# Extracting Functionally Equivalent Object-Oriented Designs from Legacy Imperative Code 

Ricky E. Sward

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Extracting Functionally Equivalent Object-Oriented Designs from<br>Legacy Imperative Code<br>DISSERTATION<br>Ricky E. Sward<br>Major, USAF<br>AFIT/DS/ENG/97-04

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# Extracting Functionally Equivalent <br> Object-Oriented Designs from <br> Legacy Imperative Code 

## DISSERTATION

Presented to the Faculty of the Graduate School of Engineering of the Air Force Institute of Technology

Air University
In Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Computer Engineering

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September, 1997

# Extracting Functionally Equivalent <br> Object-Oriented Designs from <br> Imperative Legacy Code 

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God is my strength and power: And He maketh my way perfect.

2 Samuel 22:33

Ricky E. Sward
Call to me, and I will answer you, And show you great and mighty things, Which you do not know.

Jeremiah 33:3

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## Abstract

The research presented in this document defines a methodology for automatically extracting functionally equivalent object-oriented designs from legacy imperative programs. The Parameter-Based Object Identification (PBOI) methodology is based on fundamental ideas that relate programs written in imperative languages such as C or Cobol to objects and classes written in object-oriented languages such as Ada 95 or $\mathrm{C}++$. The fundamental thesis that defines the PBOI methodology is that object attributes manifest themselves in imperative subprograms as data items passed between subprograms. The PBOI methodology converts each subprogram in an imperative design into a class and a method that implements the subprogram. During this process duplicate classes are eliminated, duplicate objects are identified, and overlapping classes are merged into one class. Transformations have been developed that formalize the PBOI methodology and a formal proof is provided showing the extracted object-oriented design is functionally equivalent to the legacy imperative system. To focus the task of re-engineering, generic models of imperative programming languages and object-oriented programming languages have been developed. The Generic Imperative Model (GIM) is programming language independent, programming construct independent, and canonicalizes simulated control flow constructs. Formal definitions for the semantics of each GIM construct have been defined using the weakest precondition notation. The Generic Object-Oriented Design model (GOM) is also programming language independent and programming construct independent. Formal definitions for the semantics of each GOM construct have also been defined using the weakest precondition notation. The formal tranformations convert imperative subprograms represented in the Generic Imperative Model into classes and objects represented in the Generic Object-Oriented Design Model. A taxonomy of imperative subprograms has also been developed which classifies all imperative subprograms into one of six categories. A proof of concept prototype has been developed and a 3000 -line FORTRAN- 77 system has been converted to an object-oriented design as a feasibility demonstration.

# Extracting Functionally Equivalent <br> Object-Oriented Designs from <br> Legacy Imperative Code 

## I. Introduction and Background

### 1.1 Introduction

1.1.1 Overview. The object-oriented paradigm with its promise of re-usability, extensibility, and maintainability has great appeal to organizations with aging legacy systems. Legacy systems are often complex, unstructured, and include no documentation. Making even the smallest change to a legacy system often creates unpredictable sideeffects. Legacy systems are valuable assets to an organization and should be preserved, not thrown away [49]. Re-engineering imperative legacy systems into object-oriented systems provides a way for organizations to modernize their aging systems without losing the investment that these systems represent [50,67].

The research fields of re-engineering and reverse engineering have recently emerged within the field of software engineering. The goal of software engineering is to improve the products and practices used to develop software. However, the majority of the software development effort is spent on maintaining legacy systems as opposed to developing new systems [61]. Boehm [8] estimates the proportion of resources and time devoted to maintenance ranges from $50 \%$ to $80 \%$. The implication is that in order to improve the software development process, the software maintenance process must be examined. Since maintaining an object-oriented system is inherently easier than maintaining an imperative system [32,33], re-engineering legacy code to the object-oriented paradigm improves the maintenance process. Furthermore, re-engineering to the object-oriented paradigm is desirable to organizations because it improves the overall software development process.

The research presented in this document defines a methodology for automatically identifying objects from legacy imperative programs. The Parameter-Based Object Identi-
fication (PBOI) methodology is based on fundamental ideas that relate programs written in imperative languages such as C or Cobol to objects and classes written in object-oriented languages such as Ada 95 or $\mathrm{C}++$. Transformations have been developed that formalize the methodology and a proof is provided showing the extracted object-oriented design is functionally equivalent to the legacy system. To focus the task of re-engineering, generic models of imperative programming languages and object-oriented programming languages have been developed. The formal transformations convert imperative subprograms represented in the imperative model into classes and objects represented in the object-oriented model. A taxonomy of imperative subprograms has also been developed which classifies all imperative subprograms into one of six categories.

The remainder of this chapter defines re-engineering terms in Section 1.2 and presents related work in the field of re-engineering in Section 1.3. A summary of research contributions is provided in Section 1.5 and brief descriptions of the following chapters is presented in Section 1.6. The author assumes the reader has fundamental knowledge about both the imperative paradigm [16] and the object-oriented paradigm [62].

### 1.2 Definitions

The following definitions are provided for the terms re-engineering, reverse engineering, and program understanding.

### 1.2.1 Re-Engineering. According to Chikofsky [13],

Re-engineering is the examination and alteration of a subject system to reconstitute it in a new form and the subsequent implementation of the new form.

Typically, a system being re-engineered is written in an outdated programming language or built specifically for an outdated hardware platform. Computer systems that were built many years ago and have undergone several maintenance modifications often become what are known as legacy systems. A legacy system is hard to maintain because of the lack of knowledge about the system and because of the side effects when making even a simple change to the system. The computer program in a legacy system is called legacy code. Re-engineering often focuses on revamping legacy systems using new programming


Figure 1 Re-Engineering Process
paradigms, languages, or hardware platforms. Figure 1 shows a generalized view of the process of re-engineering legacy code as developed by Byrne [12]. In order to do effective re-engineering, the legacy code must be expressed at a higher level of abstraction than the programming language in which it was written [75]. This process of expressing the legacy code in a higher level of abstraction is reverse engineering as shown on the left hand side of Figure 1. The different levels of abstraction in re-engineering include implementation, design, and requirements specifications.

Legacy code can be re-engineered at each of these levels of abstraction. At the implementation level, it is possible to re-code a program from one programming language to another. At the design level, it is possible to re-design a program changing the design of the legacy code into a design for the target system. At the requirements specification level, it is possible to re-specify the requirements for a program. Each of these processes is discussed in more detail in Section 1.2.2.

Once these transformations are done at the different levels of abstraction, forward engineering can be used to build the target system in the new paradigm or language. Reengineering is the overall process of reverse engineering followed by forward engineering. This process does not require the target system to be functionally equivalent to the legacy system.


Figure 2 Reverse Engineering Process
1.2.2 Reverse Engineering. Figure 2 shows the process of reverse engineering in more detail. To abstract legacy code, implementation information must be available. This is typically the programming language code such as FORTRAN or Cobol code. It is possible to restructure the implementation information during reverse engineering without re-engineering the legacy code to a new target system. A restructuring at the implementation level could possibly include eliminating GOTO statements from the legacy code.

The process of reverse design abstracts the implementation information up to the design level. This process extracts information such as structure charts showing the calling hierarchy of the legacy code, data flow diagrams showing the flow of data in and between
legacy code routines, or control flow diagrams showing the flow of control for the legacy code. Restructuring can also be done at the design level without doing re-engineering. For example, the structure chart could be reorganized to improve the number of connections between different legacy code routines, i.e., improve the coupling [64] between routines.

The design level information is abstracted up to the requirements specification level by the reverse specification process. This process extracts the specifications for the legacy code from the design information. Restructuring can also be done at this level of abstraction. For example, the specifications could be split apart and reorganized to improve understandability.

Often in reverse engineering, a system undergoes a program understanding process in order to create a more robust and understandable abstraction of the program. Program understanding is a special form of reverse engineering. An overall view of program understanding is presented in the following section.
1.2.3 Program Understanding. This section describes the current cognitive science notion of human program understanding as found in Tiemens [73]. Figure 3 shows an


Figure 3 Program Understanding Theory
overall notion of how humans understand programs. The knowledge base stores concepts the person already understands such as analysis and design techniques, programming architectures, specific algorithms, and specific programming language constructs. This knowledge ranges from abstract notions such as architecture to concrete notions such as assignment statements in FORTRAN.

The legacy code is the programming language code the person is trying to understand. This code is typically in the form of a listing the person can examine. There is typically no pre-processing of this code before the person examines it.

The assimilation process involves the person looking at the code and trying to find concepts in the knowledge base implemented in the legacy code. This process is aptly called "assimilation" because the person is trying to fit the processing being done by the legacy code into the knowledge he or she already has in the knowledge base. The process builds new knowledge that is linked to the knowledge base and stored in the mental model.

The mental model is a series of links created in the assimilation process by the person understanding the legacy code. The mental model is the person's understanding of how the processing of the legacy code fits into what he or she already knows. The links between the knowledge base and the legacy code are rich semantic links from what was known before to what is being understood now. Understanding is a process of recognition, abstraction, assimilation, and storage [73].

This overall view of program understanding is by no means complete because the area of program understanding is an open field in cognitive science. However, the ideas presented here are an adequate presentation of the current literature in this field.

### 1.3 Related Work

This section reviews previous work in the area of reverse engineering. This review includes significant contributions in program understanding, extracting software architectures, extracting objects, and maintaining functional equivalence. The overall landscape of these contributions is presented below followed by detailed descriptions of selected systems.

Research contributions thus far in the area of reverse engineering can be loosely categorized based on three criteria:

Knowledge The level of the knowledge extracted from the legacy code.
Strategy The approach used for reverse engineering.
Goal The end result or product.


Figure 4 Reverse and Re-Engineering Systems

Figure 4 shows these criteria on three axes. The axis labeled Knowledge shows the different levels of knowledge that can be extracted. Language knowledge is low-level knowledge about a programming language such as FORTRAN or Cobol. Programming knowledge includes knowledge about common ways to implement algorithms such as sorting or searching. Design knowledge is knowledge about the architecture or overall design of a legacy code. Domain knowledge is knowledge about the application domain being implemented by the legacy code. The axis labeled Strategy shows the different strategies used in reverse engineering. Plan Recognition strategies rely on pre-defined plans or clichés that describe the concepts to be recognized. Data Driven strategies rely on information from the legacy code such as data-flow, control-flow, and program slices in order to extract knowledge. Informal Information strategies extract information from documentation and comments from the legacy code.

The axis labeled Goal shows the goals of the reverse engineering systems. Program Understanding systems produce a mapping between concepts and the legacy code which represents an understanding of the legacy code. The Architecture goal refers to systems that recover the architecture used in the legacy code. This ranges from low-level products such as module interconnection diagrams to high-level products such as recognizing a clientserver architecture. The Objects goal refers to systems that recover objects and classes from the legacy code. Systems in this category recover as many objects as possible, but are not focused on duplicating the functionality of the legacy code with these objects. The Functionally Equivalent Objects systems recover objects and classes with the specific goal of maintaining the functionality of the legacy code.
1.3.1 Plan Recognition Systems. Work in the area of program understanding has been dominated thus far by the plan (or cliché) recognition systems [ $28,48,51,60,70]$. Plan recognition systems were inspired by forward engineering systems based on plans including the Programmer's Apprentice [56-59, 74]. Plan recognition systems aid in understanding legacy code by matching the code to a pre-defined plan with concepts and constraints between the concepts. Once the plan is found in the code, the concept being represented by the plan is considered to be understood. These plans can be organized into a hierarchy representing low-level programming knowledge up to high-level domain knowledge.

Figure 4 shows where plan recognition systems fit in the overall landscape of reengineering systems. As shown in the figure, some plan recognition systems (Wills, Ning, etc.) can recognize domain knowledge while others (Hartman and Johnson) recognize design knowledge. The goal of plan recognition systems has, to this point, been program understanding.
1.3.1.1 Rich's Work [60]. The Recognizer system built by Rich and Wills automatically finds all occurrences of a concept in a program. This system relies on the idea of clichés for concept recognition and uses the plan calculus [57] to represent programs and clichés. The Recognizer builds a hierarchical representation of the concepts it finds in the programs and associates a natural language description with each of the concepts recognized.

Rich addresses five problems in legacy code that are handled by using clichés to represent knowledge:

Syntactic variation The same net flow of data and control can be achieved in many ways.
Non-contiguousness Parts of a cliché can be scattered through the program text; they do not have to be contiguous.
Implementation variation An abstraction can be implemented in many ways.
Overlapping implementation Program optimization can merge the implementations of distinct abstractions.
Unrecognizable code It is hard to find "good" concepts in code if the code was not built from a template such as a cliché.

The Recognizer system transforms programs into the plan calculus to overcome the problems of syntactic variation and non-contiguousness. The Recognizer also includes a hierarchy of clichés represented in the plan calculus. By representing both programs and clichés in the plan calculus, Rich has made the matching process much simpler. The process of matching clichés to programs is treated as a graph parsing problem. The Recognizer demonstrates understanding of the program by producing documentation automatically. It does this by using generic natural language templates stored with each cliché and filling in the appropriate fields from the code. The documentation has an artificially formal flavor to it, but demonstrates shallow understanding of design decisions made in producing the code.

Overall, Rich's research raises some important issues. First, for program understanding, there must be some representation of the concepts to be recognized. Second, the representation of concepts must be compatible with the representation of the program. Using the plan calculus to represent both the program and the clichés is a good common representation for the Recognizer. Using the plan calculus overcomes the problem of syntactic variation and non-contiguousness. It also casts the problem as a graph parsing problem which taps into a rich resource for problem solving. Rich calls this approach a representation shift.
1.3.1.2 Ning's Work. Ning [26] developed the Program Analysis Tool (PAT) to aid program understanding. The system uses a plan hierarchy and rule-based
inferencing system to recognize low-level constructs in the code and combine these to form higher-level abstract concepts. Ning [39] discusses automating the process of concept recognition by transforming programs to different levels of abstraction:

Text-level transformations treat a program as a sequence of characters and transform the program using character substitution.
Syntactic-level transformations treat programs as abstract syntax trees and transform programs using rules about the abstract syntax descriptions.
Semantic-level transformations add meaning to the syntax of programs and transform programs using the semantic information found in control flow and data flow diagrams.

Concept-level transformations rely on knowledge of programming, problem solving, and application domains to transform programs at the conceptual level.

Ning [39] also discusses the general problem of concept recognition. Programs contain many different kinds of information which can be divided into language concepts and abstract concepts. Language concepts are the low-level syntactic programming concepts from the legacy code. They include modules, language statements and other concepts defined by the programming language syntax. Abstract concepts refer to language-independent concepts such as programming concepts and problem solving methods such as searching and sorting.

A programming language compiler recognizes language concepts and builds them into an Abstract Syntax Tree for a program. To recognize abstract concepts, a concept classification hierarchy called a concept model is used to define the knowledge to be recognized. This model will not be complete since abstract theories of general programming are hard to develop. Thus, a partial recognition of the program is possible. This means it may not be able to recognize the program as representing a single concept, but it may be possible to recognize islands of concepts in the program. The concept model also defines the attributes of the concepts to be recognized. Ning develops the idea of a plan to represent a concept. A plan contains components (or sub-concepts) of the concept and the constraints between the sub-concepts. This information is encoded in the form of concept recognition rules. Plans represent the abstract concepts to be recognized, and through the constraints of the plans, they represent how these concepts will be recognized.

Ning's work is important because it exemplifies plan recognition systems. In any program understanding system, all levels of abstraction need to be recognized. Ning shows how a hierarchy of concepts can be recognized using a hierarchy of plans. Ning's work was the basis for continued work in plan recognition by Letovsky [41], Quilici [51], Chin [52], and others as discussed in the next section.
1.3.1.3 Other Plan Recognition Systems. The Cognitive Program Understander (CPU) system by Letovsky [41] uses plans as correctness preserving transformations for rewriting programs into semantically equivalent yet more abstract representations. Quilici [51] enhanced Ning's plan hierarchy to include indices into the plan hierarchy, a specialization mechanism, and links to other related plans. These improvements were intended to speed up the plan recognition process. The DECODE system from Chin [52] builds up a concept base through interaction with the programmer. DECODE uses an enhanced plan recognition system to recognize as much of the legacy code as possible. It increases understanding of the program by allowing the programmer to enter new conceptual design primitives and linking them to the program. The UNPROG system by Hartman [28] uses plans to recognize control designs such as read-process loops in Cobol programs. The PROUST system from Soloway and Johnson [70] uses plans in a top-down method in order to understand novice programmer's code.

