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Exploring the Perceived Relevance of University Mathematics Studies by First-Semester Teaching Students

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Abstract

As earlier research results suggest that many mathematics teaching students criticize a missing relevance in their studies, we explore explanations and interrelationships of their relevance assessments. We aim at finding out how one could support the students in attributing relevance to their study programs. A two-fold model for relevance assessments in mathematics teacher education is proposed, consisting of relevance content and relevance reasons. We investigate students' relevance perceptions of mathematical topics and of topics' complexities, as well as their rating of individual and societal/ vocational relevance reasons, all in relation to their perception of the relevance of their overall program of study. Contrary to earlier research findings, our results suggest that mathematics teaching students already do attribute relevance to many content areas and that a preparation for the teaching profession is not the only reason for them to assign relevance. There also seem to be many students who would attribute relevance if they could develop as individuals and pursue their interests. We suggest that giving students opportunities to set individual priorities in their studies could hence support their relevance assessments. As low relevance assessments seem to be connected to students' motivational problems, students might profit from motivational support, as well.

Keywords Relevance model · Mathematics teacher education · Support measures · Study satisfaction · Secondary-tertiary transition · Cluster analysis

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Dissatisfaction and Relevance Assessments

The International Context

While mathematics majors are themselves often dissatisfied with their studies, mathematics teaching students seem even more dissatisfied (Brown & Macrae, 2005). In many countries, mathematics teaching students at the beginning of their studies have to take courses in advanced mathematics that are taught by mathematicians (Even, 2011). Many are dissatisfied with these courses and feel unprepared for the more challenging mathematics at university (Goulding et al., 2003). Oftentimes, future mathematics teachers do not recognize how university mathematics link to what they will have to teach at school (Zazkis & Leikin, 2010), at least not until they have actually started teaching (Even, 2011). To support mathematics teaching students in recognizing the usefulness of university mathematics for their future teaching, current redesigned courses try to connect school and university mathematics (Wasserman et al., 2017).

The German Context

In many countries, teacher education is organized in a *concurrent* model of teacher training involving specific education from the beginning of their studies (e.g., Austria, Poland, Sweden, Turkey). In the alternative *consecutive* model preservice teachers first finish a degree in their subject and then study specific courses for teacher education (e.g., Finland, US, France, Italy, Malaysia, Singapore; cf. Tatto et al., 2012). In Germany, where teacher education is organized in the concurrent model, teaching students enroll in two subjects, in our case mathematics and another subject. Additionally, they take courses in pedagogy. The subjects are usually studied together with subject major students to a large extent. In their first semester, where our research took place, all mathematics students take courses in linear algebra and analysis.

Dissatisfaction by all kinds of mathematics students is connected to their dropping out of university (Geisler, 2020), which is a major problem with dropout rates of 58% (Heublein et al., 2020). Dropout mainly occurs in the first two semesters (Geisler, 2020; Heublein et al., 2017). Mathematics teaching students,[1] in particular, frequently consider dropping out of university (Blömeke, 2009). Many are highly dissatisfied with their studies (e.g., Blömeke, 2016; Heublein et al., 2010; Mischau & Blunck, 2006; Pieper-Seier, 2002; Scharlach, 1992) and they sometimes express their dissatisfaction as a criticism of a lacking relevance of university mathematics (Neuhaus & Rach, 2021; Scharlach, 1992). This leads us to suppose that missing relevance assessments might be connected to dropout. We suggest that relevance assessments include a striving for certain goals similar as in motivational theories (Deci & Ryan, 1993) and that high relevance assessments might motivate students

[1] In the following, we will speak of students and refer to mathematics teaching students.

to learn even content that is perceived as challenging. Research findings that connect dropout to motivational characteristics of students and to feelings of overburdening at university (Heublein et al., 2017) then support our hypothesis that low relevance assessments are linked with dropout. To address the high dropout rates, we explore how mathematics teaching students attribute relevance to their studies.

Current support measures that address the high dropout rates mainly highlight differences between school and university mathematics (e.g., Rach, 2019). They shall facilitate the start at university by offering courses in a school-like atmosphere. Previous findings suggest that students evaluate such measures positively (Biehler et al., 2018; Hochmuth et al., 2018; Kuklinski et al., 2019; Liebendörfer et al., 2018). Some researchers explain the success with higher relevance assessments by the students that stem from their recognition of connections between high school and university mathematics (Bauer, 2013; Bauer & Partheil, 2009; Eichler & Isaev, 2016). Hence, they suggest a connection between higher relevance assessments and a perceived usefulness.

How we Conceptualize Relevance Assessments

Though many studies emphasize the importance of relevance assessments in learning, conceptualizations of relevance differ. For example, Vollstedt (2011) relates personal relevance to the concepts of value, purpose, benefit, meaning and goals. Neuhaus and Rach (2021) use the concepts of usefulness and relevance synonymously. In a literature research concerning students' dissatisfaction and lacking relevance assessments, we found that the hypothesized connection between relevance assessments and a perceived usefulness could be complemented. Relevance assessments could also be linked to an intrinsic value that is perceived in the relevance object. Hence, we enriched the construct and in particular focused on possible reasons for students' criticism. We suggest that a perceived usefulness or value can trigger positive relevance assessments but that they could also arise from explorations of interest. Students could assign relevance to content they find interesting without finding it useful (cf. paragraph "Goals that Students Might Want to Reach with their Studies"). Moreover, we are interested in how much relevance students ascribe to different contents that educational experts judge relevant for future mathematics teachers.

Like Neuhaus and Rach (2021), we want to explore how students' relevance assessments relate to student characteristics. This could help in optimizing the communication about study programs, their goals, and optimization possibilities between students and university officials.

Our research is hence based on a relevance assessments model that considers possible reasons, contents and student characteristic variables in students' relevance assessments. It was developed by the first author (Büdenbender-Kuklinski, 2021a) in her dissertation project. The project explored reasons and interrelationships behind students' relevance assessments in a quantitative study. Some of the results presented below are enriched results from the dissertation project. Firstly, we take a closer look at relationships between global relevance assessments and aspects that are modelled to explain relevance assessments. Secondly, we propose further implications of the analyses' results for future mathematics teacher education at university.

Lacking Relevance Assessments by Mathematics Teaching Students

Earlier findings suggest different reasons that might explain students' relevance assessments.

- Different contents could be seen as more or less relevant than others, also depending on their complexity (cf. paragraph "Criticism of Mathematical Contents").
- Students might attribute relevance if they can reach personal goals (cf. paragraph "Goals that Students Might Want to Reach with their Studies").
- Higher or lower relevance attributions might be connected to student characteristics (cf. paragraph "Relevance Assessments in Relation to Student Characteristics").

Criticism of Mathematical Contents

Mathematics major students criticize a lacking applicability of study contents (Brown & Macrae, 2005). In Germany, first-semester mathematics teaching students encounter that same mathematical content. Applicability for them would mean a connection to school mathematics. Hence, they might ascribe more relevance to certain contents if they recognize their applicability for school. For example, integrals and derivatives are university content students will later have to teach while contents like vector spaces are not part of the school curriculum. Consequently, when researching reasons for the students' relevance assessments, we should consider which contents relevance is ascribed to.

Moreover, some students criticize that contents are presented too complexly (Göller, 2020). If students ascribe more relevance to contents they perceive as less complex, they might ascribe little relevance to complexly presented mathematical contents at university – even if they are part of the school curriculum. For example, integrals and derivatives at university are presented in a rather formal way and students might find the presentation too complex and not applicable to their future profession. In our research, we asked students to rate the importance of contents that differed in their degree of abstraction and formalization.

Goals that Students Might Want to Reach with their Studies

Connecting to the perceived impracticality of contents, students criticize a lacking preparation for the teaching profession. This finding was made both in problem-centered interviews that aimed at gaining knowledge about how mathematics students perceived their studies (Göller, 2020) and in a quantitative study about study satisfaction with 308 mathematics teaching students (Mischau & Blunck, 2006). Apparently, students at university aim at a preparation to fulfill their future roles as teachers. Hence, they might ascribe relevance if they feel they can reach this goal. This reason relates to earlier approaches that connect relevance constructs to usefulness (cf. paragraph "How we Conceptualize Relevance Assessments"). We considered possible connections to feelings of preparedness for the teaching profession (cf. paragraph "Relevance for What? - Model of Relevance Reasons").

