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ARTICLE

Empirical approach for the residual flexural tensile strength of steel fiber-reinforced concrete based on notched three-point bending tests

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Abstract
When designing steel fiber-reinforced concrete, for example, according to Model Code 2010, the residual flexural strength values must be specified as fundamental properties. It is often difficult to establish the relationship between residual flexural strength values and the required dosage of steel fibers depending on the type of steel fibers and the concrete quality. For an estimation of the presumably necessary dosage of steel fibers, various empirical approaches exist for the approximate determination of the residual flexural strength values, which, however, are based on different tests and have been almost exclusively been derived on the basis of few or “own” test results of the respective institute. For this reason, the validity of the respective approximation approach is often limited. Using the bending beam database “Steel Fibre Reinforced Concrete”, selected approaches were systematically analyzed and an improved approach for determining the residual flexural strength values of steel fiber-reinforced concrete was developed.

KEYWORDS
concrete, database, empirical approach, notched 3-point bending test, residual flexural tensile strength, steel fiber

1 | INTRODUCTION

By the design of steel fiber-reinforced concrete, the crack-bridging effect of the steel fibers is taken into account as residual tensile strength, for example, according to Model Code 2010[1] (MC2010). Since the residual tensile strength depends not only on the steel fiber type and the dosage of the steel fibers but also on the concrete mixture and quality (see, e.g., References 2–4), in accordance with MC2010,[1] the residual tensile strength must be verified by means of three-point bending tests according to EN 14651.[5] In the course of the design, however, the designer must specify the residual tensile strength in advance. This can be done according to MC2010 via the residual strength class and the residual flexural strength values f_{R1} and f_{R3} of the steel fiber-reinforced concrete, which are then converted into residual tensile strengths. Here,

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the residual flexural strength value f_{R1} is to be used for the serviceability limit state (SLS) design and the residual flexural strength value f_{R3} for the ultimate limit state (ULS) design. Nevertheless, in practice, it is often extremely difficult to establish a correlation between residual strength class and required dosage of the steel fibers depending on steel fiber type and concrete quality. Empirical approaches for the approximate determination of the postcrack flexural tensile strength of steel fiber-reinforced concrete exist for a rough estimate of the dosage of steel fibers that will probably be required. However, they are based on only a limited number of test results and consequently have only limited validity. For this reason, these approaches were systematically analyzed with the help of the bending beam database "Steel Fibre Reinforced Concrete"[6] and a modified approach was derived on the basis of notched three-point bending tests, which is reported below. An empirical approach based on unnotched four-point bending tests, which are used, for example, in Germany according to DAfStb Guideline "Steel Fibre Reinforced Concrete,"[7–10] can be found in Reference 11.

2 | TEST SPECIFICATIONS

2.1 | General

In the early days of steel fiber-reinforced concrete and before the publication of guidelines, exclusively the steel fiber type and dosage were often only specified in the design of steel fiber-reinforced concrete. In this case, a certain steel fiber type was often assigned a fixed performance depending on the dosage of the steel fibers. In addition to the steel fiber type and dosage, however, the composition of the concrete also has a decisive influence on the performance of steel fiber-reinforced concrete.[3,4] In order to assess this, tests must therefore always be carried out with the respective concrete mixture, steel fiber type, and dosage of the steel fibers. Based on these tests, empirical approaches for estimating the performance or the postcrack flexural strength of steel fiber-reinforced concrete were already developed in the 1990s. The approaches considered in this paper (see Section 3) are partly based on different guidelines and test specifications. These are:

- Standard EN 14651.[5]
- DAfStb Guideline "Steel Fibre Reinforced Concrete."[7,8]
- DBV Bulletin "Steel Fibre Reinforced Concrete."[12]
- Standard NBN B 15-238.[13]

In order to understand the different approaches and to be able to relate the values determined with them to each other, the respective tests and evaluations are briefly explained below. Further information can be found in the References given or, for example, in Reference 14.

2.2 | Standard EN 14651

According to EN 14651,[5] the performance of steel fiber-reinforced concrete is tested on notched three-point bending tests (beam $h/b/l_{span} = 150/150/500$ mm). This test setup has the advantage that the location of the crack formation is specified by the notch, and thus, the crack width can be precisely measured. The test control and evaluation are carried out via the crack width and load–crack mouth opening displacement (CMOD) relationships (Figure 1a). However, EN 14651 also specifies a linear relationship for converting crack widths into deflections. The evaluation of the residual flexural strengths is carried out at four defined crack widths ($CMOD_1$–$CMOD_4$), whereby the value at $CMOD_1 = 0.50$ mm is used for SLS and the value at $CMOD_3 = 2.50$ mm for ULS (see Section 1).

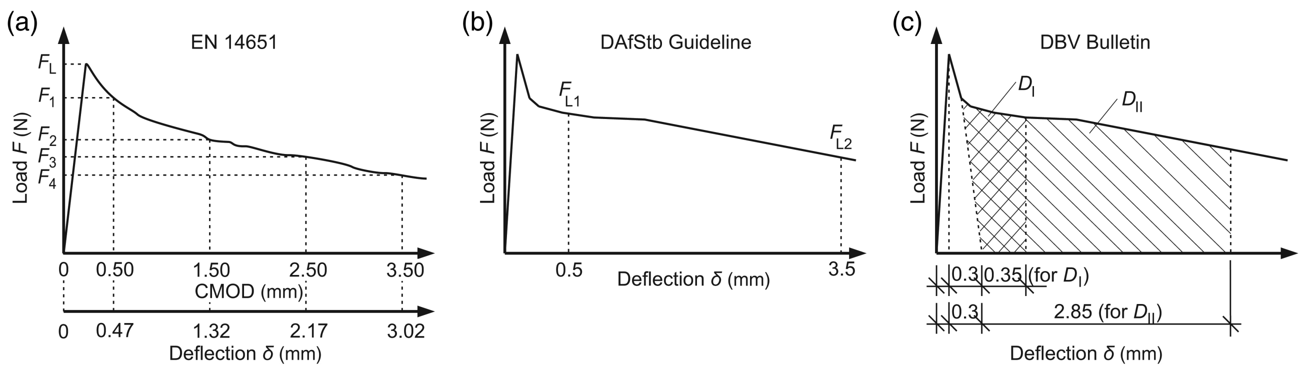

FIGURE 1 Load–CMOD or load–displacement curve to determine the residual flexural tensile strengths according to DIN EN 14651[5] (a), the residual flexural tensile strengths according to DAfStb Guideline[8] (b), and the equivalent flexural tensile strengths according to DBV Bulletin[12] (c). CMOD, Crack mouth opening displacement

