Journal of Official Statistics, Vol. 39, No. 1, 2023, pp. 123–137, http://dx.doi.org/10.2478/JOS-2023-0006

A Two-Stage Bennet Decomposition of the Change in the Weighted Arithmetic Mean

Thomas von Brasch[1], *Håkon Grini*[1], *Magnus Berglund Johnsen*[1], *and Trond Christian Vigtel*[1]

The weighted arithmetic mean is used in a wide variety of applications. An infinite number of possible decompositions of the change in the weighted mean are available, and it is therefore an open question which of the possible decompositions should be applied. In this article, we derive a decomposition of the change in the weighted mean based on a two-stage Bennet decomposition. Our proposed decomposition is easy to employ and interpret, and we show that it satisfies the difference counterpart to the index number time reversal test. We illustrate the framework by decomposing aggregate earnings growth from 2020Q4 to 2021Q4 in Norway and compare it with some of the main decompositions proposed in the literature. We find that the wedge between the identified compositional effects from the proposed two-stage Bennet decomposition and the one-stage Bennet decomposition is substantial, and for some industries, the compositional effects have opposite signs.

Key words: Index theory; weighted arithmetic mean; decomposition.

1. Introduction

What are the driving forces underlying aggregate productivity growth? Why has the labour force participation rate changed during the last two decades? What has driven the change in annual earnings over the last year and why have import prices changed? All these questions have a common feature in that statistics on productivity, the labour force participation rate, earnings and import prices are often constructed using a weighted arithmetic mean formula.

A natural starting point for answering these questions is to decompose the change in the weighted mean. A frequently used decomposition is the Bennet (1920) decomposition, often also referred to as shift-share analysis. This decomposition enables within-group growth effects to be distinguished from between-group compositional effects. For example, when examining productivity dynamics in U.S. manufacturing plants between 1972 and 1987, Baily et al. (1992) find a positive contribution to growth due to increasing output shares among high-productivity plants and decreasing output shares among low-productivity plants. Daly and Hobijn (2017) show that compositional effects due to labour market status flows are important in explaining aggregate real wage growth in the U.S.

[1] Statistics Norway, Post Box 2633, Oslo 0131, Norway. E-mails: thomas.vonbrasch@ssb.no, knut.grini@ssb.no, magnus.johnsen@ssb.no, and trond.vigtel@ssb.no

Acknowledgments: We thank Bert M. Balk, Pål Boug, W. Erwin Diewert, Terje Skjerpen and three anonymous referees for valuable comments. In particular, thanks to Bert M. Balk for pointing out that we could simplify the proof of Proposition 1 and to W. Erwin Diewert for providing us with a comprehensive analysis of the alternative and approximate decomposition that we compare with our proposed exact two-stage Bennet decomposition. The usual disclaimer applies.

Analysing the fall in the U.S. labour force participation rate, Krueger (2017) finds that the population composition has shifted toward groups with lower participation rates, and that this accounts for well over half of the decline in the labour force participation rate between 1997 and 2017. Moreover, a large body of literature has identified the deflationary effects of international trade resulting from increased import shares from low-price countries, such as China; see, for example, Kamin et al. (2006), Thomas and Marquez (2009) and Benedictow and Boug (2017, 2021).

Although the Bennet decomposition is useful for identifying the overall contribution from compositional effects, it does not identify how much of the overall compositional effect that can be attributed to a particular group or subset. To overcome this shortcoming, the Bennet decomposition is often rewritten by subtracting a scalar A from each group in the between effect, where the scalar A typically represents some measure of the weighted mean, see Huerga (2010) and Balk (2021, 137). Foster et al. (2001) analyzed productivity developments and measured the between effect as the product of changes in the plant-level output share and the deviation of average plant-level productivity from the overall industry average. If the composition of firms changes such that the output share of a low-productivity plant increases, this will lower the aggregate weighted mean productivity level and thus contribute negatively to the compositional effect. Note that in these decompositions a plant may contribute negatively to the compositional effect even if there is no change in the output of that plant. The reason is that it is the output share, and not the output of the plant, that enters the decomposition, and the output share of a given plant may change because the output of all the other plants changes. Moreover, as pointed out by Balk (2021), the choice of the scalar A is arbitrary. Since any scalar may be subtracted from the Bennet decomposition, an infinite number of possible decompositions are available, and it is therefore an open question which of the possible decompositions should be applied.

