of physical models these snowdepths can then be related to possible runout lengths. This is a typical approach in risk assessment in Switzerland (Salm, Burkard, and Gubler, 1990).
b) historical records of runout lengths. This is a typical approach in risk assessment in Norway (Lied, 1993).

In the second approach there is rarely enough historical evidence in the path under consideration for reliable estimates to be based on that alone. Thus, directly or indirectly, one is forced to relate historical evidence from avalanches in other paths to this path, i.e. to transfer recorded runout lengths in some meaningful way between paths. The purpose of this paper is to propose a classification of such transfer methods and present a comparison of different methods on a dataset of 196 Icelandic avalanches.

CLASSIFICATION OF TRANSFER METHODS

We divide transfer methods into two main groups:

1) *Topographical methods*, which only make use of information on the shape of the avalanche path.
2) *Physical methods*, which also make use of physical models in order to simulate the avalanche as it runs down the path.

Here, it must be emphasized that while the same physical models may be used for transferring avalanches between paths as in the first approach mentioned in the introduction, there is a conceptual difference between these two approaches to estimating runout lengths.

Within each group we distinguish, in turn, between three different types of methods:

i) *Type 1 methods* are based on a length scale that can be used directly to transfer runout lengths between paths, without any a priori statistical estimates. Having transferred all the avalanches from a dataset onto a particular path we can, a posteriori, estimate the statistical distribution of the transferred lengths by suitable parametric or non-parametric methods.
ii) *Type 2 methods* are based on a length scale that can be used directly to transfer runout lengths between paths, but the length scale depends on some a priori statistical estimates. The underlying assumptions behind these estimates will in turn govern the estimated statistical distribution of the transferred lengths.
iii) *Type 3 methods* make indirect use of the dataset in order to estimate the distribution of possible runout lengths in a particular path, without transferring individual runout lengths between paths. In this case we can however, a posteriori, use the probability that an avalanche will run beyond a specific location as a length scale for transferring runout lengths between paths.

This distinction will be clarified further as we consider specific examples of transfer methods below.

TOPOGRAPHICAL TRANSFER METHODS

Runout ratios, introduced by McClung and Lied (1987) amount to a *type 1 topographical method*. These ratios can be used to transfer runout lengths directly from a dataset onto a given path. Subsequently the ratios can be fitted by e.g. a Gumbel distribution in order to estimate the probability that an avalanche will run beyond a given location in that path (McClung, Mears, and Shaerer, 1989).

α/β-models, introduced by Lied and Bakkehøi (1980), amount to a *type 2 topographical method*. In this case we use a given dataset of runout lengths in order to obtain by regression a linear relationship between the expected α-angle of an avalanche, i.e. the average slope of the avalanche path to the outer end of the avalanche deposit, and the β-angle of the path, i.e. the average slope of the path to the foot of the path where the slope is 10°. The relationship is such that the variance between recorded and expected α-angles is minimized on the assumption that they are normally distributed. Having obtained this relationship we can use the deviation between expected and recorded α-angle to transfer avalanches from the dataset onto a given path, and it seems natural to retain the assumption that the deviations are normally distributed.

We are not aware of any *type 3 topographical method* apart from a method that *Guðmundur Guðmundsson* (1996), a statistician/physicist at the Central Bank of Iceland, has been developing. By solving an appropriate differential equation, with a number of path-independent parameters, along a prescribed path he obtains directly the probability distribution function of possible runout lengths in that path. The values of the free parameters are obtained by maximizing the likelihood that the runout lengths of a dataset agree with the calculated distribution functions of each path in the set.

PHYSICAL TRANSFER METHODS

We have, ourselves, been developing physical transfer methods of different types (Jónasson and Arnalds, 1997). All these methods have so far been based on the simple *PCM-model* for avalanche flow, with two free parameters, the Coulomb resistance parameter μ and the mass-to-drag parameter *M/D* (Perla, Cheng, and McClung, 1980). It should be noted, however, that we separate the curvature term, κ, from the *M/D*-term in the PCM-model, i.e. the differential equation of the model is

$$\frac{1}{2}\frac{d}{ds}(u^2) = g(\sin\theta - \mu\cos\theta) - \left(\mu\kappa + \frac{1}{M/D}\right)u^2$$

where u denotes the speed of the avalanche, s the distance along the path, θ its slope, and g the gravitational acceleration. In our numerical solution we use an appropriate smoothing procedure so that the path has a continuous curvature rather than being composed of straight line segments.