2.3 | DAfStb Guideline "Steel Fibre Reinforced Concrete"

The determination of the residual flexural tensile strengths of steel fiber concrete is carried out according to the DAfStb Guideline "Steel Fibre Reinforced Concrete"[8] using unnotched four-point bending tests (beam $h/b/l_{\text{span}} = 150/150/600$ mm). The beams are tested path controlled with monotonically increased deflections up to a deflection of $\delta \geq 3.5$ mm. At the deflections of 0.5 mm (L1 value) and 3.5 mm (L2 value), the residual flexural tensile strengths $f^{\text{f}}_{\text{cfl,L1}}$ (SLS) and $f^{\text{f}}_{\text{cfl,L2}}$ (ULS) of the cracked steel fiber-reinforced concrete are calculated with the help of the corresponding loads F_{L1} and F_{L2} (Figure 1b). The determined residual flexural strengths thus reflect the strength at the defined deflections of the test beam.

2.4 | DBV Bulletin "Steel Fibre Reinforced Concrete"

In analogy to Reference,[8] tests on path-controlled four-point bending tests (beam $h/b/l_{\text{span}} = 150/150/600$ mm) under monotonically increasing deflections are to be carried out according to DBV Bulletin "Steel Fibre Reinforced Concrete."[12] In contrast to the DAfStb Guideline, however, residual flexural tensile strengths are not determined, but the performance is defined via the equivalent flexural tensile strengths. The surface integrals D_{I} and D_{II}—the so-called energy capacities—determined under the load–displacement curve (Figure 1c), is then divided by the deflection of the area under consideration, and the equivalent flexural tensile strengths $f_{\text{eq,I}}$ (SLS) and $f_{\text{eq,II}}$ (ULS) are finally calculated with this average value. The equivalent flexural tensile strengths thus reflect averaged strengths that must be assigned to similar deflection limits (about 0.7 or 3.2 mm) as according to the previously explained DAfStb Guideline but significantly lower ("average") deflections of about 0.6 or 1.9 mm (depending on the curve progression).

2.5 | Standard NBN B 15-238

According to Belgian Standard NBN B 15-238,[13] the performance of steel fiber-reinforced concrete is to be determined by means of path-controlled four-point bending tests (beam $h/b/l_{\text{span}} = 150/150/450$ mm) under monotonically increasing deflections. Similar to the DAfStb Guideline and EN 14651, the residual flexural tensile strengths are determined at fixed deflections, whereby the evaluation is carried out at a deflection of 1.5 mm (SLS) and 3.0 mm (ULS).

2.6 | Comparative consideration of the test specifications

Table 1 summarizes the decisive parameters of the test specifications considered in Sections 2.2–2.5. It is clear that these differ—apart from different test beams—above all with regard to the deflections considered. In addition, residual values are determined according to the DAfStb Guideline, EN 14651 and NBN B 15-238, that is, residual flexural tensile strengths at fixed deflections or crack widths, whereas according to DBV Bulletin, equivalent flexural tensile strengths are determined. These differences are also reflected in the empirical approaches listed in the following Section 3, as their derivation was linked to one of these test specifications.

3 | EMPIRICAL APPROACHES FOR DETERMINATION THE RESIDUAL FLEXURAL STRENGTH VALUES

3.1 | General

As already mentioned in Section 2.1, various empirical approaches for the approximate determination of the residual flexural tensile strength of steel fiber-reinforced concrete can be found in the literature or have been developed by the steel fiber manufacturers and

TABLE 1 Compilation of the decisive parameters of the test specifications

Test specification	Test	Notch	Beam $h/b/l_{\text{span}}$ (mm)	Flexural tensile strength	Deflection considered (mm)
EN 14651	Three-point bending test	Yes	150/150/500	Residual	0.47, 1.32, 2.17, and 3.02
DAfStb Guideline	Four-point bending test	No	150/150/600	Residual	0.5 and 3.5
DBV Bulletin	Four-point bending test	No	150/150/600	Equivalent	ca. 0.6 and 1.9 (depending on the curve progression)
NBN B 15-238	Four-point bending test	No	150/150/450	Residual	1.5 and 3.0

participating testing institutes. Within the scope of this paper, the following approaches are considered:

- Bekaert.[15]
- Teutsch/Falkner/Klinkert.[16–18]
- Verband der Stahlfaserhersteller e.V. (VDS).
- Schulz/Material Testing Institute Braunschweig (MPA BS).

These approaches are listed and briefly explained below.

3.2 | Approach according to Bekaert

The approach according to Bekaert[15] was developed on the basis of NBN B 15-238[13] using unnotched four-point bending tests (cf. Table 1) and allows the estimation of the mean residual flexural tensile strength at a beam deflection of 3.0 mm (ULS). The following applies:

$$f_{\mathrm{flm},3.0} = \frac{180 \cdot V_{\mathrm{f}} \cdot \lambda_{\mathrm{f}} \cdot d_{\mathrm{f}}^{1/3} \cdot f_{\mathrm{ctm,fl}}}{\left(180 \cdot C + V_{\mathrm{f}} \cdot \lambda_{\mathrm{f}} \cdot d_{\mathrm{f}}^{1/3}\right) \cdot 100}, \tag{1}$$

where V_{f} is the dosage of the fibers (kg/m^3), λ_{f} is the aspect ratio of the fibers ($\lambda_{\mathrm{f}} = l_{\mathrm{f}}/d_{\mathrm{f}}$), l_{f} is the length of the fibers (mm), d_{f} is the diameter of the fibers (mm), C is the coefficient depending on the type of fibers ($C = 20$ for Dramix fibers), and $f_{\mathrm{ctm,fl}}$ is the flexural tensile strength of the concrete (N/mm^2).

From the coefficient C, it becomes clear that the approach is in fact only valid for Dramix fibers (hooked-end steel fibers) from Bekaert. This is due to the fact that the approach was developed by Bekaert to give the designer a reference value for the size of the residual flexural tensile strength of steel fiber-reinforced concrete with Dramix fibers.