In this article, we derive a decomposition that identifies the contribution to the overall change in the weighted mean from changes in prices and quantities. In contrast to the previously discussed one-stage Bennet decomposition, which relates to changes in weights, our proposed two-stage Bennet decomposition relates to the change in the underlying quantity variables. This difference is important because the weights can never change without a change in at least one of the underlying quantity variables. Moreover, a change in a single quantity variable will affect all the share variables. Therefore, examining the shares provides only limited information about the underlying driving forces. In the two-stage Bennet decomposition, all the terms related to the within effects are identical to those in the one-stage Bennet decomposition. Also, the overall between effect, or compositional effect, is identical to the overall between effect in the one-stage Bennet decomposition. The group-specific between effects, however, differ from those in the one-stage Bennet decomposition. The decomposition captures the intuitive property that the weighted mean increases if a group whose quantity variable is growing has a level that is above the weighted mean level. There are two ways in which compositional effects for a group will be zero: either the group-specific indicator equals the weighted mean level, and/or there is no change in the quantity variable of that group. The proposed decomposition is easy to employ and interpret and furthermore gives a better platform for comparing groups. Moreover, we show that the decomposition is invariant with respect to treatment of time, and that it therefore satisfies the difference counterpart to the index number time reversal test; see ILO et al. (2004, 411).

To provide further support for the two-stage Bennet decomposition, we also compare it with a decomposition based on a quadratic approximation of the weighted mean, while considering the weighted mean as a non-linear function of underlying prices and quantities. It is shown that this alternative decomposition has some similar features to the two-stage Bennet decomposition. In particular, it also identifies how the weighted mean price level increases if products that are growing in quantity have price levels that are higher than the mean price level. However, this alternative decomposition is not exact, and the size of the approximation error depends on how much prices, quantities, and weights change.

To illustrate the two-stage Bennet decomposition, we use data on aggregate earnings growth in Norway between 2020Q4 and 2021Q4. In this empirical example, it is shown that the wedge between the identified compositional effects from the two-stage and one-stage Bennet decompositions is substantial, and for some industries, the compositional effects are of opposite signs. We also compare the two-stage Bennet decomposition with the decomposition based on a quadratic approximation. These two decompositions yield similar group-specific contributions to the overall compositional effects.

The article is structured as follows: Section 2 outlines the weighted mean formula, some of the most standard decompositions applied in the literature, our proposed decomposition and the decomposition based on a quadratic approximation. Section 3 contrasts and compares empirically our proposed decomposition with those used in the literature and the decomposition based on a quadratic approximation, using the case of earnings growth in Norway. Section 4 provides a conclusion.

2. Decomposing the Change in the Weighted Mean

Our point of departure is the weighted mean of indicators P_{it} across units i at time t of the form:

$$P_t = \sum_{i=1}^{N} S_{it} P_{it}, \tag{1}$$

with weights $S_{it} = \frac{X_{it}}{\sum_{j=1}^{N} X_{jt}}$, where the quantity variable $X_{it} \geq 0$ and $\sum_{j=1}^{N} X_{jt} > 0$. Note that the weights sum to unity. The weighted mean in Equation (1) has numerous applications within the fields of economics and measurement theory. In some applications, the weighted mean is also referred to as a unit value. To our knowledge, unit values were first introduced by Segnitz (1870). Although the weighted mean has been applied in a variety of fields, and the indicator and quantity variables may refer to "inter alia" wages, hours worked, productivity, output prices and so on, we will henceforth refer to P_{it} and X_{it} as representing prices and quantities, respectively, and unit i as product i. Note that for the weights to have a meaningful interpretation, the quantities involved must be comparable. In the following we are concerned with identifying the contribution to the change in the weighted mean of changes in both prices and quantities. Before we present our proposed decomposition, we start by recapitulating the most widely utilized decompositions in the literature.

2.1. The One-Stage Bennet Decomposition

Bennet (1920) provided a decomposition of the nominal value change into the sum of a price change and a quantity change. This decomposition stands in contrast to traditional index theory, which focuses on decomposing a value ratio into the product of a price index and a quantity index. Diewert (2005) analyzed the axiomatic and economic properties of the Bennet decomposition. When applied to Equation (1), the decomposition yields the identity:

$$\Delta P = \sum_{i=1}^{N} \bar{S}_i \Delta P_i + \sum_{i=1}^{N} \bar{P}_i \Delta S_i, \qquad (2)$$

where Δ is the difference operator and a bar over a variable represents the moving average operator between time t and v, that is, $\Delta x = x_t - x_v$ and $\bar{x} = 1/2(x_t + x_v)$, and the time subscript is dropped when it is superfluous, for notational convenience. This one-stage Bennet decomposition is standard in productivity and shift-share analysis, see for example, Baily et al. (1992) and OECD (2018). The terms $\bar{S}_i \Delta P_i$ and $\bar{P}_i \Delta S_i$ represent the contributions to the change in the weighted mean of a change in the price of product i, and in the quantity share of product i, respectively.

