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APPLICATION OF NEUTROSOPHIC POISSON PROBABILITY DISTRIBUTION SERIES FOR CERTAIN SUBCLASS OF ANALYTIC UNIVALENT FUNCTION

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Abstract. Necessary and sufficient conditions for neutrosophic Poisson probability distribution series to be in the classes $M_{\gamma}^{n}(\theta)$ were derived by means of coefficient inequalities. The results obtained further strengthening the relationships between geometric function theory and statistics and by extension, fuzzy logic.

Keywords: Analytic, close-to-convex, fuzzy, neutrosophic, Poisson, Salagean.

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1. Introduction

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{1}$$

that are analytic in the open unit disk $D = \{z : |z| < 1\}$ with normalization condition $f(0) = 0$ and $f'(0) = 1$. Denote by S the subclass of A which consist of univalent functions of the form

$$f(z) = z - \sum_{k=2}^{\infty} |a_k|\, z^k. \tag{2}$$

The well known subclasses of S are starlike and convex functions. The classes of starlike and convex functions of order zero are respectively denoted by S^* and K. Other class of

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interest to the researcher here present is the class of close-to-convex functions with the geometrical representation

$$Re\left(e^{i\delta}\frac{f'(z)}{g(z)}\right) > 0, \quad z \in D.$$

The class of all close-to-convex functions is denoted by C and was introduced by Kaplan in [5] where g is convex. Let $g(z) \equiv z$, the class C reduces to class introduced and investigated by Mahzoon and Kargar in [13] and defined by

$$R = \left\{f \in A : Re\left(\frac{e^{i\beta}f'(z)}{z}\right) > 0, \quad for \;\; \beta \in \mathcal{R}, \quad z \in D\right\}.$$

where $\delta \in \mathcal{R}$.

Definition 1. A function f of the form (2) belongs to class $M_\gamma^n(\theta)$ if it satisfies the condition

$$Re\left\{(D^n f(z))' + \frac{1+e^{i\theta}}{2}z(D^n f(z))''\right\} > \gamma, \quad 0 \le \gamma < 1, \quad -\pi < \theta \le \pi \tag{3}$$

where $n = 0, 1, 2, 3, ...$ and D^n is the well-known Salangean derivative operator.

Remark A. If we let $n = 0$ in the Definition 1, the class introduced and investigated by in [13] is obtained. Other chains of new and existing function classes can be obtained with various choices of n and other parameters involved in the definition.

The concept of neutrosophic theory is a new branch of philosophy as a generalization for the fuzzy logic and a generalization of instrinstic fuzzy logic. The concept was introduced and investigated in [19] by Smarandache, (see also [3,4-7,20-21] it provides a new foundation for dealing with issues that have indeterminate data (may be numbers). The use of neutrosophic crisp sets theory alongside classical probability (Poisson, exponential, uniform distribution) opens a new stairway for dealing with issues that follows the classical distribution and at the same time contain data not specified accurately [15] as example. Neutrosophic Poisson distribution of discrete variable X is a classical Poisson distribution of x, its parameters are imprecise, m can be set with two or more elements and the most common of such distribution is when m is interval say [1,7].

The proposed work will addresses issues that follows classical distribution and at the same time containing data not specified accurately or indeterminate, which makes it different from the existing works which only deals with classical distribution in Geometric Function Theory. The novelty of this investigation is such that the earlier existing works in Geometric Function Theory/Univalent Function Theory addresses classical Poisson distribution whose data are specific and accurate, whereas the present investigation captured both the classical Poisson distribution and neutrosophic Poisson distribution whose data are indeterminate which in turn addresses physical problems with fluctuations in the situations of our daily living.

The current work is limited due to its restriction to solving few initial coefficients estimate of neutrosophic Poisson distribution in the unit disk, though with potential applications in telecommunication industry, signal processing and transport industry.