3.3 | Approach according to Teutsch/Falkner/Klinkert

The approach according to Teutsch/Falkner/Klinkert was developed by Teutsch[18] on the basis of an empirical approach for the approximate determination of the post-crack tensile strength of steel fiber concrete for the design of steel fiber concrete pipes according to Schnütgen[19] and validated on the basis of numerous four-point bending tests.[16,17] With this approach, the characteristic equivalent flexural tensile strength $f_{\mathrm{eqk,II}}$ (ULS) can be estimated according to DBV Bulletin "Steel Fibre Reinforced Concrete"[12]:

$$f_{\mathrm{eqk,II}} = \frac{1}{0.37} \cdot k \cdot V_{\mathrm{f}} \cdot (1 - k \cdot V_{\mathrm{f}}) \cdot \left(\frac{f_{\mathrm{ck}}}{0.78}\right)^{2/3}, \tag{2}$$

where k is the factor that depends on the type of fibers with $k = 5.0$ for steel chips, $k = 9.0$ for crimped wire strips and $k = l_{\mathrm{f}}/d_{\mathrm{f}} \cdot \chi$ for steel wire fibers; χ is the factor that depends on the anchoring of the fiber with $\chi = 0.3$ for hooked-end steel fibers and $\chi = 0.2$ for straight steel fibers; V_{f} is the volumetric dosage of the fibers (−); f_{ck} is the characteristic value of cylinder compressive strength of concrete with $f_{\mathrm{ck}} = f_{\mathrm{cm}} - 8.0\,\mathrm{N/mm^2}$.

An approximate determination of the characteristic equivalent postcrack flexural tensile strength $f_{\mathrm{eqk,I}}$ (SLS) is not provided with the approach according to Teutsch/Falkner/Klinkert.

3.4 | Approach according to VDS

The approach according to VDS modifies the approach according to Teutsch/Falkner/Klinkert by means of an additional factor α so that the following applies for the equivalent postcrack flexural tensile strength:

$$f_{\mathrm{eq,II}} = \frac{1}{0.37} \cdot k \cdot V_{\mathrm{f}} \cdot \alpha \cdot (1 - k \cdot V_{\mathrm{f}} \cdot \alpha) \cdot \left(\frac{f_{\mathrm{ck}}}{0.78}\right)^{2/3}, \tag{3}$$

where α is the modification factor, $\alpha = 1.8$.

The conversion of the equivalent flexural tensile strength $f_{\mathrm{eq,II}}$ into the mean residual flexural tensile strength $f_{\mathrm{flm,L2}}$ (L2 value for ULS) can be done via

$$f_{\mathrm{flm,L2}} = 0.95 \cdot f_{\mathrm{eq,II}}. \tag{4}$$

The factors were derived by the manufacturers, which are organized in the VDS on the basis of test results collected in the course of the regular initial tests required by the DAfStb Guideline "Steel Fibre Reinforced Concrete".[7,8] A conversion into a mean residual flexural tensile strength $f_{\mathrm{flm,L1}}$ (L1 value for SLS) is not possible with this approach, as it is based on the previously explained approach by Teutsch/Falkner/Klinkert.

3.5 | Approach according to Schulz/MPA BS

The approach according to Schulz/MPA BS is also based on the approach according to Teutsch/Falkner/Klinkert, but it was developed for three-point bending tests and validated accordingly (mean value). In the further development, an attempt was made to take into account the influence of the length of the fibers l_f with constant the aspect ratio of the fibers λ_f. Furthermore, a differentiation was made with regard to the postcrack flexural tensile strength. In the tests considered at that time, it became clear that the performance does not increase linearly with the dosage of the fibers so that this effect was also taken into account via an additional coefficient. The following applies:

$$f_{\mathrm{Rm,i}} = \frac{1}{0.37} \cdot k \cdot V_f \cdot (1 - k \cdot V_f) \cdot \frac{f_{\mathrm{ctm,fl}}}{0.393} \cdot \zeta_i \cdot \eta, \quad (5)$$

where $f_{\mathrm{ctm,fl}}$ is the flexural tensile strength of the concrete or limit of proportionality according to EN 14651 (N/mm^2), ζ_i is the coefficient for taking into account the fiber effect as a function of the length of the fibers and the CMOD considered (conversion to residual flexural tensile strength) with $\zeta_1 = 1.70 - \frac{7.5 \cdot l_f}{1000}$ for $\mathrm{CMOD}_1 = 0.5$ mm and $\zeta_4 = 0.80 + \frac{5.0 \cdot l_f}{1000}$ for $\mathrm{CMOD}_4 = 3.5$ mm, and η is the coefficient for considering the nonlinear influence of the dosage of the fibers with $\eta = 1/(0.7 + 0.01 \cdot V_f)$.

Equation (5) shows that due to the coefficient ζ_i, the approach according to Schulz/MPA BS—compared to the above-mentioned approaches—allows a differentiated determination of the residual flexural tensile strength. However, the residual flexural strength value f_{R4} (CMOD_4) is determined instead of the residual flexural strength value f_{R3} (CMOD_3) as required by MC2010[1] (see Section 1).

3.6 | Summary view

Overall, it can be said that some of the approaches are very similar, respectively, build on one another, but they are based on different tests and levels and determine different postcrack flexural tensile strengths. For a better overview, the individual approaches are summarized in the following Table 2 regarding the characteristic parameters.

4 | BENDING BEAM DATABASE "STEEL FIBRE REINFORCED CONCRETE"

4.1 | Selection criteria and data scope

For the investigations, a database with bending tests on steel fiber-reinforced concrete beams was used, which was compiled in the German DAfStb Subcommittee "Steel Fibre Reinforced Concrete" within the DAfStb Working Group "Database".[6] The database currently comprises 1123 mean values of the residual flexural tensile strengths of a series of six beams (=6738 number of specimens) from three-point bending tests according to EN 14651[5] and four-point bending tests according to DAfStb Guideline[7,8] on normal- and high-strength concrete reinforced with steel fibers.

An evaluation of the database and derivation of an empirical approach regarding the postcrack flexural tensile strength of unnotched four-point bending tests can be found in Reference 11. In this article, the focus is on the notched three-point bending test, for which the following selection criteria were applied:

- Three-point bending test,
- anchoring of the fiber: hooked-end,
- volumetric dosage of the fibers V_f between 0.1 and 2.0 vol%,
- length of the fibers l_f between 25 and 80 mm,
- diameter of the fibers d_f between 0.2 and 1.2 mm,
- aspect ratio of the fibers λ_f between 37.5 and 120.0,
- tensile strength of the fibers f_{tf} between 1100 and 3100 N/mm^2,
- cylinder compressive strength of concrete f_{cm} between 24 and 108 N/mm^2, and
- flexural tensile strength of the concrete $f_{\mathrm{ctm,fl}}$ between 2.5 and 8.5 N/mm^2.