2.2. The One-Stage Bennet Decomposition with Extended Weight

The one-stage Bennet decomposition may be rewritten to create terms that explicitly capture the fact that the weighted mean price level increases if the quantity shares of high-priced products increase. Since the weights sum to unity, we can subtract the term $\sum_{i=1}^{N}(A\Delta S_i)$, for any given scalar A, such that:

$$\Delta P = \sum_{i=1}^{N} \bar{S}_i \Delta P_i + \sum_{i=1}^{N} (\bar{P}_i - A) \Delta S_i. \qquad (3)$$

In this case, the contribution to the change in the weighted mean of a change in the weight of product i is given by the term $(\bar{P}_i - A)\Delta S_i$. For example, this term captures the increase in the weighted mean price if products whose quantity shares are growing have an average price level between time v and t that is larger than the scalar A. In the literature, the scalar A has often been chosen to represent some measure of the mean price level. For example, the choice $A = \bar{P}$, where $\bar{P} = 1/2(P_t + P_v)$, is often used, see Huerga (2010) and Balk (2021, 137). Note that when $A = \bar{P}$ is chosen, the framework above is invariant with respect to treatment of time, that is, it satisfies the difference counterpart to the index number time reversal test; see Diewert and Fox (2010). The contribution to the change in the weighted mean of a change in the share of product i is then given by $(\bar{P}_i - \bar{P})\Delta S_i$. This term captures the fact that the weighted mean price level increases if products whose quantity shares are growing have a price level that is higher than the weighted mean price level. Conversely, the term also captures the fact that the weighted mean price level will decrease if products whose quantity shares are growing have a price level that is lower than the weighted mean price level.

There are shortcomings related to the two decompositions above if the purpose is to identify how much of the overall compositional effect that can be attributed to a particular group or subset. First, and as argued by Balk (2021), the choice of the scalar A is completely arbitrary. Second, the weight of product i (S_{it}) may change even if there is no change in the quantity of product i (X_{it}), that is, if there is a change in the sum of all the other products. Since the change in the weight of product i may reflect more than just changes in the quantity of product i, neither the one-stage Bennet decomposition nor the one-stage Bennet decomposition with extended weight identifies the contributions to the change in the weighted mean that are due to changes in quantities.

2.3. *The Two-Stage Bennet Decomposition*

To create a decomposition that identifies the contributions to the change in the weighted mean from changes in quantities, we apply the Bennet decomposition also in a second stage. Using the weights $S_{it} = \frac{X_{it}}{\sum_{j=1}^{N} X_{jt}}$ and defining $Q_t = \sum_{j=1}^{N} X_{jt}$, the expression for the quantity variable X_{it} can be written as:

$$X_{it} = S_{it} Q_t, \tag{4}$$

with the change defined as $\Delta X_i = X_{it} - X_{iv}$. Although S_{it} and Q_t cannot vary independently, we apply the Bennet decomposition to the change in the quantity variable in Equation (4), which yields the identity:

$$\Delta X_i = \bar{S}_i \Delta Q + \bar{Q} \Delta S_i, \tag{5}$$

where $\Delta Q = Q_t - Q_v$ and $\bar{Q} = (Q_t + Q_v)/2$. Solving Equation (5) for ΔS_i gives:

$$\Delta S_i = \frac{1}{\bar{Q}} \left(\Delta X_i - \bar{S}_i \Delta Q \right) \tag{6}$$

Inserting Equation (6) into Equation (2) and collecting terms yields the following exact decomposition of the change in the weighted mean:

Proposition 1 (Two-Stage Bennet Decomposition of the Change in the Weighted Mean) Consider the weighted mean across units i at time t of the form: $P_t = \sum_{i=1}^{N} S_{it} P_{it}$, with weights $S_{it} = \frac{X_{it}}{\sum_{j=1}^{N} X_{jt}}$, where $X_{it} \geq 0$ and $Q_t = \sum_{j=1}^{N} X_{jt} > 0$. The change in the weighted mean between times t and v can be exactly decomposed as

$$\Delta P = \sum_{i=1}^{N} \bar{S}_i \Delta P_i + \sum_{i=1}^{N} \left(\frac{1}{\bar{Q}} \right) \left(\bar{P}_i - \bar{\bar{P}} \right) \Delta X_i \tag{7}$$

where $\bar{\bar{P}} = \sum_{i=1}^{N} \bar{S}_i \bar{P}_i$, Δ is the difference operator and a bar over a variable represents the moving average operator between times t and v, that is, $\Delta x = x_t - x_v$ and $\bar{x} = 1/2\,(x_t + x_v)$.