In another study we found another type of goals that students want to reach with their studies. We propose this type could play a role in their relevance assessments, as well. In a questionnaire study, Pieper-Seier (2002) found that teaching students were "much less likely than mathematics majors to perceive their studies as an opportunity for varied learning experiences" (p. 397). Possibly those mathematics teaching students that aim for individual development surpassing a preparation for their future profession might ascribe more relevance to their studies. Therefore, we also considered goals that relate to students' individual interests (cf. paragraph "Relevance for What? - Model of Relevance Reasons").

Relevance Assessments in Relation to Student Characteristics

Relevance assessments might moreover be linked to student characteristics. Earlier findings suggest that mathematics major students' difficult relationship with academia could hardly be separated from broader attitudes, for example, socially and emotionally (Brown & Macrae, 2005). Pieper-Seier (2002) found that mathematics teaching students lacked a "positive basic attitude" (p. 395) toward the study of mathematics. Many of them indicated a preference for their other field of study besides mathematics. A "positive basic attitude" might be necessary to attribute relevance. We suggest that such an attitude would include positive views of key practices in university mathematics like proving. Proving in its function of securing new knowledge is more important in university mathematics than in school mathematics (Fischer et al., 2009; Kempen, 2019; Tall, 2008). At university, it is both necessary to acquire proving skills (Rach, 2014) and to develop a need for proof (Hemmi, 2008; Winter, 1983). However, in interviews by Göller (2020), mathematics teaching students judged proofs as irrelevant for their aspired profession and they felt very insecure about proving. Kosiol et al. (2019), using regression analyses, found that interest in proofs was a strong predictor of study satisfaction of first-semester mathematics students. Positive attitudes towards proving might therefore explain high relevance assessments.

Research into relevance is always concerned with an object whose relevance is explored (Hernandez-Martinez & Vos, 2018, cf. paragraph "Our Relevance Assessments Model"). In our case, this object is the study of mathematics at university so we consider students' relationship with their studies. In the study by Pieper-Seier (2002), teaching students were less likely than mathematics majors to view mathematics as "'intellectually challenging,' 'aesthetically pleasing,' and a 'living science'" (p. 396). Thus, a more positive relationship with mathematics could increase students' satisfaction with their studies and possibly their relevance assessments. We propose that such a positive relationship should include resisting frustration (classified as a resource-based learning strategy; cf. Liebendörfer et al., 2021) and enjoyment of mathematics. Hence, in our research we explored whether students expressing different relevance assessments differed in their frustration resistance and interest in mathematics as one form of enjoyment of mathematics. Interest is seen as a relationship between a person and an object (Krapp, 2007, 2010; Wild et al., 2006), where the object of interest can be a concrete object or it can be a thematic area or activity (Krapp, 2007, 2010). In the present context, we are concerned with enduring individual interest rather than short-term situational interest (Hidi & Renninger, 2006; Krapp, 1992, 2007, 2010; Renninger & Hidi, 2002). In a study by Rach (2019), interest in mathematics paired with interest in university mathematics accounted for 25% of the variance in subjectively assessed study satisfaction among mathematics students. Kosiol et al. (2019) found that general interest was a positive predictor of study satisfaction in mathematics studies. As we consider relevance assessments to be closely related to study satisfaction, we suspect that students' higher relevance assessments might also be linked to higher interest.

Many motivational theories distinguish different qualities of motivation, which are roughly referred to as intrinsic motivation and extrinsic motivation (Krapp, 1999). We explored whether students that ascribe relevance differently also differed in their qualities of motivation. We follow Ryan and Deci (2000), where an activity is performed out of intrinsic motivation when it is performed because it is enjoyed, and extrinsic motivation means that an activity is performed in order to achieve a separate outcome. Extrinsic motivation is further differentiated into external regulation, introjected regulation, identified regulation, and integrated regulation, assuming that people can internalize originally external forms of regulation if stimuli come from significant reference groups (Deci & Ryan, 2002). In practice, integrated regulation is usually not surveyed because it can hardly be separated empirically from intrinsic motivation (Müller et al., 2007; Vallerand et al., 1992). A self-determination index (SDI) is formed of the four forms of motivation as the sum of the scales weighted according to their level of self-determination. Intrinsic motivation is weighted by + 2, identified regulation by + 1, introjected regulation by -1, and external regulation by -2 (Levesque et al., 2004). Vansteenkiste et al. (2018) designed a model for school lessons where a relevance construct called "self-relevance" increases with an increasing internalization of the four regulation forms. Other works also suggest that an increasing perceived relevance can lead to higher or more internal motivation in the learning process (Gaspard, 2015; Hernandez-Martinez & Vos, 2018; Kember et al., 2008; Priniski et al., 2018). Hence, we explored whether students with different self-determination indices assign relevance differently.

As we mentioned in paragraph "Criticism of Mathematical Contents" that students criticize contents as being too complex, we also assessed their self-efficacy expectation. Low relevance assessments could be related to feelings of overstraining. Students' higher self-efficacy expectations might then be linked to higher relevance assessments. Self-efficacy expectations in general describe "people's judgments of their capabilities to organize and execute courses of action required to attain designated types of performances" (Bandura, 2002, p. 391). General self-efficacy expectations, which summarize assessments of the general ability to cope with life, are distinguished from specific self-efficacy expectations that include situation-specific and domain-specific self-efficacy expectations (Schwarzer &

Jerusalem, 1999, 2002). As Neuhaus and Rach (2021) found positive correlations between students' self-efficacy expectations concerning a fixed university mathematics topic and their relevance assessments, an exploration of connections between relevance assessments and self-efficacy expectations seems worthwhile.

With students' attitudes to proving, their frustration resistance, their interest in mathematics, their motivation and their self-efficacy expectations, we have a variety of student characteristics that might be linked to their relevance assessments. They can be summarized as attitudinal characteristics. However, students' encounter with mathematics at university also concerns their handling of university mathematics. Hence, we also considered students' learning behavior and learning conduct during and between lectures.

Our Relevance Assessments Model

The literature firstly suggested that students might find different contents with differing complexity more or less relevant (cf. paragraph "Criticism of Mathematical Contents"). Secondly, if students could reach the desired goals of preparedness for their future job or of development of their interests, they might ascribe relevance (cf. paragraph "Goals that Students Might Want to Reach with their Studies"). In the conceptualization of relevance of mathematics as a learning content by Hernandez-Martinez and Vos (2018), our research then concerns two of four possible perspectives in relevance research: Hernandez-Martinez and Vos (2018) define relevance as a connection between subject matter, its usefulness, and the learner, and name four questions that play a role in statements about relevance (cf. Ernest, 2004; Jablonka, 2007; Nyabanyaba, 1999).

1. Relevance of what?
2. Relevance for whom?
3. Relevance according to whom?
4. Relevance for what?

We explore relevance for mathematics teaching students (question 2) from their own perspective (question 3), meaning that the answers to two of the four questions are fixed. Particularly, we explore the relevance that students would ascribe themselves rather than the effective relevance of university mathematics. Our research takes a closer look at the answers to questions 1 and 4. While question 1 concerns possibly relevant content (cf. paragraph "Criticism of Mathematical Contents"), question 4 deals with possible reasons for relevance (cf. paragraph "Goals that Students Might Want to Reach with their Studies").

As we aimed at exploratory-empirical research on reasons for students' relevance assessments, we first needed to define and operationalize our two perspectives in the relevance construct (Döring & Bortz, 2016). In paragraphs "Relevance of What? - Model of Relevance Content" and "Relevance for What? - Model of Relevance Reasons", we present how possibly relevant contents and reasons for relevance in mathematics teaching studies were modeled in the first author's dissertation project (Büdenbender-Kuklinski, 2021a).