Overall, the plan recognition systems are limited by the number of plans required to fully understand legacy code. If the exact plan that represents the concept being implemented by the legacy code is not present in the plan hierarchy, the plan recognition system is not able to understand the legacy code. This implies large plan hierarchies are required to understand even simple programs. The plan recognition process does not scale up well to large production-level legacy code. Quilici and Chin write [52]:

Applying this paradigm to reverse engineering real-world legacy systems is problematic, as it appears to require enormous libraries of code patterns, relies on program-understanding algorithms that are not guaranteed to scale, and fails completely on existing idiosyncratic code that does not fit well into patterns [17, $40,69]$.

The plan recognition systems suffer from problems that limit their applicability to real-world systems [52]. The methodology presented in this document does not use libraries
of code patterns. Instead, it uses a data-driven approach similar to the systems described in the next section.
1.3.2 Data-Driven Systems. Plan recognition systems are contrasted with the data-driven systems as shown in Figure 4. Data-driven systems do not rely solely on the process of matching plans for recognizing concepts in legacy code. Some data-driven systems use low-level knowledge about the legacy code to limit the amount of code being considered when matching plans to the code. In general, data-driven systems use dataflow and control-flow dependencies, program slicing, and definition/usage information to extract design and programming knowledge. The data-driven systems have varying reverse engineering goals including extracting architectures and objects. The following sections present relevant data-driven systems.
1.3.2.1 Newcomb's Work [47]. This section describes Newcomb's work on recovering functionally equivalent collections of objects from legacy code. The following terms are defined by Newcomb and help explain the process.

Scope analysis relates the occurrence of a variable to its declaration.
Program unit analysis describes the signature of routines, i.e., the number, order, and type of parameters for all routine declarations and invocations.

Collision former is a unique template of a data structure based on record length, field offset, field length and field type.
Alias analysis is the process of finding the names of the records and subfields in legacy code that match the collision former.
Alias map represents a relation whose domain is a collision former and whose range is a set of records that match the collision former. The criteria for matching the collision former can be changed.
Program slice is computed using the transitive closure of a usage to definition arc with respect to a variable.

Newcomb uses the idea of a collision former to hold the low-level declarative knowledge in his system. Each Cobol 01 level record from the legacy code becomes a collision former. The collision formers do not describe algorithms or designs, as would a plan. They hold the structure of the data in the system. The procedural knowledge is stored in control flow and data flow diagrams derived from the code.

The collision formers are used to do alias analysis, i.e. used as patterns to search through the other record structures for matches Newcomb calls aliases. Alias analysis is done on the data items in the legacy code instead of on the routines in the code (as in plan recognition systems). Newcomb creates an alias map that maps from each collision former to the set of record definitions that match the collision former.

Classes in the object model are formed by alias analysis and collision formation. The classes are built from a generalized version of the set of record definitions, and the instances of the classes are the records in the set. The fields from the 01 records are assigned to the objects as attributes. Currently, Newcomb's object modeling process does not construct class hierarchies.

Once objects are found through alias analysis, Newcomb takes a program slice focusing on one object. This indicates what procedures in the code modify or use the object. Newcomb presents a thorough explanation of how these procedures are built as methods and assigned to classes of objects.

Overall, Newcomb's system is important because of the work in extracting functionally equivalent objects. Newcomb claims the Object-Oriented Model (OOM) extracted is functionally faithful to the legacy code, but does not provide a proof of this claim.
1.3.2.2 Sneed's Work. The system developed by Sneed [67] is a pre-cursor to Newcomb's system [47] and also extracts functionally equivalent objects from legacy code. Sneed uses Cobol record structures to extract objects. Every record type is identified as an object and every field as an object attribute. The methods for these objects are extracted by using a program slicing technique Sneed calls phasing. A phase is an executable sequence of statements that follows the data flow path starting at an input from a file and ending at an output to a file. These program slices are attached to the objects to which they refer and become methods in the class that describes these objects.

An object-oriented specification is built for each extracted object. The specification language used is $\mathrm{Z}++$, an object-oriented version of the Z specification language. At the end of the reverse engineering process, there is a specification for each method and the imperative code is distributed among the objects in the target system. Sneed claims the
objects extracted are functionally equivalent to the legacy code, and uses dynamic profiling to validate this claim. Dynamic profiling is a technique that inserts statements in the legacy code to print out the values of variables at certain points in the code. The values of the variables are collected during execution of the legacy system. The values of variables in the re-engineered system are also collected during execution of the re-engineered system. The systems are functionally equivalent if the values for each of the variables in the legacy system are equal to the values for that variable in the re-engineered system.

The system from Sneed is an interesting data-driven system because of the program slicing and the extraction of functionally equivalent objects. The use of dynamic profiling requires execution of both the legacy system and the extracted object system, which limits the practicality of Sneed's proof of functional equivalence. Furthermore, errors in the transformation of the legacy system are not discovered until the re-engineered system is executed.
1.3.2.3 Nyary's Work. The system described by Nyary [68] automatically extracts object-oriented design documentation from legacy code. Nyary claims the design extracted is functionally equivalent to the legacy code, but does not prove this claim. The first step in Nyary's approach is to identify several different types of objects including user interface, information, file, record, view, work, and link objects. Nyary gives heuristics for finding each type of object. The seven types of objects are identified and stored with the names and descriptions of their attributes.

The system extracts operations for these objects by using data-flow and control-flow information. The connections between objects are identified by examining the parameters of routines in the legacy code and identifying any foreign variables in the object operations. Foreign variables are variables used by an operation that are not included in the object associated with the operation. Nyary claims the extracted operations are functionally equivalent to the legacy code. The sequence of these operations is expressed as a state driven model (as defined by Shlaer and Mellor [63]).

Nyary presents another example of a system that claims to extract functionally equivalent objects. Nyary does not provide any formal proof of this claim.
1.3.2.4 Achee and Carver's Work. The methodology developed by Achee and Carver [1] uses statistical analysis of subroutine parameters to extract objects from legacy FORTRAN systems. The frequency with which pairs of parameters are used in different subroutines is used to construct the "strongest cohesive unit" [1]. Objects are extracted by grouping these data items together (to form the cohesive unit) and then attaching methods to these objects.

Achee and Carver do not claim to extract functionally equivalent objects from legacy code. Their methodology is interesting because it uses subroutine parameters to extract objects using a statistical method rather than the methodology presented in this document. Their analysis focuses more on how parameters are used in a single subroutine than on the parameters passed between several subroutines.
1.3.2.5 Liu, Lividas, and Johnson's Work. Lividas and Johnson [43] present three techniques that use data-driven methods to extract objects from legacy code. The first two techniques were developed by Liu [42]; Lividas and Johnson introduce a third. Lividas and Johnson express all three methods for object identification using formal definitions. These definitions are presented below.

Let $P$ be an imperative program, $F$ the set of all routines in $P, T$ the set of all types in $P$, and $D$ the set of all data items in $P$. A candidate object $[43]$ is a triple $C_{M}^{P}=(\varphi, \tau, \delta)$ where $\varphi \subset F, \tau \subset T, \delta \subset D$, and $M$ indicates the method used for object identification.
Global-based Object Identification (GBOI) defines a triple $C_{g}^{P}=(\varphi, \emptyset, x)$ where $\varphi \subset F$ and $x$ is a global variable with respect to $\varphi$.
Type-based Object Identification (TBOI) defines a triple $C_{t}^{P}=(\varphi, \tau, \emptyset)$ where $\varphi \subset F$ and $\tau \subset T$. Here,

$$
\tau=\bigcup_{i=1}^{n}\left\{a_{i}\right\} \cup\{b\}
$$

where $a_{i}$ represent the types of the parameters and $b$ the type of the returned variable (if any).
Receiver-based Object Identification (RBOI) defines a triple $C_{r}^{P}=(\varphi, \tau, \emptyset)$ where $\varphi \subset F$ and $\tau \subset T$. Here,

$$
\tau=\bigcup_{i=1}^{n}\left\{r_{i}\right\} \cup\{b\}
$$

where $r_{i}$ represent the types of the parameters that are changed in $\varphi$.
These three techniques identify candidate objects in imperative programs based on relationships between the data items.
1.3.2.6 Other Data-Driven Systems. Figure 4 shows several other datadriven systems. These systems are described briefly in this section. The system proposed by Hausler [29] is a hybrid of the plan recognition systems and the data-driven systems. Hausler uses program slicing to isolate pieces of legacy code and then uses pattern matching to recognize plans in the program slices. This is interesting because the knowledge extracted by data-driven means is being used to limit the amount of code considered when matching the plans to the code. The systems from Ferrante [20] and from Wilde [77] capture dependencies in legacy code. Horwitz [30] and Weiser [76] present data-driven methods for program slicing. These four contributions provide building blocks for datadriven reverse engineering tools. The knowledge being represented is at the language level. The systems presented by Choi [14] and by Hutchens [31] use data-driven methods to analyze system structure of legacy code. Choi presents a method for extracting various module interconnection diagrams based on various criteria. The system built by Harris [27] uses program slicing and recognition rules to recognize architectural aspects of legacy code. The recognizers used are similar to plans because they structure the knowledge being recognized, but the recognizers do not require an exact match in the legacy code. Program slicing is used to limit the amount of code considered when matching recognizers to code.

The system proposed by Gall [21] uses data flow diagrams and structure charts to cluster procedures and data together into objects. Semantic knowledge is added to the objects found by comparing them to an independently developed object-oriented model of the underlying system. The OBAD system built by Yeh [79] (a sub-system of the Harris system [27]) extracts Abstract Data Types (ADTs) from legacy code. This system first builds a graph where the procedures and the data structure types are the nodes and the references from the procedures to the internal fields of the structures are the edges. The ADTs are extracted from the connected components of this graph. This contribution is included under the objects rubric because of the similarity between ADTs and objects [11].

The system developed by Ong [50] extracts objects from FORTRAN code. The system uses information from the COMMON block of the FORTRAN code to organize data and procedures into objects. The work presented by Jacobson [33] describes an incremental approach for re-engineering legacy code to the object paradigm. An interface is built from the new object-oriented part of the system to the part of the system not being re-engineered.
1.3.3 Informal Information Systems. The final group of contributions extract knowledge from the informal information found in legacy systems. This information includes any available user's manuals, programmer's manuals, or testing manuals. The informal information also includes the comments in the legacy code associated with the piece of code being analyzed. The DESIRE system from Biggerstaff [4-6] uses informal information to extract domain level knowledge, as indicated in Figure 4. DESIRE uses the informal information to aid program understanding. The work done by Lutsky [45] uses informal information in legacy system documentation to generate test cases for legacy systems. The I-DOC work by Johnson [34] automatically answers user's questions about a legacy system from a hyper-media database of knowledge. I-DOC automates the informal information associated with a software system.

### 1.4 Summary

The data-driven systems presented in this chapter use approaches for reverse engineering that are more promising than the approaches used by plan recognition systems. Newcomb's system [47], for example, has been demonstrated on legacy systems as large as 168,000 lines of Cobol code. The data-driven systems are more scalable because they either limit the amount of code being matched against plans or they don't use plans at all.

Several approaches were presented that extract objects from legacy code. Some of these systems claim to extract functionally equivalent objects, but there are no proofs of these claims. Sneed [67] validates his claim using dynamic profiling, but this is not practical because it requires the extracted object-oriented design to be implemented. Any
error introduced in the transformation would not be uncovered until the entire design was implemented.

The research presented in this document is similar to the work done by Newcomb [47], Sneed [67], and Nyary [68], as indicated in Figure 4. This research uses data-driven methods to extract an object-oriented design that includes functionally equivalent objects.

### 1.5 Summary of Contributions

Given the overview of re-engineering and the related work that has been done in this area, the research presented in this document makes the contributions shown below. Each of these contributions is discussed in subsequent chapters and a more detailed summary of the contributions is provided in Chapter IX.

1. An object identification methodology that extracts functionally equivalent objects from legacy imperative code.
2. A taxonomy of imperative subprograms.
3. Formal transformations that define the object identification methodology.
4. A proof of functional equivalence between the legacy imperative code and the extracted object-oriented code.
5. A canonical form for representing legacy imperative code.
6. A surface syntax for the imperative canonical form.
7. A canonical form for representing object-oriented designs and code.
8. A surface syntax for the object-oriented canonical form.
9. Definitions of the formal semantics for an imperative function call.
10. Definitions of the formal semantics for an object-oriented message.

### 1.6 Overview of Chapters

Chapter II defines in detail the problem of re-engineering imperative systems to the object-oriented paradigm. Chapter III presents the generic model of imperative programming languages and Chapter IV presents the generic model of object-oriented languages. Chapter V presents the taxonomy of imperative subprograms, the methodology for extracting objects from subprograms, and explains how program slicing [76] is used to simplify the methodology. Chapter VI presents transformations that formalize the methodology.

Chapter VII presents a proof that the extracted design is functionally equivalent to the legacy system. Chapter VIII presents a feasibility demonstration using a prototype implementation of the methodology. Chapter IX describes the contributions this research makes to the field of re-engineering. Chapter X discusses future research and presents the conclusions.

## II. Problem Statement

### 2.1 Introduction

This chapter presents a summary of the overall re-engineering methodology developed by this research. Each part of the methodology is explained briefly here and the more significant parts are explained in greater detail in the following chapters.

### 2.2 The Re-Engineering Methodology

The goal of this research is to develop a methodology for re-engineering legacy systems into functionally equivalent object-oriented designs. Figure 5 shows the overall view of this methodology. In the figure, the large rectangles are groupings of processes and products within certain paradigms. The large box on the left shows the re-engineering processes and products within the imperative paradigm. The large box on the right represents the forward engineering processes and products within the object-oriented paradigm. The large box in the middle represents versions of processes and products in the objectoriented paradigm that can be manipulated automatically with a computer. The "roundedrectangles" inside the large boxes represent products within a paradigm. The arrows between rounded-rectangles represent processes for converting one product to another. Each of the rounded-rectangles is discussed in the sections below. The dashed rectangle shows the scope of this research.
2.2.1 Legacy Code. The input to any re-engineering methodology is the collection of programming language code to be transformed. As explained in Chapter I, this code is termed legacy code. An assumption of this research is that the legacy code is available in some format, the least desirable of which is a program listing. The prototype developed to implement this research further assumes the legacy code can be accessed by a computer.
2.2.2 Canonical Form. The first step in the re-engineering methodology is to transform the legacy code into a canonical form. A canonical form allows code that does the same to look the same. A canonical form is language independent, programming


Figure 5 Re-Engineering Methodology

```
IF(ITYP.EQ.1) THEN
ALT = -5.71D0
ELSEIF(ITYP.EQ.2) THEN
ALT = -1.57DO
ELSEIF(ITYP.EQ.3) THEN
ALT = -9.46DO
END IF
```

Figure 6 FORTRAN elseif structure
construct independent, and control flow construct independent. Each of these aspects is explained in the following sections.
2.2.2.1 Language Independent. A canonical form is language independent, i.e., the representation is not tied to any one specific programming language. This allows the re-engineering methodology to operate at a higher level of abstraction, encompass more diverse legacy code, and be built independent of the nuances used in the legacy code.

### 2.2.2.2 Programming Construct Independent. A canonical form allows

 different programming language constructs to be recognized as the same entity. Certain constructs can have a different surface syntax but provide the same control flow. These constructs are easily identified because they have equivalent control flow graphs. Wills [60] refers to this as the syntactic variation problem as discussed in Section 1.3.1.1. For example, Figure 6 shows the FORTRAN elseif construct. Not all programming languages have this control construct, so the elseif is converted to embedded if-then-else statements in the canonical form. Embedded if-then-else statements provide the same control flow as the elseif statement, thus providing the canonical form.2.2.2.3 Simulated Control Flow Constructs. Another use of the canonical form is to recognize control flow constructs not implemented in the legacy code programming language. For example, there is no while loop in FORTRAN, but it can be simulated with the proper combination of an if-then statement and a goto statement. This is referred to as the implementation variation problem by Wills [60].

```
150 IF(C2.GT.1.ODO) THEN
    C1 = 1.ODO - E**2
    C2 = C1/C2GAM
    GO TO 150
    END IF
```

Figure 7 Simulated while control flow

Figure 7 shows FORTRAN if-then and goto statements that simulate a while loop. By recognizing these simulated control flow structures in the legacy code, they can be represented in the canonical form as the control flow entity they emulate. For example, the if-then and goto structure from Figure 7 are transformed into an equivalent while structure in the canonical form.
2.2.2.4 Generic Imperative Model. The Generic Imperative Model (GIM) has been developed as the canonical form for the re-engineering methodology, as shown in Figure 5. The GIM includes fundamental aspects of imperative programming languages such as FORTRAN, C, Pascal, Cobol, and Ada 83. Chapter III presents the GIM in detail.
2.2.3 Program Slices. Once the legacy code is converted to the canonical form, a process of program slicing is used to split the imperative legacy code into program slices based on functionality of the routines. A program slice [76] is the collection of imperative programming statements extracted from a legacy program that are required to produce a single value. Program slicing over entire legacy systems has been used in such applications as software maintenance and debugging [22] to isolate sections of code. The approach taken in this research is to use program slicing on a single legacy subprogram, as discussed in more detail in Chapter V.