Two features of the two-stage Bennet decomposition in Proposition 1 merit attention. First, the term that shows the contribution to the change in the weighted mean from the change in the quantity of product i is given by

$$\left(\frac{1}{\bar{Q}} \right) \left(\bar{P}_i - \bar{\bar{P}} \right) \Delta X_i. \tag{8}$$

This term differs from that in the one-stage Bennet decomposition. It has a natural interpretation and captures the intuitive property that the weighted mean price level increases if products that are growing in quantity have price levels that are higher than the mean price level. $\bar{P}_i - \bar{\bar{P}}$ compares the price level of product i with a measure of the weighted mean price level $\bar{\bar{P}} = \sum_{i=1}^{N} \bar{S}_i \bar{P}_i$. There are thus two ways in which the compositional effects of product i can equal zero: the price of product i equals the weighted average price level, and/or there is no change in the quantity of product i.

Second, the two-stage Bennet decomposition does not hold a time subscript. In other words, the framework is invariant with respect to treatment of time and it therefore satisfies the difference counterpart to the index number time reversal test. The time reversal test for indices states that if the data for the two time periods are interchanged, then the resulting formula should equal the reciprocal of the original index; see, for example, ILO et al. (2004, 295). This test can be rephrased in the case where the formula is in the form of differences, such as the decomposition in Proposition 1: if the data for the two time periods are interchanged, then the resulting formula should equal the negative of the original formula. To illustrate this analytically, let the function $H(\mathbf{P}_t, \mathbf{P}_v, \mathbf{X}_t, \mathbf{X}_v)$ represent the formula for decomposing the change in the weighted mean, where $\mathbf{P}_t = (\mathbf{P}_{1t}, \mathbf{P}_{2t}, \ldots, \mathbf{P}_{Nt})$ and $\mathbf{X}_t = (x_{1t}, x_{2t}, \ldots, x_{Nt})$. The function H passes the time reversal test if and only if $H(\mathbf{P}_t, \mathbf{P}_v, \mathbf{X}_t, \mathbf{X}_v) = -H(\mathbf{P}_v, \mathbf{P}_t, \mathbf{X}_v, \mathbf{X}_t)$. The proposed decomposition in Proposition 1 satisfies this counterpart to the time reversal test.

We commented above on the practice in the literature of choosing a scalar A when decomposing the change in the weighted mean, see Equation (3). Although the choice of A is arbitrary, it is nevertheless interesting to see whether it is possible to derive a value for A that is consistent with the two-stage Bennet decomposition. From Equation (3), the contribution to the change in the weighted mean due to a change in the quantity share of product i is given by the term $(\bar{P}_{it} - A)\Delta S_{it}$. In the two-stage Bennet decomposition, the contribution to the change in the weighted mean due to a change in the quantity of product i is given by the term $\left(\frac{1}{\bar{Q}}\right)(\bar{P}_{it} - \bar{\bar{P}}_t)\Delta X_i$. For these terms be equal, the scalar A must be given by:

$$A = \bar{\bar{P}} - \left(\frac{\Delta Q/\bar{Q}}{\Delta S_i/\bar{S}_i}\right)(\bar{P}_{it} - \bar{\bar{P}}) \tag{9}$$

However, the right-hand side of the equality sign is usually not a constant. This feature stands in contrast to Equation (3), where the property that A is a scalar and independent of i is central to deriving Equation (3) from Equation (2). In the case where the aggregate quantity is unchanged, that is, $\Delta Q = 0$, Equation (9) reduces to $A = \bar{\bar{P}}$, which is independent of i. Moreover, in this case the value of A is close to the choices commonly used in the literature. Several values for the scalar A have been applied, most frequently P_t, P_v and the average of the two, all of which are close to the average measure $\bar{\bar{P}}_t$. However, when the aggregate quantity changes, $\Delta Q \neq 0$, the factor $\left(\frac{\Delta Q/\bar{Q}}{\Delta S_i/\bar{S}_i}\right)$ may differ from zero, possibly leaving a sizable discrepancy between the two-stage Bennet decomposition and most common decompositions used in the literature. In particular, and as can be seen from Equation (5), ΔX_i may have the opposite sign to ΔS_i, depending on how much the aggregate quantity (Q) changes. As a result, the measured contributions from compositional effects in Equation (3) and the two-stage Bennet decomposition may have opposite signs. In the empirical section, we examine in depth how large the discrepancy between the two decompositions may be in practice when aggregate earnings growth in Norway is decomposed. Before we embark on the empirical application, we first examine an alternative way to decompose the change in the weighted mean that has many similarities with the two-stage Bennet decomposition.