A variable x is said to be Neutrosophic Poisson distributed if it takes the values $0, 1, 2, 3, ...$ the probability e^{-m_N}, $\frac{me^{-m_N}}{1!}$, $\frac{m^2e^{-m_N}}{2!}$, $\frac{m^3e^{-m_N}}{3!}$, ... respectively, where m_N is called distribution parameter which is equal to the expected value and the variance,

$$P(x = k) = m_N^k\frac{e^{-m_N}}{x!}, k = 0, 1, 2, 3, ...$$

and

$$NE(x) = NV(x) = m_N$$

where $N = d + I$ is a nuetrosophic statistical number (see [1,7]) and the reference therein. Recently, Alhabib *et al.* [1] introduced and investigated a power series of neutrosophic Poisson distribution which was further investigated by Oladipo [15] via coefficient inequalities expressed as

$$K(m_N, z) = z + \sum_{k=2}^{\infty} \frac{m_N^{k-1}}{(k-1)!} e^{-m_N} z^k, \quad z \in D, \tag{4}$$

where $m \in [1, \infty]$ and by ratio test, the radius of convergence of the above series was shown to be infinity. Further, we define a series

$$\Omega(m_N, z) = 2z - K(m_N, z) = z - \sum_{k=2}^{\infty} \frac{m_N^{k-1}}{(k-1)!} e^{-m_N} z^k, \quad z \in D. \tag{5}$$

By convolution principle (Hadamard product), we have

$$\Omega K(m_N, z) = K(m_N, z) * f(z) = z + \sum_{k=2}^{\infty} \frac{m_N^{k-1}}{(k-1)!} e^{-m_N} a_k z^k, \quad z \in D \tag{6}$$

where $*$ denote the convolution or Hadamard product of two series.
Motivated by the earlier works in [9,11,15,17,18,22] we determine the necessary and sufficient conditions for neutrosophic Poisson distribution class $\Omega(m_N, z)$ to be in the following analytic function classes $M_\gamma^0(\theta)$, $M_\gamma^1(\theta)$ and $M_\gamma^2(\theta)$.
In what follows, we state conditions for the integral

$$G(m_N, z) = \int_0^z \epsilon^{-1} K(m_N)(\epsilon) d\epsilon$$

to be in the class $M_\gamma^0(\theta)$, $M_\gamma^1(\theta)$ and $M_\gamma^2(\theta)$.

2. Preliminary and Lemmas

Lemma 1. A function f of the form (2) belongs to class $M_\gamma^n(\theta)$ if and only if

$$\sum_{k=2}^{\infty} k^n \left[2k + k(k-1)(1 + cos\theta)\right] |a_k| \leq 2(1-\gamma), \tag{7}$$

where $n = 0, 1, 2, ..., 0 \leq \gamma < 1, -\pi < \theta \leq \pi$. The result (7) is sharp for the relation

$$f(z) = z - \frac{2(1-\gamma)}{k^n \left[2k + k(k-1)(1 + cos\theta)\right]} z^k, \quad k \geq 2.$$

Proof. Suppose that the function $f(z) \in M_\gamma^n(\theta)$. Then by (3) we have

$$R\left\{1 - \sum_{k=2}^{\infty} k^n \left[\frac{2k + k(k-1)(1 + cos\theta)}{2}\right] |a_k| \, z^{k-1}\right\} > \gamma.$$

Choose z to be real and let $z \to 1^-$, we obtain

$$1 - \sum_{k=2}^{\infty} k^n \left[\frac{2k + k(k-1)(1 + cos\theta)}{2}\right] |a_k| > \gamma$$

which is equivalent to (7). Conversely, suppose that (7) is true, then, we have

$$\left|(D^n f(z))' + \frac{1+e^{i\theta}}{2} z(D^n f(z))''\right| < \sum_{k=2}^{\infty} k^n \left[2k + k(k-1)(1+cos\theta)\right]|a_k|$$
$$\leq 2(1-\gamma),$$