The rationale for using program slicing in the re-engineering methodology is to split the legacy code into separate imperative subprograms that resemble object-oriented methods. In his Object Modeling Technique, Rumbaugh [62] defines a specific type of objectoriented method called queries. A query of an object returns a specific value from the object. By using program slicing, an imperative subprogram that returns multiple val-
ues can be split into multiple subprograms that each return a single value. These new subprograms now resemble query methods and can be built into an object class.

### 2.2.4 Parameter-Based Object Identification (PBOI). This section describes the

 arrow shown in Figure 5 labeled PBOI. The goal of this step is to organize the data and associated program slices into objects and classes that comprise the extracted objectoriented design. Chapter V describes the Parameter-Based Object Identification (PBOI) methodology developed by this research that extracts objects based on the data items being passed throughout the legacy system. As each object is extracted, a class is built in the object-oriented design that defines the attributes and operations of the object. As a final step in this methodology, duplicate object instances are eliminated and overlapping classes are merged. This is explained in detail in Chapter V. In order to scope this research, generalization and specialization hierarchies are not identified for the extracted classes. Each class is built as a sub-class of an overall super-class resulting in a flat hierarchy of classes.2.2.5 Generic Object-Oriented Design Model. The Generic Object-Oriented Design Model (GOM) is a canonical form that models any object-oriented design. The GOM has been developed as part of this research and is used to represent the extracted design. The design is built by creating entities from the GOM such as objects, classes, messages, and methods. The GOM also defines a design entity that is a collection of the extracted classes. Chapter IV defines the GOM in detail.

### 2.2.6 Generic Object-Oriented Program Model. The Generic Object-Oriented

 Program Model (GOM-P) is a canonical form that models any object-oriented program. The program differs from the design in that the object-oriented methods include objectoriented programming statements. This research assumes the statements of each method are from the imperative programming language paradigm. Part of the re-engineering process is to transform GIM statements to GOM-P statements. The GOM-P models languages such as C++, Java, and Ada 95. Chapter IV defines the statements that are modeled in the GOM-P.2.2.7 Requirements Specification. Figure 5 shows a dashed arrow from the program slices to the requirements specifications. Although this specific process is outside the scope of this research, it will be addressed briefly. It may be possible to extract specifications from the partitions that describe the functionality of the imperative code. These requirements specifications might be used to create an equivalent Object-Oriented Analysis (OOA). Being able to recover accurate specifications of the legacy code would clearly allow a more extensive and robust analysis of the legacy system. However, recovering specifications from code is considered a hard problem [44] and is left as future research.
2.2.8 Generic Object-Oriented Analysis Model. The Generic Object-Oriented Analysis Model (GOM-A) is a canonical form that models any object-oriented analysis. According to Rumbaugh [62], Object-Oriented Analysis (OOA) differs from Object-Oriented Design (OOD) in that the OOA object model includes associations between objects and that the functional and dynamic models are separate in OOA. In OOD, the dynamic and functional models are incorporated into the object model. For these reasons, the GOM-A includes entities for the object, dynamic, and functional models. It also includes entities for the associations between objects. It is hypothetically possible to build a GOM-A covering any object-oriented analysis. The GOM-A is not included as part of this research. DeLoach [15] has done work in this area and developed a generic model of the Rumbaugh Object Modeling Technique (OMT) [62].

It is possible to convert a requirements specification recovered from the imperative legacy code to an object-oriented analysis as indicated in Figure 5, but this research does not include such processing. The arrow in Figure 5 from the GOM to the GOM-A shows a hypothetical abstraction process. This is left as future research.
2.2.9 Object-Oriented Program. Once the OOD has been built by the reengineering methodology, it is possible to convert it to an object-oriented program. This program can be implemented using any object-oriented language such as $\mathrm{C}++$ [71] or Ada 95 [3]. This research has done only rudimentary prototyping of this step, as explained in Chapter VIII.
2.2.10 Object-Oriented Products and Processes. The rounded-rectangles in Figure 5 labeled OOA, OOD, and OOP represent the products of Object-Oriented Analysis, Object-Oriented Design, and Object-Oriented Programming, respectively. There are currently several methodologies for analysis, design, and programming in the object paradigm $[9,10,62]$. These rounded-rectangles are included in the figure to imply a correlation between the object-oriented design produced by the re-engineering methodology and an object-oriented design produced through forward object-oriented analysis and design.
2.2.11 Maintaining Functional Equivalence. After the object extraction methodology has been used on an imperative legacy system, the question remains about the utility of the transformation. How can one judge that the object-oriented design produced from the methodology is of any value? This question can be interpreted in at least two different ways. First, the question could be asking about the quality of the design and whether the recovered design is a good object-oriented design. Second, the question could be asking about the value of the design in general and whether representing the legacy system in the object-oriented paradigm adds value.

The former view requires a metric to determine the quality of an object-oriented design. This is an open research issue in the area of object-oriented analysis and design, i.e. there is currently no metric that accurately judges the quality of an object-oriented design. There is only an informal attempt in this research to answer the question from this view.

Instead, the value of the object-oriented design recovered is judged from the latter perspective. If it can be shown that the object-oriented design recovered from the imperative legacy code returns the same output as the legacy system given the same input, then the design is quite valuable. Such an object-oriented design can replace an aging legacy system and provide the same functionality as the original system while allowing the maintenance of the system to be done in the object-oriented paradigm. Chapter VII provides a proof that the design extracted using this methodology is functionally equivalent to the legacy code.

### 2.3 Summary

This chapter has presented a summary of the re-engineering methodology defined in this research. The methodology transforms imperative legacy code into the GIM, uses program slicing on the subprograms, extracts objects from the data items in the legacy system, and represents the extracted design using the GOM and GOM-P. The rationale for each of these steps has been discussed briefly and more detailed explanations are provided in the following chapters. Several other entities, such as the requirements specifications and the GOM-A, were presented and left for future research.

## III. The Generic Imperative Model

This chapter defines the Generic Imperative Model (GIM) developed to model the variables, expressions, assignment statements, and control flow typically built into imperative programming languages. The imperative paradigm is discussed first, followed by detailed descriptions of how these traits of imperative programming languages are modeled using Abstract Syntax Trees (ASTs). Rumbaugh's notation [62] for classes and objects is used to present the objects and classes that define these ASTs. Formal semantics are defined for each GIM entity using the weakest precondition notation [18, 19, 24, 25].

### 3.1 The Imperative Paradigm

According to Dershem [16], there are currently four different programming language paradigms: imperative, logic, functional, and object-oriented. Tennent [72] and Ghezzi and Jazayeri [23] describe the imperative programming language paradigm as a style of programming based on the following concepts.

Variables Variables hold state information during execution of the program.
Data Types Data types define the acceptable values for a variable and the operations that can be done on the variable.

Expressions Expressions are combinations of variables and operators used to express temporary intermediate values.

Assignment Statements Assignment statements change state by assigning new values to variables via expression evaluation.

Input/Output Input and output statements read and write to the standard input/output devices and to files.
Sequential Control In sequential control flow, a sequence of statements executes one after another.

Selective Control In selective control flow, a choice is made, based on the result of a boolean expression, between executing one sequence of statements versus another.

Iterative Control In iterative control flow, a sequence of statements is executed repeatedly while a boolean expression is true.
Procedural Abstraction A procedural abstraction collects a sequence of statements that are executed when the abstraction is referenced by name. A procedural abstraction can be passed parameters and may return values.

Main Program In systems of imperative subprograms, there is always one subprogram that is given the flow of control as the system begins execution. This special subprogram is termed the main program.

Imperative programming languages include FORTRAN, C, Pascal, Ada, Cobol, and others. In fact, any language where the majority of the language implements the concepts presented above is considered an imperative programming language. These are the concepts that distinguish the imperative paradigm from the other programming language paradigms.

These imperative programming language constructs are modeled in the GIM by building ASTs that store knowledge about the constructs. For a specific programming language such as Ada 83, it is possible that a construct in the language is not part of the imperative paradigm. For example, the accept and entry statements implement communication between tasks in Ada 83. These statements are not modeled in the GIM because they fall outside the definition of the imperative paradigm presented in this chapter. Overall, this means certain imperative languages can not be completely modeled by the GIM. The Feasibility Demonstration Chapter (Chapter VIII) discusses a method for determining which parts of a specific language can be modeled by the GIM.

The organization of this chapter emphasizes the importance of assignment and control flow in the imperative paradigm as opposed to the issues of data storage and expressions. Unfortunately, one cannot talk about assignment without variables and expressions. The reader is referred to Section 3.14 and Section 3.20 for the explanations of how variables and expressions, respectively, are modeled in the GIM.

For each programming language construct modeled in the GIM, formal semantics are provided using the state model [19] of programs. Preconditions and postconditions are used to define the semantics for each GIM representation of an imperative construct. Specifically, given a postcondition $R$ that is guaranteed to be true after a statement $S$ is executed, the weakest precondition, $w p(S, R)$, defines the weakest set of preconditions that must hold in order for the execution of $S$ to establish $R$ [19]. This research relies heavily on the previous work done by Djikstra [18], Gries [25], and Dromey [19] to define formal semantics for imperative programming language constructs.

### 3.2 The GIM Domain Model

This section presents a brief overview of the ASTs that are included in the GIM domain model. Figure 8 shows a partial representation of the GIM domain model. The


Figure 8 GIM Domain Model
overall superclass of the domain is the imperative-domain AST. The imperative-design class models collections of imperative subprograms as defined in Section 3.8. The abstract class imperative-statement is the superclass for all imperative programming statements modeled in the GIM. The imperative-data-construct class is the superclass of imperative expressions, data types, and variables modeled in the GIM. Each of the lower-level classes in the domain model are described in the rest of this chapter.

### 3.3 Imperative Assignment

An imperative assignment statement takes the general form $x:=e$, where $x$ is a variable and $e$ is an expression of the same type. When this statement is executed, the expression $e$ is evaluated and the result is assigned to $x$ [19]. Assignment statements in the GIM are modeled using the imperative-assignment class. Figure 9 shows the

| imperative-assignment |
| :--- |
| imp-assign-lhs : imperative-variable <br> imp-assign-rhs : imperative-data-construct |

Figure 9 Imperative Assignment Class
class description for the AST that models imperative-assignment in the GIM (using Rumbaugh's notation [62]). The imp-assign-lhs attribute models the variable being assigned a value. The imp-assign-rhs attribute models the expression to be evaluated and assigned to the variable. When variables appear on the right hand side of an assignment,
they are considered expressions. The GIM is able to model both variables and expressions on the right hand side of the assignment statement.

As an example, consider the following C assignment statement.

```
mfpgrc = 0;
```

This assignment statement is modeled in the GIM by creating an instance of the class imperative-assignment, which is shown in Figure 9. Figure 10 shows the object instance


Figure 10 Imperative Assignment Object
that models this C assignment statement in the GIM. The variable mfpgrc is modeled as a variable in the GIM (see Section 3.14) and is stored in the imp-assign-lhs attribute. The value 0 is modeled as a literal expression (see Section 3.20) and is stored in the imp-assign-rhs attribute of the AST.

The semantics for the GIM assignment statement are defined formally using the weakest precondition notation. Let $R_{e}^{x}$ denote the postcondition $R$, with all free occurrences of $x$ simultaneously replaced by $e$ [19]. The semantics for the general form of imperative assignment, $x:=e$, are defined using the weakest precondition notation as shown below.

Definition III.1. $w p(x:=e, R)=R_{e}^{x}$
In the GIM, the imp-assign-lhs models $x$ and the imp-assign-rhs models $e$. By relating the general form of assignment to the specific representation of assignment in the GIM, the formal semantics for assignment in the GIM are now defined.

### 3.4 Imperative Sequential Control

The default method of program control in the imperative paradigm is sequential control flow where a sequence of statements is executed one statement after another. Program statements that are executed sequentially in an imperative programming language are
modeled in the GIM by storing the statements in a sequence. The order of the statements in the sequence corresponds to the order of the statements from the imperative program.

For example, in the collection of statements shown below, <Statement $1>$ is executed followed by <Statement 2> followed by <Statement $3>$.
<Statement 1>
<Statement 2>
<Statement 3>
This collection of statements is modeled in the GIM using the following sequence.
[<Statement 1>, <Statement 2>, <Statement 3>]
The semantics for sequential control in the GIM are based on the semantics of the composition command [25] and are defined using weakest precondition notation. Let $S 1$ and $S 2$ be imperative statements and $[S 1, S 2]$ represent the sequential composition of the two statements.

Definition III.2. $w p([S 1, S 2], R)=w p(S 1, w p(S 2, R))$
Definition III. 2 leads to the claim that the GIM representation of sequential control flow is associative. Let $S 1, S 2$, and $S 3$ be imperative statements.

Theorem III.1. $w p(\{S 1,[S 2, S 3]], R)=w p([[S 1, S 2], S 3], R)$

Proof.

$$
\begin{aligned}
w p([S 1,[S 2, S 3]], R) & =w p(S 1, w p([S 2, S 3], R)) \\
& =w p(S 1, w p(S 2, w p(S 3, R))) \\
& =w p([S 1, S 2], w p(S 3, R)) \text { (by function composition) } \\
& =w p([[S 1, S 2], S 3], R)
\end{aligned}
$$

Since the weakest precondition for the sequences $[S 1,[S 2, S 3]]$ and $[[S 1, S 2], S 3]$ are equal, the sequence $[S 1, S 2, S 3$ ] is often used in this document to represent these sequences. In addition, singleton sequences and empty sequences are used when convenient.

### 3.5 Imperative Selective Control

In the imperative paradigm, control flow consisting of a selection between two or more statements is called selective control flow. The selection between statements $S 1$ and $S 2$ is based on a boolean expression $B$. In general, selective control flow takes the following form.

$S 2$

To be more specific, $S 1$ and $S 2$ above may consist of sequentially composed statements, so they are modeled in the GIM as sequences of statements. $S 1$ must include at least one statement to execute, but $S 2$ can be an empty sequence. Selective control flow is typified in imperative programming languages by the if-then-else statement. Selective control flow where $S 2$ is empty is typified in the imperative paradigm by the if-then statement.

Selective control flow is modeled in the GIM by using the imperative-selection class. Figure 11 shows the class description for the imperative-selection class. The

| imperative-selection |
| :--- |
| imperative-exp : imperative-data-construct <br> imperative-then <br> impart $:$ seq(imperative-program-construct) <br> impetive-else-part $:$ seq(imperative-program-construct) |

Figure 11 Imperative Selection Class
imperative-exp attribute models the boolean expression that controls the selection. The imperative-then-part models the sequence of imperative statements that are executed if the boolean expression is true. The imperative-else-part attribute models the sequence of imperative statements that are executed when the boolean expression is false.

For example, the following FORTRAN if-then-else statement shows a choice between executing the statement LEXIST(I) $=1$ or the statement LERROR $=$. TRUE..

```
IF (MFPGRC .EQ. 0) THEN
    LEXIST(I) = 1
ELSE
    LERROR = .TRUE.
END IF
```

The choice is made based on the value of the boolean expression (MFPGRC .EQ. 0). This FORTRAN if-then-else statement is represented in the GIM as shown in Figure 12.


Figure 12 Imperative-Selection Object

The imperative-exp attribute holds the <imperative-equal> object which models the expression (MFPGRC .EQ. 0). The imperative-then-part attribute holds the singleton sequence containing the assignment statement from the then part. The imperative-else-part holds the singleton sequence containing the assignment statement from the else part.

The semantics for the imperative-selection statement are defined using weakest precondition notation. Let $B$ represent an imperative boolean expression and let $S 1$ and $S 2$ represent sequences of imperative statements. The general form of selective control flow presented at the beginning of this section as represented in the GIM is given by the following form.

## if $B$ then $S 1$ else $S 2$

The sequence $S 1$ is executed if $B$ is true and the sequence $S 2$ is executed if $B$ is false. The semantics of this GIM form of selective control flow are defined below.

Definition III.3. $w p($ if $B$ then $S 1$ else $S 2, R)=(B \Rightarrow w p(S 1, R)) \wedge(\neg B \Rightarrow w p(S 2, R))$

To relate these formal semantics to the GIM, note that $B$ is modeled using the imperative-exp attribute. The sequence of statements $S 1$ is modeled by the imperative-then-part attribute and the sequence $S 2$ is modeled by the imperative-else-part attribute. The formal semantics for selective control flow as modeled in the GIM are now defined.

The formal semantics for selective control flow in the GIM when the sequence $S 2$ is empty deserve special attention. If $S 2$ is empty, then no change in state occurs if the boolean expression $B$ evaluates to false. Let skip represent a statement that has no effect on the state of a program.

