Distributed Ordering and Optimization for Intersection Management with Connected and Automated Vehicles

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Abstract

Intelligent transport systems are preparing to welcome connected and automated vehicles (CAVs), although it is uncertain which algorithms should be employed for the effective and efficient management of CAV systems. Even though remarkable improvements in telecommunication technologies, such as vehicle-to-everything (V2X), enable communication and computation sharing among different agents, e.g. vehicles and infrastructures, within existing approaches, a significant part of the computation burden is still typically assigned to central units. Distributed algorithms, on the other hand, could alleviate traffic units from most, if not all, of the high dimensional calculation duties, while improving security and remaining effective. In this paper, we propose a formation-control-inspired distributed algorithm to rearrange vehicles' passing time periods through an intersection and a novel formulation of the underlying trajectory optimization problem so that vehicles need to exchange and process only a limited amount of information. We include early simulation results to demonstrate the effectiveness of our approach.

Keywords: connected vehicles, intersection management, distributed control

1 Introduction

Connected and automated vehicles (CAVs) promise a revolution in the field of traffic management, due to improved sensing and actuating capabilities. However, scientific and industrial communities are called to address the resulting challenges, such as how to integrate CAVs with human-driven vehicles, guarantee connectivity, and develop vehicle real-time planning and control strategies to name but a few [Gua18; Pap19].

Graph theory is a relevant line of research in this context [Maj20], which has already been investigated in the topic of mobile robots formation control [Laf04], but has also great potential for application in CAV systems due to its capability to simplify models for CAV interactions, even in presence of imperfect communication, relying on a strong mathematical basis. Similarly, distributed network control systems are presented in [Ge17], with an overview of their system configurations, as well as challenges in the communication, computation, and control fields. In the context of vehicular and traffic systems, while distributed or decentralized formation control methods have been applied to longitudinal control of CAVs [Li17], other traffic operations, such as traffic intersection management, have not been sufficiently explored. Nevertheless, CAVs are called to operate in various other more complex situations, such as intersections, roundabouts, or junctions [Son18]. As highlighted in [Xu21], microscopic modeling-based distributed control methods are not straightforwardly applicable to vehicular formation control because of the lack of consolidated results in constraints management. The need for appropriate and well-thought methodologies to incorporate such methods into CAV driving algorithms leads to reconsidering distributed consensus control techniques and adapting them to complex scenarios, allowing them to benefit from and fully exploit the communications and computation capabilities of CAVs.

A comprehensive survey about autonomous intersection management is provided by [Zho20], where a classification of such strategies in centralized, decentralized and distributed is discussed. While the centralized approach involves an autonomous intersection manager, representing a single point of failure and bearing the whole computation burden, both the decentralized and distributed approaches allow for a sharing of the processing load. Decentralized algorithms usually still need an autonomous intersection manager to communicate with the head vehicle of the queues approaching the intersection, which will then forward messages to their following vehicles. Distributed algorithms, on the other hand, do not need a central unit, but they typically require the head vehicle to compute a proper solution and propagate it upstream. Moreover, a typical assumption in these approaches is that vehicles pass the intersection at a given (maximum) speed once it has received (or computed) the order of vehicles; however, although convenient for planning reasons, this may not always be the most desirable behavior. Indeed, vehicle dynamics are highly nonlinear and constrained, thus, not allowing a straightforward manipulation of the speed in the intersection, considering that different requirements for the physics of the vehicle must be taken into account according to the maneuver adopted (e.g. turn left, turn right, or go straight).

To address the aforementioned challenges, in this work we propose: 1) a novel distributed reallocation algorithm, inspired by robotic formation control, applied to assign the time slots for the vehicles passing through an intersection; and 2) a reformulation of the underlying optimization control problem into a space-dependent optimization control problem to be solved by the CAVs.

2 Methodology

In this section, we present the proposed methodology for managing vehicles approaching an intersection, consisting of the following two main ingredients.

First, the ordering by which the CAVs will pass through an intersection is defined via Algorithm 1. The proposed algorithm is fully distributed and does not assume that the vehicles undertake any specific role (head, mid, or last in a queue). A preliminary version of such an approach is thoroughly described in our previous work [Vit22].

Algorithm 1 Pseudo-code of the distributed ordering algorithm run by CAV i.