Relevance of What?—Model of Relevance Content

According to our explanations in paragraph "Criticism of Mathematical Contents", students could rate different contents as more or less relevant where the contents' complexity might also play a role. To model possibly relevant contents, we used a paper published by leading German mathematical associations (DMV et al., 2008). It presents societally recognized standards and we were interested how much relevance students ascribed to contents that according to the educational policy are relevant for future mathematics teachers. For the topics of arithmetic/ algebra, geometry, linear algebra, and analysis, the paper names study contents that should be mastered by future teachers. The competencies are further subdivided into four different levels that are to be learned by teachers of different school levels and school types. The levels differ "according to content expansion, conceptual elaboration and degree of abstraction and formalization" (DMV et al., 2008, p. 2). The levels become more complex from Level 4 to Level 1: Level 4 includes the basic competencies of any teacher, regardless of the grade level they teach, and Level 1 includes competencies that a teacher teaching at the upper secondary level should still possess. Except for linear algebra, where competencies are described for levels 3 through 1 only, the paper lists competencies on all four levels for each content area (DMV et al., 2008). The high school teaching students researched below are expected to have mastered the competencies of all four levels by the end of their studies. However, they only encounter linear algebra and analysis in their first semester. We were interested whether that would influence their relevance assessments. Possibly, they would ascribe more relevance to content areas they had already encountered at university. Moreover, the complexity levels are suitable to explore in how far relevance assessments differ for contents of differing degrees of abstraction. The resulting model of relevance content (cf. Fig. 1) on the one hand comprises the dimension of topic areas and on the other hand the dimension of complexity levels.

Relevance for What?—Model of Relevance Reasons

As suggested in paragraph "Goals that Students Might Want to Reach with their Studies", students might attribute relevance if they can reach the goals of preparedness for their future profession or individual development. This answers the fourth question of Hernandez-Martinez and Vos (2018): "Relevance for what?" To model such goals, a model by Stuckey et al. (2013) on relevance in school science education was adapted for mathematics teacher education. In the original model's

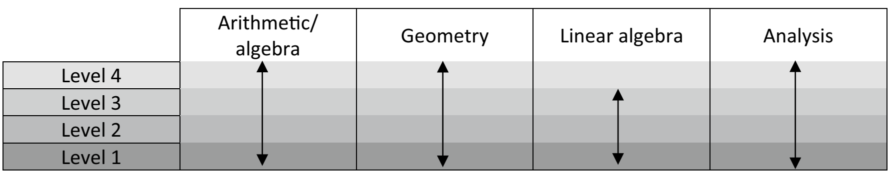

Fig. 1 Model of relevance content

definition, learning material has relevance if it has (positive) consequences for the learners' lives. Stuckey et al. (2013) classify potential consequences on an individual, societal, and vocational dimension. Consequences of the individual dimension concern the development of the subject's personality or a benefit in everyday life and consequences of the societal dimension concern a preparation for a responsible life in society. The vocational dimension includes consequences that concern an orientation in the field of occupation or a preparation for specific career paths. Consequences within each dimension are further categorized concerning their occurrence in the present or future and concerning their extrinsic or intrinsic nature. Extrinsic consequences are positive because externally imposed expectations and requirements are met and internally ones because one's own interests and motives are acted out.

Our own model of relevance reasons accordingly classifies goals as possible consequences that students might want to achieve with their studies, assuming that achieving such consequences could justify attributing relevance. The consequences are categorized as individual or societal/ vocational. While in Stuckey et al.'s (2013) original model the societal and vocational dimensions are considered separately, we suggest to merge them for the field of teacher education. Vocational consequences here can only concern the profession of teaching, which is closely tied to its societal function. Within each of the two dimensions, consequences are classified as extrinsic or intrinsic. The distinction between present and future consequences is omitted because at university, students' mentality is strongly related to their future professional life. This makes it difficult to distinguish whether students already feel like good teachers now or want to be good teachers later.

Accordingly, the model distinguishes between four dimension specifications. We grouped two aspects to the individual dimension and the other two to the societal/ vocational dimension. By definition these two dimensions together form the overall construct of relevance reasons (cf. Fig. 2).

Within each of the dimension specifications, we group various possible consequences that treat different aspects of the dimension specification. Students find content relevant for reasons given in a specific dimension if they consider one consequence of that dimension specification important. They would not have to consider any of the other possible consequences important as those treat other aspects. Based on this model, we will explore how consequences students could possibly pursue with their studies might justify relevance assessments for them. Moreover, we will explore how groups of students that focus on different dimension specifications further differ in other characteristics.

Research Questions

We first explored to what extent the aspects that we assumed played a role in students' relevance assessments (cf. paragraph "Our Relevance Assessments Model") were indeed important to students. That students actually find them important is a prerequisite for them possibly explaining students' relevance assessments.

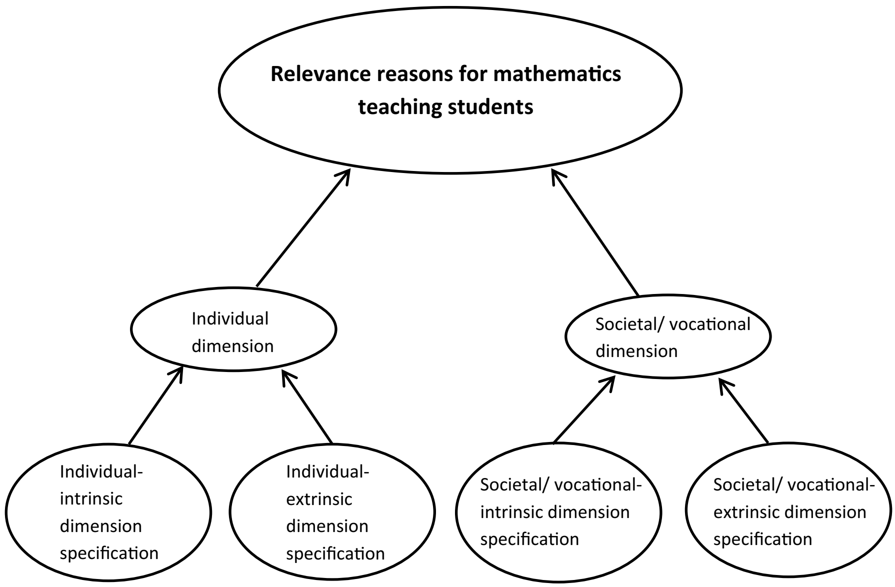

Fig. 2 The layered model of relevance reasons

Research question 1: Are the aspects assumed to play a role in students' relevance assessments indeed considered important by them?

We assumed we could explain students' relevance assessments to contents in mathematics studies with the following aspects:

- The contents students considered important were actually treated in the lectures.
- The contents of the lectures were as complex as appeared appropriate to students.
- Students were achieving consequences they were striving for with their studies.

Hence, we examined to what extent variance in a global assessment of the study contents' relevance could be explained by assessments of the importance and of the implementation of relevance contents and reasons.

Research question 2: To what extent are assessments of importance and implementation of the individual aspects of the model of relevance assessments related to the assessment of relevance of the overall study program?

As will be shown in paragraphs "Results Concerning Research Question 1" and "Results Concerning Research Question 2", our results to these first two research questions indicate that there are indeed students who want to reach consequences in the individual dimension. This stands out as earlier research has concentrated on relevance assessments via professional references (cf. paragraph "The German Context"). We were especially interested in characterizing these newly found students who want to develop individually and find out how they might differ from students that rather want to be prepared for their teaching profession. For this purpose, we tried to single these students out through a cluster analysis and then compared them to the other student groups based on student characteristics that could be linked to different relevance assessments (cf. paragraph "Relevance Assessments in Relation to Student Characteristics").

Research question 3: How can different categories of students who focus on different relevance reasons be characterized concerning their similarities and differences – also concerning other student characteristics?

Methods

Relevance describes a complex relationship between a person, content and a reason (Hernandez-Martinez & Vos, 2018, cf. paragraph "Our Relevance Assessments Model"). This complexity is mirrored in the meaning of the German word “Relevanz”. The German standard dictionary describes “Relevanz” as importance in a specific context (*Duden | Relevanz | Rechtschreibung, Bedeutung, Definition, Herkunft*, n.d.). Accordingly, in the operationalization we asked if the different aspects of our relevance model were important (“wichtig”) to the students in their mathematics studies.