With regard to the type of anchorage of the steel fibers, only hooked-end steel fibers were considered as very few test results are available so far for all other types of anchorage (cf. also Reference 6). Using the selection criteria mentioned above, a total of $n = 182$ mean values of the individual

TABLE 2 Compilation of the approaches

Approach	Postcrack flexural tensile strength(s)	Value level	Base
Bekaert	$f_{\mathrm{flm,3.0}}$ (residual)	Mean value	Four-point bending test
Teutsch/Falkner/Klinkert	$f_{\mathrm{eqk,II}}$ (equivalent)	Characteristic value	Four-point bending test
VDS	$f_{\mathrm{flm,L2}}$ (residual)	Mean value	Four-point bending test
Schulz/MPA BS	$f_{\mathrm{Rm,1}}$ and $f_{\mathrm{Rm,4}}$ (residual)	Mean value	Three-point bending test

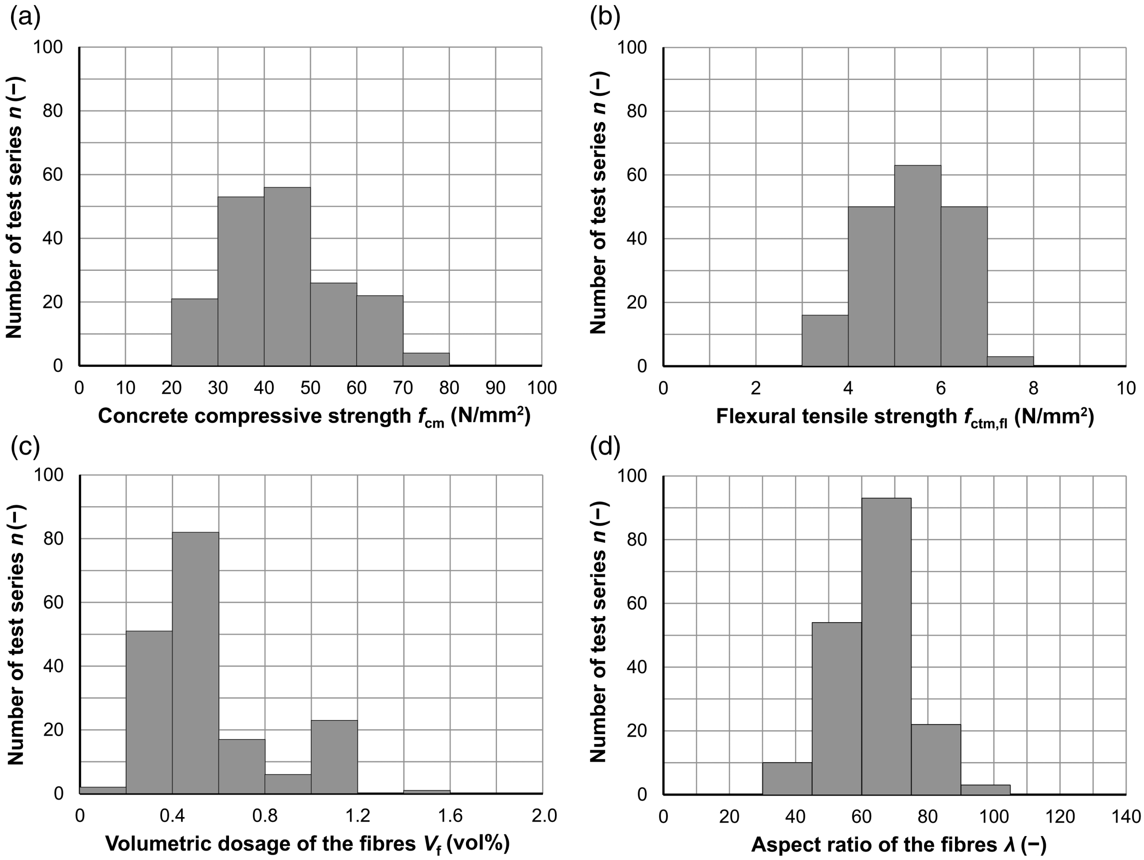

FIGURE 2 Range of parameters of the used data with regard to (a) concrete compressive strength f_{cm}, (b) flexural tensile strength of the concrete $f_{\mathrm{ctm,fl}}$, (c) volumetric dosage of the fibers V_{f}, and (d) aspect ratio of the fibers λ

series (=1092 number of specimens) for the residual flexural tensile strength f_{R1m} as well as f_{R3m} was considered.

The range of test results can be seen in Figure 2. Here, it is clear that many test results are available for normal strength concrete and only a small number for high-strength concrete ($n = 34$ mean values of the individual series [=204 number of specimens], Figure 2a,b). Figure 2c,d shows that the volumetric dosage of the fibers V_{f} varies between 0.2 and 0.8 (or 1.2) and the aspect ratio of the fibers λ_{f} between 37.5 and 82.5 (or 105), which covers the usual ranges in construction practice.

4.2 | Basis of evaluation and assumptions

For the Bekaert, respectively, Schulz/MPA BS approach, the flexural tensile strength of the concrete or the limit of proportionality according to EN 14651 ($f_{\mathrm{ctm,fl}}$) is used, but this was not available for all tests in the database. According to MC2010,[1] the flexural tensile strength of concrete can be estimated as follows:

$$f_{\mathrm{ctm,fl}} = f_{\mathrm{ctm}}/\alpha_{\mathrm{fl}}, \tag{6}$$

where f_{ctm} is the mean uniaxial tensile strength of concrete (N/mm²) with $f_{\mathrm{ctm}} = 0.30 \cdot f_{\mathrm{ck}}^{2/3}$ for $\leq$ C50/60 and $f_{\mathrm{ctm}} = 2.12 \cdot ln[1 + (f_{\mathrm{cm}}/10)]$ for $>$ C50/60, α_{fl} is the factor depending on the beam depth with $\alpha_{\mathrm{fl}} = \left(0.06 \cdot h_{\mathrm{b}}^{0.7}\right)/\left(1 + 0.06 \cdot h_{\mathrm{b}}^{0.7}\right)$, and h_{b} is the beam depth (mm).

