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# Reserved Domination Number of Line Graph 

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#### Abstract

The reserved dominating set is special up gradation of dominating set, such that some of the vertices in the vertex set have the special privilege (reserved) to appear in the Dominating set irrespective of their adjacency due to the necessity of the user. The minimum --cardinality of a reserved dominating set of $G$ is called the reserved domination number of $G$ and is denoted by $R_{(k)}-$ $\gamma(G)$ where $k$ is the number of reserved vertices. In this paper reserved domination number of $L\left(P_{n}\right), L\left(C_{n}\right), L\left(S_{n}\right)$, $L\left(B_{m, n}\right), L\left(W_{n}\right)$ and $L\left(F_{1, n}\right)$ are found


Keywords: Dominating set, reserved dominating set, reserved domination number, line graph.

## 1. Introduction

The Oystein Ore [1] defined that the dominating set of a graph. Rajasekar et al., $[3,6]$ defined the reserved dominating set (RDS) of the graph $G$ to be the subset $S$ of $V$, whose vertices are reserved in such a way that they must appear in the dominating set. The dominating RDS with the minimum cardinality is called reserved domination number of $G$ and is denoted by $R_{(k)}-\gamma(G)$ where $k$ is the number of reserved vertices. In [4] authors found the location domination number of line graph. Rajasekar et al. [3,5,6,7] have found the reserved domination number, 2-reserved domination

[^0]number of graphs and reserved domination number of complement of a graphs.
Throughout this paper we use the indexing set [initial value; final value: step value] where initial value is the first value of indexing set, step value is the incremented value of initial value and the final value is the maximum value that can be achieved by initial value by incrementing. Therefore $k \in[1 ; n: 1]$ implies $k=1,2,3, \ldots, n$ and $\left\{v_{1} ; v_{n}: 1\right\}$ implies $\left\{v_{1}, v_{2}, v_{3}, \ldots, v_{n}\right\}$. Further throughout the paper Reserved vertex is referred to as $R_{V}$, Dominating Set as DS, Reserved Dominating Set as RDS and Reserved Domination Number asRDN.

## 2. Preliminaries

Definition 2.1: [3] Reserved Domination.
Let $G=(V, E)$ be a graph. A subset $S$ of $V$ is called a Reserved Dominating Set (RDS)of $G$ if
(i) $\mu$ be any nonempty proper subset of $S$.
(ii) Every vertex in $V-S$ is adjacent to a vertex inS.

The dominating set $S$ is called a minimal reserved dominating set if no proper subset of $S$ containing $\mu$ is a dominating set. The set $\mu$ is called Reserved set. The minimum cardinality of a reserved dominating set $S$ of $G$ is called the reserved domination number of $G$ and is denoted by $R-\gamma(G)$.

Definition 2.2: [3,6] 2-Reserved Domination.
Let $G=(V, E)$ be a graph. A subset $S$ of $V$ is called a $k$-reserved dominating set (RDS)of $G$ if
(i) $\mu$ is any nonempty proper subset of $S$ with $k$ vertices.
(ii) Every vertex in $V-S$ is adjacent to a vertex inS.

The dominating set $S$ is called a minimal $k$-reserved dominating set if no proper subset of $S$ containing $\mu$ is a dominating set. The set $\mu$ is called $k$-reserved set.

The minimum cardinality of a $k$-reserved dominating set $S$ of $G$ is called the $k$-reserved domination number of $G$ and is denoted by $R_{(k)}-\gamma(G)$ where $k$ is the number of reserved vertices.

Definition 2.3: Bistar Graph.
A Bistar graph is the graph obtained by joining the centre (apex) vertices of two copies of $K_{1, n}$ by an edge and it is denoted by $B_{m, n}$.
Theorem 2.4: [3] For $P_{n}$, theRDN,
$R_{(1)}-\gamma\left(P_{n}, \mu\right)=1+\left\lceil\frac{k-2}{3}\right\rceil+\left\lceil\frac{n-(k+1)}{3}\right\rceil$, if $\mu=v_{k}(k \in[1 ; n: 1])$.
Theorem 2.5: [3] For $C_{n}$, the RDN, $R_{(1)}-\gamma\left(C_{n}, \mu\right)=\gamma\left(C_{n}\right)=\left\lceil\frac{n}{3}\right\rceil$ if $\mu=$ $v_{k}(k \in[1 ; n: 1])$.
Remark 2.6: [3] For $K_{n}, n \geq 3$ the $R D N, R_{(1)}-\gamma\left(K_{n}\right)=1$.