Operationalization of Relevance Attributions

For the relevance contents, we worked with the lists by DMV et al. (2008) of contents in each topic area and each complexity level. We created items that asked students to rate the importance of the content of various topic areas of different complexity. Items covered all four levels for all topic areas but linear algebra where items cover levels 3 through 1 (cf. paragraph "Relevance of What? - Model of Relevance Content"). Exemplary items for all four complexity levels for the topic area of arithmetic/ algebra are given in Table 1.

For the operationalization of the relevance reasons, we created items that asked students to rate the importance of consequences of the four dimension specifications. We reviewed literature seeking for diverse aspects of each dimension specification which were then turned into items (Büdenbender-Kuklinski, 2021b). We assumed

Table 1 Exemplary items for the relevance contents

	In my mathematics studies it is important to me that …
Level 4	… I have a basic understanding of the aspect variety of natural numbers, fractions and rational numbers.
	… I can formally handle the laws of basic arithmetic for natural and rational numbers.
Level 3	… I can describe the limits of the rational numbers in the theoretical solution of the measurement problem.
	… I grasp the laws of the logarithm for mathematics and its applications.
Level 2	… I can explain the completeness of the real numbers using examples.
	… I can handle the elementary-algebraic formula language.
Level 1	… I master conceptual tools such as equivalence classes for the formal foundation of number ranges.
	… I can formally describe the relationships of divisibility theory.

a formative measurement model where different items add up to form the whole index (Diamantopoulos & Winklhofer, 2001) as we needed items that treated diverse aspects which together form each dimension specification (cf. paragraph "Relevance for What? - Model of Relevance Reasons"). While this means that the items do not have to correlate, it has no further consequences for the analyses presented below. The final instrument is given in Table 2.

As we were interested in students' estimation whether they would learn the contents and reach the consequences (cf. research question 2, paragraph "Research Questions"), as well, we furthermore developed appropriate measurement instruments for this purpose. They contained the same items but with a different introductory phrase ("In my mathematics studies, it is true that…"). In all these instruments, answer choices ranged on six-point Likert scales. For the assessments of importance of relevance contents and of their implementation we gave an additional option of a neutral answer where students could indicate that they could not give a rating.

Table 2 Items of the four indices of the dimension specifications

Dimension specification	Item text In my mathematics studies it is important to me that …
Individual-intrinsic	… I have fun.
	… I am being prepared to be able to put my own goals into practice in the future.
	… I learn fascinating things.
	… I can show my own top performance.
Individual-extrinsic	… I fulfill the expectations placed on me by others.
	… I perform as well as is expected of me.
	… I acquire the necessary competencies that will be relevant to me as an individual in society in the future.
	… I am being prepared for how I can meet the demands placed on me in the future.
Societal/ vocational-intrinsic	… it prepares me for what it feels like to be a teacher.
	… it leads to a strengthening of my identification with the teaching profession.
	… I can learn everything in it that I want to learn for my profession.
	… I feel that I can be a good teacher.
Societal/ vocational-extrinsic	… I am being prepared to fulfill my societal function as a mathematics teacher well in the future.
	… I am being prepared to be able to deal with educational standards, curricula and textbooks in a reflective manner in my future profession.
	… I am being prepared for how I can be a good ambassador for the subject of mathematics in my future profession.
	… I am being prepared to lead a class competently.

Our Study

We carried out a study with two paper–pencil surveys but measured the perceived implementation of relevance contents and reasons only at the second survey point. The first survey took place in the second week of lectures of the winter semester 2018/ 19 in a course for first-semester mathematics teaching students and the second survey in the penultimate week of lectures of the same semester in the same course. 162 students, 78 of whom were female, participated in the first survey. 162 students, 91 of whom were female, participated in the second survey. Only 109 participants participated in both surveys. Participation was voluntary and anonymous. To be able to draw longitudinal conclusions despite the data's anonymity, students were instructed to construct a personal code.

We used a global item assessing the perceived relevance of the overall study program ("How relevant do you consider the totality of your mathematics studies?" – answered on a six-point Likert scale) and applied psychometric scales on affective and behavioral characteristics of students established in and adapted to the field of university mathematics (cf. Table 3). In order to prevent the surveys from becoming too time-consuming, some constructs were only measured at one survey point where this seemed reasonable. The selection of the instruments used only at one point in time was based on theory. The constructs on learning conduct during lectures and between sessions, for example, were only measured at the second survey point, as students were hardly able to make a statement about this at the first survey point where they had just started their first semester.

For the scales, internal consistencies were mostly in the acceptable to excellent range with Cronbach's alpha at 0.71 or higher (cf. Appendix). Only for three motivational styles was Cronbach's alpha below 0.70, the value that is often discussed as the lower limit (cf. Cho & Kim, 2014; Cortina, 1993; Schmitt, 1996). However, since these are established scales, the values measured with the instruments can justifiably be evaluated in this paper despite the lower consistency values. The scores for all scales and indices were computed as the means of all corresponding items.

Methods to Answer the Research Questions

To answer research question 1, we studied the means of the corresponding scales and indices. We calculated linear regressions with the item on the perceived relevance of the overall study program as the dependent variable to answer research question 2. A significance level of 10% was set. To answer research question 3, we carried out a cluster analysis on the indices concerning the importance of the four dimension specifications of relevance reasons. We subsequently compared the cluster types in their characteristics via mean value comparisons. For the cluster analysis, we first z-standardized the four indices. We identified outliers with the single-linkage procedure and used Ward's procedure on the remaining cases (cf. Backhaus et al., 2016; Bortz & Schuster, 2010; Wiedenbeck & Züll, 2001). We carried out one cluster analysis on the data from the first survey and another cluster analysis

Table 3 List of instruments for measuring the queried characteristics

Characteristics/ Constructs	Number of Items	Number of scale points	References
Perceived relevance of the overall study program	1	6	Own development
Relevance reasons & Perceived implementation[b] of the individual dimension	8 each		
Relevance reasons & Perceived implementation[b] of the societal/ vocational dimension	8 each		
Relevance contents: arithmetic/ algebra, geometry, analysis	8 each	6 + neutral option	Own development following (DMV et al., 2008)
- thereof complexity level 4	2		
- thereof complexity level 3	2		
- thereof complexity level 2	2		
- thereof complexity level 1	2		
Perceived implementation[b]: arithmetic/ algebra, geometry,	8 each		
analysis	2		
- thereof complexity level 4	2		
- thereof complexity level 3	2		
- thereof complexity level 2	2		
- thereof complexity level 1			
Relevance content linear algebra	6		
- thereof complexity level 3	2		
- thereof complexity level 2	2		
- thereof complexity level 1	2		
Perceived implementation[b] of linear algebra	6		
- thereof complexity level 3	2		
- thereof complexity level 2	2		
- thereof complexity level 1	2		
Mathematical self-efficacy expectation	4	4	Fischer (2014)
Mathematics-related interest	6	6	FSI (Krapp et al., 1993)
Frustration resistance (resource-based learning strategy)	3		LimSt (Liebendörfer et al., 2021)
Motivation[a]		5	Müller et al. (2007)

Table 3 (continued)

Characteristics/ Constructs	Number of Items	Number of scale points	References
- intrinsic motivation	5		
- identified regulation	4		
- introjected regulation	4		
- external regulation	4		
Learning conduct[b]		6	Adapted from Farah (2015)
- during lectures	4		
- between lectures	4		
Dropout tendency[b]	2		Own development
Attitudes towards proving[b]	4		Part of a scale by Kempen (2019)
Task-related self-efficacy expectancy[b]	4		Unpublished part of a scale by Liebendörfer et al. (2021)

[a]Only run in the first survey

[b]Only run in the second survey

on the data from the second survey. Instead, we could have carried out one cluster analysis over the data of the two measurement points which would have facilitated the interpretation of cluster changes of students from the first to second survey. Our approach was better suitable to link the cluster affiliations to other student characteristics measured at the same survey point.