However, it is unclear whether this approach also applies to notched beams. Figure 3 shows a comparison of the available experimental flexural tensile strength $f_{\mathrm{ctm,fl,exp}}$ of the tests in the database with the calculated flexural tensile strength $f_{\mathrm{ctm,fl,cal}}$ using Equation (6) (regarding the statistical parameters, see the next Section 5.1). In this connection, the distance between the notch tip and the top of the specimen was taken as the beam depth ($h_{\mathrm{b}} = 125$ mm). It can be seen that the flexural tensile strength of the concrete can be estimated very well with Equation (6). For this reason, the flexural tensile strength is determined and applied uniformly for all tests of the database with Equation (6) for the following evaluations.

Further calculation bases and assumptions, on which the individual values of the database are based on, can be found in Reference 6.

5 | VALIDATION OF THE APPROACHES

5.1 | General

In order to validate the individual approaches, the experimental values are compared with the calculated values of the residual flexural tensile strength. The statistical parameters listed were determined from the quotient $f_{\mathrm{Rim}}/f_{\mathrm{cal}}$ (ratio of the mean value of a series of experimental results f_{Rim} to the calculated value f_{cal}, whereby f_{cal} corresponds to the calculated residual flexural tensile strength $f_{\mathrm{flm},3,0}$, $f_{\mathrm{eqk,II}}$, $f_{\mathrm{flm,L2}}$, or $f_{\mathrm{Rm,i}}$), assuming a logarithmic normal distribution, n is the number of test series, m is the median, $E(X)$ is the expected value, s is the standard deviation, ν is the variation coefficient, and $Q_{0.05;0.95}$ denotes the 5% and 95% quantiles.

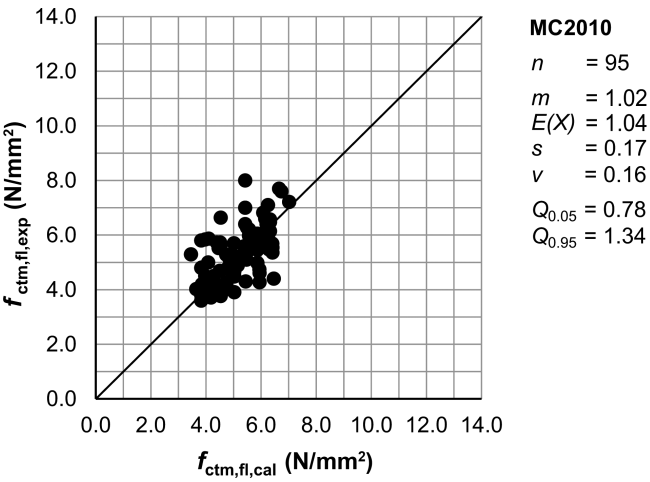

FIGURE 3 Comparison of calculated flexural tensile strength $f_{\mathrm{ctm,fl}}$ according to MC2010 with the experimental flexural tensile strength $f_{\mathrm{ctm,fl,exp}}$

5.2 | Evaluation

The following diagrams (Figure 4–7) show the evaluation of the individual approaches. The evaluation for the f_{R1m} value (R1m) is shown on the (a) and the evaluation for the f_{R3m} value (R3m) on the (b) . Even if the respective approach is only valid for an equivalent or a residual flexural tensile strength (SLS or ULS) or was derived based on four-point bending tests (cf. Table 2), it should be examined whether the approach can also represent the other value or the other test.

With the Bekaert approach, an underestimation of the experimental values in the range of large postcrack flexural tensile strengths and a (slight) overestimation of the experimental values in the range of small postcrack flexural tensile strengths can be seen both for the evaluation of the f_{R1m} value and for the evaluation of the f_{R3m} value (Figure 4). For the Teutsch/Falkner/Klinkert approach, a clear underestimation of the experimental postcrack flexural tensile strengths can be seen for the f_{R1m} and f_{R3m} values (Figure 5). This is because the approach was developed for the equivalent and not for the residual flexural tensile strength (cf. Section 3.3). The approach according to VDS, on the other hand, can represent the f_{R1m} value well to very well (Figure 6a), whereby—due to the lack of information on the conversion of the equivalent flexural tensile strength into a residual flexural tensile strength f_{R1} (cf. Section 3.4)—this is the calculated residual flexural tensile strength f_{R3}. For the f_{R3m} value, on the other hand, there is a slight overestimation in the range of small postcrack flexural

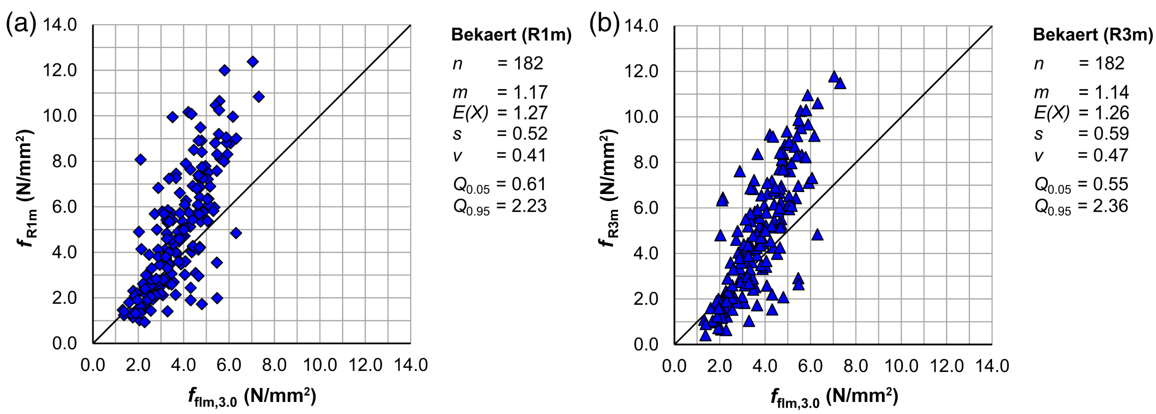

FIGURE 4 Comparison of calculated residual flexural strength $f_{\mathrm{flm},3,0}$ with the mean value of the experimental residual flexural strength f_{R1m} (a) and f_{R3m} (b) using the approach of Bekaert[15]

