GLOSA System with Uncertain Green and Red Signal Phases

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Abstract

A discrete-time stochastic optimal control problem was recently proposed to address the GLOSA (Green Light Optimal Speed Advisory) problem in cases where the next signal switching time is decided in real-time and is therefore uncertain in advance. However, there was an assumption that the traffic signal is initially red and turns to green, which means that only half traffic light cycle was considered. In this work, the aforementioned problem is extended considering a full traffic light cycle, consisting of four phases: a certain green phase, during which the vehicle can freely pass; an uncertain green phase, in which there is a probability that the traffic light will extend its duration or turn to red at any time; a certain red phase during which the vehicle cannot pass; and an uncertain red phase, in which there is a probability that the red signal may be extended or turn to green at any time. It is demonstrated, based on preliminary results, that the proposed SDP (Stochastic Dynamic Programming) approach achieves better average performance, in terms of fuel consumption, compared to the IDM (Intelligent Driver Model), which emulates human-driving behavior.

Keywords: GLOSA systems, stochastic dynamic programming, speed advisory, traffic signals

1 Introduction

A common dilemma for a vehicle approaching a traffic light, is whether it should maintain its speed or should accelerate to cross (if the current signal is green) or decelerate to avoid stopping (if the current signal is red). To this end, many systems have been developed which aim at guiding the driver (or the automated vehicles) by giving speed advise which ensures that the vehicle will cross the traffic signal during the green phase and with minimum fuel consumption and emissions. Such systems are often referred to as Green Light Optimal Speed Advisory (GLOSA) systems [Sta16].

In the case of fixed signals and hence prior knowledge of the next switching time, the signal controller may broadcast a corresponding message to approaching vehicles. Under these conditions, the problem of how to optimize the approach to the traffic signals has been addressed in different ways [Kat11; Law13; San06]. The situation becomes more complicated when real-time signals with very short (e.g. second-by-second) control update periods are present, in which case exact prior knowledge of the next switching time is not available. In this case, the best available knowledge can be presented as an estimate [Kou11] or as a probabilistic distribution for the next switching time within a future time-window; such a distribution may be obtained by use of statistics from previous signal operation [Mah12; Law13; Sun20].

In [Typ20], the problem of producing fuel-optimal vehicle trajectories for a vehicle approaching a traffic signal, for both cases of known and stochastic switching times, was considered. For the first case, the problem was formulated as an optimal control problem and was solved analytically via PMP (Pontryagin's Maximum Principle). Subsequently, the case of stochastic switching time with known probability distribution was also addressed, and the problem was cast in the format of a stochastic optimal control problem, which was solved numerically using SDP (Stochastic Dynamic Programming). However, there was an assumption that the traffic signal is initially red and turns to green, which means that only the half traffic light cycle was considered.

The present work delivers an extension of the problem by considering a full traffic light cycle. Specifically, the signal's cycle consists of four phases, i.e. a certain-switching green phase, an uncertain-switching green phase, a certain-switching red phase, and an uncertain-switching red phase (see Figure 1). As mentioned, in our previous work, the vehicle was assumed to appear during the last two phases (certain red and uncertain red), where the certain red phase has a fixed and known switching time, followed by the uncertain red phase, in which there is at any time a probability that red will be extended, or that it will turn to green. The addition of the two green phases completes the handling of the GLOSA problem, as the vehicle may now appear at any of the four phases, with the certain-green phase having a fixed and known switching time, followed by the uncertain green phase, in which there is again at any time a probability that the traffic light may extend or turn red. Preliminary results demonstrate that the proposed SDP approach achieves better performance, in terms of fuel consumption and passenger comfort, compared to the base case of IDM [Tre13], which emulates the driving behavior of a manually driven vehicle.

2 Problem Formulation and Solution

The proposed stochastic GLOSA approach aims at guiding a vehicle, starting from an initial state x_0 (comprising initial vehicle position and speed), to cross a traffic signal, located at position x_1, at green; and reach a fixed final state x_e, within a free (but penalized) time horizon t_e (Figure 1). The final state x_e comprises a specified position x_e downstream of the traffic signal and a specified, reasonably high speed v_e. Note that the SDP algorithm solves the problem for vehicle positions up to the traffic signal position, i.e. for $x_0 \leq x_1$; however, the use of a deterministic GLOSA solution is included in the problem formulation and enables the vehicle to "escape" to the final state [Typ20]. Note also that the solution of the problem via SDP delivers an optimal feedback law, i.e. for every admissible state $x(k)$, we get the corresponding optimal control (acceleration) $a(k)$.

