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IDEAL AND DOT IDEAL OF A PS ALGEBRA IN PICTURE FUZZY ENVIRONMENT

S. DOGRA[1*], M. PAL[1], §

Abstract. In this paper, the notion of picture fuzzy ideal of a PS algebra is introduced as a generalization of fuzzy ideal of a PS algebra and some important results are explored in this regard. It is highlighted that the intersection of two picture fuzzy ideals is a picture fuzzy ideal and Cartesian product of two picture fuzzy ideals is a picture fuzzy ideal. Also, it is shown by two examples that union of two picture fuzzy ideals is not necessarily a picture fuzzy ideal. The concept of picture fuzzy dot ideal is initiated here as a generalization of fuzzy dot ideal and some elementary results are investigated. A relation between picture fuzzy ideal and picture fuzzy dot ideal is developed here. It is shown that a picture fuzzy ideal is a picture fuzzy dot ideal. But the converse of this proposition is not true in general which has been clarified by an example.

Keyword: Fuzzy set, fuzzy ideal, intuitionistic fuzzy set, picture fuzzy set, picture fuzzy ideal, picture fuzzy dot ideal.

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1. Introduction

Uncertainty is a part of our life. In almost each and every problem, uncertainty arises in our daily life. In classical atmosphere, an element either belongs to a set or not. Partial belongingness of an element is not allowed in case of crisp set. Modifying the classical set theoretic view and allowing the partial membership of an element to a set, Zadeh [39] introduced fuzzy set theory. Fuzzy set theory deals only with the measure of membership. When an information is clear enough then the measure of non-membership is the complement of the measure of membership. In case of doubtful information, individual measurements of the measure of membership and the measure of non-membership are needed. To handle this type of situation successfully, the notion of intuitionistic fuzzy set was propounded by Atanassov [6]. The concepts of BCK/BCI algebra, which is a special class of abstract algebra, was presented by Iseki and his co-workers [16, 17, 18]. The study of BCK algebra under fuzzy environment was done by Xi [38]. Fuzzy set was

[1] Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore, 721102, India.
e-mail: shovansd39@gmail.com; ORCID: https://orcid.org/0000-0002-0768-1764.
[*] Corresponding author.
e-mail: mmpalvu@gmail.com; ORCID: https://orcid.org/0000-0002-6709-836X.
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applied in BCI algebra by Ahmad [1]. The notion of d-algebra (special type of algebra like BCK/BCI algebra) was introduced by Neggers and Kim [30]. The concept of d-ideal in d-algebra was investigated by Neggers, Jun and Kim [31]. Later on Akram [2] applied fuzzy set to d-algebra. A lot of works on BCK/BCI/d-algebra and related ideals under fuzzy set environment were performed by several researchers [21, 28, 34, 36]. BCK algebra and related ideals were studied by Jun and Kim [22] as an extension of fuzzy set concept applied in BCK algebra. As the time advances, research works on BCK/BCI algebra and related ideals were done by Senapati et al. [35, 37] in context of intuitionistic fuzzy set in various directions. PS algebra, a new type of logical algebra, was initiated by Priya and Ramachandran [32] as a generalization of BCI/BCK/d-algebra. Also, they [33] studied PS algebra by applying the notion of fuzzy set in PS algebra. Inclusion of measure of neutral membership and generalization of intuitionistic fuzzy set was given by Cuong [7] in the form of picture fuzzy set. As the time goes, different types of research works under picture fuzzy environment were done by the researchers [8, 19, 20]. A lot of research works on picture fuzzy set based on algebraic structures were done by Dogra and Pal [9, 10, 11, 12, 13]. Decision making problems in different types of uncertain environment (fuzzy environment, soft environment, intuitionistic fuzzy environment, picture fuzzy environment etc.) were studied by several researchers [3, 4, 5, 14, 15, 23, 24, 25, 26, 27, 29].

Ideal of PS algebra in fuzzy setting is already available in literature. Here, it arises our matter of interest to study ideal of PS algebra in an atmosphere which is more complicated than fuzzy atmosphere. In this research article, we show that how multiple components can be handled to study an algebraic structure and what is the connection of our defined algebraic structure with the real life, that is, what our defined algebraic structures actually signify in real life field. Dot ideal in picture fuzzy setting is a new concept defined by us in this research article. We study here some elementary properties of picture fuzzy ideal and picture fuzzy dot ideal of PS algebra. We establish a relationship between picture fuzzy ideal and picture fuzzy dot ideal.

List of Abbreviations:

FS : Fuzzy set
FI : Fuzzy ideal
IFS : Intuitionistic fuzzy set
PFS : Picture fuzzy set
PFSs : Picture fuzzy sets
PFI : Picture fuzzy ideal
PFIs : Picture fuzzy ideals
FDI : Fuzzy dot ideal
PFDI : Picture fuzzy dot ideal
MMS : Measure of membership
MNonMS : Measure of non-membership
MPMS : Measure of positive membership
MNeuMS : Measure of neutral membership
MNegMS : Measure of negative membership
CP : Cartesian product
AS : Algebraic structure
FAS : Fuzzy algebraic structure
IFAS : Intuitionistic fuzzy algebraic structure

PFAS : Picture fuzzy algebraic structure
SFAS : Spherical fuzzy algebraic structure

2. Preliminaries

In this section, some basic concepts of BCK/BCI algebra, d-algebra, PS algebra, ideal of PS algebra, FS, FI of PS algebra, IFS, PFS and some basic operations on PFSs are recapitulated.

The concept of BCI algebra, a special class of abstract algebra, was initiated by Iseki [18].