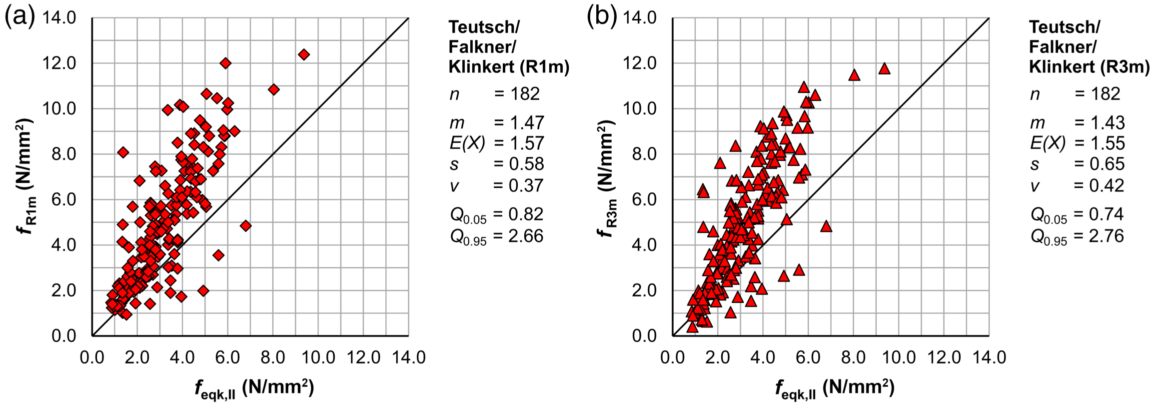

FIGURE 5 Comparison of calculated residual flexural strength $f_{eqk,II}$ with the mean value of the experimental residual flexural strength f_{R1m} (a) and f_{R3m} (b) using the approach of Teutsch/Falkner/Klinkert[16–18]

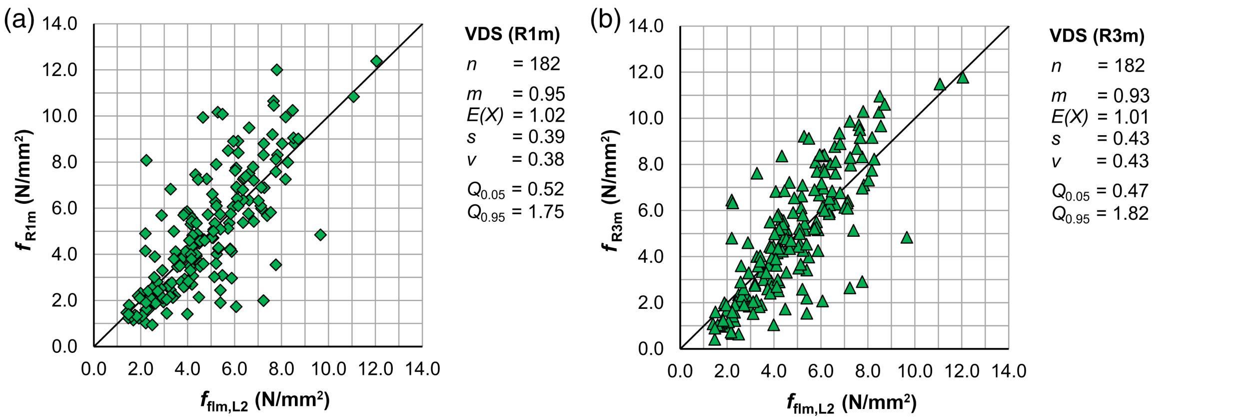

FIGURE 6 Comparison of calculated residual flexural strength $f_{flm,L2}$ with the mean value of the experimental residual flexural strength f_{R1m} (a) and f_{R3m} (b) using the approach of VDS

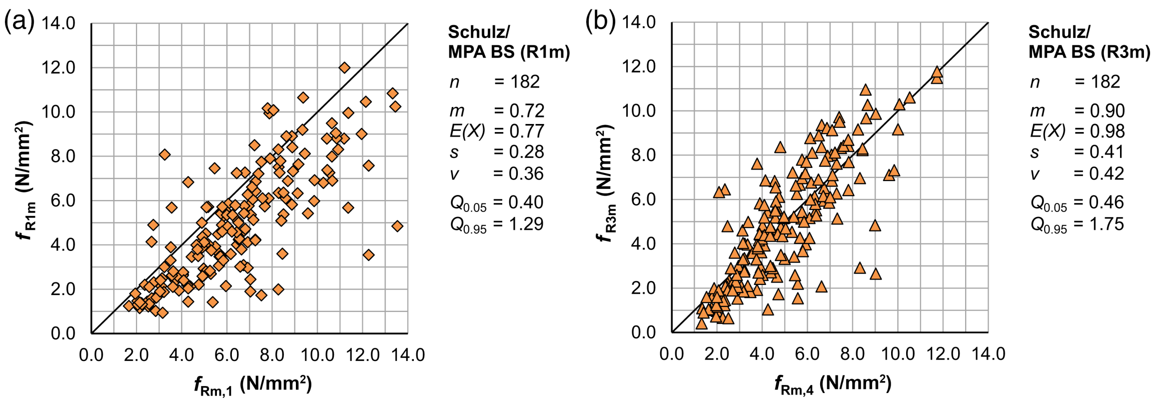

FIGURE 7 Comparison of calculated residual flexural strength $f_{Rm,1}$ and $f_{Rm,4}$ with the mean value of the experimental residual flexural strength f_{R1m} (a) and f_{R3m} (b) using the approach of Schulz/MPA BS

tensile strengths and a slight underestimation in the range of large postcrack flexural tensile strengths (Figure 6b). The same applies to the Schulz/MPA BS approach for the evaluation of the f_{R3m} value (Figure 7b). This is because the calculated value actually refers to the CMOD$_4$ value (cf. Section 3.5). For the evaluation of the f_{R1m} value, an overestimation of the experimental values can be seen (Figure 7a).

Table 3 summarizes the statistical parameters of the individual approaches separately according to the

TABLE 3 Compilation of statistical parameters

Ansatz	Statistical parameters of the evaluation f_{R1m}							Statistical parameters of the evaluation f_{R3m}						
	n	m	$E(X)$	s	v	$Q_{0.05}$	$Q_{0.95}$	n	m	$E(X)$	s	v	$Q_{0.05}$	$Q_{0.95}$
Bekaert	182	1.17	1.27	0.52	0.41	0.61	2.23	182	1.14	1.26	0.59	0.47	0.55	2.36
Teutsch/Falkner/Klinkert	182	1.47	1.57	0.58	0.37	0.82	2.66	182	1.43	1.55	0.65	0.42	0.74	2.76
VDS	182	0.95	1.02	0.39	0.38	0.52	1.75	182	0.93	1.01	0.43	0.43	0.47	1.82
Schulz/MPA BS	182	0.72	0.77	0.28	0.36	0.40	1.29	182	0.90	0.98	0.41	0.42	0.46	1.75
Mod. approach	182	0.98	1.04	0.39	0.37	0.54	1.77	182	0.99	1.08	0.46	0.43	0.51	1.95
Mod. approach NSFRC	148	0.96	1.03	0.41	0.40	0.51	1.80	148	0.96	1.04	0.45	0.43	0.48	1.89
Mod. approach HPFRC	34	1.05	1.09	0.28	0.26	0.70	1.60	34	1.18	1.25	0.45	0.36	0.67	2.09

Abbreviations: HPFRC, High-performance steel fiber-reinforced concrete; NSFRC, normal-strength steel fiber-reinforced concrete.