## 3. Reserved Domination Number of Line Graph

Proposition 3.1: For $P_{n}, R_{(1)}-\gamma\left(L\left(P_{n}\right)\right)=R_{(1)}-\gamma\left(P_{n-1}\right), \operatorname{asf} L\left(P_{n}\right)=$ $P_{n-1}$.
Proposition 3.2: For $C_{n}, L\left(C_{n}\right)=C_{n}$ and hence $R_{(1)}-\gamma\left(L\left(C_{n}\right)\right)=$ $R_{(1)}-\gamma\left(C_{n}\right)$.
Theorem 3.3: For $S_{n}=K_{1, n}, R_{(1)}-\gamma\left(L\left(S_{n}\right), \mu\right)=R_{(1)}-\gamma\left(K_{n}, \mu\right)=1$, if $\mu=e_{k}(k \in[1 ; n: 1])$.

Proof: $S_{1}=K_{2}=P_{2}$ and so by Proposition 3.1, $R_{(1)}-\gamma\left(L\left(S_{1}\right)\right)=$ $R_{(1)}-\gamma\left(L\left(P_{2}\right)\right)=R_{(1)}-\gamma\left(P_{1}\right)=1$.
For $n>1, \mathrm{f} L\left(S_{n}\right) \cong K_{n}$ and so $R_{(1)}-\gamma\left(L\left(S_{n}\right), \mu\right)=R_{(1)}-\gamma\left(K_{n}, \mu\right)=$ 1.

Theorem 3.4: For $B_{m, n}$, the $R D N$,
$R_{(1)}-\gamma\left(L\left(B_{m, n}\right), \mu\right)=\left\{\begin{array}{l}1, \text { if } \mu=e \\ 2, \text { if } \mu=\left\{\begin{array}{l}e_{u_{k^{\prime}}}, k \in[1 ; m: 1] \text { or } \\ e_{v_{k^{\prime}}}, k \in[1 ; n: 1] .\end{array}\right.\end{array}\right.$
Proof:
Case (i): When $m=1=1, B_{1,1} \cong P_{4}$ and by Proposition 3.1, $R_{(1)}-$ $\gamma\left(L\left(B_{1,1}\right), \mu\right)=\left\{\begin{array}{l}1, \text { if } \mu=e \\ 2, \text { if } \mu=e_{u_{1}}\end{array}\right.$ or $e_{v_{1}}$.
Case (ii): Either $m=1$ for $n=1$.
Without loss of generality, assume that $m>1$ and $n=1 . L\left(B_{m, 1}\right)$ is isomorphic to $L_{m+1,1}$.

Suppose $\mu=e$ is $R_{V}$. Then $e$ must be in the DS and $e$ dominates all other vertices. Hence the required $R D S$ is $\{e\}$.
Thus $R_{(1)}-\gamma\left(L\left(B_{m, 1}\right), \mu\right)=1$ where $\mu=e$.