In the analysis of the data of our explorative study, we applied informal statistical inference (Makar & Rubin, 2009; Makar et al., 2011). The explorative nature of our research includes different comparisons, for example the relevance of some content compared to other content in the same survey as well as the relevance of the same content in the other survey. A consequent procedure based on formal statistical inference would need several tests and some alpha-error correction (like Bonferroni). This would likely give no statistically significant result which should not be taken as evidence that there is no real difference. We thus used informal statistical inference, which considers the broader context of the data and makes inferences based on this context rather than statistical measures (Makar et al., 2011). The statements are accordingly made in a non-deterministic language to show that there is some uncertainty and to communicate the probabilistic character (cf. Makar et al., 2011). In our regression analyses carried out to answer research question 2, we interpret not only statistically significant coefficients of the independent variables but also the values of non-significant ones with the broader context in mind. This seemed especially worthwhile as our research is of an explorative nature so we aimed at identifying a variety of possible reasons for students' relevance attributions that should be investigated in future research.

Results

We present our results arranged by the three research questions.

Results Concerning Research Question 1

Research question 1: Are the aspects assumed to play a role in students' relevance assessments indeed considered important by them?

We studied the means of the scales and indices for the relevance contents and relevance reasons to answer our first research question. Concerning the importance of the four topic areas (cf. Fig. 3[2]) all mean values at both survey times were above the neutral value at 3.5 (theoretical mean of the scale) with little difference between the assessments for the different topic areas. This indicated that students found the topic areas rather important than unimportant. Standard deviations were also similar. However, far fewer students answered the questions of how important they found geometry (68 at T1, 92 at T2) than of the other topic

[2] A table with the standard deviations and sample sizes is given in the Appendix.

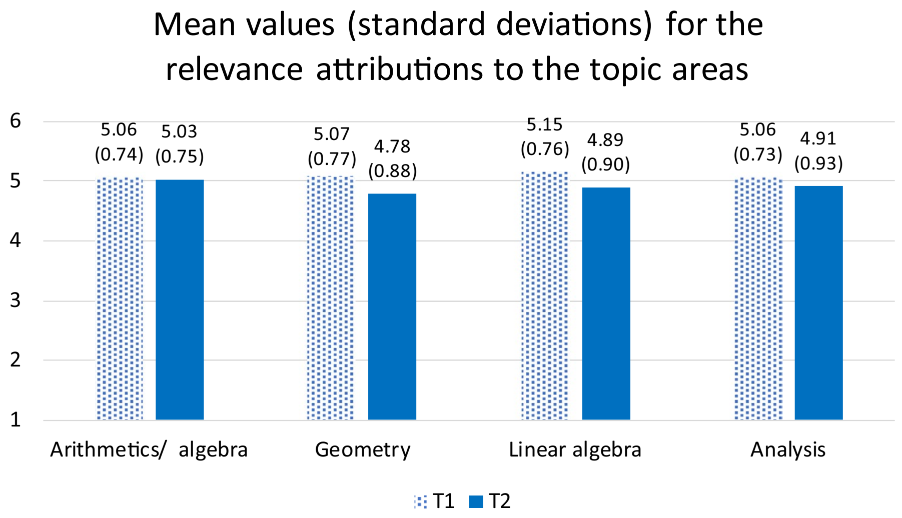

Fig. 3 Relevance attributions to the topic areas

areas. In the second survey, the assessments of importance for all four topic areas were somewhat lower than in the first survey.

On average the assessments of importance for all complexity levels were above the neutral value, as well (cf. Fig. 4). In the second survey, the more complex the content, the less importance was ascribed to it, and fewer students gave an assessment of the importance of the more complex levels. The standard deviation of the

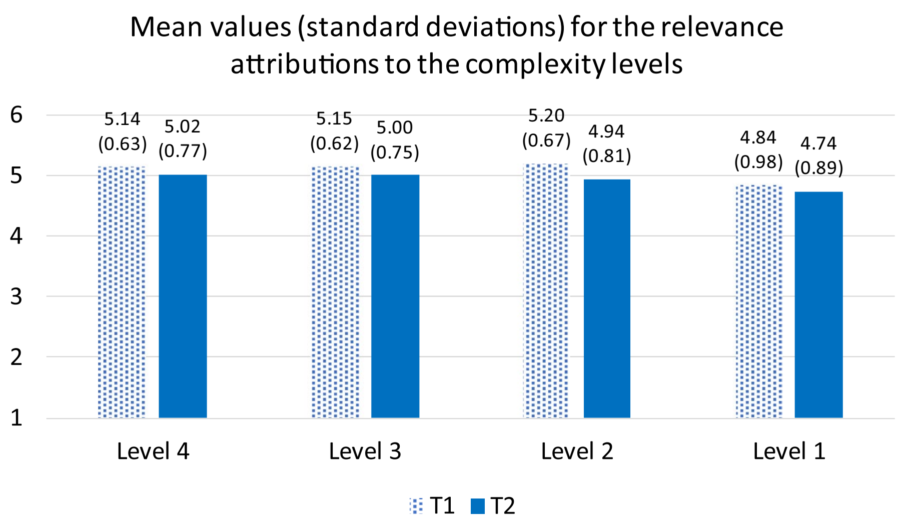

Fig. 4 Importance assessments of the complexity levels

variables increased with increasing complexity of the level. From the first to second survey, the importance assessments to content at all levels of complexity decreased.

Finally, the mean values for the assessed importance of all dimension specifications were above the neutral value in both surveys (cf. Fig. 5). Almost all of the students surveyed assessed the importance of all of the relevance reasons queried. Mean values for relevance reasons on the societal/ vocational dimension were higher than for those on the individual dimension. There was also a decrease in the importance assessments of the dimension specifications from the first to the second survey.

Results Concerning Research Question 2

Research question 2: To what extent are assessments of importance and implementation of the individual aspects of the model of relevance assessments related to the assessment of relevance of the overall study program?

To answer our second research question, we looked at six different linear regression models. In all models, the item on the perceived relevance of the overall study program formed the dependent variable. In the first three models the independent variables were given as the importance assessments of the dimension specifications, the complexity levels and the topic areas, respectively (cf. paragraph "Regressions Based on the Perceived Importance of the Relevance Aspects"). In the remaining three models, we set the assessments of the implementation of the same aspects as the independent variables (cf. paragraph "Regressions Based on the Estimated Implementation of the Relevance Aspects"). Finally, we carried out a backward elimination analysis including independent variables that had been significant predictors in the first six regression models to gain further insights which variables could best explain variance in the overall assessment of relevance (cf. paragraph "Backward Elimination").

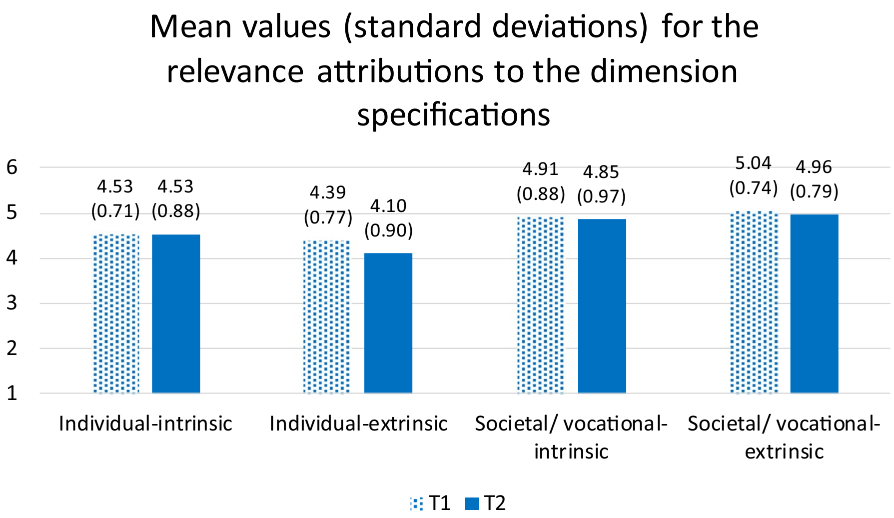

Fig. 5 Importance assessments of the dimension specifications of the relevance reasons

Regressions Based on the Perceived Importance of the Relevance Aspects

According to our model of relevance assessments, the assessment of relevance of the overall study program by students could possibly be explained by their perceived importance of the dimension specifications of the relevance reasons, by their perceived importance of contents of various complexities or of different topic areas. In the linear regressions with the perceived importance of the four dimension specifications of relevance reasons in a first model, the importance assessments of the four complexity levels in a second model and the importance assessments of the four topic areas in a third model, the R^2 values for the first survey for all models ranged between 0.03 and 0.06. This indicated that these regression models did not provide meaningful results. Only the results for the second survey are reported here and all coefficients are unstandardized.