Firstly, we have considered a *type 2 method*, analogous to the α/β-method. The only difference between the methods lies in the estimate of the expected α-angle of an avalanche in a path. Here it is obtained by running the PCM-model on the path using the values of the parameters μ and *M/D* that minimize the variance between recorded and expected a-angles on the assumption that they are normally distributed. Since calculated runout-lengths and hence α-angles are nonlinear functions of μ and *M/D*, depending on the path, the estimate of the most likely parameter values is, however, computationally more complicated than for the α/β-method. We refer to this method as the *PCM-2 method*.

Secondly, we have developed a method, where we make an assumption on the statistical distribution of the *input* parameters, μ and *M/D*, to the PCM-model, rather than on the *output* value, i.e. the runout length or the corresponding α-angle. This we consider as a more natural approach and we have thus abandoned the PCM-2 method. It is included here mainly for comparison purposes. We have assumed that the input parameters obey a truncated bivariate normal distribution. A central difficulty is that with two parameters different input pairs can account for the same runout length. This difficulty can be dealt with, at some computational expense, in a consistent manner, allowing us to obtain the parameters of the distribution by a maximum likelihood criterion for a given dataset. Having obtained this distribution we can subsequently, for a given path, calculate the corresponding distribution of possible runout lengths. Thus we have a *type 3 method* that we refer to as the *PCM-3 method*.

Finally, we have developed a method where we restrict the input parameters, μ and *M/D*, of the PCM-model to lie on a given axis in the parameter plane. With this restriction only one input pair can account for a given runout length in a path. We can use this pair to directly transfer the runout length to another path, i.e. the transferred runout length will be that calculated from the PCM-model using the same input pair. Thus we have a *type 1 method*. Having transferred all the avalanches of the dataset to the path under consideration we use a kernel method to calculate a non-parametric probability density function of possible runout lengths. We refer to this the *PCM-1 method*.

The PCM-1 method is computationally considerably more efficient than the PCM-3 method. It also has the advantage, in terms of risk assessment, that because each avalanche is assigned a particular pair of input values one may calculate the speed of the avalanche along the path using the PCM-model. On the other hand, we may have extra information on avalanches in the dataset, e.g. on the thickness in the starting zone or the speed at some point, that restricts the choice of input values that will account for the runout length according to the PCM-model. This can be taken into consideration in a consistent way in the PCM-3 method but not the PCM-1 method. The question also remains with the PCM-1 method as how to choose the parameter axis. We have adopted the approach to choose the principal axis of the bivariate normal distribution associated with the PCM-3 method.

THE STANDARD PATH AND RUNOUT INDICES

In order to make a comparison between transfer methods we introduce a *standard path* that is representative for the Icelandic avalanche paths. It is parabola shaped, 700 m high and