Definition 2.1. [18] *An algebra $(A, \diamond, 0)$ is said to be BCI-algebra if for any $a, b, c \in A$;*
(i) $[(a \diamond b) \diamond (a \diamond c)] \diamond (c \diamond b) = 0$
(ii) $[a \diamond (a \diamond b)] \diamond b = 0$
(iii) $a \diamond a = 0$
(iv) $a \diamond b = 0$ *and* $b \diamond a = 0 \Rightarrow a = b$.

A BCI algebra in which the condition $0 \diamond a = 0$ for all $a \in A$ is satisfied, is called BCK algebra. Another special class of abstract algebra namely d-algebra as a related algebraic structure of BCK/BCI-algebra was introduced by Neggers and Kim [30].

Definition 2.2. [30] *An algebra $(A, \diamond, 0)$ is said to be d-algebra if for any $a, b \in A$;*
(i) $a \diamond a = 0$
(ii) $0 \diamond a = 0$
(iii) $a \diamond b = 0$ *and* $b \diamond a = 0 \Rightarrow a = b$.

Generalization of BCK/BCI/d-algebra was given by Priya and Ramachandran [32] in the form of PS algebra.

Definition 2.3. [32] *An algebra $(A, \diamond, 0)$ is called PS algebra if for any $a, b \in A$;*
(i) $a \diamond a = 0$
(ii) $a \diamond 0 = 0$
(iii) $a \diamond b = 0$ *and* $b \diamond a \Rightarrow a = b$.

A binary relation '$\leqslant$' on A is defined as: $a \leqslant b$ holds iff $b \diamond a = 0$.

Definition 2.4. [32] *Let $(A, \diamond, 0)$ be a PS algebra and $C \subseteq A$ is non-empty. Then C is called an ideal of A if*
(i) $0 \in C$
(ii) $b \diamond a \in C$ *and* $b \in C \Rightarrow a \in C$.

Definition 2.5. [39] *A FS τ over A is defined as $\tau = \{(a, \tau_1(a)) : a \in A\}$, where $\tau_1 : A \to [0, 1]$. Here, $\tau_1(a)$ is the MMS of a in τ.*

Atanassov [6] defined IFS as a generalization of FS.

Definition 2.6. [6] *An IFS τ over the set of universe A is defined as $\tau = \{(a, \tau_1(a), \tau_2(a)) : a \in A\}$, where $\tau_1(a) \in [0, 1]$ is the MMS of a in τ and $\tau_2(a) \in [0, 1]$ is the MNonMS of a in τ with $0 \leqslant \tau_1(a) + \tau_2(a) \leqslant 1$ for all $a \in A$.*

FI of PS algebra was introduced by Priya and Ramachandran [33], which is an application of FS to ideal of a PS algebra.

Definition 2.7. [33] *Let $(A, \diamond, 0)$ be a PS algebra and $\tau = \{(a, \tau_1(a)) : a \in A\}$ be a FS in A. Then τ is called FI of A if the below stated conditions are satisfied.*
(i) $\tau_1(0) \geqslant \tau_1(a)$
(ii) $\tau_1(a) \geqslant \tau_1(b \diamond a) \wedge \tau_1(b)$ *for all* $a, b \in A$.

More possible types of uncertainty handling tool was introduced by Cuong [7] in the form of PFS which is an extension of FS and IFS.

Definition 2.8. [7] *A PFS τ over the universe A is defined as $\tau = \{(a, \tau_1(a), \tau_2(a), \tau_3(a)) : a \in A\}$, where $\tau_1(a) \in [0, 1]$ is the MPMS of a in τ, $\tau_2(a) \in [0, 1]$ is the MNeuMS of a in τ and $\tau_3(a) \in [0, 1]$ is the MNegMS of a in τ with $0 \leqslant \tau_1(a) + \tau_2(a) + \tau_3(a) \leqslant 1$ for all $a \in A$. For all $a \in A$, $1 - (\tau_1(a) + \tau_2(a) + \tau_3(a))$ is called MRefMS a in τ.*

Some basic operations on PFSs are as follows.

Definition 2.9. [7] *Let $\tau = \{(a, \tau_1(a), \tau_2(a), \tau_3(a)) : a \in A\}$ and $\tau' = \{(a, \tau'_1(a), \tau'_2(a), \tau'_3(a)) : a \in A\}$ be two PFSs over the universe A. Then*
(i) $\tau \subseteq \tau'$ iff $\tau_1(a) \leqslant \tau'_1(a)$, $\tau_2(a) \leqslant \tau'_2(a)$, $\tau_3(a) \geqslant \tau'_3(a)$ for all $a \in A$.
(ii) $\tau = \tau'$ iff $\tau_1(a) = \tau'_1(a), \tau_2(a) = \tau'_2(a), \tau_3(a) = \tau'_3(a)$ for all $a \in A$.
(iii) $\tau \cup \tau' = \{(a, \max(\tau_1(a), \tau'_1(a)), \min(\tau_2(a), \tau'_2(a)), \min(\tau_3(a), \tau'_3(a))) : a \in A\}$.
(iv) $\tau \cap \tau' = \{(a, \min(\tau_1(a), \tau'_1(a)), \min(\tau_2(a), \tau'_2(a)), \max(\tau_3(a), \tau'_3(a))) : a \in A\}$.