2.4. An Approximate Decomposition of the Change in the Weighted Mean

An alternative way to decompose the change in the weighted mean is to apply the quadratic approximation lemma (QAL) which, loosely defined, states that the average of two first order approximations is equivalent to a second order approximation. According to Theil (1975, 38), the quadratic approximation lemma provides an "approximation which is as simple as the linear approximation and as accurate as the quadratic approximation", where the term "quadratic approximation" refers to a second-order Taylor approximation, see also Diewert (2002) and references therein for applications of QAL.

Consider the weighted mean as a non-linear function of underlying prices and quantities, that is, $P_t = f(\mathbf{P}_t, \mathbf{X}_t)$, see also Von Brasch et al. (2017). The first-order Taylor series approximation around the initial period (period v) values for the price and quantity variables can be expressed as:

$$\Delta^v \approx \sum_{i=1}^{N} S_{iv}\Delta P_i + \sum_{i=1}^{N}\left(\frac{1}{Q_v}\right)(P_{iv} - P_v)\Delta X_i \tag{10}$$

The first-order Taylor series approximation around the end period (period t) values for the price and quantity variables can be expressed as:

$$\Delta^t \approx \sum_{i=1}^{N} S_{it}\Delta P_i + \sum_{i=1}^{N}\left(\frac{1}{Q_t}\right)(P_{it} - P_t)\Delta X_i \tag{11}$$

Applying QAL to the weighted mean, that is, taking the arithmetic average of the two approximations in Equation (10) and Equation (11), yields:

$$\Delta P \approx \sum_{i=1}^{N} \bar{S}_i\Delta P_i + \sum_{i=1}^{N}\frac{1}{2}\left[\frac{1}{Q_t}(P_{it} - P_t) + \frac{1}{Q_v}(P_{iv} - P_v)\right]\Delta X_i \tag{12}$$

Like the two-stage Bennet decomposition, the decomposition based on QAL also holds the intuitive property that the weighted mean price level increases if products that are growing in quantity have price levels that are higher than the mean price level. However, in contrast to the two-stage Bennet decomposition, the prices are demeaned separately for each period and separately divided by their respective Q_t.

The decomposition based on QAL in Equation (12) is not exact. The size of the approximation error, measured by the difference between the change in the weighted mean and the right-hand side of Equation (12), is given by:

$$\sum_{i=1}^{N}\frac{1}{2}\left[\frac{1}{Q_t}P_{it} - \frac{1}{Q_v}P_{iv}\right]\Delta S_i\Delta Q \tag{13}$$

The smaller the change in Q, the smaller the approximation error. If either all weights or all prices are unchanged, the approximation error is zero. In the empirical application we consider the size of this approximation error and compare the decomposition based on QAL with the two-stage Bennet decomposition.

3. Empirical Application

The data used in the empirical application are obtained through the "a-ordning", which is a collaborative digital system shared by Statistics Norway, The Norwegian Tax Administration and the Norwegian Labour and Welfare Administration (NAV). It provides information about employment, remuneration in cash and in kind and taxes (The Norwegian Tax Administration 2022). Data for all industries and individuals are compiled monthly, and this is the main source Statistics Norway utilizes for producing statistics on earnings and the labour market.

We focus on the change in monthly earnings per full-time equivalent as the price variable from 2020Q4 to 2021Q4 and allow for compositional effects across industries using the number of full-time equivalents in each industry as the quantity variable. Table 1 shows the mean monthly earnings and the number of full-time equivalents in each industry and in the aggregate for 2020Q4 and 2021Q4.