which implies that $f(z) \in M_\gamma^n(\theta)$.
A function $f \in A$ is said to be in the class $R^T(A,B), T \in \mathbb{C}/\{0\}, -1 \leq B < A \leq 1$ if

$$\left|\frac{f'(z)-1}{(A-B)T - B(f'(z)-1)}\right| < 1.$$

Lemma 2. [10] If f of the form (1) belongs to class $R^T(A,B)$, then

$$|a_k| \leq (A,B)\frac{|T|}{k}, \quad k \in N - \{1\}.$$

for $-1 \leq B < A \leq 1$. The result is sharp for

$$f(z) = \int_0^z [1 + (A+B)\frac{Tt^{t-1}}{1+Bt^{t-1}}]dt, \quad (z \in D, \quad k \in N - \{1\}).$$

3. Necessary and sufficient conditions

For easy handling, throughout the investigation, we shall refer to the following

$$\sum_{k=2}^{\infty} \frac{m_N^{k-1}}{(k-1)!} = e^{m_N} - 1 \quad and \quad \sum_{k=j}^{\infty} \frac{m_N^{k-1}}{(k-1)!} = m_N^{j-1} e^{m_N}, \quad j \geq 2. \tag{8}$$

Theorem 2.1. Let $m \in [1,\infty], \quad 0 \leq \gamma < 1$ and $-\pi < \theta \leq \pi$. Then, $\Omega(m_N, z) \in M_\gamma^0(\theta)$ if and only if

$$(1+cos\theta)m_N^2 + 2(2+cos\theta)m_N + 2(1-e^{-m_N}) \leq 2(1-\gamma). \tag{9}$$

Proof. In order to prove that $\Omega(m_N, z) \in M_\gamma^0(\theta)$, using Lemma 1 we have to show that

$$P_1 = \sum_{k=2}^{\infty} \left[2k + k(k-1)(1+cos\theta)\right] \frac{m_N^{k-1}}{(k-1)!} e^{-m_N} \leq 2(1-\gamma).$$

Setting

$$k = (k-1)+1, \quad k^2 = (k-1)(k-2) + 3(k-1) + 1, \tag{10}$$

we have

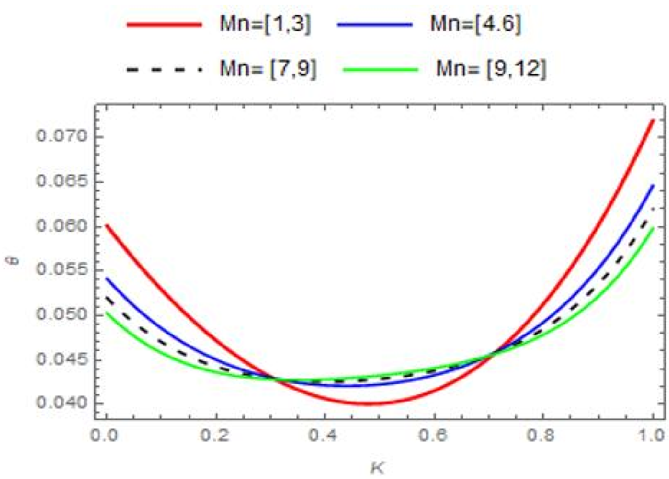
$$P_1 = \sum_{k=2}^{\infty} \left[2k + k(k-1)(1+cos\theta)\right] \frac{m_N^{k-1}}{(k-1)!} e^{-m_N}$$
$$= \sum_{k=2}^{\infty} \left[(1+cos\theta)(k-1)(k-2) + 2(2+cos\theta)(k-1) + 2\right] \frac{m_N^{k-1}}{(k-1)!} e^{-m_N}$$
$$= \sum_{k=2}^{\infty} (1+cos\theta)(k-1)(k-2)\frac{m_N^{k-1}}{(k-1)!} e^{-m_N} + \sum_{k=2}^{\infty} 2(2+cos\theta)(k-1)\frac{m_N^{k-1}}{(k-1)!} e^{-m_N}$$
$$+ \sum_{k=2}^{\infty} 2\frac{m_N^{k-1}}{(k-1)!} e^{-m_N}$$
$$= (1+cos\theta)\sum_{k=2}^{\infty} \frac{m_N^{k-1}}{(k-3)!} e^{-m_N} + 2(2+cos\theta)\sum_{k=2}^{\infty} \frac{m_N^{k-1}}{(k-2)!} e^{-m_N} + 2\sum_{k=2}^{\infty} \frac{m_N^{k-1}}{(k-1)!} e^{-m_N}$$