Definition 2.10. *Let $\tau = \{(a, \tau_1(a), \tau_2(a), \tau_3(a)) : a \in A_1\}$ and $\tau' = \{(b, \tau'_1(b), \tau'_2(b), \tau'_3(b)) : b \in A_2\}$ be two PFSs over two sets of universe A_1 and A_2 respectively. Then the CP of τ and τ' is the PFS $\tau \times \tau' = \{((a, b), \xi_1((a, b)), \xi_2((a, b)), \xi_3((a, b))) : (a, b) \in A_1 \times A_2\}$, where $\xi_1((a, b)) = \tau_1(a) \wedge \tau'_1(b)$, $\xi_2((a, b)) = \tau_2(a) \wedge \tau'_2(b)$ and $\xi_3((a, b)) = \tau_3(a) \vee \tau'_3(b)$ for all $(a, b) \in A_1 \times A_2$.*

3. Picture Fuzzy Ideals

In the current section, the notion of PFI of PS algebra is initiated and some ground properties of PFI are studied using some basic operations (intersection, CP, union etc.) on PFSs.

Definition 3.1. *Let $(A, \diamond, 0)$ be a PS algebra and τ be a PFS in A. Then P is said to be PFI if the below stated conditions are satisfied.*
(i) $\tau_1(0) \geqslant \tau_1(a)$, $\tau_2(0) \geqslant \tau_2(a)$ *and* $\tau_3(0) \leqslant \tau_3(a)$
(ii) $\tau_1(a) \geqslant \tau_1(b \diamond a) \wedge \tau_1(b)$, $\tau_2(a) \geqslant \tau_2(b \diamond a) \wedge \tau_2(b)$
and $\tau_3(a) \leqslant \tau_3(b \diamond a) \vee \tau_3(b)$ *for all* $a, b \in A$.

Example 3.1. *Let us consider a PS algebra $(A = \{0, a, b\}, \diamond, 0)$ as follows.*

$\diamond$	0	a	b
0	0	a	b
a	0	0	b
b	0	a	0

Now, let us suppose a PFS $\tau = (\tau_1, \tau_2, \tau_3)$ in A as follows.

$$\tau_1(p) = \begin{cases} 0.35, & \text{when } p = 0 \\ 0.25, & \text{when } p = a \\ 0.15, & \text{when } p = b \end{cases}$$

$$\tau_2(p) = \begin{cases} 0.3, & \text{when } p = 0 \\ 0.2, & \text{when } p = a \\ 0.15, & \text{when } p = b \end{cases}$$

and

$$\tau_3(p) = \begin{cases} 0.2, & \text{when } p = 0 \\ 0.3, & \text{when } p = a \\ 0.4, & \text{when } p = b \end{cases}$$

It can be shown that τ is a PFI of A.

Proposition 3.1. *Let $\tau = (\tau_1, \tau_2, \tau_3)$ be a PFI of a PS algebra $(A, \diamond, 0)$. Then $\tau_1(a) \geqslant \tau_1(b)$, $\tau_2(a) \geqslant \tau_2(b)$ and $\tau_3(a) \leqslant \tau_3(b)$ for $a, b \in A$ with $a \leqslant b$.*

Proof: Let $a, b \in A$ such that $a \leqslant b$. Then $b \diamond a = 0$.

$$\begin{aligned} \text{Now, } \tau_1(a) &\geqslant \tau_1(b \diamond a) \wedge \tau_1(b) \text{ [as } \tau \text{ is a PFI of } A] \\ &= \tau_1(0) \wedge \tau_1(b) \\ &= \tau_1(b) \text{ [as } \tau \text{ is a PFI of } A], \end{aligned}$$

$$\begin{aligned} \tau_2(a) &\geqslant \tau_2(b \diamond a) \wedge \tau_2(b) \text{ [as } \tau \text{ is a PFI of } A] \\ &= \tau_2(0) \wedge \tau_2(b) \\ &= \tau_2(b) \text{ [as } \tau \text{ is a PFI of } A] \end{aligned}$$

$$\begin{aligned} \text{and } \tau_3(a) &\leqslant \tau_3(b \diamond a) \vee \tau_3(b) \text{ [as } \tau \text{ is a PFI of } A] \\ &= \tau_3(0) \vee \tau_3(b) \\ &= \tau_3(b) \text{ [as } \tau \text{ is a PFI of } A]. \end{aligned}$$

Thus, it is obtained that $\tau_1(a) \geqslant \tau_1(b)$, $\tau_2(a) \geqslant \tau_2(b)$ and $\tau_3(a) \leqslant \tau_3(b)$ for $a, b \in A$ with $a \leqslant b$.

Proposition 3.2. *Let $(A, \diamond, 0)$ be a PS algebra and $\tau = (\tau_1, \tau_2, \tau_3)$ be a PFI of A. Then $\tau_1(a) \geqslant \tau_1(b) \wedge \tau_1(c)$, $\tau_2(a) \geqslant \tau_2(b) \wedge \tau_2(c)$ and $\tau_3(a) \leqslant \tau_3(b) \vee \tau_3(c)$ for $a, b, c \in A$ with $b \diamond a \leqslant c$.*

Proof: Let $a, b, c \in A$ with $b \diamond a \leqslant c$. Then $c \diamond (b \diamond a) = 0$.