For the stochastic optimal control problem, the vehicle kinematics in discrete-time, with time step T, are described as follows:

$$x(k+1) = x(k) + v(k)T + \frac{1}{2}a(k)T^2, \tag{1}$$

$$v(k+1) = v(k) + a(k)T, \tag{2}$$

where $x(k),\ v(k)$ correspond to the vehicle position and speed at discrete times $k = 0, 1, \ldots$ (where $kT = t$), while the control variable $a(k)$ is the acceleration that remains constant over each time-period k. The state and control variables are bounded within the following admissible regions

$$x(k) \in X = [x_{\min}, x_{\max}], \tag{3}$$

$$a(k) \in U = [a_{\min}, a_{\max}], \tag{4}$$

with $x_{\min},\ x_{\max}$ and $a_{\min},\ a_{\max}$ the lower and upper bounds of the states and acceleration, respectively. We consider the upper bound of the position, $x_{\max}$, to be the traffic light position x_1.

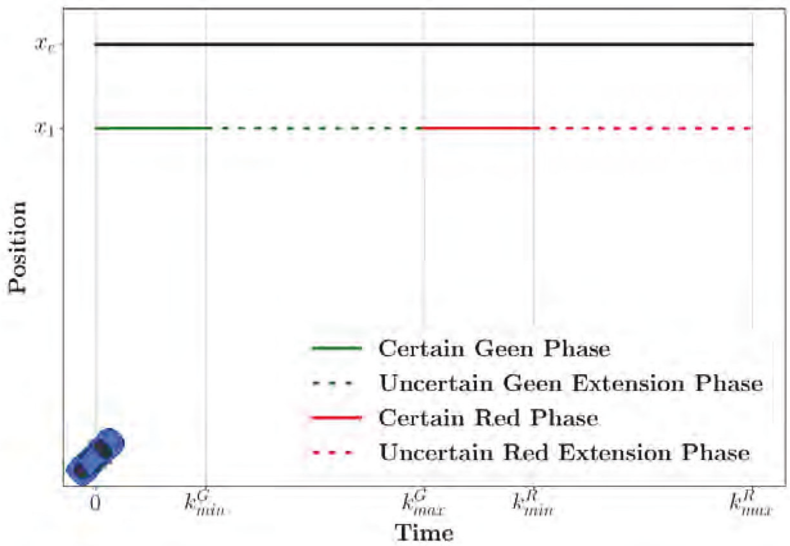

Figure 1: Traffic light phases.

A full signal cycle comprises four phases (see Figure 1): certain-switching green, lasting $[0, k^{\mathrm{G}}_{\min} - 1]$; uncertain-switching green, starting at k^{G}_{min} and lasting at most until $k^{\mathrm{G}}_{\max} - 1$;

certain-switching red, lasting from its (uncertain) initial time until $k^{\mathrm{R}}_{\min}-1$; and uncertain-switching red, starting at $k^{\mathrm{R}}_{\min}$ and lasting at most until $k^{\mathrm{R}}_{\max}$. We consider the case of fixed $k^{\mathrm{G}}_{\min}$ and $k^{\mathrm{R}}_{\min}$, whereby any uncertain-green portion that was not used, due to green-red switching at $k < k^{\mathrm{G}}_{\max}$, is added to the certain-red phase, hence the uncertain-red window $[k^{\mathrm{R}}_{\min}, k^{\mathrm{R}}_{\max}]$ does not change due to "early" green-red switching; and the same applies when we have an "early" red-green switching. The case where the certain-red or certain-green time durations are fixed, and hence the cycle duration reduces in case of "early" green-red or red-green switchings, may be treated similarly.

A vehicle may appear at any time during a signal cycle, at an admissible initial state x_0, and be guided optimally to the final state x_{e}. The green-red or red-green switching times are not known beforehand, but, when they actually occur, this is communicated to the approaching vehicles.

