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ATANASSOV'S INTUITIONISTIC FUZZY TRANSLATIONS OF INTUITIONISTIC FUZZY SUBALGEBRAS IN BG-ALGEBRAS

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Abstract. In this paper, the concepts of intuitionistic fuzzy translation to intuitionistic fuzzy subalgebras in BG-algebras are introduced. The notion of intuitionistic fuzzy extensions and intuitionistic fuzzy multiplications of intuitionistic fuzzy subalgebras are introduced and several related properties are investigated. In this paper, the relationships between intuitionistic fuzzy translations and intuitionistic fuzzy extensions of intuitionistic fuzzy subalgebras are investigated.

Keywords: Intuitionistic fuzzy subalgebra, Intuitionistic fuzzy translation, Intuitionistic fuzzy extension, Intuitionistic fuzzy multiplication.

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1. Introduction

The study of BCK/BCI-algebras [9, 10] was initiated by Imai and Iséki in 1966 as a generalization of the concept of set-theoretic difference and propositional calculus. Bej and Pal [5] introduced interval-valued doubt fuzzy ideals in BCK-algebras. Besides this, the authors have attempted to substantiate a few common features relating them. Some properties of interval-valued doubt fuzzy ideals under homomorphism are investigated and the product of interval-valued doubt fuzzy ideals in BCK-algebras is also established. Bej and Jun [6] defined fuzzy translation of doubt interval-valued fuzzy ideals in BF-algebras. Bej et al. [7] applied the intuitionistic fuzzy hyper filters to hyper BE-algebras and to

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derive the conditions for an intuitionistic fuzzy set to be a doubt intuitionistic fuzzy hyper filter in hyper BE-algebras. Neggers and Kim [14] introduced a new notion, called a B-algebras which is related to several classes of algebras of interest such as BCI/BCK-algebras. Senapati et al. [17] done lot of works on B-algebras. Kim and Kim [11] introduced the notion of BG-algebras, which is a generalization of B-algebras. Ahn and Lee [1] studied fuzzy subalgebras of BG-algebras. Saeid [15] introduced fuzzy topological BG-algebras. The authors [8] presented the concept and basic properties of intuitionistic fuzzy subalgebras, intuitionistic L-fuzzy ideals, interval-valued intuitionistic fuzzy subalgebras, interval-valued intuitionistic fuzzy closed ideals of BG-algebras. Recently, in [16], the authors have studied fuzzy translations, fuzzy extensions and fuzzy multiplications of fuzzy subalgebras in BG-algebras. The notion of intuitionistic fuzzy sets was introduced by Atanassov [4] as a generalization of the notion of fuzzy sets [18].

From these instances and discussions, we are to believe that the intuitionistic fuzzy set has an effective reliability to demonstrate the questionable and probable datum which emerge in real-world issues. Aforesaid translation theories in various fuzzy environment inspire us to a large extent to develop our present paper. The main object of this article is to exhibit translation theories in BG-algebras. In spite of the esteemed innovative approaches developed earlier in this concerned field we, earnestly have left no stone unturned to exhibit our proposed method, so that it can surpass all other earlier inconveniences to solve the real-world issued concern.

In this paper, intuitionistic fuzzy translations, intuitionistic fuzzy extensions and intuitionistic fuzzy multiplications of intuitionistic fuzzy subalgebras in BG-algebras are discussed. Relations among intuitionistic fuzzy translations and intuitionistic fuzzy extensions of intuitionistic fuzzy subalgebras in BG-algebras is also investigated.

2. Preliminaries

In this section, some elementary aspects that are necessary for this paper are included. A BG-algebra is an important class of logical algebras introduced by Kim and Kim [11] and was extensively investigated by several researchers. This algebra is defined as follows.

A non-empty set X with a constant 0 and a binary operation $*$ is called a BG-algebra [11] if it satisfies the following axioms

$F1$. $x * x = 0$

$F2$. $x * 0 = x$

$F3$. $(x * y) * (0 * y) = x$, for all $x, y \in X$.