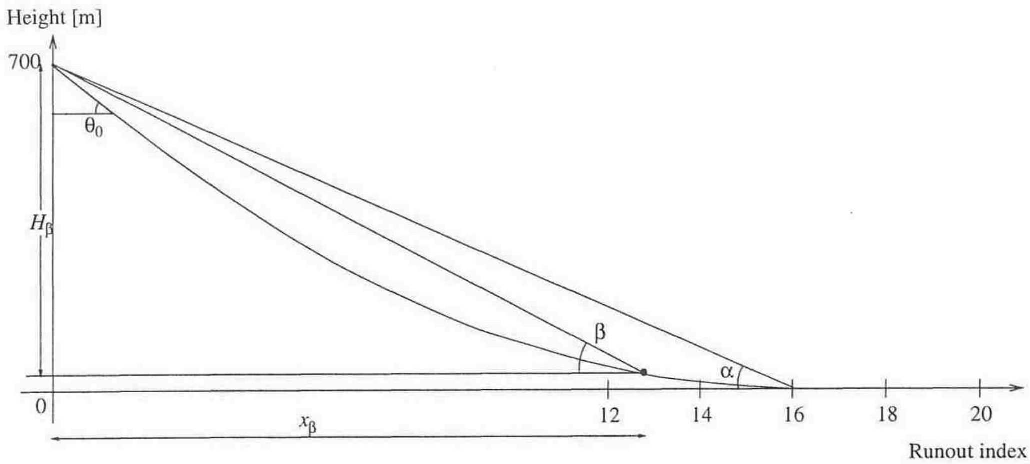

Figure 1. The standard slope. The α–angle is the expected α-angle of an avalanche according to the α/β-model.

reaches level ground at 1600 m from the starting point with 0° slope, and extends horizontally beyond that (Jónasson and Arnalds, 1997). The distance from the starting point to the β-point, x_β, is 1278 m and the β-angle is 27.7°. The height of the path from the β-point to the starting point, H_β, is 672 m. The average slope between the heights of 600 and 700 m, θ_0, is 40.1°. This slope is shown in fig. 1. We transfer all the avalanches to this standard slope by the different transfer methods.

We have used this same standard slope in connection with the physical transfer methods in order to associate with each avalanche, or each location in an avalanche path, a so-called *runout index*. It is defined as the horizontal length of the transferred avalanche in the standard slope, measured in hundreds of meters. The runout index is thus a path independent measure of the length of an avalanche, but it will depend on the transfer method and the underlying dataset. Thus we can e.g. refer to the PCM-1 runout index or the PCM-3 runout index. It is equally applicable to topographical transfer methods and we shall make use of this measure in our comparisons below.

When applying transfer methods one has to distinguish between the cases when the dataset only includes the longest recorded avalanche in each path and when it may include more than one "long" avalanche in each path. In our development of physical transfer methods we have been using a dataset of 196 "long" avalanches in Iceland in 53 different paths. 16 of the paths have 1 avalanche, 8 paths 2, 6 paths 3, 4 paths 4, 7 paths 5, 3 paths 6, 3 paths 7, and 2 paths have 8, 9 and 11 avalanches resp. Since runout ratios and α/β-models are usually connected with datasets of longest recorded avalanches we base the corresponding transfer methods on a reduced dataset of 44 such Icelandic avalanches, developed by Tómas Jóhannesson (1998). In this reduced set some paths have been omitted and information on other paths has been revised. The exceedance probability, p, based on a dataset of "long" avalanches may, however, be related to the eceedance probability, p_l, based on a corresponding dataset of longest avalanches, by the following formula

$$p_l = 1 - \sum_{i=1}^{11} \gamma_i (1-p)^i. \qquad (1)$$

if we make the (admittedly wrong) assumption that all the paths in the dataset have been observed for the same length of time, and γ_i denotes the proportion of paths in the set for which there will, on average, be i "long" avalanches in that period of time. Our reason for working with a dataset of "long" avalanches, rather than the longest avalanches, is that we find it more useful in terms of risk assessment.

One also has to note that in the Icelandic dataset there are 39 avalanches that have fallen into the sea. 20 of these are also in the reduced set. This has been accounted for in a consistent manner within the maximum likelihood framework for all the transfer methods.

COMPARISON OF TRANSFER METHODS

We compare in this section the topographical transfer methods based on runout ratios and α/β-models and the physical PCM-1 and PCM-3 models. The topographical method of Guðmundur Guðmundsson is not included since the full details of it have not been published and we exclude the PCM-2 method since we have abandoned that approach.