The total time horizon $[0, k^{\mathrm{R}}_{\max}]$ is subdivided in four parts as follows:

- **Part 1**: $[0, k^{\mathrm{G}}_{\min}-1]$ (certain green),
- **Part 2**: $[k^{\mathrm{G}}_{\min}, k^{\mathrm{G}}_{\max}-1]$ (uncertain green extension),
- **Part 3**: $[k^{\mathrm{G}}_{\max}, k^{\mathrm{R}}_{\min}-1]$ (certain red),
- **Part 4**: $[k^{\mathrm{R}}_{\min}, k^{\mathrm{R}}_{\max}]$ (uncertain red extension).

To formally address the uncertainty in Parts 2 and 4, i.e. the uncertain green and uncertain red phases, we consider binary stochastic variables $z^{\mathrm{G}}(k)$ and $z^{\mathrm{R}}(k)$, respectively. The binary variables are equal to 0 if the traffic light switches to red or green, respectively, at time $k+1$; or 1 else. We introduce virtual state variables $x^{\mathrm{G}}(k)$ and $x^{\mathrm{R}}(k)$, with initial values 1 and 0, respectively, and respective state equations

$$x^{\mathrm{G}}(k+1) = x^{\mathrm{G}}(k) z^{\mathrm{G}}(k), \tag{5}$$

$$x^{\mathrm{R}}(k+1) = \begin{cases} 1 - z^{\mathrm{G}}(k) & \text{if } x^{\mathrm{R}}(k) = 0, \\ x^{\mathrm{R}}(k) z^{\mathrm{R}}(k) & \text{else}, \end{cases} \tag{6}$$

hence

$$x^{\mathrm{G}}(k) = \begin{cases} 1 & \text{if the green light has not yet switched until time } k-1, \\ 0 & \text{if switching occurred at time } k \text{ or earlier}, \end{cases} \tag{7}$$

$$x^{\mathrm{R}}(k) = \begin{cases} 0 & \text{if the green light has not yet switched until time } k-1, \\ 1 & \text{if the green light has switched, but the red light has not,} \\ & \text{yet switched until time } k-1 \\ 0 & \text{if red-green switching occurred at time } k \text{ or earlier}. \end{cases} \tag{8}$$

The virtual states $x^{\mathrm{G}}(k),\ x^{\mathrm{R}}(k)$ are assumed measurable, which means that the system knows, at each time kT, if switching has taken place or not within the last time-period

$((k-1)T, kT]$. Note in particular for Part 2 (green extension) that the vehicle is allowed to cross the traffic light position during any time period $((k-1)T, kT]$, during which the switching may occur, since the vehicle decided its last acceleration at time $(k-1)T$, i.e. before the switching occurred. In other words, we may have $x(k) > x_{\max}$ if $z^{\mathrm{G}}(k-1) = 0$. This convention is not expected to jeopardise traffic safety, as time steps are short and accelerations and speeds are bounded.

The stochastic variables $z^{\mathrm{G}}(k)$ and $z^{\mathrm{R}}(k)$ are independent of their previous values and take values according to time-dependent probability distributions $p^{\mathrm{G}}(z|k)$ and $p^{\mathrm{R}}(z|k)$ respectively. Based on the statistics of previous signal switching activity, availability of a-priori discrete probability distributions $P^{\mathrm{G}}(k),\ k^{\mathrm{G}}_{\min} \le k \le k^{\mathrm{G}}_{\max}$ and $P^{\mathrm{R}}(k),\ k^{\mathrm{R}}_{\min} \le k \le k^{\mathrm{R}}_{\max}$, is assumed, with the involved probabilities summing up to 1. Based on these probabilities, we can calculate, using crop-and-scale, the required probabilities $p^{\mathrm{G}}(z|k)$ and $p^{\mathrm{R}}(z|k)$ for the stochastic variables $z^{\mathrm{G}}(k)$ and $z^{\mathrm{R}}(k)$ [Typ20].