A non-empty subset S of a BG-algebra X is called a subalgebra ([11]) of X if $x * y \in S$ for any $x, y \in S$.

Let X be the collection of objects denoted generally by x, then a fuzzy set [18] A in X is defined as $A = \{\langle x, \wp(x)\rangle : x \in X\}$, where $\wp(x)$ is called the membership value of x in A and $0 \leq \wp(x) \leq 1$.

A fuzzy set $\wp$ in a BG-algebra X is called a fuzzy BG-subalgebra of X if $\wp(x * y) \geq \min\{\wp(x), \wp(y)\}$ for all $x, y \in X$.

If $(L, \leq)$ is a partially ordered set (poset), and $S \subseteq L$ is an arbitrary subset, then an element $u \in L$ is said to be an upper bound of S if $s \leq u$ for each $s \in S$. A set may have many upper bounds, or none at all. An upper bound u of S is said to be its least upper bound, or join, or supremum, if $u \leq x$ for each upper bound x of S. A set need not have a least upper bound, but it cannot have more than one. Dually, $l \in L$ is said to be a lower bound of S if $l \leq s$ for each $s \in S$. A lower bound l of S is said to be its greatest lower bound, or meet, or infimum, if $x \leq l$ for each lower bound x of S. A set may have many lower bounds, or none at all, but can have at most one greatest lower bound.

Let $\wp$ be a fuzzy subset of X and let $\zeta \in [0, 1 - \sup\{\wp(x) | x \in X\}]$. A mapping $\wp_\zeta^T : X \to [0,1]$ is called a fuzzy ζ-translation [16] of $\wp$ if it satisfies $\wp_\zeta^T(x) = \wp(x) + \zeta$ for all $x \in X$.

An intuitionistic fuzzy set [4] A over X is an object having the form $A = \{\langle x, \wp_A(x), \partial_A(x)\rangle : x \in X\}$, where $\wp_A(x) : X \to [0,1]$ and $\partial_A(x) : X \to [0,1]$, with the condition $0 \leq \partial_A(x) + \partial_A(x) \leq 1$ for all $x \in X$. The numbers $\wp_A(x)$ and $\partial_A(x)$ denote, respectively, the degree of membership and the degree of non-membership of the element x in the set A.

Let $A = \{\langle x, \wp_A(x), \partial_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \wp_B(x), \partial_B(x)\rangle : x \in X\}$ be two intuitionistic fuzzy sets on X. Then the intersection and union [4] of A and B is denoted by $A \cap B$ and $A \cup B$ respectively and is given by

$A \cap B = \{\langle x, \min(\wp_A(x), \wp_B(x)), \max(\partial_A(x), \partial_B(x))\rangle : x \in X\}$,

$A \cup B = \{\langle x, \max(\wp_A(x), \wp_B(x)), \min(\partial_A(x), \partial_B(x))\rangle : x \in X\}$.

An intuitionistic fuzzy set $A = \{\langle x, \wp_A(x), \partial_A(x)\rangle : x \in X\}$ in X is called an intuitionistic fuzzy subalgebra [8] of X if it satisfies the following two conditions

(i) $\wp_A(x * y) \geq \min\{\wp_A(x), \wp_A(y)\}$ and

(ii) $\partial_A(x * y) \leq \max\{\partial_A(x), \partial_A(y)\}$ for all $x, y \in X$.

3. Translations of Intuitionistic Fuzzy subalgebras

For the sake of simplicity, we shall use the symbol $A = (\wp_A, \partial_A)$ for the intuitionistic fuzzy subset $A = \{\langle x, \wp_A(x), \partial_A(x)\rangle : x \in X\}$. Throughout this paper, we take $\Re = \inf\{\partial_A(x) : x \in X\}$ for any intuitionistic fuzzy set $A = (\wp_A, \partial_A)$ of X.