$$\begin{aligned} \text{Now, } \tau_1(a) &\geqslant \tau_1(b \diamond a) \wedge \tau_1(b) \text{ [because } \tau \text{ is a PFI of } A] \\ &\geqslant \tau_1(c \diamond (b \diamond a)) \wedge \tau_1(c) \wedge \tau_1(b) \text{ [because } \tau \text{ is a PFI of } A] \\ &= \tau_1(0) \wedge \tau_1(c) \wedge \tau_1(b) \\ &= \tau_1(b) \wedge \tau_1(c) \text{ [because } \tau \text{ is a PFI of } A], \end{aligned}$$

$$\begin{aligned} \tau_2(a) &\geqslant \tau_2(b \diamond a) \wedge \tau_2(b) \text{ [because } \tau \text{ is a PFI of } A] \\ &\geqslant \tau_2(c \diamond (b \diamond a)) \wedge \tau_2(c) \wedge \tau_2(b) \text{ [because } \tau \text{ is a PFI of } A] \\ &= \tau_2(0) \wedge \tau_2(c) \wedge \tau_2(b) \\ &= \tau_2(b) \wedge \tau_2(c) \text{ [because } \tau \text{ is a PFI of } A] \end{aligned}$$

$$\begin{aligned} \text{and } \tau_3(a) &\leqslant \tau_3(b \diamond a) \vee \tau_3(b) \text{ [because } \tau \text{ is a PFI of } A] \\ &\leqslant \tau_3(c \diamond (b \diamond a)) \vee \tau_3(c) \vee \tau_3(b) \text{ [because } \tau \text{ is a PFI of } A] \\ &= \tau_3(0) \vee \tau_3(c) \vee \tau_3(b) \\ &= \tau_3(b) \vee \tau_3(c) \text{ [because } \tau \text{ is a PFI of } A]. \end{aligned}$$

Thus, it is obtained that $\tau_1(a) \geqslant \tau_1(b) \wedge \tau_1(c)$, $\tau_2(a) \geqslant \tau_2(b) \wedge \tau_2(c)$ and $\tau_3(a) \leqslant \tau_3(b) \vee \tau_3(c)$ for $a, b, c \in A$ with $b \diamond a \leqslant c$.

Proposition 3.3. *Let $\tau = (\tau_1, \tau_2, \tau_3)$ and $\tau' = (\tau_1', \tau_2', \tau_3')$ be two PFIs of a PS algebra $(A, \diamond, 0)$. Then $\tau \cap \tau'$ is a PFI of A.*

Proposition 3.4. *Let $\tau = (\tau_1, \tau_2, \tau_3)$ and $\tau' = (\tau_1', \tau_2', \tau_3')$ be two PFIs of a PS algebra $(A, \diamond, 0)$. Then $\tau \times \tau'$ is a PFI of $A \times A$.*

Here, we have noticed that intersection of two PFIs of a PS algebra is a PFI. But this is not necessarily true in case of union which can be proved by two examples. If τ, τ' are two PFIs of a PS algebra A then Example 3.2 shows that $\tau \cup \tau'$ is not a PFI of A while the Example 3.3 shows that $\tau \cup \tau'$ is a PFI of A.

Example 3.2. *Let us consider a PS algebra $(A = \{0, p, q, r\}, \diamond, 0)$ as follows.*

$\diamond$	0	p	q	r
0	0	p	q	r
p	0	0	r	q
q	0	p	0	p
r	0	p	0	0

Now, let us suppose two PFIs $\tau = (\tau_1, \tau_2, \tau_3)$ and $\tau' = (\tau_1', \tau_2', \tau_3')$ of A as follows.

$$\tau_1(a) = \begin{cases} 0.45, & \text{when } a = 0 \\ 0.2, & \text{when } a = p \\ 0.1, & \text{when } a = q, r \end{cases}$$

$$\tau_2(a) = \begin{cases} 0.4, & \text{when } a = 0 \\ 0.25, & \text{when } a = p \\ 0.1, & \text{when } a = q, r \end{cases}$$

$$\tau_3(a) = \begin{cases} 0.15, & \text{when } a = 0 \\ 0.3, & \text{when } a = p \\ 0.4, & \text{when } a = q, r \end{cases}$$

and

$$\tau_1'(a) = \begin{cases} 0.47, & \text{when } a = 0 \\ 0.09, & \text{when } a = p, r \\ 0.2, & \text{when } a = q \end{cases}$$

$$\tau_2'(a) = \begin{cases} 0.43, & \text{when } a = 0 \\ 0.09, & \text{when } a = p, r \\ 0.25, & \text{when } a = q \end{cases}$$

$$\tau_3'(a) = \begin{cases} 0.1, & \text{when } a = 0 \\ 0.45, & \text{when } a = p, r \\ 0.3, & \text{when } a = q \end{cases}$$

Thus, $\tau \cup \tau' = \xi = (\xi_1, \xi_2, \xi_3)$ is given by

$$\xi_1(a) = \begin{cases} 0.47, & \text{when } a = 0 \\ 0.2, & \text{when } a = p \\ 0.2, & \text{when } a = q \\ 0.1, & \text{when } a = r \end{cases}$$

$$\xi_2(a) = \begin{cases} 0.4, & \text{when } a = 0 \\ 0.09, & \text{when } a = p \\ 0.1, & \text{when } a = q \\ 0.09, & \text{when } a = r \end{cases}$$

$$\xi_3(a) = \begin{cases} 0.1, & \text{when } a = 0 \\ 0.3, & \text{when } a = p \\ 0.3, & \text{when } a = q \\ 0.4, & \text{when } a = r \end{cases}$$

It is observed that $0.1 = \xi_1(r) \ngeq \xi_1(q \diamond r) \wedge \xi_1(q) = \xi_1(p) \wedge \xi_1(q) = 0.2 \wedge 0.2 = 0.2$, $0.4 = \xi_3(r) \nleq \xi_3(q \diamond r) \vee \xi_3(q) = \xi_3(p) \vee \xi_3(q) = 0.3 \vee 0.3 = 0.3$, *but* $0.09 = \xi_2(r) \geq \xi_2(q \diamond r) \wedge \xi_2(q) = \xi_2(p) \wedge \xi_2(q) = 0.09 \wedge 0.1 = 0.09$. *So,* $\tau \cup \tau'$ *is not a PFI of* A.