The cost criterion of the stochastic problem is the same as in the deterministic GLOSA problem in [Typ20]. However, in the stochastic case, the exact value of the criterion depends on the stochastic variables' realizations, and therefore we consider minimization of the expected value

$$J = E\left\{wt_{\mathrm{e}} + \frac{1}{2}\int_0^{t_{\mathrm{e}}} a^2 dt\right\}, \tag{9}$$

where the expectation refers to the stochastic variables $z^{\mathrm{G}}(k), z^{\mathrm{R}}(k), k = 0, \ldots, k_{\max} - 1$. Note that, when the vehicle crosses the traffic signal at state $x(k)$, the problem instantly becomes a deterministic GLOSA problem, and the corresponding optimal cost-to-go is $J^*_{DG}(x(k), k)$, as described in [Typ20].

To obtain a formally proper cost criterion, the stochastic variables $z^{\mathrm{G}}(k), z^{\mathrm{R}}(k)$ and virtual variables $x^{\mathrm{G}}(k), x^{\mathrm{R}}(k)$ are used, and, similarly to [Typ20], this yields the objective function as follows

$$J = E\left\{\sum_{k=0}^{k_{\max}-1}\left[\frac{1}{2}a(k)^2 + \left(x^{\mathrm{G}}(k) + (1 - z^{\mathrm{R}}(k))x^{\mathrm{R}}(k)\right) J^{\sigma *}_{DG}[x(k), a(k), k+1]\right]\right\}. \tag{10}$$

The recursive Stochastic Bellman Equation (SBE) has four corresponding parts. Starting from $k^{\mathrm{R}}_{\max}$, we need to move backwards, calculating the function V (optimal cost-to-go) step-by-step. The SBE for the generalized problem reads as follows

$$
\begin{aligned}
V[x(k), x^{\mathrm{G}}(k), x^{\mathrm{R}}(k), k] = & \min_{u(k)\in U} E\left\{\frac{1}{2}a(k)^2\right. \\
& + \left(x^{\mathrm{G}}(k) + (1 - z^{\mathrm{R}}(k))x^{\mathrm{R}}(k)\right) J_{DG}^{\sigma *}(x(k+1), k+1) \qquad (11) \\
& \left. + V[x(k+1), x^{\mathrm{G}}(k+1), x^{\mathrm{R}}(k+1), k+1]\right\} \\
= & \min_{u(k)\in U} E\left\{\frac{1}{2}a(k)^2\right. \\
& + \left(x^{\mathrm{G}}(k) + (1 - z^{\mathrm{R}}(k))x^{\mathrm{R}}(k)\right) J_{DG}^{\sigma *}(x(k), k+1). \qquad (12) \\
& \left. + V[x(k), a(k), x^{\mathrm{G}}(k)z^{\mathrm{G}}(k), x^{\mathrm{R}}(k)z^{\mathrm{R}}(k), k+1]\right\} \\
= & \min_{u(k)\in U} \left(\frac{1}{2}a(k)^2 + (x^{\mathrm{G}}(k) + p^{\mathrm{R}}(k)x^{\mathrm{R}}(k))J_{DG}^{\sigma *}(x(k), k+1)\right. \\
& + (p^{\mathrm{G}}(k)x^{\mathrm{G}}(k) + (1 - p^{\mathrm{R}}(k))x^{\mathrm{R}}(k))V[x(k), a(k), 0, 1, k+1] \\
& \left. + ((1 - p^{\mathrm{G}}(k))x^{\mathrm{G}}(k))V[x(k), a(k), 1, 0, k+1]\right),
\end{aligned}
\tag{13}
$$

where

$$
J_{DG}^{\sigma *}(x(k+1), k+1) = \sigma(x(k+1))J_{DG}^{\sigma *}(x(k+1), k+1), \tag{14}
$$

$$
\sigma(x(k+1)) = \begin{cases} 1 & \text{if } x(k+1) > 0 \text{ and } v(k+1) \leq v_{\max} \text{ and } k < k_{\max}^{\mathrm{G}}, \\ \infty & \text{if } x(k+1) > 0 \text{ and } v(k+1) > v_{\max} \text{ and } k < k_{\max}^{\mathrm{G}}, \\ 1 & \text{if } x(k+1) \in X \text{ and } k \geq k_{\max}^{\mathrm{G}}, \\ \infty & \text{if } x(k+1) \notin X \text{ and } k \geq k_{\max}^{\mathrm{G}}, \\ 0 & \text{else.} \end{cases} \tag{15}
$$

The details of SBE for each traffic signal phase are not explained here, due to limited space.