Definition III.4. $w p(\operatorname{skip}, R)=R$
Selective control flow in the GIM when $S 2$ is empty takes the following form.

## if $B$ then $S 1$ else skip

The formal semantics for this form of selective control in the GIM is defined below.

Definition III.5. $w p($ if $B$ then $S 1$ else skip, $R)=(B \Rightarrow w p(S 1, R)) \wedge(\neg B \Rightarrow R)$

### 3.6 Imperative Iterative Control

The imperative paradigm also includes a control mechanism for repeating a sequence of statements known as iterative control flow. With iterative control flow, the sequence of statements is repeated while some boolean expression is true. Let $B$ represent the boolean expression that controls the iteration and $S 1$ represent the sequence of imperative statements that are repeated. The general form of iteration in the imperative paradigm is defined as
while $B$
S1

Execution of the sequence $S 1$ continues while the boolean $B$ is true.

Iteration in the GIM is modeled using ASTs built from the imperative-iteration class. Figure 13 shows the class description for the imperative-iteration class. The

| imperative-iteration |
| :--- |
| iter-exp : imperative-expression <br> iter-body : seq(imperative-program-construct) |

Figure 13 Imperative-Iteration Class
iter-exp attribute models the boolean expression $B$ and the iter-body attribute models the sequence of statements $S 1$.

For example, the following while statement from Pascal provides an example of imperative iteration.

```
WHILE Destination > Distance2 DO
    BEGIN
        Time := Time + Interval;
        Distance2 := Distance2 + Increment
    END
```

This WHILE statement is modeled using the imperative-iteration object shown in Figure 14. Other imperative programming languages include statements that implement


Figure 14 Imperative-Iteration Object
imperative iteration, for example, the while statement in C and the loop statement in Ada. These statements and statements in other programming languages provide several options for implementing imperative iteration. The GIM imperative-iteration class provides one canonical form to represent these looping mechanisms. Chapter VIII demonstrates the conversion of the FORTRAN DO loops, FOR loops, and loops formed through the structured use of the GOTO statement.

The formal semantics of iteration in the GIM are defined using a weakest precondition notation based on the guarded loop mechanism. Given a precondition $P$ and a postcondition $R$, let $H_{k}(R)$ represent the set of all states in which
while $B$ do $S 1$
will establish $R$ in at most $k$ iterations [19].
The formal semantics for iterative control flow in the GIM are defined below.

Definition III.6. $w p($ while $B$ do $S 1, R)=\left(\exists k: 0 \leq k \wedge H_{k}(R)\right)$
To tie this general definition to the specific model of iterative control flow in the GIM, recall that $B$ is modeled by the iter-exp attribute and $S 1$ is modeled by the iter-body attribute of the imperative-iteration class.

Admittedly, the task still remains to define $H_{k}(R)$. There is no attempt in my research to define loop invariants [19] for iteration in the GIM. What is provided is a general definition of the formal semantics for iterative control flow.

### 3.7 Imperative Subprograms

The imperative paradigm allows a programmer to group a sequence of statements together into a suitable abstraction unit that may be referenced by name. This mechanism is known as the subprogram [23], which gives a name to a sequence of statements. A subprogram call invokes the abstraction unit, i.e. forces control to transfer to the called unit, which upon completion returns control to the calling point [23].

Parameter passing conventions allow explicit communication between a subprogram and a call to the subprogram [23]. A formal parameter appears in the definition of a subprogram, and an actual parameter appears in the call of a subprogram. Note that the names of the identifiers used as actual parameters need not match the names of the
identifiers of the formal parameters of a subprogram. In fact, an actual parameter need not be an identifier at all, but can be an expression as well ${ }^{1}$.

The following definitions are needed to define subprograms more fully.
Definition III.7. A data item has a definition [2] in an imperative subprogram when the subprogram includes an assignment statement or an input statement that assigns a value to the data item.

Definition III.8. A subprogram parameter is an output parameter if the subprogram includes a definition of the parameter. A parameter is also an output parameter if the subprogram invokes another subprogram and the parameter is an output parameter of the called subprogram.

Definition III.9. A data item has a use [2] in an imperative subprogram when any of part of a statement in the subprogram references the data item and obtains its value.

Definition III.10. A subprogram parameter is an input parameter if $a$ use of the parameter occurs before a definition of the parameter.

Specific exemplars of the subprogram in the GIM include procedures and functions. A procedure in the GIM is a subprogram that does not explicitly return a value at the end of its processing. Instead, all values returned from procedures are returned via output parameters. For this research, the following restriction applies to procedures in the GIM.

Restriction III.1. A formal parameter of a procedure must not be both an input and an output parameter.

Appendix D provides a process for converting procedures with a parameter that is both an input and output parameter into a procedure that has no such parameters. A function in the GIM is a subprogram abstraction that does return a value at the end of its processing. For this research, the following restriction applies to functions in the GIM.

Restriction III.2. All functions in the GIM return a single value at the end of their execution and have no output parameters.

[^0]This research provides no process for converting a function with output parameters into a function with no output parameters.

### 3.8 Imperative Designs

In order to model the collection of subprograms as a whole, the subprograms are stored in an AST built from the GIM class imperative-design. Figure 15 shows the class

| imperative-design |
| :--- |
| imperative-programs : seq(imperative-subprogram) <br> imperative-files : seq(imperative-file) |

Figure 15 GIM Imperative Design Class
description of the imperative-design class. The imperative-programs attribute holds the collection of subprograms that comprise the design. The imperative-files attribute holds the collection of input/output files being modeled for this design. See Section 3.21 and Section 3.22 for discussions of how input and output statements and files are modeled in the GIM.

### 3.9 Imperative Procedures

Many imperative languages include a construct for declaring procedures. In FORTRAN, the SUBROUTINE statement is used to define the name, formal parameters, and statements of a procedure.

For example, the FORTRAN code shown in Figure 16 defines the procedure VADD. The sequence of statements included in the body of this procedure will be executed anytime the VADD procedure is called. A call to this procedure must include actual parameters that match the types of IOP, R1, R2, and R3. Other examples of programming language constructs that implement procedures are the procedure constructs in Pascal and Ada.

The declaration of a procedure is modeled in the GIM using ASTs built from the imperative-procedure class. Figure 17 shows the class description for the imperativeprocedure class. The imp-proc-identifier attribute is used to model the name of the procedure. The imp-proc-formals attribute models the sequence of formal parameters

## SUBROUTINE VADD(IOP,R1,R2,R3)

INCLUDE 'bdincl.f'
C
C THIS ROUTINE PERFORMS VECTOR ADDITION FOR THREE DIMENSIONAL
C VECTORS
C
DIMENSION R1(3),R2(3),R3(3)
$\mathrm{F}=\mathrm{FLOAT}(\mathrm{IOP})$
$\mathrm{R} 3(1)=\mathrm{R} 1(1)+\mathrm{F} * \mathrm{R} 2(1) \quad 10$
$\mathrm{R} 3(2)=\mathrm{R} 1(2)+\mathrm{F} * \mathrm{R} 2(2)$
$\mathrm{R} 3(3)=\mathrm{R} 1(3)+\mathrm{F} * \mathrm{R} 2(3)$
RETURN
END

Figure 16 FORTRAN Subroutine VADD

| imperative-procedure |
| :--- |
| imp-proc-identifier : symbol |
| imp-proc-formals : seq(imperative-variable) |
| imp-proc-statements : seq(imperative-program-construct) |

Figure 17 GIM Procedure Class
defined for the procedure. The imp-proc-statements attribute holds the sequence of statements from the procedure. Figure 18 shows the AST object that models the FORTRAN procedure VADD shown in Figure 16.

The declaration of a subprogram does not change the state of a program. It is the invocation of the subprogram that may affect the state. For this reason, the formal semantics for subprogram declarations defined in this section serve only as a foundation for the definition of the formal semantics of subprogram invocations.

Let $S 1$ represent the sequence of statements declared in a subprogram. Let $P$ represent the precondition of $S 1$ and let $R$ represent the postcondition of $S 1$ such that the following holds.


Figure 18 Modeling a Procedure
For a subprogram $p$, let $\bar{x}$ be a vector representing the input parameters of $p$. Let $\bar{z}$ be a vector representing the output parameters of $p$. Each procedure declaration in the GIM takes the following form.

$$
\begin{aligned}
& \text { procedure } p(\bar{x}, \bar{z}) \\
& \quad\{P\} \\
& \quad S 1 \\
& \quad\{R\}
\end{aligned}
$$

### 3.10 Imperative Functions

Many imperative languages also provide a mechanism for declaring functions. For example, the FORTRAN FUNCTION statement declares a function that returns a specific type of value from the execution of the statements in the function.

As an example, consider the FORTRAN function declaration found in Figure 19. This function returns a value of type DOUBLE PRECISION after the statements in the body of the function are executed. A call to this function must include actual parameters that match the data type of BETA, XLAMDA, and DIAM.

Functions in the imperative paradigm are modeled in the GIM using the imperativefunction class. Figure 20 shows the class description for the imperative-function class. The imp-func-identifier attribute holds the name of the function being modeled. The imp-func-formals attribute models the sequence of formal parameters defined for the

## DOUBLE PRECISION function PRDIV(BETA,XLAMDA,DIAM)

INCLUDE 'bdincl.f'
C
C CalCULATES THE PROJECTOR BEAM DIVERGENCE FOR A GIVEN OPTICS
C SIZE THIS IS BASED ON THE RAYLEIGH (AIRY DISK) CRITERION.
C
DATA FAC/1.220D0/

```
QUAL = MAX(BETA,1.0D0)
PDIAM = MAX(DIAM,0.1D0)
WAVELN = XLAMDA*1.D-06
PRDIV = QUAL *WAVELN*FAC/PDIAM
RETURN
END
```

Figure 19 FORTRAN Function PRDIV

| imperative-function |
| :--- |
| imp-func-identifier $:$ symbol <br> imp-func-formals $:$ seq(imperative-variable) <br> imp-func-statements $:$ <br> impq(imperative-program-construct) <br> imp-return-type : imperative-data-type |

Figure 20 GIM Function Class
function. The imp-func-statements attribute holds the sequence of statements from the function. The imp-function-return-type attribute models the data type of the value returned from the function. The FORTRAN function PRDIV shown in Figure 19 is modeled in the GIM using the imperative-function object shown in Figure 21.

Because of Restriction III.2, the formal semantics of a function declaration are defined more restrictively than the formal semantics of a procedure declaration. For a function $f$, let $\bar{x}$ be a vector representing the input parameters in $f$. Each function declaration in the GIM takes the following form.

$$
\begin{gathered}
\text { function } f(\bar{x}) \\
\quad\{P\} \\
S 1 \\
\{R\}
\end{gathered}
$$



Figure 21 Modeling a Function

In order to define the semantics for the value that is returned from the function $f$, a modified representation of imperative functions is used. Let $y$ represent the value that is returned from $f$. Let $f^{\prime}$ be a procedure that takes the same input parameters as $f$, and includes $y$ as the single output parameter. The statements in $f^{\prime}$ are the statements from $f$, viz. $S 1$. The procedure $f^{\prime}$ takes the following form.

$$
\begin{aligned}
& \text { procedure } f^{\prime}(\bar{x}, y) \\
& \{P\} \\
& S 1 \\
& \{R\}
\end{aligned}
$$

This modified representation of imperative functions is used to define the semantics of a function invocation, as described in Section 3.12.

### 3.11 Imperative Procedure Invocation

If an imperative language provides a way to define a subprogram, it will also provide a way to invoke the subprogram. This subprogram call mechanism refers to the called subprogram by name and passes any actual parameters required by the subprogram. This section defines one of the specific implementations of the subprogram call, viz. procedure calls.

An imperative procedure is invoked using an imperative procedure call, which is a kind of statement in the GIM. The call to a procedure is modeled in the GIM using ASTs built from the imp-procedure-call class. Figure 22 shows the class description for the

| imp-procedure-call |
| :--- |
| imp-proc-call-identifier $:$ symbol <br> imp-proc-call-actuals $:$ seq(imperative-variable) |

Figure 22 Imperative Procedure Call Class
imp-procedure-call class. The imp-call-identifier attribute holds the name of the procedure being invoked. The imp-call-actuals attribute holds the actual parameters used in the procedure call.

For example, in FORTRAN, the invocation of a defined SUBROUTINE is implemented using the CALL statement. The FORTRAN procedure VADD shown in Figure 16 is invoked with the following CALL statement.

CALL VADD (1, R1, R2, R3)
This procedure call invokes the procedure VADD and passes in the parameters 1, R1, R2, and R3.


Figure 23 Imperative Procedure Invocation Object

Figure 23 shows how this invocation of VADD is modeled in the GIM using an instance of the imp-procedure-call class. Because of the nature of the object extraction methodology, actual parameters used in subprogram calls must be variables. This leads to the following restriction on subprogram calls.

Restriction III.3. All actual parameters in subprograms calls must be variables.

For this reason, actual parameters such as the integer 1 in Figure 23 are replaced by a temporary variable and an assignment statement is added to assign this variable the value of the parameter. For example, the original GIM call to VADD is shown below.

```
VADD ( 1, R1, R2, R3 )
```

The updated call to VADD with the integer actual parameter replaced by a variable is shown below.

```
TEMP-29 := 1;
VADD ( TEMP-29, R1, R2, R3);
```

The formal semantics of a procedure call are defined as follows. Let $p$ be a procedure and let $S 1$ be the sequence of statements declared in $p$. Let $\bar{a}$ be a vector representing the actual parameters that correspond to input parameters in $p$. Let $\bar{c}$ be a vector representing the actual parameters that correspond to output parameters in $p$. An invocation of the procedure $p$ takes the following form. ${ }^{2}$

$$
p(\bar{a}, \bar{c})
$$

To invoke $p$, the actual parameters are copied to the formal parameters and control flow transfers to $p$. Executing the procedure call is equivalent to executing the following sequence.

$$
[\bar{x}:=\bar{a}, S 1, \bar{c}:=\bar{z}]
$$

Using weakest precondition notation, the semantics for a procedure call are defined below.
Definition III.11. $w p(p(\bar{a}, \bar{c}), R)=w p([\bar{x}:=\bar{a}, S 1, \bar{c}:=\bar{z}], R)$

### 3.12 Imperative Function Invocation

This section defines the other specific implementation of the subprogram call, viz. the function call. Most imperative languages invoke declared functions by using the name of the function and passing in the required parameters. A function call is a kind of expression.

[^1]Because of Restriction III.3, expressions do not appear in procedure calls but do appear in assignment, selective, and iterative statements. This implies function calls do not appear in procedure calls in the GIM, but can appear in the other statements. The call to a function is modeled in the GIM using the imp-function-call class. Figure 24 shows

| imp-function-call |
| :--- |
| imp-fun-call-identifier $:$ symbol <br> imp-fun-call-actuals $:$ seq(imperative-variable) |

Figure 24 Imperative Function Call Class
the class description for the imp-function-call class. The imp-fun-call-identifier attribute holds the name of the procedure being invoked. The imp-fun-call-actuals attribute holds the actual parameters used in the function call.

An important difference between the invocation of procedures and functions in the imperative paradigm is that procedure calls can only appear as whole statements and not part of another statement. Function calls can only appear as part of an expression that is part of another statement and not as whole statements themselves.

For example, the FORTRAN function PRDIV shown in Figure 19 is invoked as part of the following statement.

$$
\text { SIGPR }=\text { PRDIV(BETA, XLAMDA, DIAM) }
$$

This function call invokes the function PRDIV passing in the parameters BETA, XLAMDA, and DIAM. The result of this function call is used as the expression assigned to the variable SIGPR. Figure 25 shows how the invocation of PRDIV is modeled in the GIM using an


Figure 25 Imperative Function Invocation Object
instance of the imp-function-call class.
The formal semantics for function calls in the GIM are defined as follows. Let $f$ be a function and let $S 1$ be the sequence of statements declared in $f$. Let $\bar{a}$ be a vector representing the actual parameters that correspond to the input parameters in $f$. Since every function appears as part of an expression in some statement, let $S$ represent the statement in which $f$ appears. Recall the modified representation of a function presented in Section 3.10 includes one output parameter, $y$, which represents the value returned from the function $f$. Let $f^{\prime}$ be the procedure that represents $f$ with the input parameters from $f$ and the additional parameter $y$. Let $b$ represent the actual parameter that corresponds to $y$.

Using these definitions, an invocation of the function $f$ takes the following form.