and by (8), we note that

$$P_1 = (1 + cos\theta)m_N^2 + 2(2 + cos\theta)m_N + 2(1 - e^{-m_N}).$$

The last expression is bounded above by $2(1-\gamma)$, if and only if (9) is satisfied. Using MATHEMATICA to solve for (9) for selected interval m_N, the table and the figure below are obtained. The values obtained are in agreement with existing literature with consideration for $cos\theta$

γ	θ	$\exp^{-(1,3)}$	$\exp^{-(4,6)}$	$\exp^{-(7,9)}$	$\exp^{-(10,12)}$	$\exp^{-(13,15)}$
0.0	-170°	0.742368	3.305782	3.71256	1.62463	5.191065
0.1	-150°	0.765961	3.315391	3.664105	1.333853	5.191065
0.2	-120°	0.739958	3.328393	3.652495	0.958093	5.191065
0.3	-90°	0.840355	3.344756	3.344756	0.457078	5.191065
0.4	-60°	0.806345	3.85672	3.234632	0.758511	4.651781
0.5	-30°	0.765961	1.073055	3.108090	0.855382	0.86886
0.6	0°	0.596623	3.344756	3.344756	0.657287	-0.10607
0.7	30°	0.768936	4.810744	3.572167	0.734085	-4.33072
0.8	60°	0.749724	5.703564	5.133456	0.945536	-1.48213
0.9	90°	0.748955	5.007446	4.191590	1.050707	1.95339
0.1	120°	0.656740	3.999445	3.344756	0.720507	2.72255
0.2	150°	0.638521	3.344756	2.074505	0.742961	2.69613
0.3	170°	0.623452	3.325782	1.054101	0.442614	2.31235



Theorem 2.2. Let $m \in [1, \infty], 0 \leq \gamma < 1$ and $-\pi < \theta \leq \pi$. Then, $\Omega(m_N, z) \in M_\gamma^1(\theta)$ if and only if

$$(1 + cos\theta)m_N^3 + (7 + 5cos\theta)m_N^2 + 2(5 + 2cos\theta)m_N + 2(1 - e^{-m_N}) \leq 2(1-\gamma). \quad (11)$$

Proof. Let

$$TK(m_N, z) = z - \sum_{k=2}^{\infty} \frac{m_N^{k-1}}{(k-1)!} e^{-m_N} z^k$$

and by Lemma 1 we have to prove that

$$P_2 = \sum_{k=2}^{\infty} k\left[2k + k(k-1)(1+cos\theta)\right] \frac{m_N^{k-1}}{(k-1)!} e^{-m_N} \leq 2(1-\gamma)e^{m_N}.$$

Using (10) we have

$$k^3 = (k-1)(k-2)(k-3) + 6(k-1)(k-2) + 7(k-1) + 1,$$

we have

$$P_2 = \sum_{k=2}^{\infty} \left[2k + k(k-1)(1+cos\theta)\right] \frac{m_N^{k-1}}{(k-1)!} e^{-m_N}$$

$$= ((1+cos\theta)) \sum_{k=2}^{\infty} \frac{m_N^{k-1}}{(k-4)!} e^{-m_N} + (7+5cos\theta) \sum_{k=2}^{\infty} \frac{m_N^{k-1}}{(k-3)!} e^{-m_N}$$

$$+2(5+2cos\theta) \sum_{k=2}^{\infty} \frac{m_N^{k-2}}{(k-2)!} e^{-m_N} + 2 \sum_{k=2}^{\infty} \frac{m_N^{k-1}}{(k-1)!} e^{-m_N}$$

and by (8), we observe that

$$P_2 = (1+cos\theta)m_N^3 e^{m_N} + (7+5cos\theta)m_N^2 e^{m_N} + 2(5+2cos\theta)m_N e^{m_N} + (e^{m_N} - 1).$$