For the runout-ratios we use a Gumbel distribution with the probability distribution function

$$D(r) = \exp(\exp(-(r-0.2)/0.17))$$

obtained by Tómas Jóhannesson (1998) from the reduced Icelandic dataset. Our α/β-model is

$$\alpha = 0.85 \cdot \beta \quad \text{with} \quad \sigma_{\Delta\alpha} = 2.3°$$

also obtained by Tómas Jóhannnesson (1998) from the same set. The centre of the bivariate normal distribution in the PCM-3 method is at

$$\mu = 0.23 \text{ and } M/D = 322 \quad \text{with} \quad \sigma_\mu = 0.096 \text{ and } \sigma_{M/D} = 121$$

and the equation of its principal axis is

$$(\mu, M/D) = (0.23, 322) + t \cdot (1, -711).$$

This is also the parameter axis of the PCM-1 method.

Firstly, we show in tables 1 and 2 those avalanches from the Icelandic dataset that have the ten highest runout indices according to each transfer method. The tables include topographical information i.e. the horizontal length of the avalanche, x, information on whether the avalanche has reached

Table 1. Icelandic avalanches with the ten highest runout indices according to four different transfer methods. Location and topographical parameters. Two avalanches that are virtually identical to avalanches no. 76 and 88 have been omitted from the table.

Aval. no.	Year	Location	Path	x	Sea	α	β	x_β	h_β	θ
125	1995	Flateyri	Skollahvilft	1927		18	24	1375	604	39
102	1974	Flateyri	Innra-Bæjargil	1558		22	28	1146	596	39
103	1953	Flateyri	Skollahvilft	1732		20	24	1375	604	39
41	1994	Ísafjörður	Tunguskógur	2027		17	19	1807	615	36
127	1936	Neskaupstaður	Innri-Sultarbotnagjá	1980		19	25	1376	610	36
117	1995	Flateyri	Innra-Bæjargil	1478		23	28	1146	596	39
29	1995	Súðavík	Traðargil	1712	x	18	21	1398	518	32
76	1916	Hnífsdalur	Bakkagil	1285		24	30	1061	562	40
126	1990	Neskaupstaður	Gunnólfsskarð	1713		18	22	1259	516	40
30	1995	Súðavík	Súðavíkurhlíð	1282		24	30	973	530	37
130	1885	Neskaupstaður	Ytri-Sultarbotnagjá	1942	x	21	26	1442	665	35
62	1962	Ísafjörður	Kubbi	549		22	28	501	215	36
88	1906	Patreksfjörður	Vatneyri	666	x	24	30	554	272	43

Table 2: Icelandic avalanches with the ten highest runout indices according to four different transfer methods. Runout indices and ranking order.

Aval. no.	Runout index (ranking order) according to: α/β-model	Runout r.	PCM-3	PCM-1
125	18.0 (2)	18.2 (2)	18.3 (1)	18.6 (1)
102	17.6 (3)	17.8 (6)	16.8 (2)	17.0 (2)
103	16.4 (13)	16.4 (21)	16.6 (3)	16.8 (3)
41	15.5 (31)	14.7 (69)	16.5 (4)	16.7 (4)
127	17.4 (4)	19.0 (1)	16.0 (8)	16.4 (5)
117	16.8 (10)	16.8 (10)	16.2 (5)	16.3 (6)
29	16.0 (21)	16.4 (23)	15.9 (10)	16.2 (7)
76	17.0 (8)	16.7 (11)	16.0 (6)	16.1 (8)
126	16.4 (14)	17.4 (9)	15.6 (15)	16.1 (10)
30	17.4 (5)	17.6 (8)	15.9 (9)	16.1 (12)
130	16.7 (11)	17.8 (5)	15.5 (17)	15.9 (15)
62	18.0 (1)	17.7 (7)	14.3 (49)	14.3 (57)
88	17.0 (6)	17.8 (3)	13.7 (76)	13.7 (81)

the sea, the α-angle of the avalanche, the β-angle of the path, the distance to the β-point, x_β, the height of the path from the β-point to the starting point, H_β, and the initial slope, θ_0. This is followed by the runout index of the avalanche for each transfer method, and, within parentheses, its order in length according to these indices.