Example 3.3. *Let us consider the PS algebra given in Example 3.1. Now, let us suppose two PFIs* $\tau = (\tau_1, \tau_2, \tau_3)$ *and* $\tau' = (\tau_1', \tau_2', \tau_3')$ *in* A *as follows.*

$$\tau_1(a) = \begin{cases} 0.5, & \text{when } a = 0 \\ 0.35 & \text{when } a \neq 0 \end{cases}$$

$$\tau_2(a) = \begin{cases} 0.4, & \text{when } a = 0 \\ 0.2, & \text{when } a \neq 0 \end{cases}$$

$$\tau_3(a) = \begin{cases} 0.1, & \text{when } a = 0 \\ 0.3, & \text{when } a \neq 0 \end{cases}$$

and

$$\tau_1'(a) = \begin{cases} 0.45, & \text{when } a = 0 \\ 0.4 & \text{when } a \neq 0 \end{cases}$$

$$\tau_2'(a) = \begin{cases} 0.35, & \text{when } a = 0 \\ 0.3, & \text{when } a \neq 0 \end{cases}$$

$$\tau_3'(a) = \begin{cases} 0.2, & \text{when } a = 0 \\ 0.25, & \text{when } a \neq 0 \end{cases}$$

Thus, $\tau \cup \tau' = \xi = (\xi_1, \xi_2, \xi_3)$ *is given by*

$$\xi_1(a) = \begin{cases} 0.5, & \text{when } a = 0 \\ 0.4 & \text{when } a \neq 0 \end{cases}$$

$$\xi_2(a) = \begin{cases} 0.35, & \text{when } a = 0 \\ 0.2, & \text{when } a \neq 0 \end{cases}$$

$$\xi_3(a) = \begin{cases} 0.1, & \text{when } a = 0 \\ 0.25, & \text{when } a \neq 0 \end{cases}$$

Here, clearly $\xi = \tau \cup \tau'$ is a PFI of A.

Proposition 3.5. *Let* $\tau = (\tau_1, \tau_2, \tau_3)$ *and* $\tau' = (\tau_1', \tau_2', \tau_3')$ *be two PFIs of a PS algebra* $(A, \diamond, 0)$. *Then* $\tau \times \tau'$ *is a PFI of* $A \times A$.

Proof: Let $\tau \times \tau' = \xi = (\xi_1, \xi_2, \xi_3)$. Then $\xi_1((a,b)) = \tau_1(a) \wedge \tau_1'(b)$, $\xi_2((a,b)) = \tau_2(a) \wedge \tau_2'(b)$ and $\xi_3((a,b)) = \tau_3(a) \vee \tau_3'(b)$ for all $(a,b) \in A \times A$.

$$\begin{aligned} \text{Now, } \xi_1((0,0)) &= \tau_1(0) \wedge \tau_1'(0) \\ &\geqslant \tau_1(a) \wedge \tau_1'(b) \text{ [as } \tau, \tau' \text{ are PFIs of } A] \\ &= \xi_1((a,b)), \end{aligned}$$

$$\begin{aligned} \xi_2((0,0) &= \tau_2(0) \wedge \tau_2'(0) \\ &\geqslant \tau_2(a) \wedge \tau_2'(b) \text{ [as } \tau, \tau' \text{ are PFIs of } A] \\ &= \xi_2((a,b)) \end{aligned}$$

$$\begin{aligned}\text{and } \xi_3((0,0)) &= \tau_3(0) \vee \tau_3'(0)\\ &\leqslant \tau_3(a) \vee \tau_3'(b) \text{ [as } \tau, \tau' \text{ are PFIs of } A]\\ &= \xi_3((a,b)) \text{ for all } (a,b) \in A \times A.\end{aligned}$$

$$\begin{aligned}\text{Also, } \xi_1((a,b)) &= \tau_1(a) \wedge \tau_1'(b)\\ &\geqslant (\tau_1(c \diamond a) \wedge \tau_1(c)) \wedge (\tau_1'(d \diamond b) \wedge \tau_1'(d)\\ &\text{[as } \tau, \tau' \text{ are PFIs of } A]\\ &= (\tau_1(c \diamond a) \wedge (\tau_1'(d \diamond b)) \wedge (\tau_1(c) \wedge \tau_1'(d))\\ &= \xi_1((c,d) \diamond (a,b)) \wedge \xi_1((c,d)),\end{aligned}$$

$$\begin{aligned}\xi_2((a,b)) &= \tau_2(a) \wedge \tau_2'(b)\\ &\geqslant (\tau_2(c \diamond a) \wedge \tau_2(c)) \wedge (\tau_2'(d \diamond b) \wedge \tau_2'(d))\\ &\text{[as } \tau, \tau' \text{ are PFIs of } A]\\ &= (\tau_2(c \diamond a) \wedge (\tau_2'(d \diamond b)) \wedge (\tau_2(c) \wedge \tau_2'(d))\\ &= \xi_2((c,d) \diamond (a,b)) \wedge \xi_2((c,d))\end{aligned}$$

$$\begin{aligned}\text{and } \xi_3((a,b)) &= \tau_3(a) \vee \tau_3'(b)\\ &\leqslant (\tau_3(c \diamond a) \vee \tau_3(c)) \vee (\tau_3'(d \diamond b) \vee \tau_3'(d))\\ &\text{[as } \tau, \tau' \text{ are PFIs of } A]\\ &= (\tau_3(c \diamond a) \vee (\tau_3'(d \diamond b)) \vee (\tau_3(c) \vee \tau_3'(d))\\ &= \xi_3((c,d) \diamond (a,b)) \vee \xi_3((c,d)) \text{ for all } (a,b), (c,d) \in A \times A.\end{aligned}$$

Consequently, $\xi = \tau \times \tau'$ is a PFI of $A \times A$.