An invocation of the procedure, $f^{\prime}$, takes the following form.

$$
f^{\prime}(\bar{a}, b)
$$

The difference between the invocations of $f$ and $f^{\prime}$ is that the invocation of $f$ is part of $S$ and the invocation of $f^{\prime}$ is a single statement. Invoking $f$ is equivalent to invoking $f^{\prime}$ and then substituting $b$ in $S$ for any invocations of $f$. This provides a formal representation of the value returned from $f$. The substitution of $b$ for the call to $f$ in $S$ is represented by the following notation.

$$
S_{b}^{f(\bar{a})}
$$

In this way, the call to $f$ is equivalent to the following sequence of statements.

$$
\left[f^{\prime}(\bar{a}, b), S_{b}^{f(\bar{a})}\right]
$$

As defined in Section 3.11, when the procedure $f^{\prime}$ is invoked, the actual parameters in $\bar{a}$ are copied to the formal parameters in $\bar{x}$ and control flow transfers to $f^{\prime}$. After execution of the function, the value of $y$ is copied to $b$. Hence, executing the call to $f^{\prime}$ is equivalent to executing the following sequence.

$$
[\bar{x}:=\bar{a}, S 1, b:=y]
$$

Using weakest precondition notation, the formal semantics of a call to function $f$, where the statement $S$ contains $f(\bar{a})$, are defined below.

Definition III.12. $w p(S, R)=w p\left(\left[\bar{x}:=\bar{a}, S 1, b:=y, S_{b}^{f(\bar{a})}\right], R\right)$

### 3.13 Recursion

In the imperative paradigm, it is possible for a subprogram to invoke itself in a process known as recursion. Knuth [37] claims that any recursive algorithm can be transformed into an iterative algorithm. For this reason, the GIM does not model recursion. This leads to the following restriction.

Restriction III.4. Subprograms to be modeled in the GIM are not allowed to make calls to themselves.

Even more restrictively, the call tree consisting of the graph of subprogram calls made in a collection of imperative subprograms is assumed to be a directed acyclic graph or DAG.

Restriction III.5. The call tree of a collection of imperative subprograms must be a directed acyclic graph.

### 3.14 Imperative Variables

Each variable that is set or used in an imperative program is modeled in the GIM AST by building an imperative-variable object. Figure 26 shows the class description for the imperative-variable object. The imp-data-scope attribute models the name of the procedural abstraction in which this variable is defined. The imp-data-identifier attribute models the name of the imperative variable. The imp-data-type attribute is an

| imperative-variable |
| :--- |
| imp-data-scope $:$ symbol |
| imp-data-identifier : symbol |
| imp-data-type : imperative-data-type |
| imp-data-indices : seq(imperative-data-construct) |
| imp-constant-value : any-type |

Figure 26 Imperative-Variable Class
object that describes the data type of the imperative variable. The imp-data-indices attribute stores the GIM representations of the indices for this variable, if any. The imp-constant-value attribute stores the value assigned to the variable if the variable is a constant.


Figure 27 Imperative-Variable Object

For example, Figure 27 shows how the C integer variable mfpgrc, defined in a procedure wc, is represented in the GIM. Here, the variable mfpgrc is modeled by building an instance of the imperative-variable class representing an integer variable with no indices and no constant value. The imp-data-scope attribute is set to the symbol WC since this is the name of the procedure in which this variable is defined. Scoping issues are discussed in more detail in Section 3.16. The data type of mfpgrc is modeled using the imperative-integer object in the GIM. Certain data types are modeled in the GIM as discussed in Section 3.15.

### 3.15 Imperative Data Types

In order to model variables in the GIM, certain data types are also modeled. The data types being modeled are:

Integer zero and positive and negative whole numbers
Real rationals and irrationals
Boolean true and false logical values
Character single alpha characters
String sequences of characters
Array homogeneous collections of elements accessed with an index.
Each of the classes representing these data types has an attribute imp-type-size that indicates the number of bytes associated with the data type. Some imperative languages allow the programmer to specify the size of the data type at the declaration of a variable. Other languages do not allow this, but include types of different sizes to meet data storage requirements. Programmer-defined data type sizes are modeled in the GIM by storing the specified size in the imp-type-size attribute. For languages that do not allow the programmer to define the size of the data type, the size of the built-in type must be determined and stored into the imp-type-size attribute.

For example, in the FORTRAN programming language, the programmer is allowed to declare the size of the data type when declaring variables. The MFPGRC variable from Section 3.14 can be declared as shown below.

INTEGER*4 MFPGRC
The integer data type is represented using the imperative-integer object. Figure 28

| imperative-integer |
| :---: |
| imp-type-size $=$ integer |

Figure 28 Imperative-Data-Type Class
shows the class description for imperative-integer. The imp-type-size attribute indicates the size of the data type. The four byte FORTRAN integer data type used to declare MFPGRC in the example above is represented using the imperative-integer object shown in Figure 29.

In the C programming language, the programmer must use intrinsic data types to specify the size of data storage locations. The int data type in $\mathbf{C}$ is normally the natural

## Figure 29 Four Byte Integer Object

size for a particular host machine: either 16 or 32 bits [35]. The short or long qualifier can be applied to the basic $C$ types to change the number of bytes used for the variable. A short integer is often 2 bytes and a long integer is often 4 bytes [35]. In order to model these data types in the GIM, the size of the intrinsic data types and qualifiers must be determined for a specific compiler in order to store the correct value for the imp-type-size attribute of the imperative data type objects.

In C, the MFPGRC variable can be declared as follows:

```
short MFPGRC;
```

Here, an imperative-integer object with the imp-type-size attribute set to 2 correctly models the data type of the MFPGRC variable, as shown in Figure 30.

```
(imperative-integer)
imp-type-size =2
```

Figure 30 Two Byte Integer Object

### 3.16 Imperative Scoping Issues

In the imperative paradigm, the "visibility" a variable has to subprograms is known as the scope [72] of the variable. A subprogram that defines a variable has visibility to it, but different imperative programming languages provide different rules for the visibility of the variable to other subprograms.

Each subprogram in the GIM has a local scope in which the variables declared are not visible to any other subprograms. The GIM subprogram that declared the variable can share its value by passing the variable to other subprograms as a parameter. Passing a local variable to another subprogram as an actual parameter has the effect of making the variable visible to the called subprogram. In the imperative paradigm, this is termed pass by reference parameter passing [16]. All parameters in the GIM are passed by reference.

Variables declared in the main program are considered to be in the global scope. Some imperative programming languages allow subprograms to directly access variables declared in the global scope, i.e. without passing the variable to the subprogram via a parameter. This is not modeled in the GIM and leads to the following restriction on variable scoping in the GIM.

Restriction III.6. All variables in a subprogram are either declared locally or are formal parameters of the subprogram.

In the imperative paradigm, some programming languages allow subprograms to be declared inside another subprogram. This leads to a nesting of variable scopes where variables in the "outer" subprogram are visible to the "inner" subprogram. Declaring a subprogram inside another subprogram is not modeled in the GIM which leads to the following restriction on declaring subprograms in the GIM.

Restriction III.7. Subprograms can not be declared inside of another subprogram. They are all declared in the main program's global scope.

### 3.17 Imperative Homogeneous Data Structures

In the imperative paradigm, collections of homogeneous elements can be built using constructs such as lists and arrays. The only such collection modeled in the GIM is the array. An array is an indexable collection of homogeneous elements. Variables in the GIM can be of the data type array. The indices used to access an array element are modeled using the imp-indices attribute of the imperative-variable class.

For example, an access to the first element of a FORTRAN array R used in a subprogram VADD is modeled in the GIM as shown in Figure 31 The imp-data-indices attribute models the expression or variable being used to access the element of the array. In the figure, the access to the first element of the array $R$ is modeled by storing the literal value 1 as the index into the array.


Figure 31 Imperative Array Access Object

### 3.18 Imperative Heterogeneous Data Structures

Some programming languages in the imperative paradigm, allow the programmer to build collections of heterogeneous data items, e.g. records in Pascal and structs in C. These heterogeneous data structures are not modeled in the GIM, which leads to the following restriction.

Restriction III.8. The GIM does not model heterogeneous data structures.

### 3.19 Imperative Pointers

Some imperative programming languages allow the programmer to store the address of a variable and then access the variable via the address. This mechanism is typically referred to as using pointers to variables. Pointers are not modeled in the GIM, which leads to the following restriction.

Restriction III.9. The GIM does not model pointers.

### 3.20 Imperative Expressions

In the imperative paradigm, expressions can be literal values, unary or binary expressions with operators, function calls, or even variables. This section describes how expressions are modeled in the GIM.

Literal values in the GIM are modeled by the imperative-literal-constant class. Figure 32 shows the class description for the imperative-literal-constant class. The

| imperative-literal-constant |
| :--- |
| imperative-literal-value : any-type |

Figure 32 Modeling Imperative Literal Constants
imperative-literal-value attribute models the value of the literal. Figure 33 shows all imperative-literal-boolean imperative-literal-integer imperative-literal-real imperative-literal-string

Figure 33 List of Imperative Literal Constants
the literal constants currently modeled in the GIM. Each literal shown is modeled as an AST defined as a subclass of the imperative-literal-constant class. Literal constants other than those listed in Figure 33 are not modeled in the GIM, but could easily be modeled by defining a new subclass under the imperative-literal-constant class that describes the new literal constant.

Unary expressions in the GIM are modeled using the imperative-unary-expression class. Figure 34 shows the class description for the imperative-unary-expression class.

| imperative-unary-expression |
| :---: |
| imp-unary-operand : imperative-expression |

Figure 34 Modeling Imperative Unary Expressions

The imp-unary-operand attribute models the single operand of the unary expression. Figure 35 shows all the unary expressions currently modeled in the GIM. Each unary expression shown is modeled as an AST defined as a subclass of the imperative-unaryexpression class. As with literal constants, any unary expressions other than those listed in Figure 35 are not modeled in the GIM but can be by building a new subclass that defines the new unary expression.

Binary expressions in the GIM are modeled by the imperative-binary-expression class. Figure 36 shows the class description for imperative-binary-expression class. Binary expressions in the imperative paradigm either have two operands or multiple operands.

Figure 35 List of Imperative Unary Expressions

| imperative-binary-expression |
| :--- |
| imp-bin-exp-operand-1 : imperative-name |
| imp-bin-exp-operand-2 : imperative-data-construct |
| imp-bin-exp-seq : seq(imperative-expression) |

Figure 36 Modeling Imperative Binary Expression

For this reason, the imperative-binary-expression class has three attributes for modeling the operands of the expression. The imp-bin-exp-operand-1 and imp-bin-exp-operand-2 attributes hold the two operands if the binary expression has only two operands. When the same binary expression is repeated for multiple operands, such as in the expression $A+B+C$, the multiple operands are modeled in the imp-bin-exp-seq attribute. Certain binary expressions can be repeated in this way, e.g. addition and subtraction. Others can not be repeated in this way, e.g. $<,<=$, and $>$. Figure 37 lists all the binary ex-
> imperative-division imperative-equal imperative-exponent imperative-greater-than-or-equal imperative-greater-than imperative-less-than-or-equal imperative-less-than imperative-not-equal imperative-subtraction
imperative-addition imperative-and imperative-concat imperative-multiplication imperative-or
(a) Two operand.
(b) Sequence.

Figure 37 List of Binary Expressions Modeled in the GIM
pressions that are modeled in the GIM. The binary expressions that can not be repeated are shown in Figure $37(\mathrm{a})$. These expressions are modeled using the imp-bin-exp-operand-1 and imp-bin-exp-operand-2 attributes. The binary expressions that can be repeated are shown in Figure 37(b). These expressions are modeled using the imp-bin-exp-seq attribute.

As with literal constants and unary expressions, any binary expressions not shown in Figure 37 are not modeled by the GIM. This new binary expression could be modeled by adding a subclass to the imperative-binary-expression class that defines the new binary expression.

### 3.21 Imperative Input

Programming languages in the imperative paradigm allow the programmer to interact with the user via input and output statements. These statements, if directed towards a file, allow the programmer to implement persistent storage of data. This section describes how input statements in imperative languages are modeled in the GIM.

In order to receive data values from outside an imperative program, imperative programming languages provide the programmer with an input statement. This statement can be used to communicate with the user or used to input data from a file. The Cobol programming language actually has two statements: one for inputting low-volume data and one for inputting data from a file. For example, the following Cobol ACCEPT statement is used to input a program name from the user using the console typewriter.
aCCEPT PROGRAM-NAME FROM CONSOLE.
Input statements are modeled in the GIM using the imperative-input object. Figure 38 shows the class description for this object. The imp-in-logical-file attribute

| imperative-input |
| :---: |
| imp-in-logical-file $:$ imperative-data-construct <br> imp-input-list $:$ seq(imperative-data-construct) |

Figure 38 Imperative Input Class
is used to model from where the input is coming. When the input comes from the user, the attribute is set to an accepted value that models that fact. The imp-input-list attribute holds the sequence of data items being input. Figure 39 shows how the Cobol ACCEPT statement is modeled in the GIM, assuming the PROGRAM-NAME variable is defined in a procedure called GET-INFO. The imp-in-logical-file attribute, in this example, has been arbitrarily set to the symbol console in order to represent input from the user


Figure 39 Imperative Input Object
console. As long as this attribute is set consistently, receiving input from the user can be modeled in this way.

In imperative languages, when the programmer wants to input from a file, the file must first be "opened". The act of opening a file establishes the link between the physical file on disk and the logical file in the program. The status of the file, e.g. opened for input or output, and the access mode of the file, e.g. direct or sequential, are also established when the file is opened. This information about the file is modeled in the GIM using the imperative-file object. Figure 40 shows the class description for this object. The

| imperative-file |
| :--- |
| imp-designator : imperative-data-construct |
| imp-file-name : imperative-data-construct |
| imp-access : symbol |
| imp-status : symbol |

## Figure 40 Imperative File Class

imp-designator attribute holds name of the logical file as referenced in the program. The imp-access attribute holds the access type, direct or sequential, for the file. The imp-status attribute holds the status of the file, opened either for input or output. In Cobol, the OPEN statement is used to open a file. The following Cobol OPEN statement opens a file MASTER-FILE for input. The access mode is assumed to be sequential.

OPEN INPUT MASTER-FILE
Figure 41 shows how this file is modeled in the GIM. The name of the file being opened is used as the file designator.

```
    (imperative-file)
imp-designator = MASTER-FILE
imp-status = INPUT
imp-access = SEQUENTIAL
```

Figure 41 Modeling MASTER-FILE

Once files are opened, input statements are used to input data from the files. In Cobol, the READ statement is used to input data from a file. For example, the following READ statement reads the next record from the file MASTER-FILE and stores it into the variable MASTER-WORK.

```
READ MASTER-FILE INTO MASTER-WORK
    AT END PERFORM END-DATA-MASTER.
```

Figure 42 shows how this READ statement is modeled in the GIM. The imp-in-logical-


Figure 42 Modeling the COBOL Read
file attribute, in this example, has been set to an imperative name object referring to the file MASTER-FILE in order to indicate the input is coming from this specific file.

The semantics of the GIM input statement are similar to the semantics of the GIM assignment statement. Both statements define values for variables. Abstractly, an input statement in the GIM is equivalent to calling a subprogram where all the parameters are output from the subprogram.

For example, let $I$ represent an input statement and let $\bar{a}$ represent a vector holding the variables that appear in $I$. Let $\bar{x}$ represent a vector of variables returned from the input statement after the values are read from the indicated file (or the console). Then,
execution of the GIM input statement is equivalent to the following sequence:

$$
[I, \bar{a}:=\bar{x}]
$$

The formal semantics for the input statement $I$ are defined below.
Definition III.13. $w p(I(\bar{a}), R)=w p([I(\bar{a}), \bar{a}:=\bar{x}], R)$

### 3.22 Imperative Output

This section describes how output statements in imperative languages are modeled in the GIM. In order to communicate the values of variables outside a program, imperative programming languages provide the programmer with an output statement. This statement is used to communicate with the user or store values in data files. The output statement is modeled in the GIM using the imperative-output object. Figure 43 shows

| imperative-output |
| :---: |
| imp-out-logical-file $:$ imperative-data-construct <br> imp-output-list $:$ seq(imperative-data-construct) |

Figure 43 Imperative-Output Class
the class description for this object. The imp-out-logical-file attribute is used to store the logical link to a file opened for output. The imp-output-list attribute holds the list of data items to be output. The data items are modeled using imperative-output-item objects. Figure 44 shows the class description for the imperative-output-item object.

| imperative-output-item |
| :---: |
| imp-out-item : imperative-expression <br> imp-format : imperative-format |

Figure 44 Imperative Output Item

The imp-out-item attribute holds the expression to be output. The imp-format attribute holds formatting information for the item to be output. This attribute models the output formatting implemented in some imperative languages, e.g. total number of spaces in the
output or number of digits to output after the decimal point. If the output item is not being formatted in any way, the imp-format attribute is empty.