The last expression is bounded above by $2(1-\gamma)$, if and only if (11) is satisfied. $(1+cos\theta)M_n^2 + 2(2+cos\theta)M_n + (1-\exp^{-M_n})$

Theorem 2.3. Let $m \in [1,\infty], 0 \leq \gamma < 1$ and $-\pi < \theta \leq \pi$. Then, $\Omega(m_N, z) \in M_\gamma^2(\theta)$ if and only if

$$(1+cos\theta)m_N^4 + (11+9cos\theta)m_N^3 + (31+19cos\theta)m_N^2 + 2(11+4cos\theta)m_N + 2(1-e^{-m_N}) \leq 2(1-\gamma). \tag{12}$$

Proof. Let

$$\Omega(m_N, z) = z - \sum_{k=2}^{\infty} \frac{m_N^{k-1}}{(k-1)!} e^{-m_N} z^k$$

and by Lemma 1 we have to show that

$$P_3 = \sum_{k=2}^{\infty} \left[2k^3 + k^3(k-1)(1+cos\theta)\right] \frac{m_N^{k-1}}{(k-1)!} e^{-m_N} \leq 2(1-\gamma)e^{m_N}. \tag{13}$$

Using (10) and setting

$k^4 = (k-1)(k-2)(k-3)(k-4) + 10(k-1)(k-2)(k-3) + 25(k-1)(k-2) + 15(k-1) + 1,$

in equation (13) and by simple computation we have the desired result.

Making use of Lemma 2, we shall study the effect of neutrsophic Poisson distribution series on the classes $M_\gamma^0(\theta)$, $M_\gamma^1(\theta)$ and $M_\gamma^2(\theta)$.

Theorem 2.4. Let $m \in [1,\infty]$, $0 \leq \gamma < 1$ and $-\pi < \theta \leq \pi$ and $f \in R^T(A,B)$. Then, $\Omega(m_N, z) \in M^0\gamma(\theta)$ if and only if

$$(A-B)\left|T\right|\left[(1+cos\theta)m_N + 2(1-e^{-m_N})\right] \leq 2(1-\gamma). \tag{14}$$

Proof. In view of Theorem 1 it suffices to prove that

$$P_4 = \sum_{k=2}^{\infty} k\left[2 + (k-1)(1+cos\theta)\right] \frac{m_N^{k-1}}{(k-1)!} e^{-m_N} \leq 2(1-\gamma)$$

and since $f \in R^T(A,B)$ from Lemma 2 we have

$$P_4 \leq (A-B)\left|T\right| \left[\sum_{k=2}^{\infty}\left[2+k(k-1)(1+cos\theta)\right]\frac{m_N^{k-1}}{(k-1)!}e^{-m_N}\right].$$

Thus

$$P_4 \leq (A-B)\left|T\right| \left[\sum_{k=2}^{\infty}(1+cos\theta)(k-1)\frac{m_N^{k-1}}{(k-1)!}e^{-m_N}+2\sum_{k=2}^{\infty}\frac{m_N^{k-1}}{(k-1)!}e^{-m_N}\right].$$

$$\leq (A-B)\left|T\right| \left[\sum_{k=2}^{\infty}(1+cos\theta)\frac{m_N^{k-1}}{(k-2)!}e^{-m_N}+2\sum_{k=2}^{\infty}\frac{m_N^{k-1}}{(k-1)!}e^{-m_N}\right]$$

$$\leq (A-B)\left|T\right|\left[(1+cos\theta)m_N+2(1-e^{-m_N})\right].$$

The last expression is bounded above by $2(1-\gamma)$, if and only if (14) is satisfied.