Table 2 shows the results from using the one-stage Bennet decomposition in Equation (2), the one-stage Bennet decomposition with extended weight in Equation (3), the approximation in Equation (12) and our proposed two-stage Bennet decomposition in Proposition 1. As expected, the contribution to the change in the weighted mean from the change in earnings of each industry (and the aggregate) is identical across the four decompositions, as is the total compositional effect. We find that the wedge between the identified compositional effects from (1) the two-stage Bennet decomposition and (2) the one-stage Bennet decomposition and one-stage Bennet decomposition with extended weight is considerable, and for some industries, such as mining and quarrying and wholesale and retail trade, the compositional effects are of opposite signs. Such divergence in the signs of the compositional effects between the methods can be attributed to the changes in the share and quantity variables being of opposite signs (see Figure 2). Furthermore, the identified compositional effects in the one-stage Bennet decomposition generally have a greater absolute value than the compositional effects identified using the one-stage Bennet decomposition with extended weight and the two-stage Bennet decomposition. This is attributable to the use of $\bar{P}_i$ in the one-stage Bennet decomposition, compared to the use of the relative earnings level in the other two decompositions $\left(\left(\bar{P}_i - A\right) \text{ and } \left(\bar{P}_i - \bar{\bar{P}}\right)\right)$. The intuition for why the one-stage Bennet decomposition with extended weight and the two-stage Bennet decomposition yield somewhat similar results can be seen by rewriting the two decompositions in a manner that is easier to compare. The one-stage Bennet decomposition with extended weight may be written as

$$\Delta P = \sum_{i=1}^{N} \bar{S}_i \Delta P_i + \sum_{i=1}^{N} \left(\bar{P}_i - \bar{\bar{P}}\right) \left[\frac{X_{it}}{\Sigma_{j=1}^{N} X_{jt}} - \frac{X_{iv}}{\Sigma_{j=1}^{N} X_{jv}} \right]$$

Table 1. Monthly earnings per full-time equivalent and number of full-time equivalents, 2020Q4 and 2021Q4.

	2020Q4		2021Q4	
	Monthly earnings (NOK)	Number of full-time equivalents	Monthly earnings (NOK)	Number of full-time equivalents
All industries	48,750	2,242,706	50,790	2,320,214
Agriculture, forestry and fishing	41,880	24,838	43,830	25,664
Mining and quarrying	74,290	59,890	76,310	61,292
Manufacturing	49,090	197,800	50,940	201,393
Electricity, water supply, sewerage, waste management	54,450	32,012	57,430	32,846
Construction	46,110	221,718	47,730	228,745
Wholesale and retail trade; repair of motor vehicles and motorcycles	44,220	262,080	46,320	267,773
Transportation and storage	47,860	110,862	49,350	114,705
Accommodation and food service activities	33,340	60,593	34,380	72,623
Information and communication	63,270	93,626	65,980	100,882
Financial and insurance activities	70,360	45,928	73,670	46,890
Real estate, professional, scientific and technical activities	60,060	146,836	63,460	155,418
Administrative and support service activities	41,000	115,691	42,760	122,725
Public administration and defence; compulsory social security	52,210	157,396	54,240	159,831
Education	46,750	195,500	48,710	197,832
Human health and social work activities	44,860	446,720	47,010	455,729
Other service activities	43,830	70,212	45,360	74,466
Unspecified	63,440	1,003	68,490	1,403

Source: Statbank Table 11419, Statistics Norway.

In comparison, the two-stage Bennet decomposition may be written as

$$\Delta P = \sum_{i=1}^{N} \bar{S}_i \Delta P_i + \sum_{i=1}^{N} \left(\bar{P}_i - \bar{\bar{P}}\right) \left[\frac{X_{it}}{\left(\Sigma_{j=1}^{N} X_{jt} + \Sigma_{j=1}^{N} X_{jv}\right)/2} - \frac{X_{iv}}{\left(\Sigma_{j=1}^{N} X_{jt} + \Sigma_{j=1}^{N} X_{jv}\right)/2} \right]$$

The difference between the contributions to the overall compositional effects in these two decompositions is thus caused by changes in quantities (full-time equivalents), not changes in prices. The compositional effects from each industry, identified using each of the four different decompositions, are illustrated in Figure 1. An add-in for carrying out

Table 2. Decomposition of change in monthly earnings (NOK), from 2020Q4 to 2021Q4.

	All decompositions	One-stage Bennet		One-stage Bennet with extended weight		Approximation		Two-stage Bennet	
Industry	Earnings contribution	Compositional effect	Total	Compositional effect	Total	Compositional effect	Total	Compositional effect	Total
All industries	2,074	-33	2,041	-33	2,041	-33	2,041	-33	2,041
Agriculture, forestry and fishing	22	-1	21	0	22	-3	19	-3	19
Mining and quarrying	54	-22	32	-7	47	16	69	16	69
Manufacturing	162	-70	92	1	163	0	162	0	162
Electricity, water supply, sewerage, waste management	42	-7	36	-1	42	2	45	2	45
Construction	160	-13	147	1	161	-9	151	-9	151
Wholesale and retail trade: repair of motor vehicles and motorcycles	244	-66	178	8	252	-11	233	-11	233
Transportation and storage	74	0	74	0	74	-2	72	-2	72
Accommodation and food service activities	30	145	175	-73	-42	-84	-54	-84	-54
Information and communication	115	112	227	24	139	47	163	47	163