In COBOL, there are two output statements: one for low-volume output and the other for output to a data file. The DISPLAY statement is used in COBOL to output data to the user. For example, the following COBOL DISPLAY statement is used to prompt the user for the program name outputting to the console screen.

```
DISPLAY 'ENTER PROGRAM NAME'
    UPON CONSOLE.
```

Figure 45 shows how the COBOL DISPLAY statement is modeled in the GIM. In this


Figure 45 Modeling the COBOL Display
example, the imp-out-logical-file attribute has been set to the symbol STD-OUT to indicate output to the standard output port. As with input, as long as the standard I/O ports are modeled consistently, output to the user can be modeled in this way. The imp-format attribute is empty indicating this output is not formatted.

In COBOL, the WRITE statement is used when the programmer wants to output data values to a file. After the file has been opened for output, a data record is associated with the file and this data record can be output to the file. While this mechanism is more complicated than the mechanism in COBOL for reading from a file, writing to a file can still be modeled in the GIM by determining with which file a record is associated and storing this information in the imperative-output object. For example, the following COBOL program, ADDRESS-LISTING, declares a file ADDRESS-LIST, declares a record OUTPUT-RECORD associated with the file, opens the file for output, and writes a record to the file.

```
PROGRAM-ID.
    PROGRAM-ADDRESSES.
FILE-CONTROL.
    SELECT ADDRESS-LIST ASSIGN TO UR-1403-S-OUTFILE.
FILE SECTION.
FD ADDRESS-LIST
O1 OUTPUT-RECORD.
    O5 LAST-NAME
PROCEDURE DIVISION.
MAIN-ROUTINE.
    OPEN OUTPUT ADDRESS-LIST
    ACCEPT OUTPUT-RECORD FROM CONSOLE.
    WRITE OUTPUT-RECORD.
```

Figure 46 shows how the COBOL WRITE statement is modeled in the GIM. In this example,


Figure 46 Modeling the COBOL Write
the connection between OUTPUT-RECORD and the file ADDRESS-LIST is used to determine the value of the imp-out-logical-file attribute. Even though the logical file is not explicitly referenced in the WRITE statement, the indirect reference can be used to model output to the file.

The semantics of the GIM output statement are defined using the weakest precondition notation. Semantically, the output statement is not allowed to change the state of the program. Let $O$ represent an output statement in the GIM. The formal semantics of the output statement are defined as

Definition III.14. $w p(O, R)=R$

This means the GIM output statement can have no side-effects that would change the state. In this way, the input and output statements provided by imperative programming languages are modeled in the GIM.

The following sections provide formal definitions for imperative subprograms and subprogram calls. These definitions are used throughout the rest of this document to build formal transformations.

### 3.23 Formalizing GIM Statements and Expressions

This section presents formal representation for GIM statements and expressions. These formal representations are used in the formal transformations presented in Chapter VI. Let $x$ represent a GIM variable and let $e$ represent a GIM expression. A GIM assignment statement is represented formally by the following tuple.

$$
<x,:=, e>
$$

This tuple indicates that $x$ is the variable to be assigned the value of the expression $e$. The $:=$ symbol is a syntactic token that distinguishes this tuple as an assignment statement.

Let $e$ represent a GIM expression and let $S_{1}$ and $S_{2}$ represent sequences of GIM statements. A GIM selection statement is represented by the following tuple.

$$
<\text { if, } e, \text { then, } S_{1}, \text { else, } S_{2}>
$$

This tuple indicates that the sequence of statements $S_{1}$ will be executed if the expression $e$ evaluates to true otherwise the $S_{2}$ sequence will be executed. The if, then, and else symbols are syntactic tokens used to identify this tuple.

Similarly, let $e$ represent a GIM expression and let $S_{3}$ represent a sequence of GIM statements. The GIM iteration statement is represented by the following tuple.

$$
<\text { while, } e, S_{3}>
$$

This indicates that the sequence of statements $S_{3}$ will be executed while the expression $e$ is true. The while symbol is a syntactic token used to identify this tuple.

Let $E$ represent a sequence of expressions. The GIM input and output statements are represented formally by the following tuples.

$$
\begin{gathered}
\langle\text { input, iport, } E\rangle \\
\langle\text { output, oport, } E\rangle
\end{gathered}
$$

These tuples indicate the expressions in the sequence $E$ are either input from the input port iport or output to the output port oport.

Expressions in the GIM are represented formally by the following tuples. Let $e_{1}$ and $e_{2}$ be GIM expressions.

$$
\begin{array}{r}
<e_{1},+, e_{2}> \\
<e_{1}, \text { and }, e_{2}> \\
<e_{1}, \text { concat, } e_{2}> \\
<e_{1}, /, e_{2}> \\
<e_{1},=, e_{2}> \\
<e_{1}, * *, e_{2}> \\
<e_{1},>=, e_{2}> \\
<e_{1},>, e_{2}> \\
<e_{1},<=, e_{2}> \\
<e_{1},<, e_{2}> \\
<e_{1}, *, e_{2}> \\
<e_{1},<>, e_{2}> \\
<e_{1}, \text { or }, e_{2}> \\
<e_{1},-, e_{2}> \\
<-, e_{1}> \\
<\text { not }, e_{1}>
\end{array}
$$

### 3.24 Formalizing Imperative Subprograms

This section explains how subprograms modeled by the GIM imperative-procedure and imperative-function ASTs can be represented more mathematically.

The following definitions are used to formally define subprograms.
$i d_{S}$ - A symbol storing the subprogram's name.
$P_{i n}$ - The sequence of input parameters.
$P_{\text {out }}$ - The sequence of output parameters.
$P_{\text {ret }}$ - The sequence of returned values.
$P_{l o c}$ - The sequence of local variables.
$\Sigma$ - The sequence of statements from the subprogram.
Thus, a subprogram is defined by the following tuple.

$$
<i d_{S}, P_{i n}, P_{o u t}, P_{r e t}, P_{l o c}, \Sigma>
$$

Several of the attributes of imperative subprograms are sequences of imperative variables, viz. $P_{\text {in }}, P_{\text {out }}, P_{r e t}$, and $P_{l o c}$. In order to manipulate these sequences formally, the four special sequence operators, $\oplus, \ominus, \sqsubseteq$, and pos are defined below.
$S_{1} \oplus S_{2}=S_{1}$ concat $\left[x \mid x \in S_{2} \wedge x \notin S_{1}\right]$
$S_{1} \ominus S_{2}=\left[x \mid x \in S_{1} \wedge x \notin S_{2}\right]$
$S_{1} \sqsubseteq S_{2}=\forall x \in S_{1} \Rightarrow x \in S_{2}$
$\operatorname{pos}(x, \Sigma)$ Let the pos operation indicate the ordinal position of the element $x$ in the sequence $\Sigma$.

For example, Figure 47 shows the imperative subprogram UPLREQ. This subprogram is formally defined by the following tuple.

## DOUBLE PRECISION function UPLREQ(ZCOS,XLAMDA,SIGTRB)

```
    INCLUDE 'bdincl.f'
C
    DATA SIGT0/3.75D-06/
C
    TMP1 = XLAMDA*ZCOS*ZCOS*ZCOS
    TMP2 = TMP1**0.200
C
C SIGUP IS THE UPLINK BEAM DIVERGENCE DUE TO TURBULENCE FOR
C A BEAM WITH NO PHASE CORRECTION.
C
    SIGUP = SIGTO/TMP2
C
    IF(SIGTRB.GT.0.0) THEN
    UPLREQ = SIGUP/SIGTRB
    ELSE
    UPLREQ = 1000000.0D0
    END IF
    RETURN 20
    END
```

Figure 47 FORTRAN Function UPLREQ
$<U P L R E Q, P_{\text {in }}, P_{\text {out }}, P_{\text {ret }}, P_{\text {loc }}, \Sigma>$ where
$P_{\text {in }}=[\mathrm{ZCOS}, \mathrm{XLAMDA}, \mathrm{SIGTRB}]$ and $P_{\text {out }}=$ [] and $P_{\text {ret }}=$ [UPLREQ] and $P_{l o c}=[$ SIGT0, TMP1, TMP2, SIGUP] and $\Sigma=$ statements between DATA and END

The input parameters of UPLREQ are the data items ZCOS, XLAMDA, and SIGTRB. UPLREQ has no output parameters and the data item returned from the execution of the UPLREQ function is represented by including the identifier UPLREQ in the $P_{\text {ret }}$ sequence. The local variables of UPLREQ are the data items SIGT0, TMP1, TMP2, and SIGUP. The sequence of imperative statements between the BEGIN and the END of UPLREQ comprise $\Sigma$.

### 3.25 Formalizing Imperative Subprogram Calls

This section provides formal definitions of the GIM imp-procedure-call entity, a kind of statement, and the GIM imperative-function-call entity, a kind of expression.

The calling relationship between any two subprograms can be defined formally as follows. Let $S_{1}$ be the calling subprogram and let $S_{2}$ be the called subprogram. Let $i d_{S_{2}}$ be a symbol that holds the name of the called subprogram. Let $P_{\text {form }}^{S_{1}}$ be a sequence of variables that are formal parameters of the calling subprogram, i.e. $\left(P_{\text {in }_{S_{1}}} \oplus P_{\text {out }_{S_{1}}}\right)$. Let $P_{\text {act }_{S_{1}}}$ be a sequence of variables that are actual parameters in the calling subprogram, where $P_{\text {act }_{S_{1}}} \sqsubseteq\left(P_{\text {form }_{S_{1}}} \oplus P_{l o S_{1}}\right)$. Thus, a call to the subprogram $S_{2}$ by subprogram $S_{1}$ is represented by the following tuple.

$$
<i d_{S_{2}}, P_{a c t_{S_{1}}}>
$$

In the imperative paradigm, the number, order, and type of parameters in $P_{\text {act }_{S_{1}}}$ must match the number, order, and type of the parameters in $P_{\text {form }}^{S_{2}}$ of the called subprogram.

This link between an actual parameter in a call to a subprogram and the formal parameter in the called subprogram is important in re-engineering. Even though one subprogram can have multiple calls to another subprogram or can call many other subprograms, a static mapping between actuals and formals of each call can be built. For each call to a subprogram, the order of the actual parameters determines to which formal parameter in the called subprogram the actual parameter is mapped.

Definition III.15. The $\mu$ map links an actual parameter from the calling program to a single formal parameter in the called subprogram.

Let $l$ be a call from subprogram $S_{1}$ to subprogram $S_{2}$. Let $p_{l}$ be an actual parameter in $l$. Let $p_{2}$ be a formal parameter in the subprogram $S_{2}$. Let $\operatorname{pos}\left(p_{l}, P_{a c t_{l}}\right)$ indicate the position the parameter $p_{l}$ takes in the sequence of actual parameters $P_{\text {act }}^{l}$. Let $\operatorname{pos}\left(p_{2}, P_{\text {form }}\right)$ indicate the position of $p_{2}$ in the sequence of formal parameters $P_{\text {form }_{2}}$.

The $\mu$ relation is defined formally as follows.

$$
\begin{aligned}
& \mu\left(p_{l}\right)=p_{2} \text { where } \\
& l=<i d_{S_{2}}, P_{\text {act }}>\text { and } \\
& S_{2}=\left\langle i d_{S_{2}}, P_{\text {in }}, P_{\text {out }}, P_{r e t}, P_{l o c}, \Sigma\right\rangle \text { and } \\
& p_{\text {form }}=P_{\text {in }} \oplus P_{\text {out }} \text { and } \\
& p_{l} \in P_{\text {act }}^{l}{ }^{\text {and }} \\
& p_{2} \in P_{\text {form }} \text { and } \\
& \operatorname{pos}\left(p_{l}, P_{a c t_{l}}\right)=\operatorname{pos}\left(p_{2}, P_{\text {form }}\right)
\end{aligned}
$$

DOUBLE PRECISION function PRDIV(BETA,XLAMDA,DIAM)

```
    INCLUDE 'bdincl.f'
C
C CALCULATES THE PROJECTOR BEAM DIVERGENCE FOR A GIVEN OPTICS
C SIZE THIS IS BASED ON THE RAYLEIGH (AIRY DISK) CRITERION.
C
    DATA FAC/1.220D0/
    QUAL = MAX(BETA,1.0D0)
PDIAM = MAX(DIAM,0.1D0)
WAVELN = XLAMDA*1.D-06
PRDIV = QUAL*WAVELN*FAC/PDIAM
RETURN
END
```

Figure 48 FORTRAN Function PRDIV

For example, Figure 48 shows the declaration of the imperative subprogram PRDIV that has three formal parameters, BETA, XLAMDA, and DIAM.

Figure 49 shows the partial declaration of the subprogram RELAY and the call to the subprogram PRDIV. In this example, let

$$
l=\langle\operatorname{PRDIV},[\mathrm{B}, \mathrm{X}, \mathrm{DIAM}]\rangle
$$

Thus, the actuals in $l$ and the formals from relay are defined as follows.

$$
\begin{aligned}
& P_{\text {act }}^{l}=[\mathrm{B}, \mathrm{X}, \mathrm{DIAM}] \text { and } \\
& P_{\text {formprdiv }}=[\text { BETA, XLAMDA, DIAM }]
\end{aligned}
$$

```
    SUBROUTINE RELAY(B,X,DIAM)
    INCLUDE 'bdincl.f'
C
C DESE RESEARCH AND ENGINEERING, INC. (21 MAY 1985)
C
C
C FIRST THE PROJECTOR DIVERGENCE
C
    SIGPR = PRDIV(B,X,DIAM)
    SIGPSQ = SIGPR*SIGPR
C
RETURN
END
```

Figure 49 FORTRAN Subroutine RELAY

Let $\xrightarrow{\mu}$ represent a $\mu$ mapping from an actual parameter to a formal parameter. Hence, the following mappings are included in the $\mu$ relation.

$$
\begin{aligned}
\text { RELAY::B } & \xrightarrow{\mu} \text { PRDIV::BETA } \\
\text { RELAY::X } & \xrightarrow{\mu} \text { PRDIV::XLAMDA } \\
\text { RELAY::DIAM } & \xrightarrow{\longrightarrow} \text { PRDIV::DIAM }
\end{aligned}
$$

Note that the identifiers used to name the parameters may or may not be the same. In the example, the scope of the parameter is used to differentiate between actuals and formals using the same identifier. For example, the PRDIV: : prefix indicates the scope of a variable is PRDIV. In this way, the DIAM variable from PRDIV is distinguished from the DIAM variable in RELAY.

### 3.26 Formalizing Imperative Designs

This section provides a formal representation for imperative designs. Formal transformations that add subprograms to a design and remove subprograms from a design are also provided.

The formal definition of an imperative design is provided below. Let $S^{*}$ represent the set of subprograms included in the design. An imperative design, $S D$, is represented
formally by the following tuple.

$$
S D=S^{*}
$$

An imperative design consists of the set of imperative subprograms to be included in the design.

Let $T_{s}^{+}$be a transformation of an imperative design that adds a new subprogram to an existing design and returns a new imperative design. Let $S D$ represent an imperative design and let $S$ represent the subprogram to be added to the design. The transformation is defined below.

$$
\begin{gathered}
T_{s}^{+}(S D, S)=S D^{\prime} \text { where } \\
S D^{\prime}=S D \cup\{S\}
\end{gathered}
$$

Let $T_{s}^{-}$be a transformation of an imperative design that removes a subprogram from an existing design and returns a new imperative design. Let $S D$ represent an imperative design and let $S$ represent the subprogram to be removed from the design. The transformation is defined below.

$$
\begin{gathered}
T_{s}^{-}(S D, S)=S D^{\prime} \text { where } \\
S D^{\prime}=S D-\{S\}
\end{gathered}
$$

### 3.27 Summary

This chapter has defined the Generic Imperative Model (GIM) that is used to model assignment, variables, expressions, and control flow constructs in imperative programming languages.

Several restrictions have been placed on the GIM. Some of these restrictions are meant to canonicalize the representations in the GIM. Others place limits on which imperative entities can possibly be modeled using the GIM. Specifically, Restriction III. 1 states that a parameter of a procedure can not be both an input and an output parameter. This
restriction is meant to simplify the way parameters are modeled. This restriction is not unreasonable since any procedures that do not meet the restriction can be converted using the process presented in Appendix D. Similarly, Restriction III. 2 does not allow functions to include output parameters or return more than a single data item. It is hypothetically possible to convert such functions into procedures and then represent the procedures in the GIM. Restriction III. 3 requires all parameters in subprogram calls to be variables. Parameters that are not variables are easily converted to a temporary variable, so this is a reasonable restriction of the GIM. Restrictions III. 4 and III. 5 limit subprogram calls to non-recursive calls. This restriction is reasonable because Knuth [37] argues any recursive algorithm can be represented using an iterative algorithm. Restriction III. 6 does not allow global variables to appear in subprograms. This means subprograms with variables other than formal parameters or local variables must first be converted into the proper form. It is hypothetically possible to convert global variables into formal parameters throughout the call tree of a legacy system, so this restriction is not unreasonable. Restriction III. 7 does not allow one subprogram to be declared inside another subprogram. The scoping issues involved in this nesting of subprograms is more complicated than that of imperative variables, but it is hypothetically possible to convert the nested subprograms into subprograms declared at the global level.