Theorem 2.5. Let $m \in [1,\infty]$, $0 \leq \gamma < 1, -\pi < \theta \leq \pi$ and if $f \in R^T(A,B), -1 \leq B < A \leq 1$. Then, $\Omega(m_N,z) \in M^1_\gamma(\theta)$ if and only if

$$(A-B)\left|T\right|\left[(1+cos\theta)m_N^2+2(2+cos\theta)m_N+2(1-e^{-m_N})\right] \leq 2(1-\gamma). \tag{15}$$

Proof. The prove follow the same process as in the last Theorem.

Theorem 2.6. Let $m \in [1,\infty]$, $0 \leq \gamma < 1 -\pi < \theta \leq \pi$ and if $f \in R^T(A,B), -1 \leq B < A \leq 1$. Then, $\Omega(m_N,z) \in M^2_\gamma(\beta,\theta)$ if and only if

$$(A-B)\left|T\right|\left[(1+cos\theta)m_N^3+(7+5cos\theta)m_N^2+2(5+2cos\theta)m_N+2(1-e^{-m_N})\right]$$
$$\leq 2(1-\gamma). \tag{16}$$

4. Integral Operator

In this section we obtain necessary and sufficient conditions for the integral operator

$$G(m)(z) = \int_0^z \frac{F(m)()\varepsilon}{\varepsilon}d\varepsilon$$

to be in $M^0_\gamma(\theta)$, $M^1_\gamma(\theta)$ $M^2_\gamma(\theta)$.

Theorem 4.1. Let m $in[1,\infty]$, $0 \leq \gamma < 1$ and $-\pi < \theta \leq \pi$, then, $G(m_N,z) \in M^0_\gamma(\theta)$, $M^1_\gamma(\theta)$ $M^2_\gamma(\theta)$ if and only if the conditions

$$(1+cos\theta)m_N+2(1-e^{-m_N}) \leq 2(1-\gamma), \tag{17}$$

$$(1+cos\theta)m_N^2+2(2+cos\theta)m_N+2(1-e^{-m_N}) \leq 2(1-\gamma) \tag{18}$$

and

$$(1+cos\theta)m_N^3+(7+5cos\theta)m_N^2+2(5+2cos\theta)m_N+2(1-e^{-m_N}) \leq 2(1-\gamma) \tag{19}$$

are respectively satisfied.

Example for case study 1

In a company, phone employee receives phone calls, the calls arrive with rate of $[1,3]$ calls per minute, we will estimate the probability that the employee will not receive any call within a minute.

Corollary 4.1. Let $m\ in[1,3]$, $0 \leq \gamma < 1$ and $-\pi < \theta \leq \pi$. Then, $K(m_N, z) \in [M_\gamma^0(\theta),\ M_\gamma^1(\theta)\ \ M_\gamma^2(\theta)]$ if and only if

$$(1 + cos\theta)[1, 3]^2 + 2(2 + cos\theta)[1, 3] + 2(1 - e^{-[1,3]}) \leq 2(1 - \gamma),$$

$$(1 + cos\theta)[1, 3]^3 + (7 + 5cos\theta)m[1, 3]^2 + 2(5 + 2cos\theta)[1, 3] + 2(1 - e^{-[1,3]}) \leq 2(1 - \gamma)$$

and

$$(1 + cos\theta)[1, 3]^4 + (11 + 9cos\theta)[1, 3]^3 + (31 + 19cos\theta)[1, 3]^2$$
$$+2(11 + 4cos\theta)[1, 3] + 2(1 - e^{-[1,3]}) \leq 2(1 - \gamma)$$

respectively.