Definition 3.1. *Let $A = (\wp_A, \partial_A)$ be an intuitionistic fuzzy subset of X and let $\zeta \in [0, \Re]$. An object having the form $A_\zeta^T = ((\wp_A)_\zeta^T, (\partial_A)_\zeta^T)$ is called an intuitionistic fuzzy ζ-translation of A if $(\wp_A)_\zeta^T(x) = \wp_A(x) + \zeta$ and $(\partial_A)_\zeta^T(x) = \partial_A(x) - \zeta$ for all $x \in X$.*

Theorem 3.1. *Let $A = (\wp_A, \partial_A)$ be an intuitionistic fuzzy subalgebras of X and let $\zeta \in [0, \Re]$. Then the intuitionistic fuzzy ζ-translation A_ζ^T of A is an intuitionistic fuzzy subalgebra of X.*

Proof. Let $x, y \in X$. Then

$$\begin{aligned} (\wp_A)_\zeta^T(x * y) &= \wp_A(x * y) + \zeta \geq \min\{\wp_A(x), \wp_A(y)\} + \zeta \\ &= \min\{\wp_A(x) + \zeta, \wp_A(y) + \zeta\} \\ &= \min\{(\wp_A)_\zeta^T(x), (\wp_A)_\zeta^T(y)\} \\ \text{and} \quad (\partial_A)_\zeta^T(x * y) &= \partial_A(x * y) - \zeta \leq \max\{\partial_A(x), \partial_A(y)\} - \zeta \\ &= \max\{\partial_A(x) - \zeta, \partial_A(y) - \zeta\} \\ &= \max\{(\partial_A)_\zeta^T(x), (\partial_A)_\zeta^T(y)\}. \end{aligned}$$

Hence, the intuitionistic fuzzy ζ-translation A_ζ^T of A is an intuitionistic fuzzy subalgebra of X. □

Theorem 3.2. *Let $A = (\wp_A, \partial_A)$ be an intuitionistic fuzzy subset of X such that the intuitionistic fuzzy ζ-translation A_ζ^T of A is an intuitionistic fuzzy subalgebra of X for some $\zeta \in [0, \Re]$. Then $A = (\wp_A, \partial_A)$ is an intuitionistic fuzzy subalgebra of X.*

Proof. Assume that $A_\zeta^T = ((\wp_A)_\zeta^T, (\partial_A)_\zeta^T)$ is an intuitionistic fuzzy subalgebra of X for some $\zeta \in [0, \Re]$. Let $x, y \in X$, we have

$$\begin{aligned} \wp_A(x*y)+\zeta &= (\wp_A)_\zeta^T(x*y) \geq \min\{(\wp_A)_\zeta^T(x), (\wp_A)_\zeta^T(y)\} \\ &= \min\{\wp_A(x)+\zeta, \wp_A(y)+\zeta\} \\ &= \min\{\wp_A(x), \wp_A(y)\}+\zeta \\ \text{and} \quad \partial_A(x*y)-\zeta &= (\partial_A)_\zeta^T(x*y) \leq \max\{(\partial_A)_\zeta^T(x), (\partial_A)_\zeta^T(y)\} \\ &= \max\{\partial_A(x)-\zeta, \partial_A(y)-\zeta\} \\ &= \max\{\partial_A(x), \partial_A(y)\}-\zeta \end{aligned}$$

which implies that $\wp_A(x*y) \geq \min\{\wp_A(x), \wp_A(y)\}$ and $\partial_A(x*y) \leq \max\{\partial_A(x), \partial_A(y)\}$ for all $x, y \in X$. Hence, $A = (\wp_A, \partial_A)$ is an intuitionistic fuzzy subalgebra of X. □

Definition 3.2. *Let $A = (\wp_A, \partial_A)$ and $B = (\wp_B, \partial_B)$ be intuitionistic fuzzy subsets of X. If $A \leq B$, i.e, $\wp_A(x) \leq \wp_B(x)$ and $\partial_A(x) \geq \partial_B(x)$ for all $x \in X$, then we say that B is an intuitionistic fuzzy extension of A.*

Definition 3.3. *Let $A = (\wp_A, \partial_A)$ and $B = (\wp_B, \partial_B)$ be intuitionistic fuzzy subsets of X. Then B is called an intuitionistic fuzzy S-extension of A if the following assertions are valid:*

(i) B is an intuitionistic fuzzy extension of A.