None of the restrictions discussed above preclude the representation of imperative entities in the GIM. However, Restriction III. 8 states that heterogeneous data types are not modeled in the GIM. Restriction III. 9 states that pointers are not modeled in the GIM. These two restrictions limit the entities that can be modeled by the GIM. If the legacy system includes either heterogeneous data types or pointers, the system can not be represented using the GIM. This is not unreasonable for the FORTRAN language, but is a limitation for languages such as C or Pascal. The GIM is easily extended by adding new ASTs to represent the new constructs and by defining formal semantics for them using weakest preconditions.

Overall, the GIM provides a canonical form for representing imperative programs. The GIM is used throughout the following chapters to represent subprograms that are being converted from the imperative paradigm to the object-oriented paradigm.

## IV. The Generic Object Model

### 4.1 Introduction

This chapter defines the Generic Object-Oriented Design Model (GOM) which has been developed to model the objects, classes, methods, and messages typically built as part of an object-oriented programming language. A general discussion of the object-oriented paradigm is presented first, followed by detailed descriptions of how the entities in this paradigm are modeled using abstract syntax trees (ASTs). The classes that define the GOM ASTs are presented using Rumbaugh's OMT notation [62]. The formal semantics of the program constructs are defined using the weakest precondition notation.

In order to scope this research, the GOM was built to model OO languages that use sequences of imperative statements in the methods. Usually these languages started in the imperative paradigm and OO extensions were added in new versions of the languages, e.g. Ada 95 and $\mathrm{C}++$. For this reason, imperative assignment, sequential control, selective control, and iterative control constructs are included in the GOM. These constructs must be adapted for use in the GOM because GOM expressions now include accesses to object attributes (as explained in Section 4.6) and possibly messages to other objects (as explained in Section 4.8).

Since the GOM provides a canonical form for representing object-oriented languages, OO languages such as Ada 95, C++, and Java can be modeled using the GOM. Since the focus of this research was to produce an OO design, no attempt was made to recover GOM ASTs from OO code. Instead, rudimentary transformations from the GOM to Ada 95 were developed as a proof-of-concept prototype. This prototype research is discussed with the feasibility demonstration in Chapter VIII. For these reasons, throughout the rest of this chapter, examples are given that have more of a forward engineering flavor (from the GOM to a specific language) than a reverse engineering flavor (from an OO language to the GOM).

In order to visualize the ASTs built in the GOM, a surface syntax has been developed for each entity in the GOM. The surface syntax is termed the Generic Object-Oriented

Language (GOL) and is shown in Appendix B. The GOL was built only as a tool for viewing the GOM ASTs and is not intended as a new object-oriented programming language.

### 4.2 The Object-Oriented Paradigm

As discussed in Chapter III, the object-oriented paradigm is one of the four programming language paradigms [16]. Korson and McGregor [38] characterize the object-oriented paradigm using the following concepts:

Classes A class is a template that defines the attributes and operations for each instance of the class.

Objects An object is an instance of a class. Objects model real-world entities that have state, behavior, and identity.
Methods A method is a sequence of object-oriented statements that implement a specific behavior.
Messages A message invokes a specific method in an object. Messages are sent to a target object that must be able to execute the method being invoked.
Inheritance The classes in an object-oriented design are organized in a class hierarchy where certain classes inherit the attributes and operations from other classes in the hierarchy.

Polymorphism In an object-oriented design, it is possible to have methods (from different classes) with the same name. Polymorphism means the appropriate method will be executed based on the class of an object instance.

The GOM provides a canonical form for modeling each of these aspects of objectoriented languages. The object-oriented statements in each method in the GOM are assumed to be based on imperative control flow constructs. It is also assumed that one special method is given the flow of control as the object-oriented system begins execution. This special method is termed the main method. The following sections define the ASTs that store the knowledge about these entities. Where applicable, the formal semantics of these entities is also defined.

Because of the intertwining of the aspects of the OO paradigm, it is hard to present them in an order that does not include any forward references to other aspects. Because of this, it is assumed that the reader has a rudimentary understanding of the Object-Oriented
paradigm as aspects of the GOM are discussed. There are several good references that explain the OO paradigm in great detail, and the reader is referred to them $[9,10,38,62]$.

### 4.3 The GOM Domain Model

This section presents a brief overview of the ASTs that are included in the GOM domain model. Figure 50 shows a partial representation of the GOM domain model. The


Figure 50 GOM Domain Model
overall superclass of the domain is the GOM-entity AST. The GOM-design class models collections of classes as defined in Section 4.5. The abstract class GOM-statement is the superclass for all object-oriented programming statements modeled in the GOM. The GOM-data-construct class is the superclass of object-oriented expressions, data types, attribute accesses, functional messages, and variables modeled in the GOM. Each of the lower-level classes in the GOM domain model are described in the rest of this chapter.

### 4.4 Classes

In the OO paradigm, classes define templates for objects. They define the attributes each instance of the class will have and store the behavior each instance is capable of executing. Classes are modeled in the GOM using the gom-class class. Figure 51 shows

| gom-class |
| :--- |
| gom-name : symbol |
| gom-attrs : set(gom-attribute) |
| gom-opers : set(gom-method) |
| gom-super : symbol |

Figure 51 GOM Class Class
the class description for the gom-class class. The gom-name attribute holds a symbol that represents the name of the class. The gom-attrs attribute of this class holds the attributes of classes being modeled in the GOM. The gom-opers attribute of this class holds the methods for the class being modeled. The gom-super attribute holds the symbol representing the superclass from which this class inherits. The symbol USER-OBJECT is provided to represent the overall superclass at the root of the class hierarchy. As an example,

```
class CLASS-5 attributes RHOSTD, LAMBDA, ANGFAC
    method RHO ( C-5 ): real
        begin
            RHO := GET-RHOSTD ( C-5)
        end
    superclass USER-OBJECT
```

Figure 52 Class CLASS-5
Figure 52 shows the class CLASS-5 (using GOL syntax). This class has three attributes named RHOSTD, LAMBDA, and ANGFAC and one method named RHO. The class inherits from the super class USER-OBJECT. Figure 53 shows how the class CLASS- 5 is modeled in the


Figure 53 GOM Class Object
GOM. The attributes RHOSTD, LAMBDA, and ANGFAC are stored in the gom-attrs attribute. The method RHO is stored in the gom-opers attribute. The USER-OBJECT symbol is stored in the gom-super attribute to represent the fact that this class inherits from the class named USER-OBJECT.

Each class in the OO paradigm must be able to instantiate instances of the class. The gom-instantiate object is used to model this ability of a class. Since almost every OO language has a unique way to instantiate an instance of a class, the gom-instantiate object provides a canonical form for modeling object instantiation. Figure 54 shows the

| gom-instantiate |
| :--- |
| gom-inst-class : symbol |

Figure 54 GOM Instantiate Class
class description for the gom-instantiate object. The gom-inst-class attribute holds a symbol representing the name of the class being requested to instantiate a new instance. The GOL surface syntax for this class is the keyword new followed by the value of the gom-inst-class attribute. The gom-instantiate class models the intrinsic behavior of a class to create new instances. As with OO objects being modeled, there is no need to define formal semantics for OO classes being modeled. Classes do not directly affect the state of the OO system.

### 4.5 Modeling Object-Oriented Designs

In order to model the collection of classes in an OO design as a whole, classes are stored in an AST built from the GOM class gom-design. An instance of the gom-design class stores all the classes in the class hierarchy defined for a specific object-oriented design. Figure 55 shows the class description of the gom-design class. The gom-classes attribute

| gom-design |
| :--- |
| gom-classes : seq(gom-class) <br> gom-files : seq(gom-file) |

## Figure 55 GOM Design Class

holds the collection of classes that comprise the design. The gom-files attribute holds the collection of input/output files being modeled for this design. See Section 4.26 and Section 4.27 for discussions of how input and output statements and files are modeled in the GOM.

### 4.6 Objects

The basic building block for the object-oriented paradigm is the object. An object has state, behavior, and identity [9]. Objects are modeled in the GOM by using variables whose data type refers to a class. The variable is the handle into the object and is used to access the attributes of the object and to receive messages from other objects. Variables are discussed in Section 4.18, but an example of a variable used to model an object is presented here for clarity. Figure 56 shows an example of a variable that models an object


Figure 56 An Object Instance Variable
instance built from class CLASS-1. This variable is modeled by building an AST from the class gom-variable. In the figure, the gom-name attribute holds the symbol C-15 which models the identifier for the variable. The variable is referenced by this identifier in GOM statements to provide access to the object and its attributes. The gom-var-scope in this example holds the symbol PRDIV to represent the fact that this variable is declared in the scope of the PRDIV method. Scoping issues for GOM variables are discussed in Section 4.21. The gom-var-type attribute is set to the data type gom-instance which indicates the variable is an instance of CLASS-1. For simplicity, the fact that C-15 is an instance of CLASS-1 is represented in the figure by using a symbol.

Each object from an OO program is an instance of a class, which defines the attributes of the object. Each object has a copy of these attributes defining a separate state space for each object instance. Each OO programming language provides a way to access these attributes of the object. The access of an object attribute is modeled in the GOM by the gom-attr-access class. Figure 57 shows the class definition for the gom-attr-access class. The gom-tar-object attribute holds the variable representing the target object whose attribute is being accessed. The gom-attrib attribute holds the variable representing the name of the attribute to be accessed. For example, Figure 58 shows an access

| gom-attr-access |
| :--- |
| gom-tar-object : gom-variable <br> gom-attrib : gom-variable |

Figure 57 Attribute Access Class


Figure 58 Attribute Access Object.
of the attribute $A$ of the object C-1. The GOL surface syntax for this attribute access is $C-1 . A$, thus $C-1$ is the GOM variable holding the target object and $A$ is the name of the attribute being accessed.

There is no need to define formal semantics for OO objects being modeled in the GOM. Objects hold pieces of the state of an OO system, but they do not change the state directly. The programming language statements in the methods of objects do change the state and formal semantics for these statements are defined in the sections that describe how these statements are modeled.

### 4.7 Methods

A method in the OO paradigm is a named sequence of object-oriented programming language statements that is stored in a class. The statements that are modeled in the GOM include assignment, sequential control flow, selective control flow, iterative control flow, and subprogram invocation of non-user-defined subprograms. A method has formal parameters and may or may not return a value. Methods are modeled in the GOM using ASTs built from the gom-method class. Figure 59 shows the class description for the gom-method class. The gom-name attribute stores a symbol representing the name of the method. The

| gom-method |
| :--- |
| gom-name : symbol |
| gom-params : seq(gom-parameter) |
| gom-stmts : seq(gom-program-construct) |
| gom-return-type : gom-data-type |

## Figure 59 GOM Method Class

gom-params attribute stores the formal parameters of the method. Each formal parameter is a GOM variable. The number, order, and type of these parameters is important for consistency and is always maintained. The gom-stmts attribute models the sequence of programming language constructs that comprise the method. The gom-return-type attribute models the return type (if any) of the method. If the method does not return a value, then this attribute is undefined.

```
class CLASS-5 attributes RHOSTD, LAMBDA, ANGFAC
    method RHO ( C-5 ): real
        begin
            RHO := GET-RHOSTD ( C-5)
        end
    superclass USER-OBJECT
```

Figure 60 Class CLASS-5

For example, Figure 60 shows the class CLASS-5 (using GOL syntax). CLASS-5 has a single method named RHO. This method has one formal parameter, C-5, which is the target object of the method and an instance of CLASS-5. The RHO method returns a value of type gom-real and has one statement in the body of the method. Figure 61 shows how the RHO method is represented using a gom-method AST. The name of the method, RHO, is stored in the gom-name attribute, the formal parameter C-5 is stored as the only element of the gom-params attribute ${ }^{1}$. The single statement is stored in the gom-stmts attribute.

Methods in the OO paradigm are analogous to imperative subprograms. Methods that return a value after execution are similar to imperative functions. This kind of method

[^2]

Figure 61 GOM Method Object
is referred to in the rest of this document as a functional method. Methods that do not return a value after execution, but, instead, communicate through input and output parameters are similar to imperative procedures. This kind of method is referred to in the rest of this document as a procedural method. Both methods and subprograms define formal parameters where the number, order, and type of the parameters must be matched by the actual parameters in a message or a subprogram call, respectively.

Because methods in the GOM are built from subprograms in the GIM, certain properties of GIM subprograms are carried forward in the transformation and now apply to GOM methods. Specifically, for both functional and procedural GOM methods, the following restriction applies.

Restriction IV.1. A formal parameter of a method must not be both an input and an output parameter.

For functional methods, the following restriction applies.
Restriction IV.2. All functional methods must return a single value at the end of their execution and have no output parameters.

As with GIM subprograms, the declaration of a GOM method does not change the state of a program. A message that invokes the method is allowed to change the state, thus the formal semantics for method declarations provided in this section serve as a foundation for the definition of the formal semantics of messages.

Given a precondition $P$, a postcondition $R$, and a sequence of OO statements $S 1$, it is assumed that $P$ and $R$ exist such that the following holds.
$\{P\}$
$S 1$
$\{R\}$

For a method $m$, let $\bar{x}$ be a vector representing the input parameters of $m$. Let $\bar{z}$ be a vector representing the output parameters of $m$. A procedural GOM method takes the following general form.

$$
\begin{align*}
& \text { method } m(\bar{x}, \bar{z}) \\
& \quad\{P\} \\
& S 1 \\
& \{R\}
\end{align*}
$$

A functional GOM method takes the following general form.

$$
\begin{aligned}
& \text { method } m(\bar{x}) \\
& \{P\}
\end{aligned}
$$

S1
$\{R\}$

In order to define the semantics for the value that is returned from the method $m$, a modified representation is used. Let $y$ represent the value that is returned from $m$. Let $m^{\prime}$ be a method that takes the same input parameters as $m$, and includes $y$ as the single output parameter. The statements in $m^{\prime}$ are the statements from $m$, viz. $S 1$. The method $m^{\prime}$ takes the following form.

$$
\begin{aligned}
& \operatorname{method} m^{\prime}(\bar{x}, y) \\
& \quad\{P\} \\
& \quad S 1 \\
& \quad\{R\}
\end{aligned}
$$

These general forms define the formal semantics for method declarations in the GOM. To tie this general form to the specific representation in the GOM, recall that the sequence of OO statements, $S 1$, in the method is modeled by the gom-stmts attribute in the GOM.

### 4.8 Messages

In the OO paradigm, objects communicate by sending messages. Messages are requests from one object (the sending object) to another object (the target object). If the target object has a method that corresponds to the message received, then the target object executes this method. Messages have actual parameters that must match in number, order, and type to the formal parameters of the requested method.

Rumbaugh [62] describes the abstract notion of flow of control in the OO paradigm as a collection of concurrently-active objects sending messages to each other. Rumbaugh uses state diagrams in his dynamic model to represent this interaction of objects. Since the GOM was built to model OO languages that use sequences of imperative statements in their methods, flow of control in the GOM is modeled using a procedure-driven approach versus using concurrent tasks. Specifically, flow of control is passed from the sending object to the target object when the message is sent, and it is returned to the sending object when the method completes execution.

Messages are modeled in the GOM by using instances of the gom-message class. Figure 62 shows the class description for the gom-message class. The gom-call attribute

| gom-message |
| :--- |
| gom-call : symbol <br> gom-actuals : seq(gom-variable) |

Figure 62 GOM Message Class
stores a symbol representing the name of the method to be executed in the target object. The gom-actuals attribute stores the sequence of GOM variables that represent the actual parameters of the method. The first parameter in the sequence of actual parameters is always the target object. For example, consider the following assignment statement (shown in GOL syntax).
SIGTRB := RHO ( C-7);

This assignment statement includes a message that invokes the RHO method shown in Figure 60. Figure 63 shows how the message is represented using an AST built from the


Figure 63 GOM Message Object
gom-message class. The gom-call attribute is set to the symbol RHO to indicate the name of the method to be invoked. The gom-actuals attribute holds the sequence with the single actual parameter C-7. This parameter is represented as a GOM variable which is named C-7, is in the scope of PRDIV, and is an instance of CLASS- $5^{2}$. Since C-7 is the first parameter of the message, it is the target object.