Corollary 4.2. Let $m \in [1, 3]$, $\gamma = 0$ and $-\pi < \theta \leq \pi$. Then, $\Omega(m_N, z) \in [M_0^0(\theta),\ M_0^1(\theta)\ \ M_0^2(\theta)]$ if and only if

$$(1 + cos\theta)[1, 3]^2 + 2(2 + cos\theta)[1, 3] + 2(1 - e^{-[1,3]}) \leq 2,$$

$$(1 + cos\theta)[1, 3]^3 + (7 + 5cos\theta)[1, 3]^2 + 2(5 + 2cos\theta)[1, 3] + 2(1 - e^{-[1,3]}) \leq 2$$

and

$$(1 + cos\theta)[1, 3]^4 + (11 + 9cos\theta)[1, 3]^3 + (31 + 19cos\theta)[1, 3]^2$$
$$+2(11 + 4cos\theta)[1, 3] + 2(1 - e^{-[1,3]}) \leq 2$$

respectively.

Example for case study 2

In a company, phone employee receives phone calls, the calls arrive with rate of $[1, 3]$ calls per minute, we will estimate the probability that the employee will not receive any call within 5 minutes.

Corollary 4.3. Let $m \in [1, 3]$, $0 \leq \gamma < 1$ and $-\pi < \theta \leq \pi$. Then, $K(m_N, z) \in [M_\gamma^0(\theta),\ M_\gamma^1(\theta)\ \ M_\gamma^2(\theta)]$ if and only if

$$(1 + cos\theta)[5, 15]^2 + 2(2 + cos\theta)[5, 15] + 2(1 - e^{-[5,15]}) \leq 2(1 - \gamma),$$

$$(1 + cos\theta)[5, 15]^3 + (7 + 5cos\theta)[5, 15]^2 + 2(5 + 2cos\theta)[1, 3] + 2(1 - e^{-[5,15]}) \leq 2(1 - \gamma)$$

and

$$(1 + cos\theta)[5, 15]^4 + (11 + 9cos\theta)[5, 15]^3 + (31 + 19cos\theta)[5, 15]^2$$
$$+2(11 + 4cos\theta)[5, 15] + 2(1 - e^{-[1,3]}) \leq 2(1 - \gamma)$$

respectively.

Corollary 4.4. Let $m \in [5, 15]$, $0 \leq \gamma < 1$ and $-\pi < \theta \leq \pi$. Then, $K(m_N, z) \in [M_0^0(\theta),\ M_0^1(\theta)\ \ M_0^2(\theta)]$ if and only if

$$(1 + cos\theta)[5, 15]^2 + 2(2 + cos\theta)[5, 15] + 2(1 - e^{-[1,3]}) \leq 2,$$

$$(1 + cos\theta)[5, 15]^3 + (7 + 5cos\theta)[5, 15]^2 + 2(5 + 2cos\theta)[1, 3] + 2(1 - e^{-[5,15]}) \leq 2$$

and

$$(1 + cos\theta)[5, 15]^4 + (11 + 9cos\theta)[5, 15]^3 + (31 + 19cos\theta)[5, 15]^2$$
$$+2(11 + 4cos\theta)[5, 15] + 2(1 - e^{-[5,15]}) \leq 2.$$

respectively. Using MATLAB we generate the following table and figures for specific values of m_N.