Table 2. Continued

	All decompositions	One-stage Bennet		One-stage Bennet with extended weight		Approximation		Two-stage Bennet	
Industry	Earnings contribution	Compositional effect	Total	Compositional effect	Total	Compositional effect	Total	Compositional effect	Total
Financial and insurance activities	67	-19	48	-6	62	9	77	9	77
Real estate, professional, scientific and technical activities	225	93	319	17	242	45	270	45	270
Administrative and support service activities	92	55	147	-12	80	-24	68	-24	68
Public adm., defence, soc. security	141	-69	72	-3	138	4	145	4	145
Education	169	-91	78	6	175	-2	167	-2	167
Human health and social work activities	425	-127	298	13	439	-15	410	-15	410
Other service activities	49	35	84	-5	44	-10	39	-10	39
Unspecified	3	10	13	2	5	3	5	3	5

Source: Authors' own calculations using data from Statistics Norway.

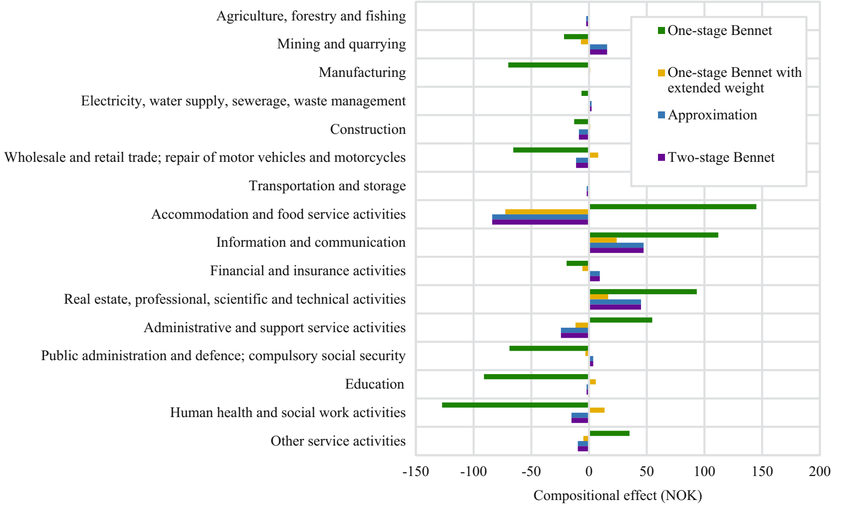

Fig. 1. Compositional effects across decompositions.
Note: See Table 2 for precise magnitudes of compositional effects for each industry and decomposition method. Note that "Unspecified" from Table 2 is not shown in this figure. Source: Authors' own calculations using data from Statistics Norway.

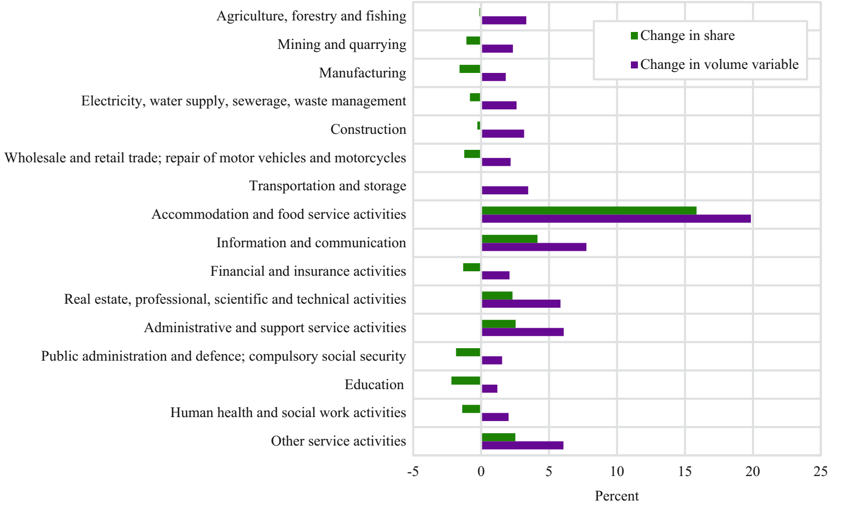

Fig. 2. Change in share and quantity variable, from 2020Q4 to 2021Q4.
Note: Change in share and quantity variable for each industry from 2020Q4 to 2021Q4, measured in percent. Note that "Unspecified" from Table 2 is not shown in this figure. Source: Authors' own calculations using data from Statistics Norway.

this decomposition in EViews, and a Stata replication code to generate the results in Table 2 and Figure 1, are available from the authors upon request.