For further clarification we show in table 3, for runout indices varying from 12 to 20, corresponding to locations in the standard path varying from 1200 to 2000 m, the α-angle of the location and the probability of an avalanche running beyond the location for each transfer method. For the physical transfer methods we also show the corresponding exceedance probability for a dataset of longest avalanches according to formula (1), making the somewhat crude approximation of calculating the γ_i-coefficients directly from the actual frequency values, given above, for the paths in the Icelandic data set.

Secondly, we show in table 3 crossplots between runout indices of all the avalanches in the Icelandic dataset for all pairs of transfer methods. We omit crossplots between runout indices for the PCM-3 method on one hand and the topographical methods on the other because they are essentially the same as the corresponding crossplots for the PCM-1 method.

DISCUSSION

Not surprisingly, table 2 and figure 2 reveal a close agreement between the PCM-1 and the PCM-3 method and also a reasonable agreement between runout ratios and α/β-methods. The difference in exceedance probabilities between the PCM-1 method and the PCM-3 method brought out in table 3 can to

Table 3. Probability of an avalanche exceeding a given runout index according to four different transfer methods. Values of first two methods based on a reduced set of 44 Icelandic longest avalanches. Values of last two methods based on a set of 196 Icelandic "long" avalanches. Values within parentheses, transformed values comparable to values based on longest avalanches.

Runout index	α	α/β-model	Runout ratio	PCM-3 method	PCM-1 method
12.0	28.7	0.987	0.990	0.731 (0.905)	0.753 (0.914)
12.5	28.1	0.975	0.975	0.643 (0.866)	0.674 (0.880)
13.0	27.5	0.954	0.947	0.548 (0.816)	0.589 (0.838)
13.5	26.8	0.922	0.903	0.448 (0.751)	0.501 (0.787)
14.0	26.2	0.874	0.843	0.351 (0.671)	0.418 (0.729)
14.5	25.6	0.808	0.770	0.261 (0.574)	0.340 (0.661)
15.0	24.9	0.723	0.689	0.183 (0.461)	0.268 (0.582)
15.5	24.3	0.622	0.604	0.120 (0.342)	0.202 (0.492)
16.0	23.6	0.510	0.521	0.071 (0.225)	0.144 (0.391)
16.5	23.0	0.400	0.443	0.038 (0.128)	0.097 (0.289)
17.0	22.4	0.302	0.372	0.018 (0.065)	0.061 (0.197)
17.5	21.8	0.221	0.309	0.008 (0.030)	0.037 (0.125)
18.0	21.3	0.157	0.254	0.003 (0.011)	0.022 (0.076)
18.5	20.7	0.108	0.208	0.001 (0.004)	0.013 (0.046)
19.0	20.2	0.073	0.169	0.000 (0.001)	0.007 (0.027)
19.5	19.7	0.048	0.137	0.000 (0.000)	0.004 (0.015)
20.0	19.3	0.031	0.110	0.000 (0.000)	0.002 (0.007)