m_N	$\theta = -150\,^\circ$	$\theta = -120\,^\circ$	$\theta = -90\,^\circ$	$\theta = -60\,^\circ$	$\theta = -30\,^\circ$	$\theta = 0\,^\circ$
1.00	3.662	4.7642	6.2642	7.7642	8.8623	9.2642
2.00	6.8011	9.7293	13.7293	17.7293	20.6575	21.7293
3.00	9.9100	15.4004	22.9000	30.4004	38.8908	37.9004
4.00	13.1788	21.9634	33.9634	45.9634	54.7480	57.9634
5.00	16.6756	29.4865	46.9865	64.4865	77.2974	81.9865
6.00	20.4258	37.9950	61.9950	85.9950	103.5643	109.9950
7.00	24.4386	47.4982	78.9982	110.4982	133.5578	141.9982
8.00	28.7173	57.9993	97.9993	137.9993	167.2814	177.9993
9.00	33.2632	69.4998	118.9998	168.4998	204.7363	217.9998
10.00	38.0769	81.9999	141.9999	201.9999	245.9230	261.9999
11.00	43.1583	95.5000	167.0000	238.5000	290.8416	310.0000
12.00	48.5077	110.0000	194.0000	278.0000	339.4923	362.0000
13.00	54.1250	125.5000	223.0000	320.5000	291.8749	418.0000
14.00	60.0103	142.000	254.0000	366.0000	447.9897	478.0000
15.00	66.1635	159.5000	287.0000	414.5000	507.8365	542.0000
16.00	72.5847	178.0000	322.0000	466.0000	571.4153	610.0000
17.00	79.2738	197.5000	359.0000	520.5000	638.7662	682.0000

m_N	$\theta = 30\,^\circ$	$\theta = 60\,^\circ$	$\theta = 90\,^\circ$	$\theta = 120\,^\circ$	$\theta = 150\,^\circ$	$\theta = 180\,^\circ$
1.00	8.8623	7.7642	6.2642	4.7642	3.6662	3.2642
2.00	20.6575	17.7293	13.7293	9.7293	6.8011	5.7293
3.00	35.8908	30.4004	22.9000	15.4004	9.9100	7.9004
4.00	54.7480	45.9634	33.9634	21.9634	13.1788	9.9634
5.00	77.2974	64.4865	46.9865	29.4865	16.6756	11.9865
6.00	103.5643	85.9950	61.9950	37.9950	20.4258	13.9950
7.00	133.5578	110.4982	78.9982	47.4982	24.4386	15.9982
8.00	167.2814	137.9993	97.9993	57.9993	28.7173	17.9993
9.00	204.7363	168.4998	118.9998	69.4998	33.2632	19.9998
10.00	245.9230	201.9999	141.9999	81.9999	38.0769	21.9999
11.00	290.8416	238.5000	167.0000	95.5000	43.1583	24.0000
12.00	339.4923	278.0000	194.0000	110.0000	48.5077	26.0000
13.00	391.8748	320.5000	223.0000	125.5000	54.1250	28.0000
14.00	447.9897	366.0000	254.0000	142.0000	60.0103	30.0000
15.00	507.8365	414.5000	287.0000	159.5000	66.1635	32.0000
16.00	571.4153	466.0000	322.0000	178.0000	72.5847	34.0000
17.00	863.7262	520.5000	359.0000	197.5000	79.2738	36.0000

5. Conclusion

Neutrosophic statistics is the extension of classical statistics and is applied when the data is coming from a complex process or from an uncertain environment. The current study can be extended using neutrosophic statistics as future research.

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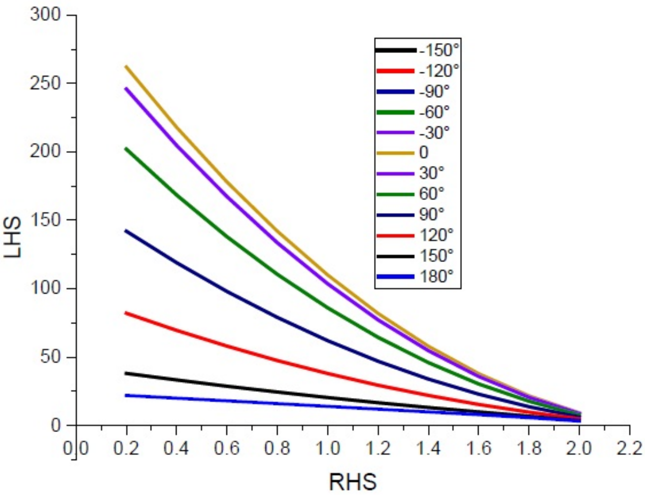

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