(ii) If A is an intuitionistic fuzzy subalgebra of X, then B is an intuitionistic fuzzy subalgebra of X.

From the definition of intuitionistic fuzzy ζ-translation, we get $(\wp_A)_\zeta^T(x) = \wp_A(x) + \zeta$ and $(\partial_A)_\zeta^T(x) = \partial_A(x) - \zeta$ for all $x \in X$. Therefore, we have the following theorem.

Theorem 3.3. *Let $A = (\wp_A, \partial_A)$ be an intuitionistic fuzzy subset of X and $\zeta \in [0, \Re]$. Then the intuitionistic fuzzy ζ-translation $A_\zeta^T = ((\wp_A)_\zeta^T, (\partial_A)_\zeta^T)$ of A is an intuitionistic fuzzy S-extension of A.*

The converse of the Theorem 3.3 is not true in general as seen in the following example.

Example 3.1. *Let $X = \{0, 1, 2, 3, 4, 5\}$ be a BCK-algebra with the following Cayley table:*

$*$	0	1	2	3	4	5
0	0	5	4	3	2	1
1	1	0	5	4	3	2
2	2	1	0	5	4	3
3	3	2	1	0	5	4
4	4	3	2	1	0	5
5	5	4	3	2	1	0

Let $A = (\wp_A, \partial_A)$ be an intuitionistic fuzzy subset of X defined by

X	0	1	2	3	4	5
$\wp_A$	0.7	0.4	0.7	0.4	0.7	0.4
∂_A	0.2	0.5	0.2	0.5	0.2	0.5

Then $A = (\wp_A, \partial_A)$ is an intuitionistic fuzzy subalgebra of X. Let $B = (\wp_B, \partial_B)$ be an intuitionistic fuzzy subset of X defined by

X	0	1	2	3	4	5
$\wp_B$	0.73	0.51	0.73	0.51	0.73	0.51
∂_B	0.18	0.46	0.18	0.46	0.18	0.46

Then B is an intuitionistic fuzzy S-extension of A. But it is not the intuitionistic fuzzy ζ-translation $A^T_\zeta = ((\wp_A)^T_\zeta, (\partial_A)^T_\zeta)$ of A for all $\zeta \in [0, \Re]$.

Clearly, the intersection of intuitionistic fuzzy S-extensions of an intuitionistic fuzzy subalgebra A of X is an intuitionistic fuzzy S-extension of A. But the union of intuitionistic fuzzy S-extensions of an intuitionistic fuzzy subalgebra A of X is not an intuitionistic fuzzy S-extension of A as seen in the following example.

Example 3.2. *Let $X = \{0, 1, 2, 3, 4, 5\}$ be a BG-algebra with the following Cayley table:*

$*$	0	1	2	3	4	5
0	0	2	1	3	4	5
1	1	0	2	4	5	3
2	2	1	0	5	3	4
3	3	4	5	0	2	1
4	4	5	3	1	0	2
5	5	3	4	2	1	0

Let $A = (\wp_A, \partial_A)$ be an intuitionistic fuzzy subset of X defined by

X	0	1	2	3	4	5
$\wp_A$	0.6	0.2	0.2	0.2	0.2	0.2
∂_A	0.4	0.7	0.7	0.7	0.7	0.7

Then $A = (\wp_A, \partial_A)$ is an intuitionistic fuzzy subalgebra of X. Let $B = (\wp_B, \partial_B)$ and $C = (\wp_C, \partial_C)$ be intuitionistic fuzzy subsets of X defined by

X	0	1	2	3	4	5
$\wp_B$	0.7	0.4	0.4	0.7	0.4	0.4
∂_B	0.2	0.6	0.6	0.2	0.6	0.6

and

X	0	1	2	3	4	5
$\wp_C$	0.8	0.5	0.5	0.5	0.8	0.5
∂_C	0.1	0.4	0.4	0.4	0.1	0.4

respectively.