Proposition 3.6. *Let $(A_1, \diamond, 0)$ and $(A_2, \circ, 0)$ be two PS algebras and $\tau = (\tau_1, \tau_2, \tau_3)$, $\tau' = (\tau_1', \tau_2', \tau_3')$ be two PFIs of A_1 and A_2 respectively. Then $\xi_1((0,0)) \geqslant \xi_1((a,b))$, $\xi_2((0,0)) \geqslant \xi_2((a,b))$ and $\xi_3((0,0)) \leqslant \xi_3((a,b))$ for all $(a,b) \in A_1 \times A_2$.*

Proof: Here,
$$\begin{aligned}\xi_1((0,0) &= \tau_1(0) \wedge \tau_1'(0)\\ &\geqslant \tau_1(a) \wedge \tau_1'(b) \text{ for all } a \in A_1 \text{ and } b \in A_2\\ &\text{[because } \tau, \tau' \text{ are PFIs of } A_1 \text{ and } A_2 \text{ respectively]}\\ &= \xi_1((a,b)) \text{ for all } (a,b) \in A_1 \times A_2.\end{aligned}$$

$$\begin{aligned}\text{Also, } \xi_2((0,0) &= \tau_2(0) \wedge \tau_2'(0)\\ &\geqslant \tau_2(a) \wedge \tau_2'(b) \text{ for all } a \in A_1 \text{and } b \in A_2\\ &\text{[because } \tau, \tau' \text{ are PFIs of } A_1 \text{ and } A_2 \text{ respectively]}\\ &= \xi_2((a,b)) \text{ for all } (a,b) \in A_1 \times A_2.\end{aligned}$$

$$\begin{aligned}\text{And } \xi_3((0,0) &= \tau_3(0) \vee \tau_3'(0)\\ &\leqslant \tau_3(a) \vee \tau_3'(b) \text{ for all } a \in A_1 \text{ and } b \in A_2\\ &\text{[because } \tau, \tau' \text{ are PFIs of } A_1 \text{ and } A_2 \text{ respectively]}\\ &= \xi_3((a,b)) \text{ for all } (a,b) \in A_1 \times A_2.\end{aligned}$$

Thus, it is obtained that $\xi_1((0,0)) \geqslant \xi_1((a,b))$, $\xi_2((0,0)) \geqslant \xi_2((a,b))$ and $\xi_3((0,0)) \leqslant \xi_3((a,b))$ for all $(a,b) \in A_1 \times A_2$.

Proposition 3.7. *Let $(A_1, \diamond, 0)$ and $(A_2, \circ, 0)$ be two PS algebras and $\tau = (\tau_1, \tau_2, \tau_3)$, $\tau' = (\tau_1', \tau_2', \tau_2')$ be two PFIs of A_1 and A_2 respectively. Then one of the below stated conditions must hold.*

(i) $\tau_1(a) \leqslant \tau_1'(0)$, $\tau_2(a) \leqslant \tau_2'(0)$ *and* $\tau_3(a) \geqslant \tau_3'(0)$

(ii) $\tau_1'(b) \leqslant \tau_1(0)$, $\tau_2'(b) \leqslant \tau_2(0)$ *and* $\tau_3'(b) \geqslant \tau_3(0)$

for all $a \in A_1$ and for all $b \in A_2$.

Proof: Let none of the conditions be hold. Then there exists some $a \in A_1$ and $b \in A_2$ such that $\tau_1(a) > \tau_1'(0)$, $\tau_2(a) > \tau_2'(0)$, $\tau_3(a) < \tau_3'(0)$ and $\tau_1'(b) > \tau_1(0)$, $\tau_2'(b) > \tau_2(0)$, $\tau_3'(b) < \tau_3(0)$.

$$\begin{aligned}\text{Now, } \xi_1((a,b)) &= \tau_1(a) \wedge \tau_1'(b) \\ &> \tau_1'(0) \wedge \tau_1(0) \\ &= \xi_1((0,0)),\end{aligned}$$

$$\begin{aligned}\xi_2((a,b)) &= \tau_2(a) \wedge \tau_2'(b) \\ &> \tau_2'(0) \wedge \tau_2(0) \\ &= \xi_2((0,0))\end{aligned}$$

$$\begin{aligned}\text{and } \xi_3((a,b)) &= \tau_3(a) \vee \tau_3'(b) \\ &< \tau_3'(0) \vee \tau_3(0) \\ &= \xi_3((0,0)).\end{aligned}$$

Now, it is obtained that $\xi_1((a,b)) > \xi_1((0,0))$, $\xi_2((a,b)) > \xi_2((0,0))$ and $\xi_3((a,b)) < \xi_3((0,0))$. But it is known from Proposition 3.6 that $\xi_1((a,b)) \leqslant \xi_1((0,0))$, $\xi_2((a,b)) \leqslant \xi_2((0,0))$ and $\xi_3((a,b)) \geqslant \xi_3((0,0))$. Hence, one of the stated conditions must hold.

4. Picture fuzzy dot ideals

In the current section, the notion of PFDI of a PS algebra is initiated as an extension of FDI of a PS algebra and some basic results are investigated.