Fig. 1: (a) $B_{m, 1}$ and (b) $L\left(B_{m, 1}\right)$
Suppose $\mu=e_{u_{k}}(k \in[1 ; m: 1])$ is the $R_{V}$. Then $e_{u_{k}}$ must be in the DS and $e_{u_{k}}$ dominates the vertices $\left\{e_{u_{1}}, e_{u_{2}}, \ldots, e_{u_{k-1}}, e_{u_{k+1}}, \ldots, e_{u_{m}}\right\} \cup$ $\{e\}$. The remaining vertex which is not dominated by $e_{u_{k}}$ is $e_{v_{1}}$. So $e_{v_{1}}$ must be in the DS.
Hence the required RDS is $\left\{e_{u_{k}}, e_{v_{1}}\right\}$.
Thus $R_{(1)}-\gamma\left(L\left(B_{m, 1}\right), \mu\right)=2$ where $\mu=e_{u_{k}}(k \in[1 ; m: 1])$.
Suppose $\mu=e_{v_{1}}$ is the $R_{V}$. Then $e_{v_{1}}$ must be in the DS and $e_{v_{1}}$ dominates only the vertex $e$. The remaining vertices which aren't dominated by $e_{u_{k}}$ are $\left\{e_{u_{1}} ; e_{u_{m}}: 1\right\}$. To dominate the remaining vertices, choose any one of the vertex say $e_{u_{1}}$ from $\left\{e_{u_{1}} ; e_{u_{m}}: 1\right\}$.
Hence the required $R D S$ is $\left\{e_{v_{1}}, e_{u_{1}}\right\}$.
Thus $R_{(1)}-\gamma\left(L\left(B_{m, 1}\right), \mu\right)=2$ where $\mu=e_{v_{1}}$.
Case (iii): Line graph of $B_{m, n}$ when $m, n>1$ is isomorphic to the graph obtained due to single vertex fusion of $K_{m+1}$ and $K_{n+1}$.


Fig. 2: (a) $B_{m, n}$ and (b) $L\left(B_{m, n}\right)$
Suppose $\mu=e$ is $R_{V}$. Then $e$ must be in the DS and $e$ dominates all other vertices. Hence the required RDS set is $\{e\}$.
Thus $R_{(1)}-\gamma\left(L\left(B_{m, n}\right), \mu\right)=1$ where $\mu=e$.
Suppose $\mu=e_{u_{k}}(k \in[1 ; m: 1])$ is the $R_{V}$. Then $e_{u_{k}}$ must be in the DS and $e_{u_{k}}$ dominates the vertices $\left\{e_{u_{1}}, e_{u_{2}}, \ldots, e_{u_{k-1}}, e_{u_{k+1}}, \ldots, e_{u_{m}}\right\} \cup$ $\{e\}$. The remaining vertices which are not dominated by $e_{u_{k}}$ are $\left\{e_{v_{1}} ; e_{v_{n}}: 1\right\}$.

To dominate the remaining vertices, choose any one of the vertices say $e_{v_{1}}$ from $\left\{e_{v_{1}} ; e_{v_{n}}: 1\right\}$.
Hence the required RDS is $\left\{e_{u_{k}}, e_{v_{1}}\right\}$.
Thus $R_{(1)}-\gamma\left(L\left(B_{m, n}\right), \mu\right)=2$ where $\mu=e_{u_{k}}(k \in[1 ; m: 1])$.
Similarly, one can prove for the $R_{V} \mu=e_{v_{k}}$ where $k \in[1 ; n: 1]$.

Theorem 3.5: For $W_{n}, R_{(1)}-\gamma\left(L\left(W_{n}\right), \mu\right)=\left\lceil\frac{n+1}{3}\right\rceil$ if $\mu=e_{k}$ or $e_{v_{k}}(k \in$ [1;n:1]).

Proof: Let $V\left(W_{n}\right)=\left\{v,\left\{v_{1} ; v_{n}: 1\right\}\right\}$ where $\operatorname{deg} v=n$ and $\operatorname{deg} v_{k}=3$ for all $k \in[1 ; n: 1]$. Label the edge $v v_{k}$ as $e_{v_{k}}$ edge $v_{k} v_{k+1}$ as $e_{k}$ for $k \in[1 ; n-1: 1]$ and $v_{1} v_{n}$ as $e_{n}$ as represented in the Fig. 3 .


Figure 3: $W_{n}$
$L\left(W_{n}\right)$ is constructed as shown in Fig. 4.