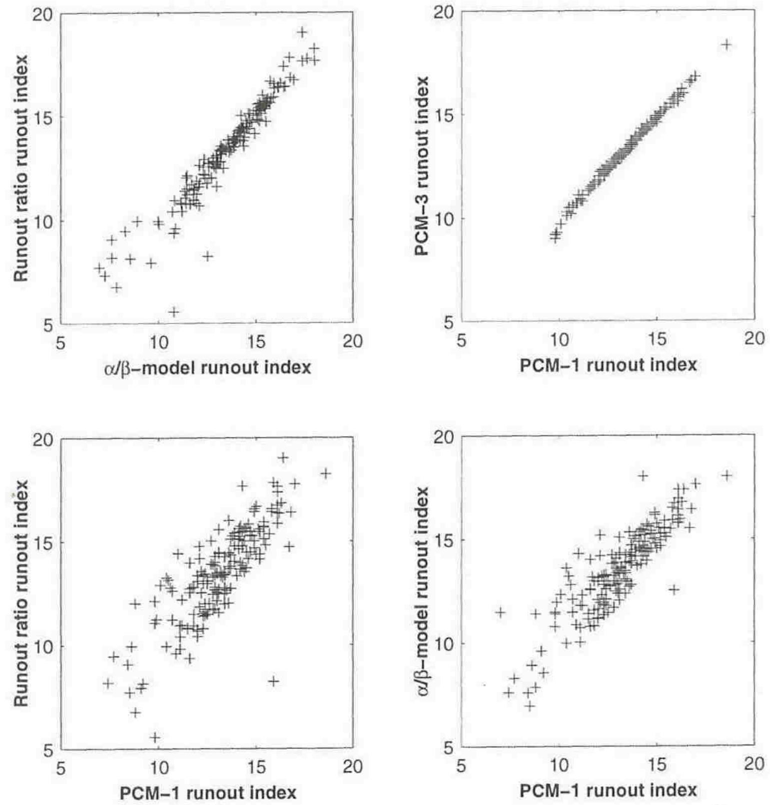

Figure 2. Crossplots between runout indices of all the avalanches in the set of 196 Icelandic "long" avalanches according to different transfer methods.

a large extent be explained by the choice of the truncated bivariate normal distribution for the PCM-3 method and the choice of kernels in the PCM-1 method. The difference in the exceedance probabilities between the transfer methods based on runout ratios and those based on α/β-models relates more to the form of the underlying dataset (cf. Jóhannesson, 1998).

There is a considerable discrepancy between the topographical transfer methods and the physical ones, both when one looks at the relative order of runout indices and at the exceedance probabilities, which is a matter of some concern in terms of risk assessment. Admittedly such assessment is to some extent more dependent on how quickly the exceedance probability falls as a function of distance or runout index rather than on the actual values themselves, but table 3 shows that even in this respect there is a considerable discrepancy between the methods. It should be noted, however, in tables 1 and 2 that avalanches no. 62 and 88 which are high in the ranking order according to the topographical transfer methods compared to the physical transfer methods are avalanches in low paths and avalanche no. 41 for which the converse is true is in a path with a distinctive plateau.

It is tempting to try to devise a maximum likelihood criterion that allows one to measure the merits of different transfer methods for a given dataset. There is an inherent difficulty related to the fact that such measures will depend on the choice of length scale, which is in turn an integral part of each transfer method. Despite this difficulty we believe that such measures would prove to be important in choosing between transfer methods. It is even possible that further investigation will yield a natural length scale. For the time being, however, the conclusion is that in risk assessment based on transfer methods it is advisable to take more than one method into account.

REFERENCES

Guðmundsson, G. 1996. *A topographical model for avalanches.* Talk presented at a work meeting on snow avalanches at the Icelandic Meteorological office 11–12 January 1996. Unpublished.

Jóhannesson, T. 1998. *A topographical model for Icelandic avalanches.* Icelandic Meteorological Office, Report VÍ-G98003-ÚR03, Reykjavík.

Jónasson, K. and Þ. Arnalds. 1997. *A method for avalanche risk assessment.* Short description. Icelandic Meteorological Office. Report VÍ-G97036-ÚR28, Reykjavík.

Lied, K. and S. Bakkehøi. 1980. Empirical calculations of snow-avalanche run-out distance based on topographical parameters. *J. Glacio.*, **26**(94), 165–177.

Lied, K. 1993. *Snow avalanche experience through 20 years.* Laurits Bjerrums Minnefond. Norwegian Geotechnical Institute. Oslo.

McClung, D. and K. Lied. 1987. Statistical and geometrical definition of snow avalanche runout. *Cold Reg. Sci. and Technol.*, **13**(2), 107–119.

McClung, D. M., A. I. Mears and P. Schaerer. 1989. Extreme avalanche run-out: data from four mountain ranges. *Ann. of Glaciol.*, **13**, 180–184.

Perla, R., T. T. Cheng and D. M. McClung. 1980. A two-parameter model of snow-avalanche motion. *J. Glaciol.*, **26**(94), 197–207.

Salm, B., A. Burkard and H. Gubler. 1990. *Berechnung von Fliesslawinen. Eine Anleitung für Praktiker mit Beispielen.* Mitteilungen des Eidgenossischen Instituts für Schnee- und Lawinenforschung, No. 47.