Definition 4.1. *Let $(A, \diamond, 0)$ be a PS algebra and $\tau = (\tau_1, \tau_2, \tau_3)$ be a PFS in A. Then τ is said to be PFI of A if the below stated conditions are meet.*

(i) $\tau_1(0) \geqslant \tau_1(a)$, $\tau_2(0) \geqslant \tau_2(a)$ *and* $\tau_3(0) \leqslant \tau_3(a)$

(ii) $\tau_1(a) \geqslant \tau_1(b \diamond a) \cdot \tau_1(b)$, $\tau_2(a) \geqslant \tau_2(b \diamond a) \cdot \tau_2(b)$ *and* $\tau_3(a) \leqslant \tau_3(b \diamond a) + \tau_3(b) - \tau_3(b \diamond a) \cdot \tau_3(b)$ *for all $a, b \in A$.*

Proposition 4.1. *Let $(A, \diamond, 0)$ be a PS algebra and $\tau = (\tau_1, \tau_2, \tau_3)$ be a PFDI of A. Then $\tau_1(a) \geqslant (\tau_1(b))^2$, $\tau_2(a) \geqslant (\tau_2(b))^2$ and $\tau_3 \leqslant 2\tau_3(b)$ for $a, b \in A$ with $a \leqslant b$.*

Proof: Let $a, b \in A$ such that $a \leqslant b$. Then $b \diamond a = 0$.

$$\begin{aligned}\text{Now, } \tau_1(a) &\geqslant \tau_1(b \diamond a) \cdot \tau_1(b) \text{ [as } \tau \text{ is a PFDI of } A] \\ &= \tau_1(0) \cdot \tau_1(b) \\ &\geqslant \tau_1(b) \cdot \tau_1(b) \text{ [as } \tau \text{ is a PFDI of } A] \\ &= (\tau_1(b))^2,\end{aligned}$$

$$\begin{aligned}\tau_2(a) &\geqslant \tau_2(b \diamond a) \cdot \tau_2(b) \text{ [as } \tau \text{ is a PFDI of } A] \\ &= \tau_2(0) \cdot \tau_2(b) \\ &\geqslant \tau_2(b) \cdot \tau_2(b) \text{ [as } \tau \text{ is a PFDI of } A] \\ &= (\tau_2(b))^2\end{aligned}$$

$$\begin{aligned}\text{and } \tau_3(a) &\leqslant \tau_3(b \diamond a) + \tau_3(b) - \tau_3(b \diamond a) \cdot \tau_3(b) \text{ [as } \tau \text{ is a PFDI of } A] \\ &= \tau_3(0) + \tau_3(b) - \tau_3(0) \cdot \tau_3(b) \\ &\leqslant \tau_3(0) + \tau_3(b) \\ &\leqslant \tau_3(b) + \tau_3(b) \text{ [as } \tau \text{ is a PFDI of } A] \\ &= 2\tau_3(b).\end{aligned}$$

Thus, it is obtained that $\tau_1(a) \geqslant (\tau_1(b))^2$, $\tau_2(a) \geqslant (\tau_2(b))^2$ and $\tau_3(a) \leqslant 2\tau_3(b)$ for $a, b \in A$ with $a \leqslant b$.

Proposition 4.2. *Let $(A, \diamond, 0)$ be a PS algebra and $\tau = (\tau_1, \tau_2, \tau_3)$ be a PFI of A. Then τ is a PFDI of A.*

Proof: Since τ is a PFI of A therefore $\tau_1(a) \geqslant \tau_1(b \diamond a) \wedge \tau_1(b)$, $\tau_2(a) \geqslant \tau_2(b \diamond a) \wedge \tau_2(b)$ and $\tau_3(a) \leqslant \tau_3(b \diamond a) \vee \tau_3(b)$ for all $a, b \in A$.
It is observed that each of $\tau_1(b \diamond a)$ and $\tau_1(b) \geqslant \tau_1(b \diamond a) \cdot \tau_1(b)$,
each of $\tau_2(b \diamond a)$ and $\tau_2(b) \geqslant \tau_2(b \diamond a) \cdot \tau_2(b)$,
each of $\tau_3(b \diamond a)$ and $\tau_3(b) \leqslant \tau_3(b \diamond a) + \tau_3(b) - \tau_3(b \diamond a) \cdot \tau_3(b)$.
Thus, it follows that

$$\begin{aligned}\tau_1(a) &\geqslant \tau_1(b \diamond a) \wedge \tau_1(b) \\ &\geqslant (\tau_1(b \diamond a) \cdot \tau_1(b)), \\ \tau_2(a) &\geqslant \tau_2(b \diamond a) \wedge \tau_2(b) \\ &\geqslant (\tau_2(b \diamond a) \cdot \tau_2(b)) \\ \text{and } \tau_3(a) &\leqslant \tau_3(b \diamond a) \vee \tau_3(b) \\ &\leqslant (\tau_3(b \diamond a) + \tau_3(b) - \tau_3(b \diamond a) \cdot \tau_3(b)) \text{ for all } a, b \in A.\end{aligned}$$

Consequently, τ is a PFDI of A.

Thus, it is noticed that every PFI is a PFDI but the converse of the proposition is not necessarily true, which can be proved by the following example.