Then B and C are intuitionistic fuzzy S-extensions of A. Obviously, the union $B \cup C$ is an intuitionistic fuzzy extension of A, but it is not an intuitionistic fuzzy S-extension of A since $\wp_{B\cup C}(4 * 3) = \wp_{B\cup C}(1) = 0.5 \ngeq 0.7 = \min\{0.8, 0.7\} = \min\{\wp_{B\cup C}(4), \wp_{B\cup C}(3)\}$ and $\partial_{B\cup C}(4 * 3) = \partial_{B\cup C}(1) = 0.4 \nleq 0.2 = \max\{0.1, 0.2\} = \max\{\partial_{B\cup C}(4), \partial_{B\cup C}(3)\}$.

For an intuitionistic fuzzy subset $A = (\wp_A, \partial_A)$ of X, $\zeta \in [0, \Re]$ and $t, s \in [0, 1]$ with $t \geq \zeta$, let

$$\begin{aligned} U_\zeta(\wp_A; t) &:= \{x \in X : \wp_A(x) \geq t - \zeta\} \\ \text{and} \quad L_\zeta(\partial_A; s) &:= \{x \in X : \partial_A(x) \leq s + \zeta\}. \end{aligned}$$

If A is an intuitionistic fuzzy subalgebra of X, then it is clear that $U_\zeta(\wp_A; t)$ and $L_\zeta(\partial_A; s)$ are subalgebra of X for all $t \in \mathrm{Im}(\wp_A)$ and $s \in \mathrm{Im}(\partial_A)$ with $t \geq \zeta$. But, if we do not give a condition that A is an intuitionistic fuzzy subalgebra of X, then $U_\zeta(\wp_A; t)$ and $L_\zeta(\partial_A; s)$ are not subalgebra of X as seen in the following example.

Example 3.3. *Let $X = \{0, 1, 2, 3, 4, 5\}$ be a BG-algebra in Example 3.2 and $A = (\wp_A, \partial_A)$ be an intuitionistic fuzzy subset of X defined by*

X	0	1	2	3	4	5
$\wp_A$	0.73	0.51	0.51	0.68	0.68	0.68
∂_A	0.24	0.43	0.43	0.27	0.27	0.27

Since $\wp_A(3*5) = 0.51 \not\geq 0.68 = \min\{\wp_A(3), \wp_A(5)\}$ and $\partial_A(4*5) = 0.43 \not\leq 0.27 = \max\{\partial_A(4), \partial_A(5)\}$, therefore, $A = (\wp_A, \partial_A)$ is not an intuitionistic fuzzy subalgebra of X.

For $\zeta = 0.15$ and $t = 0.75$, we obtain $U_\zeta(\wp_A; t) = \{0, 3, 4, 5\}$ which is not a subalgebra of X since $3 * 4 = 2 \notin U_\zeta(\wp_A; t)$.

For $\zeta = 0.15$ and $s = 0.25$, we obtain $L_\zeta(\partial_A; s) = \{0, 3, 4, 5\}$ which is not a subalgebra of X since $5 * 4 = 1 \notin L_\zeta(\partial_A; s)$.

Theorem 3.4. *For $\zeta \in [0, \Re]$, let $A^T_\zeta = ((\wp_A)^T_\zeta, (\partial_A)^T_\zeta)$ be the intuitionistic fuzzy ζ-translation of $A = (\wp_A, \partial_A)$. Then the following assertions are equivalent:*
(i) $A^T_\zeta = ((\wp_A)^T_\zeta, (\partial_A)^T_\zeta)$ is an intuitionistic fuzzy subalgebra of X.
(ii) $U_\zeta(\wp_A; t)$ and $L_\zeta(\partial_A; s)$ are subalgebras of X for $t \in \mathrm{Im}(\wp_A)$, $s \in \mathrm{Im}(\partial_A)$ with $t \geq \zeta$.