Figure 4: $L\left(W_{n}\right)$

The induced subgraph of the sets $\left\{e_{v_{1}} ; e_{v_{n}}: 1\right\}$ and $\left\{e_{1} ; e_{n}: 1\right\}$ is $K_{n}$ and $C_{n}$ respectively.
Case (i): Suppose $\mu=e_{v_{k}}(k \in[1 ; n: 1])$ is $R_{V}$. Then $e_{v_{k}}$ must be in the DS and $e_{v_{k}}$ dominates the vertices $\left\{e_{v_{1}}, e_{v_{2}}, \ldots, e_{v_{k-1}}, e_{v_{k+1}}, \ldots, e_{v_{n}}\right\} \cup$ $\left\{e_{k-1}, e_{k}\right\}$. The remaining vertices which are not dominated by $e_{v_{k}}$ are $\left\{e_{1}, e_{2}, \ldots, e_{k-2}, e_{k+1}, \ldots, e_{n}\right\}$.
Now it is enough to find the DS for the vertices $\left\{e_{1}, e_{2}, \ldots, e_{k-2}, e_{k+1}, \ldots, e_{n}\right\}$. The $L\left(W_{n}\right)\left[V_{1}\right]$ with $V_{1}=$ $\left\{e_{1}, e_{2}, \ldots, e_{k-2}, e_{k+1}, \ldots, e_{n}\right\}$ is $P_{n-2}$.
Hence $R_{(1)}-\gamma\left(L\left(W_{n}\right), \mu\right)=\left|\left\{e_{v_{k}}\right\}\right|+\gamma\left(P_{n-2}\right) \quad=1+\left\lceil\frac{n-2}{3}\right\rceil=\left\lceil\frac{n+1}{3}\right\rceil$ where $\mu=e_{v_{k}}(k \in[1 ; n: 1])$.
Case (ii): Suppose $\mu=e_{k}(k \in[1 ; n: 1])$ is $R_{V}$. Then $e_{k}$ must be in the DS and $e_{k}$ dominates the vertices $\left\{e_{k-1}, e_{k+1}\right\} \cup\left\{e_{v_{k}}, e_{v_{k+1}}\right\}$. The remaining vertices which are not dominated by $e_{k}$ are $\left\{e_{1}, e_{2}, \ldots, e_{k-2}, e_{k+2}, \ldots, e_{n}\right\} \cup\left\{e_{v_{1}}, e_{v_{2}}, \ldots, e_{v_{k-1}}, e_{v_{k+2}}, \ldots, e_{v_{n}}\right\}$.
To dominate the remaining vertices from the set $\left\{e_{v_{1}}, e_{v_{2}}, \ldots, e_{v_{k-1}}, e_{v_{k+2}}, \ldots, e_{v_{n}}\right\}$, choose the vertex $e_{v_{k+3}}$ or $e_{v_{k-3}}$. Consider the vertex $e_{v_{k+3}}$ which dominates $e_{k+2}$ and $e_{k+3}$.
Now it is enough to find the DS for the vertices $\left\{e_{1}, e_{2}, \ldots, e_{k-2}, e_{k+4}, \ldots, e_{n}\right\}$. The $L\left(W_{n}\right)\left[V_{2}\right]$ with $V_{2}=$ $\left\{e_{1}, e_{2}, \ldots, e_{k-2}, e_{k+4}, \ldots, e_{n}\right\}$ is $P_{n-5}$.
Hence $\quad R_{(1)}-\gamma\left(L\left(W_{n}\right), \mu\right)=\left|\left\{e_{k}\right\}\right|+\left|\left\{e_{v_{k+3}}\right\}\right|+\gamma\left(P_{n-5}\right)=1+1+$ $\left\lceil\frac{n-5}{3}\right\rceil=\left\lceil\frac{n+1}{3}\right\rceil$ where $\mu=e_{k}(k \in[1 ; n: 1])$.

Theorem 3.6: For the fan graph $F_{1, n}$, the reserved domination number for the different values of $n$ is summarized as follows:
i. For $n \equiv 0(\bmod 3)$

$$
\begin{aligned}
& R_{(1)}-\gamma\left(L\left(F_{1, n}\right), \mu\right) \\
& =\left\{\begin{array}{l}
{\left[\frac{n+1}{3}\right\rceil, \text { if } \mu=\left\{\begin{array}{l}
e_{v_{k}}(k=1,3,4,6, \ldots, n-5, n-3, n-2, n) \text { or } \\
e_{k}(k=1,3,6,9, \ldots, n-9, n-6, n-3, n-1)
\end{array}\right.} \\
{\left[\frac{n}{3}\right\rceil, \text { if } \mu=\left\{\begin{array}{l}
e_{v_{k}}(k=2,5,8,11, \ldots, n-10, n-7, n-4, n-1) \text { or } \\
e_{k}(k=2,4,5,7, \ldots, n-7, n-5, n-4, n-2)
\end{array}\right.}
\end{array} .\right.
\end{aligned}
$$