Example 4.1. *Let us consider a PS algebra $(A = \{0, a, b\}, \diamond, 0)$ given in the following tabulated form.*

$\diamond$	0	a	b
0	0	a	b
a	0	0	a
b	0	a	0

Now, let us suppose a PFS $\tau = (\tau_1, \tau_2, \tau_3)$ in A as follows.

$$\tau_1(p) = \begin{cases} 0.4, & \text{when } p = 0 \\ 0.35, & \text{when } p = a \\ 0.2, & \text{when } p = b \end{cases}$$

$$\tau_2(p) = \begin{cases} 0.4, & \text{when } p = 0 \\ 0.25, & \text{when } p = a \\ 0.2, & \text{when } p = b \end{cases}$$

and

$$\tau_3(p) = \begin{cases} 0.1, & \text{when } p = 0 \\ 0.2, & \text{when } p = a \\ 0.3, & \text{when } p = b \end{cases}$$

It can be shown that P is a PFDI of A.
It is observed that $0.2 = \tau_1(b) \ngeq \tau_1(a \diamond b) \wedge \tau_1(a) = \tau_1(a) \wedge \tau_1(a) = \tau_1(a) = 0.35$*,* $0.2 = \tau_2(b) \ngeq \tau_2(a \diamond b) \wedge \tau_2(a) = \tau_2(a) \wedge \tau_2(a) = \tau_2(a) = 0.25$ *and* $0.3 = \tau_3(b) \nleq \tau_3(a \diamond b) \vee \tau_3(a) = \tau_3(a) \vee \tau_3(a) = \tau_3(a) = 0.2$*. So, τ is not a PFI of A.*

5. Application:

PS algebra is an important class of abstract algebra (logical algebra). In the definition of ideal of PS algebra $(A, \diamond, 0)$, we have observed that $b \in A$ and $b \diamond a \in A \Rightarrow a \in A$. This can be interpreted as : absorption of b occurs from the elements b and $b \diamond a$ by the element a. In fact, the definition of ideal of PS algebra in picture fuzzy setting gives a relation by which one can understand how much this absorption process affects on the elements of the set of universe and this is given in terms of picture fuzzy membership values. In picture fuzzy sense, only elements have no significance but their membership values are much important to analyze something. It is necessary to mention that absorption is a phenomenon in which a liquid gets soaked into the surface of other substance. Physically, it has a lot of applications in real life. Water soaked by towel is a real life example of absorption. Distribution is an important term related to our real life. The pattern of distribution of an element $a \diamond b$ in PS algebra over 'dot/dot-plus' in terms picture fuzzy membership values is given by PFDI. So our work actually expresses real life facts mathematically in higher level uncertain environment.

PFS is the generalization of FS and IFS. So PFI and PFDI are the generalized versions of FI and FDI respectively. By choosing some particular values of the measures of membership, one can easily verify whether our established results hold in fuzzy environment as a special case of picture fuzzy environment. This study can be viewed as a platform to study ideal and dot ideal of PS algebra in more complicated uncertain environment, such as picture fuzzy interval valued environment, spherical fuzzy environment etc. In fact, algebraic structure has a lot of applications in different areas of Computer Science such as error correction, coding theory etc. This is enough to supply motivation among the researchers to re-establish different notions of abstract algebra (logical algebra) in amplified structure of fuzzy and advanced fuzzy atmosphere. Classical logic only works in certain cases, but non-classical logic (fuzzy logic) works in uncertain cases. Certain case can be easily derived from uncertain cases. So fuzzy logic is stronger than classical logic.

6. Comparative Study:

PFS is a higher level uncertainty handling tool where FS and IFS are not able to handle uncertainty in some special cases. FS deals with only one component but IFS deals with two components. When we do not consider neutral component and replace MPMS by MMS and MNegMS by MNonMS then IFS is obtained from PFS. If we do not consider neutral and negative component and replace MPMS by MMS then FS is obtained. Algebraic structures become complicated when the number of components increases. Here, we have been able to establish a special type of PFAS by which one can easily deduce FAS and IFAS. By this study one can easily verify that how the properties of AS hold in higher level uncertain environmemt. This study can also be extended in other types of uncertain environment or to other types of AS. Since PFAS is the generalized form of FAS and IFAS therefore our obtained results in this case are better than the results available in previous literature (fuzzy literature and intuitionistic fuzzy literature). PS algebra is an important type of logical algebra and study of PS algebra and related ideal in fuzzy setting is already available in literature and that study was done by Priya and Ramachandran [33]. But the study of PS algebra and related ideal in advanced fuzzy environment is not available still now. This study has been done by us in this research article. The study has been done by taking MPMS and MNeuMS in the same sense and MNegMS in the opposite sense.

7. Conclusion and Future Studies:

In this article, the notions of PFI and PFDI of a PS algebra have been introduced. Some results related to these have been studied by propositions and suitable examples. A relation between PFI and PFDI has been investigated. It has been proved that every PFI is a PFDI but the converse is not necessarily true which has been clarified through a suitable example. Sometimes, uncertainty occurs with high imprecision. PFS is not capable to handle this type of situation. In those cases, spherical fuzzy set is a powerful tool to handle such information with high imprecision. Spherical fuzzy set is a special type of advanced fuzzy set such that the square sum of three types of measure of membership lies in the interval [0,1]. For instance, let consider an information with measures of membership (0.6, 0.4, 0.2). Here, $0.6+0.4+0.2 = 1.2 > 1$ but $0.6^2+0.4^2+0.2^2 = 0.36+0.16+0.40 = 0.92 < 1$. So, in this case, spherical fuzzy set is more powerful than PFS. From the concept of PFAS, one can easily define SFAS. The working rule to define SFAS is same as PFAS. In case of SFAS, the square of membership values plays the key role. The properties of SFAS is same as PFAS. It is necessary to mention that each PFAS is a SFAS but each SFAS is not necessarily a PFAS. We expect that it will be easy for the researchers to generalize these concepts in context of some other types of set environment in future.

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