In the first model (cf. Table 4), the only significant predictor was the index for the perceived importance of the individual-intrinsic dimension specification. However, the predictive power with an R^2 value of 0.07 is poor so we inferred that probably none of these independent variables was very conclusive in explaining variance in the dependent variable.

In the second model (cf. Table 5), almost one third of the variance in the dependent variable was explained. The perceived importance of the most complex level was the only significant predictor and the value of the coefficient to this variable is more than double that of each of the other coefficient values. This suggests that the importance assessments concerning level 1 content are probably the only ones that are related to the perceived relevance of the overall study program in a meaningful way. As the algebraic signs of the coefficients to the other three independent variables change back and forth, we cannot find a clear tendency that would suggest, for example, that importance assessments of content at different levels are more meaningful in explaining the relevance assessments to the overall study program with increasing or decreasing level of formalization of content.

The third model (cf. Table 6) was at least more conclusive than the one with the importance assessments of the dimension specifications as independent variables. The perceived importance of analysis represented the only statistically significant

Table 4 Regression results for the elucidation of variance in the global relevance assessment by the importance assessments of the dimension specifications of the model of relevance reasons

Dependent variable: Perceived relevance of the overall study program **Independent variables**: Importance assessments of the dimension specifications		
$F(4,149) = 2.62$, $p = 0.037$ $R^2 = 0.07$		
Constant	3.14	
Individual-intrinsic	0.39	p = 0.002
Individual-extrinsic	-0.10	n.s
Societal/ vocational-intrinsic	-0.17	n.s
Societal/ vocational-extrinsic	0.08	n.s

Table 5 Regression results to elucidate variance in the global relevance assessment by the importance assessments of the complexity levels

Dependent variable: Perceived relevance of the overall study program **Independent variables**: Importance assessments of the complexity levels		
F(4,77) = 7.69, p < 0.001 R^2=0.29		
Constant	1.54	
Level 4	-0.29	n.s
Level 3	0.31	n.s
Level 2	-0.45	n.s
Level 1	0.98	p < 0.001

predictor. However, the coefficient concerning the variable of the importance assessment of linear algebra was almost as high as that for analysis which might indicate that importance assessments of linear algebra are also meaningfully connected to the perceived relevance of the overall study program. As students in their first semester attend lectures of linear algebra and analysis, this might suggest that relevance assessments to content that is actually encountered are more conclusive in explaining the relevance assessments to the overall study program. As the coefficient concerning the variable of importance of geometry is much lower than the values of all other coefficients, geometry might be the least meaningful topic area in explaining relevance assessments of the overall study program. The coefficient to the independent variable concerning arithmetics/ algebra is the only one with a negative sign, suggesting that students who find this subject area less important make higher relevance assessments concerning the overall study program.

In comparison of the three models, the model with the importance assessments of the complexity levels as independent variables elucidated the highest variance in the global assessment of the relevance of the study program for the second survey. In particular, the model that took a closer look at the relevance reasons was hardly meaningful.

Table 6 Regression results to elucidate variance in the global relevance assessment by the importance assessments of the topic areas

Dependent variable: Perceived relevance of the overall study program **Independent variables**: Importance assessments of the topic areas		
F(4,77) = 5.31, p = 0.001 R^2=0.22		
Constant	1.33	
Arithmetics/ algebra	-0.28	n.s
Geometry	0.17	n.s
Linear algebra	0.33	n.s
Analysis	0.35	p = 0.094

Regressions Based on the Estimated Implementation of the Relevance Aspects

In another interpretation compatible with our model of relevance assessments the assessment of relevance of the overall study program could be explained by students' assessment of the implementation of the three aspects of dimension specifications, complexity levels and topic areas. Hence, in a second step, we examined to what extent students' assessments of the implementation of the aspects in our relevance models could clarify variance in the global assessment. By assessments of the implementation of the relevance aspects we understand students' assessments of the extent to which they felt they were able to achieve the consequences from the model of relevance reasons and their assessments of the implementation of the content aspects. The independent variables were the perceived implementation of the four dimension specifications of relevance reasons in a first model, the perceived implementation of the four complexity levels in a second model and the perceived implementation of the four topic areas in a third model. All coefficients we report are unstandardized.

With the first model (cf. Table 7), almost one third of the variance in the dependent variable was explained, more variance than in all models considered so far. The perceived implementation of the individual-intrinsic and societal/ vocational-extrinsic dimension were significant positive predictors and the perceived implementation of the societal/ vocational-intrinsic a significant negative predictor. As the coefficient concerning the variable of the individual-extrinsic dimension specification is much smaller than all other coefficients, the perceived implementation of this dimension specification seems not to play an important role in the reasons for the perceived relevance of the overall study program.

For the second model (cf. Table 8), the predictive power was somewhat lower than in the last model considered. The perceived implementation of the least complex level formed a significant positive predictor and as this variable's coefficient value is at least three times higher than those of all other independent variables, we would hypothesize that this is the only variable that meaningfully contributes to explaining reasons for the relevance attributions to the overall study program. As the arithmetic signs of the coefficients to the variables concerning the two less formal

Table 7 Regression results for the elucidation of variance in the global relevance assessment by the assessments of the implementation of the dimension specifications of the model of relevance reasons

Dependent variable: Perceived relevance of the overall study program
Independent variables: Perceived implementation of the dimension specifications

$F(4,141) = 15.34$, $p < 0.001$
R^2=0.30

Constant	1.76	
Individual-intrinsic	0.31	p = 0.021
Individual-extrinsic	0.12	n.s
Societal/ vocational-intrinsic	-0.27	p = 0.061
Societal/ vocational-extrinsic	0.48	p = 0.003

Table 8 Regression results to elucidate variance in the global relevance assessment by the assessments of complexity level implementation

Dependent variable: Perceived relevance of the overall study program **Independent variables**: Perceived implementation of the complexity levels		
F(4,60) = 5.28, p = 0.001 R^2=0.26		
Constant	4.44	
Level 4	0.68	p = 0.022
Level 3	0.19	n.s
Level 2	-0.08	n.s
Level 1	-0.20	n.s

levels are positive and those of the coefficients to the variables concerning the two more formal levels are negative, relevance might rather be attributed by students who recognize less formal content in their studies but not by those who recognize more formal content.

Finally, the model with the lowest predictive power was the third one, and for none of the topic areas did the corresponding assessment represent a statistically significant predictor in the sample considered (cf. Table 9). However, the values of the coefficients to the variables of the perceived implementation of arithmetics/ algebra and geometry are much higher than the other two, suggesting a tendency that students who recognize content of these subject areas in their studies are more willing to ascribe a relevance to the overall study program.

Overall, the model with the assessments of implementation related to the dimension specifications as independent variables resolved most variance in the dependent variable of all the models considered.

Backward Elimination

In the regression analyses so far, we found seven variables that were statistically significant predictors. These were also the predictors whose regression weight values were higher than those of the non-significant predictors whose regression weights

Table 9 Regression results to elucidate variance in the global relevance assessment by the topic area implementation ratings

Dependent variable: Perceived relevance of the overall study program **Independent variables**: Perceived implementation of the topic areas		
F(4,60) = 3.50, p = 0.012 R^2=0.19		
Constant	1.63	
Arithmetics/ algebra	0.27	n.s
Geometry	0.26	n.s
Linear algebra	0.14	n.s
Analysis	-0.08	n.s

were mostly close to zero. Hence, these variables also seem to be able to explain the most variance in the overall assessment of relevance in the study program. To distinguish further which of these variables has the highest predictive power, we carried out a backward elimination analysis. However, as the topic areas and complexity levels were measured by the same variables, we could not include both constructs in one regression model.