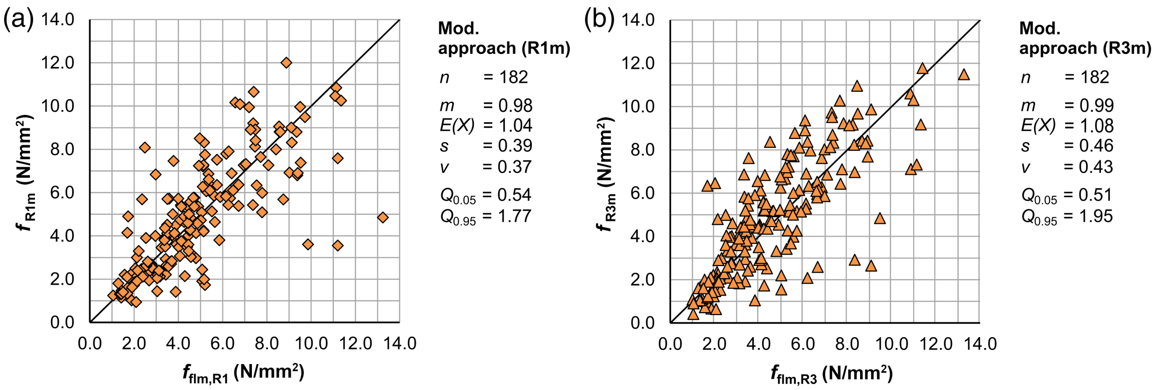

FIGURE 8 Comparison of calculated residual flexural strength $f_{flm,R1}$ and $f_{flm,R3}$ with the mean value of the experimental residual flexural strength f_{R1m} (a) and f_{R3m} (b) using the modified approach

evaluations for the f_{R1m} and f_{R3m} values (Figures 4–7). Since the approaches are supposed to predict the residual flexural tensile strength, the median m and the expected value $E(X)$ should be close to 1.00, but there should also be a small standard deviation s and a small variations coefficient ν as well as a narrow band in $Q_{0.05}$ and $Q_{0.95}$. It can be seen that the approach according to VDS and Schulz/MPA BS provides the best statistical parameters (mainly m and $E(X)$) for f_{R1m} and f_{R3m}, whereby the approach according to VDS is (marginally) better. However, with the Schulz/MPA BS approach, a more differentiated and direct and thus less interdependent determination of the residual flexural tensile strength f_{R1m} and f_{R3m} is possible. This is due to the coefficient ζ_i to take into account the influence of the fiber length, especially for larger CMOD respectively deflections, and the coefficient η to take into account the nonlinear increase in performance as a function of the dosage of the fibers (cf. Section 3.5). However, this approach has—especially for f_{R1m}—relatively low median values and expected values with $m = 0.72$ and $m = 0.90$ or $E(X) = 0.77$ and $E(X) = 0.98$. For this reason, the approach according to Schulz/MPA BS is modified as below to achieve an even better agreement.

5.3 | Modified approach

To modify the approach according to Schulz/MPA BS, an extensive sensitivity analysis was carried out, primarily to obtain a median m and an expected value $E(X)$ close to 1.00 (however, there should also be a small standard deviation s and a small variations coefficient ν as well as a narrow band in $Q_{0.05}$ and $Q_{0.95}$.). On this basis, the factor for the flexural tensile strength of the concrete $f_{ctm,fl}$, the coefficients ζ_{Li} and the coefficient η were adjusted as follows:

$$f_{flm,Ri} = \frac{1}{0.37} \cdot k \cdot V_f \cdot (1 - k \cdot V_f) \cdot \frac{f_{ctm,fl}}{0.39} \cdot \zeta_{Li} \cdot \eta_V, \qquad (7)$$

where ζ_i is the coefficient for taking into account the fiber effect as a function of the length of the fibers and the CMOD considered with $\zeta_1 = 1.18 - \frac{7.5 \cdot l_f}{1000}$ for $CMOD_1 =$

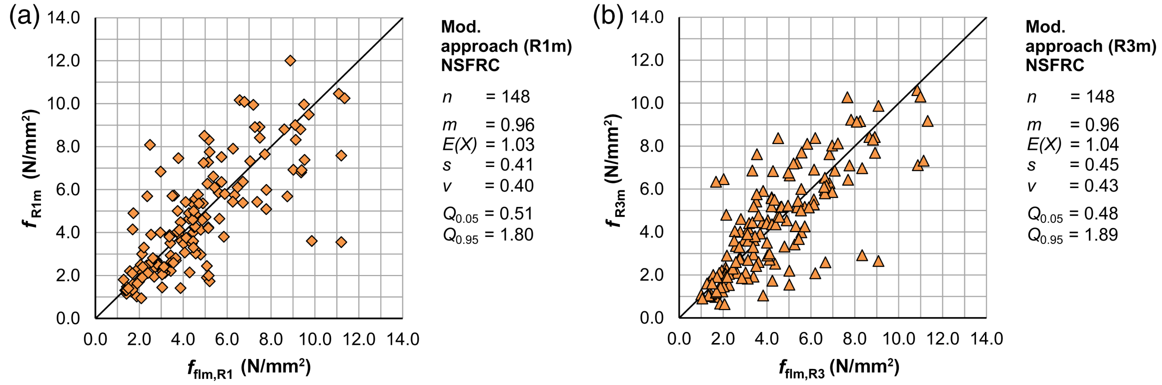

FIGURE 9 Comparison of calculated residual flexural strength $f_{\mathrm{flm,R1}}$ and $f_{\mathrm{flm,R3}}$ with the mean value of the experimental residual flexural strength f_{R1m} (a) and f_{R3m} (b) using the modified approach for NSFRC. NSFRC, Normal-strength steel fiber-reinforced concrete