ii. For $n \equiv 1(\bmod 3)$

$$
R_{(1)}-\gamma\left(L\left(F_{1, n}\right), \mu\right)=\left\lceil\frac{n}{3}\right\rceil, \text { if } \mu=\left\{\begin{array}{l}
e_{v_{k}}(k \in[1 ; n: 1]) \text { or } \\
e_{k}(k \in[1 ; n-1: 1])
\end{array}\right.
$$

iii. For $n \equiv 2(\bmod 3)$

$$
\begin{aligned}
& R_{(1)}-\gamma\left(L\left(F_{1, n}\right), \mu\right)= \\
& \left\{\begin{array}{l}
{\left[\frac{n+2}{3}\right\rceil, \text { if } \mu=e_{v_{k}}(k=3,6,9,12, \ldots, n-11, n-8, n-5, n-2)} \\
{\left[\frac{n}{3}\right], \text { if } \mu=\left\{\begin{array}{l}
e_{v_{k}}(k=1,2,4,5, \ldots, n-4, n-3, n-1, n) \text { or } \\
e_{k}(k \in[1 ; n-1: 1])
\end{array}\right.}
\end{array}\right.
\end{aligned}
$$

Proof: $F_{1,1}$ and $F_{1,2}$ are isomorphic to $P_{2}$ and $C_{3}$ respectively.
Therefore $R_{(1)}-\gamma\left(L\left(F_{1,1}\right)\right)=1$ and $R_{(1)}-\gamma\left(L\left(F_{1,2}\right)\right)=1$.
For $n>2$, let $V\left(F_{1, n}\right)=\left\{v,\left\{v_{1} ; v_{n}: 1\right\}\right\}$ where $\operatorname{deg} v=n, \operatorname{deg} v_{1}=$ $\operatorname{deg} v_{n}=2$ and $\operatorname{deg} v_{k}=3$ for all $2 \leq k \leq n-1$. Label the edge $v_{k} v_{k+1}$ as $e_{k}$ and $v v_{k}$ as $e_{v_{k}}$ as shown in Fig. 5.


Figure 5: $F_{1, n}$
In the graph $F_{1, n}$, edge adjacency is given as follows:
i. $\quad e_{1}$ is adjacent to $e_{v_{1}}, e_{v_{2}}$ and $e_{2}$.
ii. $\quad e_{n-1}$ is adjacent to $e_{v_{n-1}}, e_{v_{n}}$ and $e_{n-2}$.
iii. For $1<k<n-1, e_{k}$ is adjacent to $e_{v_{k}}, e_{v_{k+1}}, e_{k-1}$ and $e_{k+1}$.
iv. $\quad e_{v_{1}}$ is adjacent to $e_{v_{2}}, e_{v_{3}}, \ldots, e_{v_{n-1}}, e_{v_{n}}$ and $e_{1}$.
v. $\quad e_{v_{n}}$ is adjacent to $e_{v_{1}}, e_{v_{2}}, \ldots, e_{v_{n-2}}, e_{v_{n-1}}$ and $e_{n-1}$.
vi. For $1<k<n, \quad e_{v_{k}}$ is adjacent to $e_{v_{1}}, e_{v_{2}}, \ldots, e_{v_{k-1}}, e_{v_{k+1}}, \ldots, e_{v_{n-1}}, e_{v_{n}}, e_{k-1}$ and $e_{k}$.

Since adjacency matrix of line graph is nothing but the incidence matrix of the given graph, the graph $L\left(F_{1, n}\right)$ is obtained from edge adjacency of $F_{1, n}$ as shown in Fig. 6.