Focusing on our proposed decomposition, the above results illustrate that there were both positive and negative compositional effects present in aggregate earnings growth in

Norway from 2020Q4 to 2021Q4. In the aggregate, these effects were negative, which is largely attributable to developments in the industry accommodation and food service activities. This industry was to a considerable extent impacted by the Norwegian government's actions to curb the spread of the coronavirus, as these measures particularly affected industries where social interaction is a necessity. However, the gradual softening of these measures throughout 2021 was accompanied by an increase in activity in these industries. As the industry accommodation and food service activities has a level of earnings lower than the weighted mean earnings level (see Table 1), the compositional effect becomes negative when the quantity variable of the industry increases. Conversely, the industries information and communication and real estate, professional, scientific and technical activities, in which the earnings level is higher than the aggregate earnings level, contributed to aggregate earnings growth with a noteworthy positive compositional effect, as these industries had an increase in the quantity variable. At the same time, several industries had a compositional effect that was close to zero.

Comparing the results of our proposed decomposition with the results produced by the other two exact methods, a few comments are warranted. First, the proposed decomposition provides an exact and intuitive description of the contribution to the overall compositional effect from each industry, in terms of both the size and the direction of the contribution. For example, Table 2 shows a large difference between the one-stage Bennet decomposition on the one hand and the one-stage Bennet decomposition with extended weight and two-stage Bennet decomposition on the other in the contribution from accommodation and food service activities. The compositional effect is far greater with the one-stage Bennet decomposition, and it has an opposite sign from the other two. Furthermore, in the industries mining and quarrying and wholesale and retail trade, amongst others, the two-stage Bennet decomposition and the one-stage Bennet decomposition with extended weight produce compositional effects with opposite signs, and in these cases, we argue that the two-stage Bennet decomposition provides a more intuitive result. For mining and quarrying, where the average earnings are higher than the average earnings for all industries, as shown in Table 1, and the quantity of full-time equivalents increases from 2020Q4 to 2021Q4, it is natural that the contribution from compositional effects should be positive, as the two-stage Bennet decomposition shows. The comparison yields the same conclusion for wholesale and retail trade, but with opposite signs. With a lower level of earnings than the aggregate earnings level and an increase in the number of full-time equivalents, it is apparent that this industry, all else being equal, should contribute with a negative compositional effect, just as provided by the two-stage Bennet decomposition. Figure 1 also shows the approximation from the decomposition based on QAL in Equation (12), which is very similar to the two-stage Bennet decomposition. The approximation error is small, at 0.03 percent relative to the size of the compositional effect from the two-stage Bennet decomposition, due to modest changes in the aggregate quantity, as well as the prices and weights (cf. Equation (13)). It should be noted that the choice of dimension along which the change in the weighted mean is decomposed is by no means arbitrary. When choosing this dimension, it is critical that it inherently captures the nature of the changes in the quantity variable.

4. Conclusion

In this article, we have derived an exact additive decomposition of the change in the weighted mean. Our proposed two-stage Bennet decomposition is easy to employ and interpret. We also show that it satisfies the difference counterpart to the index number time reversal test. The fundamental difference between our proposed decomposition and many of the decompositions used in the literature is that our measure of the contribution to compositional changes of a given product is based on the change in the quantity of that product. If there is no change in the quantity of a product, then that product does not contribute to a compositional change in the weighted mean. In contrast, in other decompositions, such as the one-stage Bennet decomposition, the measure of the contribution to compositional changes of a given product is based on the change in the quantity share of that product. Since the quantity share of a product may change because the quantities of other products change, this may lead to compositional changes stemming from a product whose quantity level is unchanged. We have also outlined an alternative decomposition based on a quadratic approximation of the weighted mean, when the weighted mean is regarded as a non-linear function of underlying prices and quantities. Although this alternative decomposition is not exact, it has some similar features to the two-stage Bennet decomposition.

When comparing our proposed decomposition to the standard decomposition used in the literature in the case of aggregate earnings growth in Norway from 2020Q4 to 2021Q4, we find that the wedge between the identified compositional effects is substantial, and for some industries the compositional effects are of opposite signs. We also compared the two-stage Bennet decomposition with the decomposition based on a quadratic approximation and found that these two decompositions generated similar group-specific contributions to the overall compositional effects.

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Received October 2021
Revised April 2022
Accepted June 2022