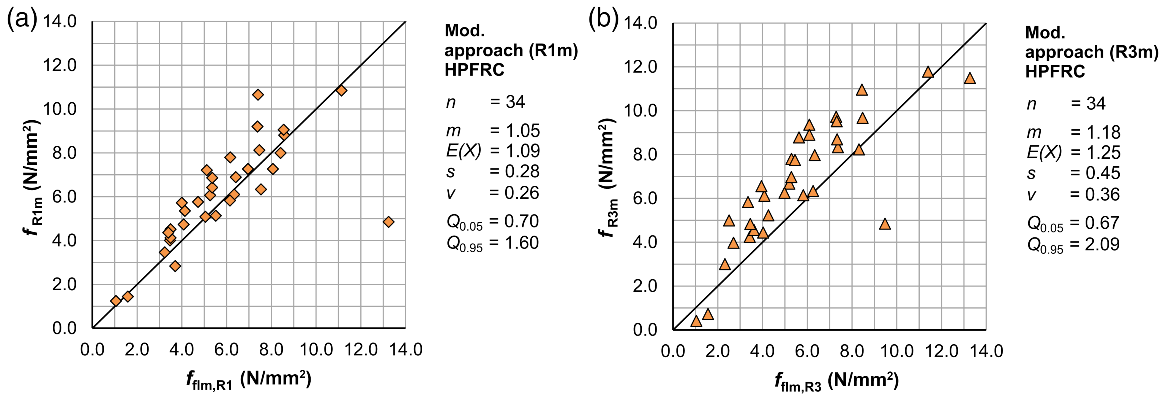

FIGURE 10 Comparison of calculated residual flexural strength $f_{\mathrm{flm,R1}}$ and $f_{\mathrm{flm,R3}}$ with the mean value of the experimental residual flexural strength f_{R1m} (a) and f_{R3m} (b) using the modified approach for HPFRC. HPFRC, High-performance steel fiber-reinforced concrete

0.5 mm and $\zeta_3 = 0.42 + \frac{7.5 \cdot l_f}{1000}$ for $CMOD_3 = 2.5$ mm, and η_V is the coefficient for considering the nonlinear influence of the dosage of the fibers with $\eta_V = 1/(0.7 - 0.2 \cdot V_f)$.

Figure 8 shows the comparison of experimental and calculated values of the residual flexural tensile strengths f_{R1m} and f_{R3m} using the modified approach. It can be seen that the modification results in a significantly improved agreement with a median of $m = 0.98$ (f_{R1m}) and $m = 0.99$ (f_{R3m}), respectively, and an expected value of $E(X) = 1.04$ and $E(X) = 1.08$, respectively.

As already mentioned in Section 4.1, only a few test results on high-strength steel fiber-reinforced concrete ($n = 34$ mean values of individual series [=204 number of specimens]) are available so far. For this reason, the selection criteria with regard to the concrete compressive strength were adjusted to $24\,\mathrm{N/mm^2} \leq f_{\mathrm{cm}} \leq 58\,\mathrm{N/mm^2}$ (normal-strength concrete) and $59\,\mathrm{N/mm^2} \leq f_{\mathrm{cm}} \leq 108\,\mathrm{N/mm^2}$ (high-performance concrete) for further considerations. Figures 9 and 10 show the evaluations, this time separately for normal-strength steel fiber-reinforced concrete (NSFRC) and high-performance steel fiber-reinforced concrete (HPFRC).

The evaluations for NSFRC with $n = 148$ (=888 number of specimens) in Figure 9 show a good to very good agreement between the experimental and calculated f_{R1m} and f_{R3m} values, each with a median of $m = 0.96$ and an expected value of $E(X) = 1.03$ and $E(X) = 1.04$. The occurring scatter can (for the most part) be attributed to the usual scatter of the residual flexural tensile strengths of steel fiber concrete. In contrast, the evaluations for HPFRC in Figure 10 show that the experimental values are largely underestimated. However, the small number of test series of $n = 34$ (=204 number of specimens) must be taken into account here.

Table 3 summarizes the statistical parameters of the modified approach according to the evaluations for the f_{R1m} and f_{R3m} values (Figure 9,10).

Based on the present investigations, it is recommended to use the modified approach according to Equation (7) for the present only for the approximate determination of the residual flexural tensile strength f_{R1m} and f_{R3m} of notched three-point bending tests made of NSFRC with hooked-end steel fibers.

6 | CONCLUSIONS AND OUTLOOK

In this paper, empirical approaches for the approximate determination of the postcrack flexural strengths of steel fiber concrete were investigated. For this purpose, the residual postcrack flexural tensile strengths of three-point bending tests of the bending beam database "Steel Fibre Reinforced Concrete" were compared with calculated postcrack flexural tensile strengths in according with the approaches according to Bekaert, Teutsch/Falkner/Klinkert, VDS, and Schulz/MPA BS. As these approaches are partly based on very different assumptions and postcrack flexural tensile strengths, different agreements with the experimental values of the postcrack flexural tensile strengths f_{R1m} and f_{R3m} according to EN 14651[5] were achieved correspondingly. The best agreement was achieved with the Schulz/MPA BS approach, although even this did not provide sufficiently satisfactory results and was therefore modified on the basis of a comprehensive sensitivity analysis. With the modified approach according to Equation (7), a good to very good estimation of the postcrack flexural tensile strengths f_{R1m} and f_{R3m} of NSFRC with hooked-end steel fibers can be made at the mean value level of notched three-point bending tests. When using the modified approach, however, the limits according to Section 4.1 must be considered, and it must be taken into account that due to unfavorable concrete compositions, such as discontinuous grading curves, unfavorable ratio of fiber length to maximum grain size, or also a lack of adjustment between the strengths of the concrete and the steel fibers used, significant deviations can occur in practice. However, the approach is very well suited as a tool for the development of concrete mixtures for initial tests and for the rough estimation of the performance of steel fiber-reinforced concrete in existing structures.

The modified approach can also be used for analyses and investigations of components reinforced with steel fibers in order to develop new design models or to further develop existing models (e.g., References 20–24). In older literature reporting on structural tests with steel fiber-reinforced concrete, there is often insufficient or no information on the postcrack flexural tensile strength of the steel fiber-reinforced concrete used. With the modified approach, a realistic estimation of the postcrack flexural tensile strength can be made—considering concrete technology aspects—and thus, those tests can be taken into account for further analyses.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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