Require: R number of iterations, H horizon, δ time period shifting amount, s intended starting time, and e intended ending time in the intersection
Ensure: Updated s and e
  for $r \leftarrow 1$ to R **do**
    for $h \leftarrow 1$ to H **do**
      Let $B \leftarrow 0$
      Announce s and e
      Gather Announcements from neighbors
      Check which neighbor is the previous vehicle $prev_i$ intending to pass immediately before i
      Check which neighbor is the next vehicle $next_i$ intending to pass immediately after i
      Check if time overlapping with either of $prev_i$ or $next_i$
      if no overlapping **then**
        if $prev_i$ is in *danger* and $next_i$ is not in *danger* **then**
          Set $B \leftarrow \delta$
        if $next_i$ is in *danger* and $prev_i$ is not in *danger* **then**
          Set $B \leftarrow -\delta$
      else if overlapping with both **then**
        Send *danger* signal to both
      else if overlapping only with $prev_i$ **then**
        if $next_i$ is not in *danger* **then**
          Set $B \leftarrow \delta$
      else
        if $prev_i$ is not in *danger* **then**
          Set $B \leftarrow -\delta$
      Shift s and e by B

Second, an optimization problem is constructed, where its constraints are designed according to the ordering obtained from the algorithm presented above. The problem is formulated in order to exchange a minimum amount of information and solve the speed optimization problem depending on the position on the road. As a consequence, even different dynamic models can be employed for different parts of the road and intersection, to describe more accurately the maneuver to be performed. A straightforward formulation of the optimization problem is:

$$\begin{aligned}
\min_{\tilde{\mathbf{z}},\tilde{\mathbf{u}}} \quad & \frac{1}{2}\tilde{\mathbf{z}}^{\intercal} Q \tilde{\mathbf{z}} + \frac{1}{2}\tilde{\mathbf{u}}^{\intercal} R \tilde{\mathbf{u}}, \\
\text{s.t.} \quad & \mathbf{z} \in \mathcal{Z}, \quad \mathbf{u} \in \mathcal{U}, \\
& \mathbf{z}_{k+1} = f(\mathbf{z}_k, \mathbf{u}_k), \ \forall k = 1, \dots, H-1, \\
& \text{collision avoidance constraints,}
\end{aligned} \tag{1}$$

where $\tilde{\mathbf{z}} = \mathbf{z} - \mathbf{z}_{\text{des}}$ is the error between the CAV state vector and the desired state trajectory weighted by matrix Q, $\tilde{\mathbf{u}} = \mathbf{u} - \mathbf{u}_{\text{des}}$ is the error between the CAV input vector and the desired input vector weighted by matrix R, subject to feasibility constraints, dynamic model constraint, and the collision avoidance constraints. The collision avoidance constraints are typically in the form: 1) $|p_{i-1}^k - p_i^k| \geq d \ \forall i = 1, \dots, N$ and $k = 1, \dots, H$, namely the distance between vehicle i and its preceding vehicle $i-1$ must be at least d at each time k; or 2) $p_{i-1}^e - p_i^s \geq d \ \forall i = 1, \dots, N$, namely the position of a vehicle i at the intended starting time s must be at least d behind its preceding vehicle $i-1$ at the intended ending time e. In the former case, an optimization procedure would adjust opportunely the starting and ending time of the vehicles, but exchanging the full trajectory would be necessary. In the latter case, only the position at a specific time (starting or ending) should be exchanged with the previous and following vehicles, but the optimized trajectory does not allow to adjust the starting and ending time instants.

We propose, instead, a reformulation of the problem so that it is space-dependent rather than time-dependent. Let us consider the state vector $\boldsymbol{\zeta} := \begin{bmatrix}\mathbf{t} & \boldsymbol{\nu}\end{bmatrix}^{\intercal}$, where $\mathbf{t} \in \mathbb{R}^S$ is the time and $\boldsymbol{\nu} \in \mathbb{R}^S$ is the inverse of the speed, both dependent on space, and S is the number of space points we are considering. Notice that we are applying the derivative with respect to the space and, hence, $t' := dt/ds = v^{-1} = \nu$ which explains why we consider $\boldsymbol{\nu}$ to be the inverse of the speed. Then, Problem (1) can be reformulated as:

$$\begin{aligned}
\min_{\tilde{\boldsymbol{\zeta}},\tilde{\boldsymbol{\eta}}} \quad & \frac{1}{2}\tilde{\boldsymbol{\zeta}}^{\intercal} \bar{Q} \tilde{\boldsymbol{\zeta}} + \frac{1}{2}\tilde{\boldsymbol{\eta}}^{\intercal} \bar{R} \tilde{\boldsymbol{\eta}}, \\
\text{s.t.} \quad & \boldsymbol{\zeta} \in \mathcal{Z}, \quad \boldsymbol{\eta} \in \mathcal{H}, \\
& \boldsymbol{\zeta}_{\sigma+1} = f(\boldsymbol{\zeta}_\sigma, \boldsymbol{\eta}_k), \ \forall \sigma = 1, \dots, S-1, \\
& t_i^{sI} - t_{i-1}^{eI} \geq \tau \ \forall i = 1, \dots, N,
\end{aligned} \tag{2}$$