Our results suggested that the complexity levels were more conclusive in explaining variance in the global assessment of relevance so we decided to focus on the complexity levels rather than topic areas in the linear regression backward elimination. We started with all significant predictors of the above models concerning complexity levels and dimension specifications as independent variables (grayed in the tables above). As dependent variable we set the perceived relevance of the overall study program. After eliminating one by one those predictors that were not significant, we found that the most significant predictor was the importance assessment of level 1. As expected, this model has the highest predictive power (cf. Table 10).

Results Concerning Research Question 3

Research question 3: How can different categories of students who focus on different relevance reasons be characterized concerning their similarities and differences – also concerning other student characteristics?

To answer our third research question, we first carried out a cluster analysis on the indices concerning the importance of the four dimension specifications of relevance reasons. In this manner we found student groups that focused on different relevance reasons. We then compared these groups concerning various characteristics studying the means.

The cluster analysis yielded a four-cluster solution in both the first and second surveys. Students of the first type strive for consequences on all four dimension specifications in their mathematics studies. Students of the second type mainly strive for consequences in the individual-intrinsic relevance domain and find all other dimension specifications less important. Students of type 3 aspire to consequences in the

Table 10 Regression results of the backward elimination

Dependent variable: Perceived relevance of the overall study program		
$F(4,92) = 17.29$, $p < 0.001$ R^2=0.43		
Constant	0.207	
Importance assessment of level 1	0.453	$p < 0.001$
Perceived implementation of the individual-intrinsic dimension specification	0.253	$p = 0.065$
Perceived implementation of the societal/ vocational-intrinsic dimension specification	-0.314	$p = 0.075$
Perceived implementation of the societal/ vocational-extrinsic dimension specification	0.522	$p = 0.010$

societal/ vocational domain, and students of type 4 do not strive for pronounced consequences on any of the four dimension specifications, relative to the other three types (cf. Fig. 6).

To analyze the four types further, we compared the mean values obtained with respect to various variables for the four types (cf. Fig. 7[3]). Particularly in the second survey, Type 4, who does not strive for consequences on any of the dimension specifications, assigned the least relevance to the study program. Especially high relevance assessments were made by Type 1, who strives for consequences of all dimension specifications, and Type 2, who strives for consequences primarily in the individual domain. The highest dropout tendency was shown by Type 4, the lowest by Type 2. Type 1, who makes quite high relevance attributions, nevertheless showed a relatively high dropout tendency. Moreover, Type 1 and Type 2 seemed more intrinsically motivated than Type 3 and Type 4. Attitudes toward proving tended to be rather dismissive for all types, but Type 1 and Type 2 were at least less negative about proving than Type 3 and Type 4. Both self-efficacy expectations and mathematics-related interest were stronger among the types who at least also found individual relevance reasons important, and these also showed a more active learning conduct – especially during the period of independent study between lecture sessions. In contrast, the students who primarily want to be prepared for their societal function as teachers became active mainly during the lecture and thus showed a more school-like learning behavior. Frustration tolerance was higher, especially toward the end of the first semester, among the groups of students who do find some consequences of the model on relevance reasons important.

Discussion and Outlook

Interpretation of the Results

In this paper, we dealt with three research questions. First, we explored how important students consider the aspects depicted in the models of relevance content and relevance reasons (cf. paragraph "Results Concerning Research Question 1"). Our study of the mean values of the importance of the four topic areas indicated that the mathematics teaching students considered these contents that leading mathematical organizations proclaim to be relevant to be rather important. The students surveyed also tended to assign a high importance to the content of the various complexity levels. Students of mathematics teaching already seem to assess much of the content of their studies as important, though there seems to be greater agreement that less complex content is more important. The mean values concerning the perceived importance of the relevance reasons suggest that the students perceived all modeled specifications as rather important. However, it was more important to them to achieve relevance reasons on the societal/ vocational dimension than on the individual dimension. While

[3] A table with all the values is given in the Appendix. For a presentation where differences were statistically significant compare Büdenbender-Kuklinski (2021a).

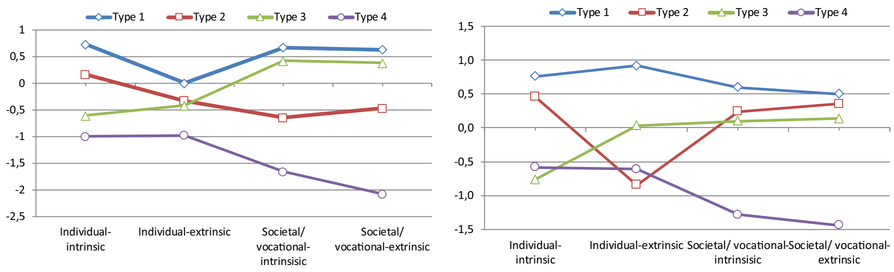

Fig. 6 z-standardized mean values in the first (left) and second survey (right)

a perceived relevance may well be related to a career connection, students studying mathematics also want to develop individually.

With our second research question, we explored to what extent the assessments of importance and implementation of the aspects of the model of relevance attributions were related to the assessment of relevance of the overall study program. Our results suggest that an overall assessment of the relevance of the study program can be explained by various reasons (cf. paragraph "Results Concerning Research Question 2"). Concerning the assessments of importance, students who want to develop individually in their studies of their own accord and, for example, deepen their interests seem to rate the study content as relevant, as well as students who consider content of the highest complexity level to be important. Concerning the assessments of implementation, we found that different types of students tended to perceive the study program as relevant overall. For one, students who believed that they were

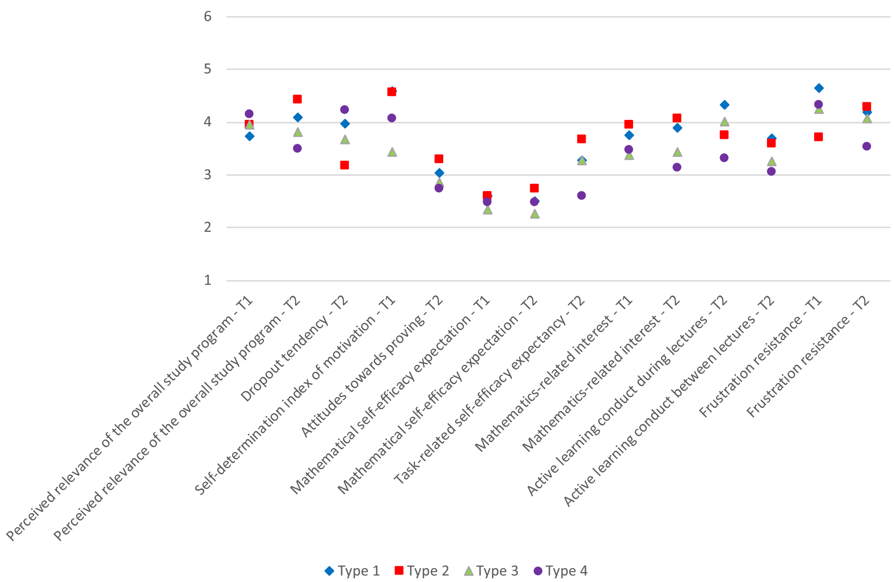

Fig. 7 Mean value comparison for the manifestations of the types on different characteristics

prepared for their societal roles as teachers in accordance with external requirements or who found that they could develop further as individuals in view of their own motives tended to perceive the study program as relevant overall. The same held for students who recognized less complex content in their studies. We could not infer from the results whether students were more likely to believe that the course content was relevant when they recognized the implementation of content of a particular topic area. However, our results suggest that students differentiate between perceived importance and perceived implementation of topic areas in their overall relevance assessments: We found a positive correlation between the perceived relevance of the overall program with the perceived importance of analysis but not with its perceived implementation. The results of our backward elimination analysis suggest that we can best infer that students perceive relevance of the overall study program when we know that they find even complex contents important. However, also the perceived implementation of the individual-intrinsic dimension specification remains a significant positive predictor. An opinion by students that they were prepared for the teaching profession according to their own ideas was rather connected to lower relevance assessments of the entire study program. This might suggest that there are other reasons for this group of students that they would ascribe relevance for.