Proof. Assume that A^T_ζ is an intuitionistic fuzzy subalgebra of X. Then $(\wp_A)^T_\zeta$ and $(\partial_A)^T_\zeta$ are fuzzy subalgebras of X. Let $x, y \in X$ such that $x, y \in U_\zeta(\wp_A; t)$ and $t \in \mathrm{Im}(\wp_A)$ with $t \geq \zeta$. Then $\wp_A(x) \geq t - \zeta$ and $\wp_A(y) \geq t - \zeta$, i.e., $(\wp_A)^T_\zeta(x) = \wp_A(x) + \zeta \geq t$ and $(\wp_A)^T_\zeta(y) = \wp_A(y) + \zeta \geq t$. Since $(\wp_A)^T_\zeta$ is a fuzzy subalgebra of X, therefore, we have

$$\wp_A(x*y) + \zeta = (\wp_A)^T_\zeta(x*y) \geq \min\{(\wp_A)^T_\zeta(x), (\wp_A)^T_\zeta(y)\} \geq t,$$

that is, $\wp_A(x*y) \geq t - \zeta$ so that $x * y \in U_\zeta(\wp_A; t)$. Again let $x, y \in X$ such that $x, y \in L_\zeta(\partial_A; s)$ and $s \in \mathrm{Im}(\partial_A)$. Then $\partial_A(x) \leq s + \zeta$ and $\partial_A(y) \leq s + \zeta$, i.e., $(\partial_A)^T_\zeta(x) = \partial_A(x) - \zeta \leq s$ and $(\partial_A)^T_\zeta(y) = \partial_A(y) - \zeta \leq s$. Since $(\partial_A)^T_\zeta$ is a fuzzy subalgebra of X, it follows that

$$\partial_A(x*y) - \zeta = (\partial_A)^T_\zeta(x*y) \leq \max\{(\partial_A)^T_\zeta(x), (\partial_A)^T_\zeta(y)\} \leq s,$$

that is, $\partial_A(x*y) \leq s + \zeta$ so that $x * y \in L_\zeta(\partial_A; s)$. Therefore, $U_\zeta(\wp; t)$ and $L_\zeta(\partial_A; s)$ are subalgebras of X.

Conversely, suppose that $U_\zeta(\wp_A; t)$ and $L_\zeta(\partial_A; s)$ are subalgebras of X for $t \in \mathrm{Im}(\wp_A)$, $s \in \mathrm{Im}(\partial_A)$ with $t \geq \zeta$. If there exists $a, b \in X$ such that $(\wp_A)^T_\zeta(a*b) < \beta \leq \min\{(\wp_A)^T_\zeta(a), (\wp_A)^T_\zeta(b)\}$, then $\wp_A(a) \geq \beta - \zeta$ and $\wp_A(b) \geq \beta - \zeta$ but $\wp_A(a*b) < \beta - \zeta$. This shows that $a \in U_\zeta(\wp_A; t)$ and $b \in U_\zeta(\wp_A; t)$ but $a * b \notin U_\zeta(\wp_A; t)$. This is a contradiction, and therefore $(\wp_A)^T_\zeta(x*y) \geq \min\{(\wp_A)^T_\zeta(x), (\wp_A)^T_\zeta(y)\}$ for all $x, y \in X$.

Again assume that there exist $c, d \in X$ such that $(\partial_A)^T_\zeta(c*d) > \delta \geq \max\{(\partial_A)^T_\zeta(c), (\partial_A)^T_\zeta(d)\}$. Then $\partial_A(c) \leq \delta + \zeta$ and $\partial_A(d) \leq \delta + \zeta$ but $\partial_A(c*d) > \delta + \zeta$. Hence, $c \in L_\zeta(\partial_A; s)$ and $d \in L_\zeta(\partial_A; s)$ but $c * d \notin L_\zeta(\partial_A; s)$. This is impossible and therefore $(\partial_A)^T_\zeta(x*y) \leq \max\{(\partial_A)^T_\zeta(x), (\partial_A)^T_\zeta(y)\}$ for all $x, y \in X$. Consequently $A^T_\zeta = ((\wp_A)^T_\zeta, (\partial_A)^T_\zeta)$ is an intuitionistic fuzzy subalgebra of X. □

