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Gravity field modelling for optical clock comparisons

Abstract. A coordinated programme of clock comparisons is carried out within the EMRP-funded project "International Timescales with Optical Clocks" (ITOC), aiming at a validation of the uncertainty budgets of the new optical clocks with regard to an optical redefinition of the SI second. Based on Einstein's general relativity theory, clocks are affected by the gravitational field and the velocity of the clocks. For an Earth-bound clock at rest, the corresponding relativistic redshift effect is directly related to the (geodetic) gravity potential, which includes a gravitational and a centrifugal component. As optical clocks are now targeting a relative accuracy of 10^{-18}, corresponding to a sensitivity of about 0.1 m^2/s^2 in terms of the geopotential or 0.01 m in height, precise knowledge of the gravity potential is required at the respective clock sites. Alternatively, optical clocks may also be employed for deriving the gravity potential (denoted as "chronometric levelling" or "relativistic geodesy") and hence offer completely new options for geodetic height determination.

The ITOC project involves clock sites at the national metrological institutes (NMIs) in France (OBSPARIS, LNE-SYRTE), Germany (PTB), Italy (INRIM), the United Kingdom (NPL), and an underground laboratory in France near the Italian border (LSM, Laboratoire Souterrain de Modane). In order to determine the gravity potential with best possible accuracy at these sites, two approaches are considered, namely geometric levelling and GNSS ellipsoidal heights in combination with a gravimetric (quasi)geoid model. Additional absolute and relative gravity observations were carried out around the clock sites and then used to compute an updated quasigeoid model, i.e. the European Gravimetric (Quasi)Geoid 2015 (EGG2015; see separate poster). The general strategy and the gravity potential results are discussed.

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Relativistic redshift effect

$$\frac{\Delta f}{f_{rec}} = \frac{f_{rec} - f_{em}}{f_{rec}} = \frac{W_{rec} - W_{em}}{c^2} + O\left(\frac{W^2}{c^4}\right)$$

- f_{rec}, f_{em} = clock frequencies (rec = receiver; em = emitter)
- W_{rec}, W_{em} = Earth gravity potential values
- equation holds for two Earth-bound clocks at rest
- equation is accurate to about 5 parts in 10^{19} if tidal and other time-variable effects are considered in W
- new optical clocks aim at fractional accuracies of about 1 part in 10^{18}
- absolute potentials W are needed for contributions to international timescales (require reference to conventional W_0, e.g., W_0 (IERS2000) = 62,636,856.0 m^2/s^2, which is also used implicitly in the IAU definition of terrestrial time TT)
- potential differences ΔW are sufficient for clock comparisons

Geodetic approaches for gravity potential determination

(A) Levelling approach

$$W_P = W_0^{(i)} - C^{(i)}; \qquad C^{(i)} = W_0^{(i)} - W_P = \int_{P_{0(i)}}^{P} g\,dn = \bar{g}H^{(i)} = \bar{\gamma}H^{N(i)}$$

- $H^{(i)}$, $H^{N(i)}$ = orthometric and normal height, respectively
- geometric levelling is a differential technique and gives only potential differences
- geometric levelling can deliver sub-mm accuracy over short distances, but is susceptible to systematic errors over large distances (may accumulate to the dm level over distances of 1,000 km)
- zero potential $W_0^{(i)}$ is typically unknown, but may be determined by approach (B)

(B) GNSS/geoid approach

$$W_P = U_P + T_P; \qquad T = \mathrm{M}(\Delta g) \qquad (\mathrm{M} = \text{Molodensky operator})$$

$$W_P = U_0 - \bar{\gamma}(h - \zeta) = U_0 + \underbrace{(W_0^{(i)} - U_0)}_{\delta W_0^{(i)}} - \bar{\gamma}(h - \zeta^{(i)})$$

$$h = H^{N(i)} + \zeta^{(i)} = H^{(i)} + N^{(i)}$$

$$\zeta^{(i)} = h - H^{N(i)} = \frac{T}{\gamma} - \frac{\delta W_0^{(i)}}{\gamma} = \zeta + \zeta_0^{(i)}$$

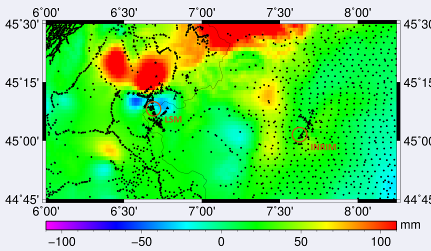
- ellipsoidal heights h and (gravimetric) disturbing potential T (or height anomalies ζ) are required
- GNSS/geoid approach can deliver absolute potential values W (assuming that the gravitational potential is regular at infinity)
- accuracy of the absolute potential values is about 0.25 m^2/s^2 (equivalent to 2.5 cm in height; best case scenario, i.e., sufficient terrestrial data and state-of-the-art satellite model employed)
- $\delta W_0^{(i)}$ can be derived from GNSS/levelling and (gravimetric) disturbing potential T

ITOC GNSS/levelling observations

ITOC gravity campaigns

- absolute (AG) and relative gravity (RG) measurements were performed around each NMI (INRIM: 1 AG, 35 RG; LSM: 1 AG, 122 RG; NPL: 2 AG, 64 RG; OBSPARIS: 3 AG, 97 RG; PTB: 1 AG, 45 RG) to
  - evaluate the existing (largely historic) gravity database (*consistency check*)
  - fill areas void of gravity data (*coverage improvement*)
  - *improve the infrastructure* of the NMIs, e.g., the gravity data (especially the absolute points) can be used for geodynamic and metrological purposes
- no significant biases detected between new and old gravity data sets

(Quasi)geoid update (EGG2008 → EGG2015)



(EGG2015 includes the new ITOC gravity observations around the NMIs)

*Fig. Differences EGG2015 minus EGG2008 around INRIM and LSM plus gravity stations (▲: new ITOC pt.; •: old pt.; *: old pt. with error flag)*

Table. Statistics of differences EGG2015 minus EGG2008 (in m) around NMIs

NMI	#	Mean	Std.dev.	RMS	Min.	Max
INRIM	638	0.027	0.013	0.030	+0.007	+0.073
LSM	999	0.039	0.039	0.055	−0.053	+0.166
NPL	416	−0.009	0.002	0.009	−0.014	−0.004
OBSPARIS	888	−0.007	0.002	0.008	−0.014	−0.002
PTB	782	−0.003	0.001	0.003	−0.006	0.000

ITOC gravity potential results

- EVRF2000 employed for the "levelling approach"
- $W_0^{(i)}$ derived from EUVN_DA GNSS/levelling data and EGG2015
- max. quasigeoid changes at clock sites (found at LSM and INRIM) are less than 3 cm
- differences between approaches (A) and (B) are +0.93 m^2/s^2 (INRIM), -0.18 m^2/s^2 (LSM), -2.96 m^2/s^2 (NPL), -1.12 m^2/s^2 (OBSPARIS), and -0.29 m^2/s^2 (PTB), respectively

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