Lastly, we asked how different categories of students that focus on different relevance reasons could be characterized concerning their similarities and differences. The results of our cluster analysis with the subsequent characterization of the cluster types (cf. paragraph "Results Concerning Research Question 3") indicate that a rather low relevance in mathematics teaching studies is attributed by students who want to be prepared for their societal function as teachers or who do not find any of the relevance reasons from the considered model particularly important. At the same time, these students show less interest, are less motivated, and their learning behavior is more reminiscent of school-based learning.

Comparing Our Findings with the Literature

Our finding that many students already ascribe importance to many contents in their mathematics studies suggests that the criticism of a lacking relevance by students discussed in the literature (Blömeke, 2016; Pieper-Seier, 2002; Scharlach, 1992) might only affect a minority of the student body. The diverging results might be due to different research methods. In our research we gave students predetermined contents that they should evaluate. Students might give high importance assessments in such a case but they might not be able to name relevant content by themselves and then criticize a lack thereof. This might indicate that students lack indicators for relevance but are willing to ascribe relevance if indicators are provided to them. The diverging results might also indicate that students' relevance assessments have changed in a positive direction since the time when the earlier studies were carried out. This might be connected to changes that were introduced in the organization of university teacher education, for example the introduction of the bachelor-master system (Hischer, 2007).

The results to our first as well as second research question moreover suggest that mathematics teaching students want to be prepared for the teaching profession as well as develop individually in their studies. Both kinds of consequences, if reached, seem to be connected to higher relevance assessments. Hence, current support measures that show connections between school and university mathematics (e.g., Bauer, 2013; Bauer & Partheil, 2009; Eichler & Isaev, 2016) seem to be able to support higher relevance assessments in one possible way. However, they might be complemented by additional support measures designed to foster students' interests and consider them as individuals rather than just future teachers. This is a new finding that should be researched further in the future.

In our regression analyses, a perceived relevance of the overall study program by students was closely linked to students finding complex contents important. Possibly the study program has so many complex contents that it is necessary to consider such content important in order to see relevance in the program. Moreover, the finding leads us to hypothesize that students' criticism of lacking relevance may be connected to personal overload. Our characterization of the types found with a cluster analysis suggests that in particular students who also show motivational problems ascribe little relevance. Low relevance attributions hence seem to be connected to various kinds of insecurity by mathematics teaching students. This assumption fits with results of Wenzl et al. (2018). In their analysis of teaching students' statements about their studies, they found that the students criticized their studies but did not have any suggestions as to how their studies could be improved. Wenzl et al. (2018) conclude that students' criticism is rather "an empty 'slogan' (…) that (…) can be used by students to express quite heterogeneous fears and insecurities about teacher studies and to hide them at the same time" (p.4f.). Other works also assume that teaching students' criticism of their studies is rather an expression of discomfort (Makrinus, 2012; Wernet & Kreuter, 2007).

Practical Implications

We found that mathematics teaching students' relevance assessments are not only connected to a feeling of being prepared for the teaching profession as earlier research leads us to believe, but also to a feeling of individual development in the mathematics studies. This suggests that it is advisable not only to justify the relevance of their mathematics studies in terms of a career reference, but also in terms of individual development opportunities. The finding also indicates possible ways to support relevance attributions of mathematics teaching students. Students who focus on individual relevance reasons could be supported by allowing them to develop as individuals in their studies. It could be helpful to use assignments in which students can still set individual priorities according to their own interests, or to leave them more choices in their studies. However, our research to date does not provide an answer as to what these individual choices could or should look like. Newer teaching concepts like the flipped classroom (Milman, 2012) or inquiry-based learning (Ernst et al., 2017) for example with students generating own questions in their learning process (Jessen, 2017) might be helpful here. For students who want to achieve consequences in the societal/ vocational area, support for relevance attributions could consist of showing them connections between school and university mathematics as has been done currently (e.g., Bauer, 2013; Bauer & Partheil, 2009; Eichler & Isaev, 2016, cf. paragraph "The German Context"). That teacher education in Germany is organized in the concurrent model (cf. paragraph "The German Context") might be profitable in supporting both students that focus on individual relevance reasons and students that want to achieve consequences in the societal/ vocational area: As students study mathematics and pedagogy simultaneously, students have opportunities to exploit their mathematical interests but also to gain insights into educational theories.

One idea behind this research was to generate ideas how one could counteract the high dropout rates based on a knowledge about how students could attribute a higher relevance to their studies. Of the four student types we found in our cluster analysis, Type 3 and Type 4 are most likely to need support in this regard. Type 4 does not focus on any of the modelled relevance reasons and shows low motivation. For these students we cannot derive considerations for suitable support measures with regard to the relevance reasons but one could try to support them in affective terms. For example, they might profit from support to feel more self-efficient which could be done via support during task-solving processes in terms of minimal help. Mathematics learning support centers (Schürmann et al., 2021) might be suitable to give such students a place where they can find help. Again, taking students' interests seriously and letting them set individual priorities to deepen interests might be helpful, as well. Such support would also help Type 3 as these are also students who show motivational problems and whose criticism could be related to a feeling of being overwhelmed. Moreover, Type 3 could be supported by showing these students how the content in their mathematics studies relates to their later professional life. In both respects, measures that show mathematics teaching students links between school and university mathematics seem profitable. Such measures explicitly show how university mathematics content is relevant for school teaching (Bauer, 2013; Bauer & Partheil, 2009; Eichler & Isaev, 2016) and previous research suggests that students' affective characteristics are more positively expressed in such innovative courses than in traditional courses (Biehler et al., 2018; Kuklinski et al., 2018, 2019; Liebendörfer et al., 2018, cf. paragraph "The German Context").

Limitations and Outlook

In our research, we modelled reasons and contents that could play a role in relevance assessments and then asked students to rate these. Thus, we can neither tell which reasons students would give for ascribing relevance if asked in an open question, nor can we make conclusions about contents students would name as relevant spontaneously. Additionally, we only considered relevance of mathematics to students personally or professionally and left out other possible relevance aspects like connections between mathematics and real-world topics of interest, such as climate change.

Moreover, our study was of an explorative nature. We used several quantitative measures on data of a restricted sample and applied informal statistical reasoning to find new perspectives on students' perceived relevance of university mathematics. Particularly, we interpreted the theoretical mean of Likert scales as a neutral value and drew conclusions from a comparison between the means and this neutral value. This method bears the danger that a higher mean could be misinterpreted as a higher agreement by respondents but really result from a misinterpretation of the answer choices on their part or from them trying to answer in a socially conforming manner. Future research needs to show if our perspectives on students' perceived relevance of university mathematics lead to general findings.

In the regression analyses in this paper, we reported only significant connections between dependent and independent variables. Of course, statistical significance must not be confused with meaningfulness. Variables that did not show up as significant predictors might still have a meaningful contribution in explaining variance in the global assessment of relevance. They might, for example, be non-linearly connected to the dependent variable. In line with the exploratory nature of our research, further possible relationships between the variables should be researched in the future.

In the future the measurement instruments for the relevance attributions could be developed further. For the societal/ vocational dimension in particular, more precise research could be conducted into the specific consequences that students pursue here, and the measurement instrument could then be adapted accordingly. Existing qualitative research into students perception of the benefits of mathematical course content in relation to their future careers (Becher & Biehler, 2015) should be considered.

Our finding that students who want to be prepared for the teaching profession attributed little relevance suggests that they understand career preparation differently than the university. Research is necessary to investigate the extent of these different interpretations of career preparation.

Since we assume that support measures could enhance students' relevance attributions, research into relevance attributions of students in support measures and their differences to those of students in traditional courses seems promising. Moreover, the construct of relevance attributions could be researched in other fields of study and courses, especially to learn more about the reasons and contexts behind it. For example, one could do research in teacher training programs with subjects other than mathematics.

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Declarations

Conflict of Interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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