Theorem 3.5. *Let $A = (\wp_A, \partial_A)$ be an intuitionistic fuzzy subalgebra of X and let $\zeta, \beta \in [0, \Re]$. If $\zeta \geq \beta$, then the intuitionistic fuzzy ζ-translation $A^T_\zeta = ((\wp_A)^T_\zeta, (\partial_A)^T_\zeta)$ of A is an intuitionistic fuzzy S-extension of the intuitionistic fuzzy β-translation $A^T_\beta = ((\wp_A)^T_\beta, (\partial_A)^T_\beta)$ of A.*

Proof. Straightforward. □

For every intuitionistic fuzzy subalgebra $A = (\wp_A, \partial_A)$ of X and $\beta \in [0, \Re]$, the intuitionistic fuzzy β-translation $A^T_\beta = ((\wp_A)^T_\beta, (\partial_A)^T_\beta)$ of A is an intuitionistic fuzzy subalgebra of X. If $B = (\wp_B, \partial_B)$ is an intuitionistic fuzzy S-extension of A^T_β, then there exists $\zeta \in [0, \Re]$ such that $\zeta \geq \beta$ and $B \geq A^T_\zeta$, that is, $\wp_B(x) \geq (\wp_A)^T_\zeta$ and $\partial_B(x) \leq (\partial_A)^T_\zeta$ for all $x \in X$. Hence, we have the following theorem.

Theorem 3.6. *Let $A = (\wp_A, \partial_A)$ be an intuitionistic fuzzy subalgebra of X and let $\beta \in [0, \Re]$. For every intuitionistic fuzzy S-extension $B = (\wp_B, \partial_B)$ of the intuitionistic fuzzy β-translation $A^T_\beta = ((\wp_A)^T_\beta, (\partial_A)^T_\beta)$ of A, there exists $\zeta \in [0, \Re]$ such that $\zeta \geq \beta$ and B is an intuitionistic fuzzy S-extension of the intuitionistic fuzzy ζ-translation $A^T_\zeta = ((\wp_A)^T_\zeta, (\partial_A)^T_\zeta)$ of A.*

Let us illustrate the Theorem 3.6 using the following example.

Example 3.4. *Let $X = \{0, 1, 2, 3, 4, 5\}$ be a BG-algebra and $A = (\wp_A, \partial_A)$ be an intuitionistic fuzzy subset of X defined in Example 3.1. Then $\Re = 0.2$. If we take $\beta = 0.11$, then the intuitionistic fuzzy β-translation $A^T_\beta = ((\wp_A)^T_\beta, (\partial_A)^T_\beta)$ of A is given by*

X	0	1	2	3	4	5
$(\wp_A)^T_\beta$	0.81	0.51	0.81	0.51	0.81	0.51
$(\partial_A)^T_\beta$	0.09	0.39	0.09	0.39	0.09	0.39

Let $B = (\wp_B, \partial_B)$ be an intuitionistic fuzzy subset of X defined by

X	0	1	2	3	4	5
$\wp_B$	0.89	0.57	0.89	0.57	0.89	0.57
∂_B	0.03	0.32	0.03	0.32	0.03	0.32

Then B is clearly an intuitionistic fuzzy subalgebra of X which is an intuitionistic fuzzy S-extension of the intuitionistic fuzzy β-translation A^T_β of A. But B is not an intuitionistic fuzzy ζ-translation of A for all $\zeta \in [0, \Re]$. If we take $\zeta = 0.15$ then $\zeta = 0.15 > 0.11 = \beta$ and the intuitionistic fuzzy ζ-translation $A^T_\zeta = ((\wp_A)^T_\zeta, (\partial_A)^T_\zeta)$ of A is given as follows:

X	0	1	2	3	4	5
$(\wp_A)^T_\zeta$	0.85	0.55	0.85	0.55	0.85	0.55
$(\partial_A)^T_\zeta$	0.05	0.35	0.05	0.35	0.05	0.35

Note that $B(x) \geq A^T_\zeta(x)$, that is, $\wp_B(x) \geq (\wp_A)^T_\zeta(x)$ and $\partial_B(x) \leq (\partial_A)^T_\zeta(x)$ for all $x \in X$, and hence B is an intuitionistic fuzzy S-extension of the intuitionistic fuzzy ζ-translation A^T_ζ of A.