3 Preliminary Results and Discussion

In this section, preliminary results of the proposed generalized GLOSA problem considering both green and red phases are reported. Two scenarios are considered, and the obtained results are compared with those derived from IDM. The two scenarios differ in the actual switching time of the uncertain green phase, i.e. in Scenario 1 the uncertain green is extended until the latest possible time, while in Scenario 2 the green switches to red earlier. The scenarios chosen have the following set up: $x_0 = 0\,\mathrm{m}$, $v_0 = 5\,\mathrm{m/s}$, $x_{\mathrm{e}} = 370\,\mathrm{m}$, $v_{\mathrm{e}} = 11\,\mathrm{m/s}$ and $x_1 = 300\,\mathrm{m}$. Moreover, the bounds for the states and control are set to $[x_{\min}, x_{\max}] = [0, 300]\,\mathrm{m}$, $[v_{\min}, v_{\max}] = [0, 16]\,\mathrm{m/s}$ and $[a_{\min}, a_{\max}] = [-2, 1]\,\mathrm{m/s}^2$. The time step T is $1\,\mathrm{s}$ and the switching time windows for the traffic signal are $[k_{\min}^{\mathrm{G}}, k_{\max}^{\mathrm{G}}] = [10, 30]\,\mathrm{s}$ and $[k_{\min}^{\mathrm{R}}, k_{\max}^{\mathrm{R}}] = [40, 60]\,\mathrm{s}$ with uniform a-priori probability distributions. The switching from green to red for Scenarios 1 and 2 occurs at $k = k_{\max}^{\mathrm{G}}$ and $k = 20$, respectively. The switching from red to green at the uncertain red phase is considered to be at $k = k^{\mathrm{R}}_{\max}$, for both scenarios.

Figure 2 displays the state (position and speed) and control (acceleration) trajectories resulting from the SDP algorithm (blue lines) and the obtained trajectories using IDM (magenta lines), for both Scenarios 1 and 2. In Figures 2a – 2c (Scenario 1), both vehicles, guided by the SDP and the IDM, cross the traffic signal before the red phase. On the other hand, in Figures 2d – 2f (Scenario 2), the vehicles do not have the time to cross before the switching and they wait until the end of the uncertain red phase. In both scenarios, the SDP algorithm, is more conservative, as it incorporates all the available knowledge, including the probabilistic distribution of the switching times and its updates over time. The proposed approach performs better in terms of fuel consumption compared to IDM, as the fuel consumption derived from the ARRB fuel consumption model [Akç87] is 44.2 ml and 46.3 ml for the SDP and IDM, respectively, for Scenario 1; and 65.3 ml and 75.4 ml for Scenario 2. This outcome is expected, especially in Scenario 2, where, in contrast to IDM, SDP avoids the full stop at the traffic light position. The resulting final times are $t_{\mathrm{e}} = 35.2$ s and 32.0 s and $t_{\mathrm{e}} = 68.5$ s and 74.0 s for SDP and IDM for Scenarios 1 and 2, respectively.

4 Conclusions

In recent work [Typ20], a stochastic GLOSA methodology was developed, by optimizing, using SDP techniques, the vehicle kinematic trajectories subject to the stochastic traffic signal switching, with fixed final state and free final time. However, the problem considered only half traffic light cycle, i.e. the traffic signal is initially red and turns to green. As an extension to that work, we consider here a full traffic light cycle, which consists of four phases, consisting of a certain green phase, an uncertain green phase, a certain red phase, and an uncertain red phase. Preliminary results illustrate the superiority of the proposed stochastic GLOSA on average, when compared with any other approach, e.g. the IDM.

Future work:

- Investigation of scenarios with multiple consecutive traffic lights and different switching times,
- Implementation of different Dynamic Programming algorithms which enable faster solutions.
- Compare the proposed SDP approach with more sophisticated approached than IDM.