Figure 6: $L\left(F_{1, n}\right)$
The induced subgraph of $\left\{e_{v_{1}} ; e_{v_{n}}: 1\right\}$ is a complete graph with $n$ vertices and the induced subgraph of $\left\{e_{1} ; e_{n-1}: 1\right\}$ is a path of length $n-1$.

Case $(i)$ : For $n \equiv 0(\bmod 3)$.
Sub case (i): Suppose $e_{v_{1}}$ is the $R_{V}$. Then $e_{v_{1}}$ must be in the DS and $e_{\nu_{1}}$ dominates the vertices $\left\{e_{\nu_{2}} ; e_{v_{n}}: 1\right\} \cup\left\{e_{1}\right\}$. The remaining vertices which are not dominated by $e_{v_{1}}$ are $\left\{e_{2} ; e_{n-1}: 1\right\}$. Now it is enough to find the DS for the vertices $\left\{e_{2} ; e_{n-1}: 1\right\}$.
The $L\left(F_{1, n}\right)\left[V_{1}\right]$ with $V_{1}=\left\{e_{2} ; e_{n-1}: 1\right\}$ is $P_{(n-1)-1}=P_{n-2}$.
Hence $R_{(1)}-\gamma\left(L\left(F_{1, n}\right), e_{v_{1}}\right)=\left|\left\{e_{v_{1}}\right\}\right|+\gamma\left(P_{n-2}\right)=1+\left\lceil\frac{n-2}{3}\right\rceil=\left\lceil\frac{n+1}{3}\right\rceil$.
Similarly, the same result is obtained for the $R_{V} e_{v_{k}}$ where $k=$ $3,4,6, \ldots, n-5, n-3, n-2, n$.
Sub case (ii): Suppose $e_{v_{2}}$ is the $R_{V}$. Then $e_{v_{2}}$ must be in the DS and $e_{v_{2}}$ dominates the vertices $\left\{e_{v_{1}}, e_{v_{3}}, \ldots, e_{v_{n-1}}, e_{v_{n}}\right\} \cup\left\{e_{1}, e_{2}\right\}$. The remaining vertices which are not dominated by $e_{v_{2}}$ are $\left\{e_{3} ; e_{n-1}: 1\right\}$. Now it is enough to find the DS for the vertices $\left\{e_{3} ; e_{n-1}: 1\right\}$.
The $L\left(F_{1, n}\right)\left[V_{2}\right]$ with $V_{2}=\left\{e_{3} ; e_{n-1}: 1\right\}$ is $P_{(n-1)-2}=P_{n-3}$.
Hence $R_{(1)}-\gamma\left(L\left(F_{1, n}\right), e_{v_{2}}\right)=\left|\left\{e_{v_{2}}\right\}\right|+\gamma\left(P_{n-3}\right)=1+\left\lceil\frac{n-3}{3}\right\rceil=\left\lceil\frac{n}{3}\right]$.

The same result holds for $R_{V} e_{v_{k}}$ where $k=5,8,11, \ldots, n-10, n-$ $7, n-4, n-1$.

Sub case (iii): Suppose $e_{1}$ is $R_{V}$. Then $e_{1}$ must be in the DS and $e_{1}$ dominates the vertices $e_{2}, e_{v_{1}}$ and $e_{v_{2}}$. The remaining vertices which are not dominated by $e_{1}$ are $\left\{e_{v_{3}} ; e_{v_{n}}: 1\right\} \cup\left\{e_{3} ; e_{n-1}: 1\right\}$. To dominate the remaining vertices from the $\operatorname{set}\left\{e_{v_{3}} ; e_{v_{n}}: 1\right\}$, choose the vertex $e_{v_{4}}$ which also dominated $e_{3}$ and $e_{4}$.

Now it is enough to find the DS for the vertices $\left\{e_{5} ; e_{n-1}: 1\right\}$.
The $L\left(F_{1, n}\right)\left[V_{3}\right]$ with $V_{3}=\left\{e_{5} ; e_{n-1}: 1\right\}$ is $P_{(n-1)-4}=P_{n-5}$.
Hence $\quad R_{(1)}-\gamma\left(L\left(F_{1, n}\right), e_{1}\right)=\left|\left\{e_{1}\right\}\right|+\left|\left\{e_{v_{4}}\right\}\right|+\gamma\left(P_{n-5}\right)=2+\left\lceil\frac{n-5}{3}\right\rceil$ $=\left\lceil\frac{n+1}{3}\right\rceil$.