Definition 3.4. *Let $A = (\wp_A, \partial_A)$ be an intuitionistic fuzzy subset of X and $\xi \in [0, 1]$. An object having the form $A^m_\xi = ((\wp_A)^m_\xi, (\partial_A)^m_\xi)$ is called an intuitionistic fuzzy ξ-multiplication of A if $(\wp_A)^m_\xi(x) = \wp_A(x) \cdot \xi$ and $(\partial_A)^m_\xi(x) = \partial_A(x) \cdot \xi$ for all $x \in X$.*

For intuitionistic fuzzy subset $A = (\wp_A, \partial_A)$ of X, an intuitionistic fuzzy 0-multiplication $A^m_0 = ((\wp_A)^m_0, (\partial_A)^m_0)$ of A is an intuitionistic fuzzy subalgebra of X.

Theorem 3.7. *If $A = (\wp_A, \partial_A)$ is an intuitionistic fuzzy subalgebra of X, then the intuitionistic fuzzy ξ-multiplication of A is an intuitionistic fuzzy subalgebra of X for all $\xi \in [0, 1]$.*

Proof. Straightforward. □

Theorem 3.8. *If $A = (\wp_A, \partial_A)$ is any intuitionistic fuzzy subset of X, then the following assertions are equivalent:*

(i) A is an intuitionistic fuzzy subalgebra of X.

(ii) for all $\xi \in (0, 1]$, A^m_ξ is an intuitionistic fuzzy subalgebra of X.

Proof. Necessity follows from Theorem 3.7. For sufficient part let $\xi \in (0,1]$ be such that $A_\xi^m = ((\wp_A)_\xi^m, (\partial_A)_\xi^m)$ is an intuitionistic fuzzy subalgebra of X. Then for all $x, y \in X$, we have

$$\begin{aligned} \wp_A(x*y)\cdot\xi &= (\wp_A)_\xi^m(x*y) \geq \min\{(\wp_A)_\xi^m(x), (\wp_A)_\xi^m(y)\} \\ &= \min\{\wp_A(x)\cdot\xi, \wp_A(y)\cdot\xi\} \\ &= \min\{\wp_A(x), \wp_A(y)\}\cdot\xi \end{aligned}$$

and

$$\begin{aligned} \partial_A(x*y)\cdot\xi &= (\partial_A)_\xi^m(x*y) \leq \max\{(\partial_A)_\xi^m(x), (\partial_A)_\xi^m(y)\} \\ &= \max\{\partial_A(x)\cdot\xi, \partial_A(y)\cdot\xi\} \\ &= \max\{\partial_A(x), \partial_A(y)\}\cdot\xi. \end{aligned}$$

Therefore, $\wp_A(x*y) \geq \min\{\wp_A(x), \wp_A(y)\}$ and $\partial_A(x*y) \leq \max\{\partial_A(x), \partial_A(y)\}$ for all $x, y \in X$ since $\xi \neq 0$. Hence, $A = (\wp_A, \partial_A)$ is an intuitionistic fuzzy subalgebra of X. □

4. Conclusions and Future Work

In this paper, intuitionistic fuzzy translations of intuitionistic fuzzy subalgebras in BG-algebras are introduced and investigated some of their useful properties. The relationship between intuitionistic fuzzy translations and intuitionistic fuzzy extensions of intuitionistic fuzzy subalgebras has been constructed. It is our hope that this work would other foundations for further study of the theory of BG-algebras.

When it comes to future research, we will expand the uses of the translation theories to different areas, as for example Γ-hyperideals over left almost Γ-semihypergroups [2], Ternary semigroups [3], multi-objective optimization problem, clustering, pattern recognition and supply chain management. Moreover, we will stretch out the suggested methodology to integrate of ICTs for pedagogy in Indian private high schools [12] and analysis of teachers' perception on pedagogical successes and challenges of digital teaching practice during new normal [13], etc.

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