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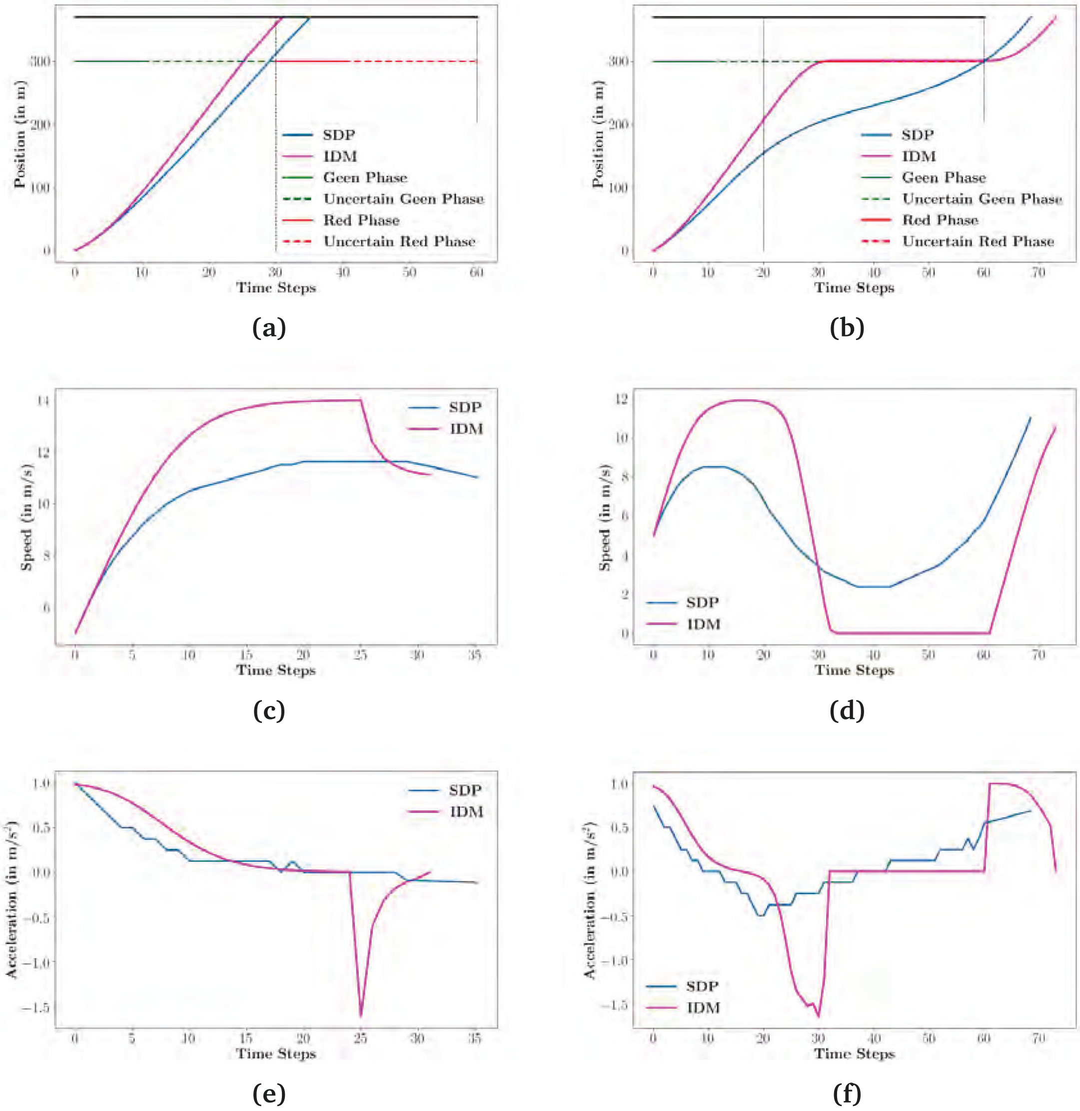

Figure 2: Optimal state and control trajectories of SDP (blue lines) versus IDM (magenta lines). In (a) and (d), the actual switching times are indicated with vertical dashed lines for Scenarios 1 and 2, respectively.

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