Similarly, we get same result for the $R_{V} e_{k}$ where $k=3,6,9, \ldots, n-$ $9, n-6, n-3, n-1$.

Sub case (iv): Suppose $e_{2}$ is $R_{V}$. Then $e_{2}$ has to be in the DS and $e_{2}$ dominates the vertices $e_{1}, e_{3}, e_{v_{2}}$ and $e_{v_{3}}$. The remaining vertices which are not dominated by $e_{2}$ are $\left\{e_{v_{1}}, e_{v_{4}}, \ldots, e_{v_{n-1}}, e_{v_{n}}\right\} \cup$ $\left\{e_{4} ; e_{n-1}: 1\right\}$. To dominate the remaining vertices from the set $\left\{e_{v_{1}}, e_{v_{4}}, \ldots, e_{v_{n-1}}, e_{v_{n}}\right\}$, choose the vertex $e_{v_{5}}$ which also dominates $e_{4}$ and $e_{5}$.

Now it is enough to find the DS for the vertices $\left\{e_{6} ; e_{n-1}: 1\right\}$.
The graph $L\left(F_{1, n}\right)\left[V_{4}\right]$ with $V_{4}=\left\{e_{6} ; e_{n-1}: 1\right\}$ is $P_{(n-1)-5}=P_{n-6}$.
Hence $R_{(1)}-\gamma\left(L\left(F_{1, n}\right), e_{2}\right)=\left|\left\{e_{2}\right\}\right|+\left|\left\{e_{v_{5}}\right\}\right|+\gamma\left(P_{n-6}\right)=2+\left\lceil\frac{n-6}{3}\right\rceil=$ $\left\lceil\frac{n}{3}\right\rceil$.

The same result holds for $R_{V} e_{k}$ where $k=4,5,7, \ldots, n-7, n-5, n-$ $4, n-2$.

Hence
$R_{(1)}-\gamma\left(L\left(F_{1, n}\right), \mu\right)$
$=\left\{\begin{array}{l}\left\lceil\frac{n+1}{3}\right\rceil \text { where } \mu=\left\{\begin{array}{l}e_{v_{k}}(k=1,3,4,6, \ldots, n-5, n-3, n-2, n) \text { or } \\ e_{k}(k=1,3,6,9, \ldots, n-9, n-6, n-3, n-1)\end{array}\right. \\ \left\lceil\frac{n}{3}\right\rceil \text { where } \mu=\left\{\begin{array}{l}e_{v_{k}}(k=2,5,8,11, \ldots, n-10, n-7, n-4, n-1) \text { or } \\ e_{k}(k=2,4,5,7, \ldots, n-7, n-5, n-4, n-2)\end{array}\right.\end{array}\right.$
Case (ii): For $n \equiv 1(\bmod 3)$.

$$
R_{(1)}-\gamma\left(L\left(F_{1, n}\right), \mu\right)=\left\lceil\frac{n}{3}\right\rceil \text { where } \mu=\left\{\begin{array}{l}
e_{v_{k}}(k \in[1 ; n: 1]) \text { or } \\
e_{k}(k \in[1 ; n-1: 1])
\end{array}\right.
$$

Case (iii): For $n \equiv 2(\bmod 3)$.
$R_{(1)}-\gamma\left(L\left(F_{1, n}\right), \mu\right)$
$=\left\{\begin{array}{l}\left\lceil\frac{n+2}{3}\right\rceil \text { where } \mu=e_{v_{k}}(k=3,6,9,12, \ldots, n-11, n-8, n-5, n-2) \\ \left\lceil\frac{n}{3}\right\rceil \text { where } \mu=\left\{\begin{array}{l}e_{v_{k}}(k=1,2,4,5, \ldots, n-4, n-3, n-1, n) \text { or } \\ e_{k}(k \in[1 ; n-1: 1])\end{array}\right.\end{array}\right.$

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