where $\tilde{\boldsymbol{\zeta}} = \boldsymbol{\zeta} - \boldsymbol{\zeta}_{\text{des}}$ is the error between the CAV state vector and the desired state trajectory weighted by matrix $\bar{Q}$, $\tilde{\boldsymbol{\eta}} = \boldsymbol{\eta} - \boldsymbol{\eta}_{\text{des}}$ is the error between the CAV input vector and the desired input vector weighted by matrix $\bar{R}$, subject to feasibility constraints, dynamic model constraint, and the collision avoidance constraint. The latter imposes a minimum time gap τ between the time at which a vehicle $i-1$ reaches the end of the intersection zone eI and the time at which its following vehicle i enters the intersection zone sI. Matrices $\bar{Q}$ and $\bar{R}$

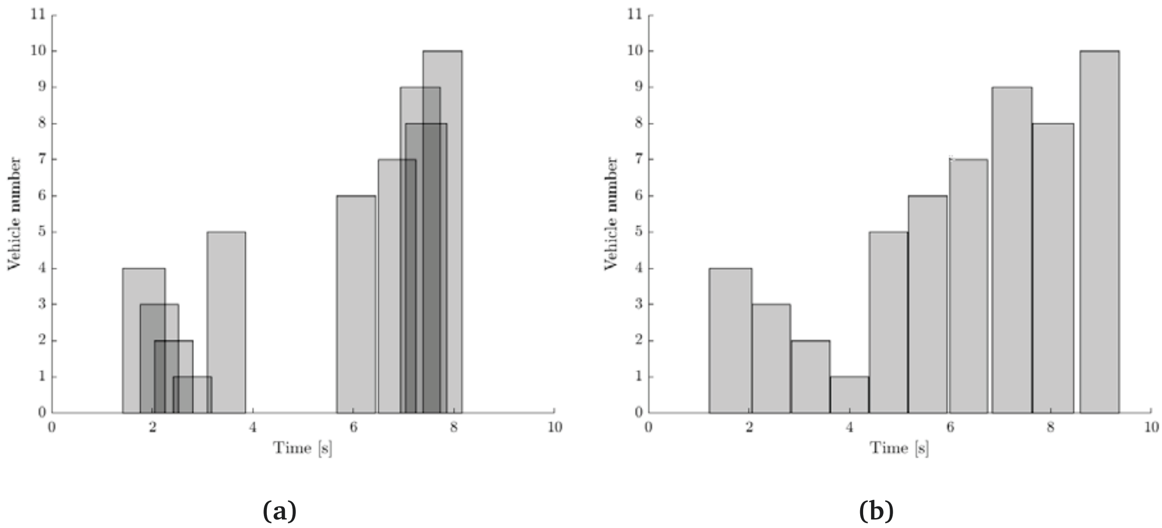

Figure 1: Distributed ordering algorithm for ten CAVs. a) Before running the algorithm, CAVs intended intersection passing times are overlapping. b) After running the algorithm, all CAVs intended intersection passing times are ordered rearranged so that they do not overlap.

differ, in general, from Q and R from Problem 1, and $\boldsymbol{\eta}$ can be used to define the derivative of $\boldsymbol{\nu}$, e.g. $\nu' := d\nu/ds = \eta$.

This new optimization problem allows to 1) exchange only a small amount of data, namely t_i^{sI} and t_i^{eI}; and 2) utilize different dynamic models and requirements for the speed, to appropriately describe the different maneuvers according to the different points on the road.

3 Results and Discussion

We provide a demonstration of our distributed ordering methodology, obtained via applying Algorithm 1. The simulation includes ten CAVs negotiating the time interval in which they will occupy the intersection. Time intervals are defined according to a CAV's own speed, maneuver at the intersection, and physical properties (length of the vehicle, etc.). For this experiment, the desired speed is 13 m/s, the maneuver is going straight, and the physical properties are random values. Before running the algorithm (Figure 1a), the time intervals overlap and, hence, collisions would occur at the intersection. After running the algorithm (Figure 1b), not only time intervals are not overlapping, but the original first-in-first-out order for the CAVs is also maintained. The integration of the optimization problem results is still ongoing work.

4 Conclusions

In this work, we presented a novel intersection management approach for CAVs. The appeal of such a strategy lies in the fact that: 1) thanks to the distributed ordering algorithm, no central unit is strictly needed to set the order by which the CAVs pass the intersection, and 2) thanks to the reformulation of the optimization problem in a space-dependent setting, it is possible to deal with vehicles complex requirements and different dynamic models according to the part of the road they are and the maneuver they are performing.

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