Messages in the OO paradigm are analogous to subprogram calls in the imperative paradigm. A message that invokes a functional method is similar to a function call. This kind of message is referred to in the rest of this document as a functional message. Functional messages are a kind of OO expression, i.e. they do not comprise a statement on their own. A message that invokes a procedural method is similar to a procedure call. This kind of message is referred to in the rest of this document as a procedural message. Procedural messages are a kind of OO statement.

The semantics of messages in the GOM are defined using the weakest precondition notation as well as the general form of method declarations defined in Section 4.7. Let $m$ be a procedural method, and let $g$ be a procedural message that invokes $m$. Let $\bar{a}$ be a vector representing the actual parameters that correspond to input parameters in $m$. Let $\bar{c}$ be a vector representing the actual parameters that correspond to output parameters in $m$. The message $g$ takes the form

$$
m(\bar{a}, \bar{c})
$$

[^3]As defined in Section 4.7, the gom-stmts attribute of a GOM method declaration models a sequence of GOM statements. Let $S 1_{m}$ be this sequence of statements from the method $m$. Recall from Section 4.7 that $\bar{x}$ is a vector representing the input parameters of $m$, and $\bar{z}$ is a vector representing the output parameters of $m$. When a message is sent to an object, the formal parameters of the method being requested are set to the values of the actual parameters from the message. The method corresponding to the message sent is then executed. Formally, this is equivalent to executing the following sequence.

$$
\left[\bar{x}:=\bar{a}, S 1_{m}, \bar{c}:=\bar{z}\right]
$$

Using weakest precondition notation, the semantics for the message $g$ is defined as follows.

Definition IV.1. $w p(m(\bar{a}, \bar{c}), R)=w p\left(\left[\bar{x}:=\bar{a}, S 1_{m}, \bar{c}:=\bar{z}\right], R\right)$
The formal semantics for functional messages are defined more restrictively as follows. Let $m$ be a functional method, let $S 1$ be the sequence of statements declared in $m$, and let $g$ be a functional message that invokes $m$. Let $\bar{a}$ be a vector representing the actual parameters that correspond to input parameters in $m$. Since every such method appears as part of an expression in some statement, let $S$ represent the statement in which $m$ appears. Recall the modified representation of a method presented in Section 4.7 includes one output parameter, $y$, which represents the value returned from $m$. Let $m^{\prime}$ be the method that represents $m$ with the formal parameters from $m$ and the additional parameter $y$. Let $b$ represent the actual parameter that corresponds to $y$.

Using these definitions, the invocation of $m$ takes the following form.

$$
m(\bar{a})
$$

The invocation of the method $m^{\prime}$ takes the following form.

$$
m^{\prime}(\bar{a}, y)
$$

The difference between the invocations of $m$ and $m^{\prime}$ is that the invocation of $m$ is part of $S$ and the invocation of $m^{\prime}$ is a single statement. Invoking $m$ is equivalent to invoking $m^{\prime}$ and
then substituting $b$ in $S$ for any invocations of $m$. This provides a formal representation of the value returned from $m$. The substitution of $b$ for the call to $m$ in $S$ is represented by the following notation.

$$
S_{b}^{m(\bar{a})}
$$

In this way, the call to $m$ is equivalent to the following sequence of statements.

$$
\left[m^{\prime}(\bar{a}, b), S_{b}^{m(\bar{a})}\right]
$$

When the method $m^{\prime}$ is invoked, the formal parameters of $m^{\prime}$ are set to $\bar{a}$ and the statements in $S 1$ are executed. After execution, the parameter $b$ is set to the value $y$. Formally, the invocation of $m^{\prime}$ is equivalent to executing the following sequence.

$$
[\bar{x}:=\bar{a}, S 1, b:=y]
$$

Using weakest precondition notation, the formal semantics for the message $g$ are defined as follows.

Definition IV.2. $w p(m(\bar{a}), R)=w p\left(\left[\bar{x}:=\bar{a}, S 1, b:=y, S_{b}^{m(\bar{a})}\right], R\right)$

## 4.9 "GET-" and "SET-" Messages

In order to provide a better encapsulation of extracted objects, the following restriction applies.

Restriction IV.3. Object attributes are accessed and assigned values only through "GET-" and "SET-" messages sent to the object.

Figure 64 shows the GET-ZCOS and SET-ZCOS methods used for accessing and assigning a value to the ZCOS attribute.

The general form of a "SET-" message is

$$
\text { SET- } a(\tau, e)
$$

```
method GET-ZCOS ( C-10 ): real
    begin
        GET-ZCOS := C-10.ZCOS
    end
method SET-ZCOS ( C-11, VAL-10 )
    begin
        C-11.ZCOS := VAL-10
    end
```

Figure 64 "SET-" and "GET-" Methods for ZCOS
where $a$ is the name of the attribute being assigned a value, $\tau$ is the target object, and $e$ is the expression to which the attribute $a$ is set. The formal semantics for a "SET-" message are defined using the weakest precondition notation. Given a postcondition $R$, let $R_{e}^{\tau . a}$ represent $R$ with all free occurrences of the attribute $a$ from $\tau$ simultaneously replaced with $e$. The semantics for the general form of the "SET-" message are defined below.

Definition IV.3. $w p(S E T-a(\tau, e), R)=R_{e}^{\tau . a}$
The general form of a "GET-" message is shown below.

$$
\operatorname{GET}-a(\tau)
$$

Here, $a$ is the name of an attribute being accessed and $\tau$ is the target object. The "GET-" message for an attribute returns the value of the attribute $a$ from the target object $\tau$. Since the "GET-" message is returning a value, it is similar to a function invocation from the imperative paradigm. Because of this similarity, the formal semantics for a "GET-" message are defined using a modified version of the "GET-" method. The modified version includes a single output parameter, $y$, which represents the value returned from the "GET" method. Let $b$ represent the actual parameter that corresponds to the formal parameter $y$. The invocation of the modified "GET-" method takes the following form.

$$
\operatorname{GET}-a(\tau, b)
$$

The execution of this modified "GET-" message is equivalent to executing the following sequence.

$$
[y:=\tau . a, b:=y]
$$

As with function invocation, the value of $b$ replaces the original "GET-" message in the statement that includes it. Let $S$ be the statement that includes the "GET-" message. Let $S_{b}^{\operatorname{GET}-a(\tau)}$ represent the textual substitution of $b$ for each occurrence of the "GET-a" message in the statement $S$. The formal semantics of a "GET-" message are defined as follows.

Definition IV.4. $w p(\operatorname{GET}-a(\tau), R)=w p\left(\left[y:=\tau \cdot a, b:=y, S_{b}^{G E T-a(\tau)}\right], R\right)$
The "GET-" and "SET-" methods for each attribute of each class in the objectoriented design must be provided before the design can be implemented. This research provides a transformation, $\kappa$, in Section 6.11.2 which formalizes the automatic generation of these methods and their addition to the set of operations for each class in the design.

### 4.10 Inheritance

In the GOM, the collection of classes in an object-oriented design is modeled as a class hierarchy where certain classes inherit the attributes and operations from one or more other classes. This concept of inheritance is represented in the GOM by the gom-super attribute of ASTs built from the gom-class class. The gom-super attribute indicates from which class a specific class inherits.

```
class CLASS-5 attributes RHOSTD, LAMBDA, ANGFAC
    method RHO ( C-5 ): real
        begin
            RHO := GET-RHOSTD ( C-5)
        end
    superclass USER-OBJECT
```


## Figure 65 CLASS-5 inherits from USER-OBJECT

For example, Figure 65 shows the class CLASS-5 (using GOL syntax). CLASS-5 inherits from the class named USER-OBJECT. To represent inheritance, the gom-super attribute holds the symbol that represents the name of the class from which this class inherits.

### 4.11 Polymorphism

There is support for modeling polymorphism in the GOM, i.e. declaring methods with the same name in different classes is allowed. This support is rather rudimentary because the concept of an abstract class is not supported by the GOM. By including abstract classes in the GOM, it would be possible to model class-wide operations as implemented by the Ada 95 language.

### 4.12 Object-Oriented Assignment

Several imperative programming language constructs appear in the GOM including the assignment construct. As discussed in Section 3.3, assignment takes the general form of $x:=e$, where $x$ is a variable and $e$ is an expression. The object-oriented version of the assignment construct now allows $x$ to be an attribute access, which models the way attributes of objects are assigned values. The expression $e$ is also allowed to be an attribute access or a message to another object. OO assignment statements are modeled in the GOM using instances of the gom-assignment class. Figure 66 shows the class description for the

| gom-assignment |
| :--- |
| gom-assign-lhs : gom-variable <br> gom-assign-rhs : gom-data-construct |

Figure 66 GOM Assignment Class
gom-assignment class. The gom-assign-lhs attribute models the variable or attribute being assigned a value, i.e. $x$. The gom-assign-rhs attribute models the expression $e$.

For example, consider the following assignment statement (shown using GOL surface syntax).

```
SIGTRB := RHO (C-7)
```

This assignment statement assigns the value returned from the RHO message. Figure 67


Figure 67 GOM Assignment Object
shows the AST built to model this assignment statement. The gom-assign-lhs attribute models the GOM variable SIGTRB, which is in the scope of PRDIV, and is of type gom-real. The gom-assign-rhs attribute models the RHO message which returns a value of type gom-real (see Figure 63).

The formal semantics for an OO assignment statement are defined using the weakest precondition notation. Given a postcondition $R$, let $R_{e}^{x}$ represent $R$ with all free occurrences of $x$ simultaneously replaced with $e$ [19]. The semantics for the general form of the GOM assignment, $x=e$ are defined below.

Definition IV.5. $w p(x:=e, R)=R_{e}^{x}$
To relate these semantics to the GOM, note that the gom-assign-1hs attribute models $x$ and the gom-assign-rhs models $e$.

### 4.13 Object-Oriented Sequential Control

As in the imperative paradigm, the default control mechanism for executing statements in the OO paradigm is sequential control. When methods are being executed, the statements are executed one after another in a sequential manner. As in the GIM, sequential control flow is modeled in the GOM by storing the statements to be executed in a sequence. The order of the sequence indicates the order in which the statements are to be executed.

For example, in the collection of statements shown below, <00 Statement 1> is executed first followed by <00 Statement 2> followed by <00 Statement 3>.

```
<00 Statement 1> <00 Statement 2> <00 Statement 3>
```

This collection of statements is modeled in the GOM using the following sequence.
[ <00 Statement 1>, <00 Statement 2>, <00 Statement 3>]
The formal semantics for sequential control flow are defined using the weakest precondition notation. Let $S 1$ and $S 2$ be statements, then the semantics for the sequential composition of these two statements in the GOM is defined below.

Definition IV.6. $w p([S 1, S 2], R)=w p(S 1, w p(S 2, R))$
The formal semantics of sequential control flow in the GOM are identical to the formal semantics of sequential control flow in the GIM. As in the GIM, the representation of sequential control flow in the GOM is based on function composition [25], which is associative, so the GOM representation of sequential control flow is associative.

Theorem IV.1. $w p([S 1,[S 2, S 3]], R)=w p([[S 1, S 2], S 3], R)$

Proof.

$$
\begin{aligned}
w p([S 1,[S 2, S 3]], R) & =w p(S 1, w p([S 2, S 3], R)) \\
& =w p(S 1, w p(S 2, w p(S 3, R))) \\
& =w p([S 1, S 2], w p(S 3, R)) \text { (by function composition) } \\
& =w p([[S 1, S 2], S 3], R)
\end{aligned}
$$

Since the weakest precondition for the sequences $[S 1,[S 2, S 3]]$ and $[[S 1, S 2], S 3]$ are equal, the sequence $[S 1, S 2, S 3]$ is often used in this document to represent these sequences. In addition, singleton sequences and empty sequences are also used when convenient.

### 4.14 Object-Oriented Selective Control

As defined in the imperative paradigm (see Section 3.5), selective control flow consists of a selection between two or more statements. Since the statements to be executed represent one or more sequentially composed statements, they are modeled as sequences of statements in the GOM. Selective control flow constructs are modeled in the GOM

| gom-selection |
| :--- |
| gom-exp : gom-data-construct <br> gom-then-part : seq(gom-program-construct) <br> gom-else-part : seq(gom-program-construct) |

Figure 68 GOM Selection Class
using ASTs built from the gom-selection class. Figure 68 shows the class description for the gom-selection class. The gom-exp attribute models the OO expression that controls which statements will be executed by the selective programming construct. The gom-then-part models the sequence of OO statements that are executed when the OO expression in gom-exp is true. The gom-else-part models the sequence of statements that will be executed if the gom-exp is false. As in the GIM, $S 1$ must contain at least one statement, but $S 2$ may be empty. ASTs where $S 1$ and $S 2$ have at least one statement model object-oriented if-then-else statements. ASTs where $S 2$ is empty model objectoriented if-then statements.

For example, the if-then-else statement shown below (using GOL syntax) provides a good example of selective control flow in the GOM.

```
if GET-SIGTRB ( C-12) > 0.0 then
    UPLREQ := SIGUP / GET-SIGTRB ( C-12)
else
    UPLREQ := 1000000.0d0
endif
```

This statement makes a selection based on the result of the GET-SIGTRB message, which is sent to the target object C-12. The statement selects between two assignment statements that assign values to the variable UPLREQ. Figure 69 shows the object instance that mod-


Figure 69 GOM Selective Control Flow Example
els this GOM if-then-else statement. The gom-exp attribute models the greater-than
boolean expression that controls the selection and the two assignment statements are modeled by the gom-then-part and the gom-else-part attributes.

The formal semantics of selective control flow in the GOM are defined using the weakest precondition notation. Let $B$ represent an OO boolean expression and let $S 1$ and $S 2$ represent sequences of OO statements. The selective control flow constructs in the GOM take the following form

## if $B$ then $S 1$ else $S 2$

Recall that $S 1$ is executed if $B$ is true and $S 2$ is executed if $B$ is false. The semantics of this GOM form of selective control flow are defined below.

Definition IV.7. $w p($ if $B$ then $S 1$ else $S 2, R)=$

$$
(B \Rightarrow w p(S 1, R)) \wedge(\neg B \Rightarrow w p(S 2, R))
$$

To relate these formal semantics to the GOM, note that $B$ is modeled using the gom-exp attribute. The sequence of statements $S 1$ is modeled by the gom-then-part attribute and the sequence $S 2$ is modeled by the gom-else-part attribute. The formal semantics for GOM selection is now defined.

As in the imperative paradigm, the formal semantics for selective control flow when the sequence $S 2$ is empty are defined using the skip command. Recall from Section 3.5 that the formal semantics of the skip command are $w p(\operatorname{skip}, R)=R$.

Selective control flow in the GOM when $S 2$ is empty takes the form

## if $B$ then $S 1$ else skip

The formal semantics for this form of selective control flow in the GOM is defined below. Definition IV.8. $w p($ if $B$ then $S 1$ else skip, $R)=(B \Rightarrow w p(S 1, R)) \wedge(\neg B \Rightarrow R)$

### 4.15 Object-Oriented Iterative Control

Iterative control flow, as defined in the imperative paradigm (see Section 3.6) is a mechanism for repeating sequences of statements. The execution of a sequence of state-
ments continues while some boolean expression is true. Iterative control flow in the GOM is modeled using ASTs built from the gom-iteration class. Figure 70 shows the class de-

| gom-iteration |
| :--- |
| gom-iter-exp : gom-expression <br> gom-iter-body : seq(gom-program-construct) |

Figure 70 GOM Iteration Class
scription for the gom-iteration class. The gom-iter-expr attribute models the boolean expression that controls the iteration. The gom-iter-body holds the sequence of statements that are executed each time through the iteration.

For example, the while loop shown below (using GOL syntax).

```
IL := 1;
while IL <= GET-NLASER ( C-14) do
    begin
    LLAS ( IL) := 0;
    TLAS ( IL) := 0;
    IL := IL + 1
    end
```

This while loop is an example of iterative control flow in the object paradigm. Figure 71


Figure 71 GOM Iteration Object
shows the AST that models this while loop in the GOM. The gom-ter-exp attribute holds the less-than-or-equal expression that controls the iteration. The gom-iter-body holds the three assignment statements in the body of the loop.

The GOM model of iteration provides a canonical form for representing imperativestyle iteration in the object paradigm. Several OO programming languages provide pro-
gramming constructs that implement iteration including the while statement in $\mathrm{C}++$ and the loop statement in Ada 95. The feasibility demonstration (see Chapter VIII) provides rudimentary examples of converting certain GOM entities into Ada 95 code.

As with the other control flow constructs, the formal semantics for iterative control flow are defined using the weakest precondition notation. Given a precondition $P$ and a postcondition $R$, let $H_{k}(R)$ represent the set of all states in which

## while $B$ do $S 1$

will establish $R$ in at most $k$ iterations [19]. The formal semantics for iterative control flow in the GOM are defined below.

Definition IV.9. $w p($ while $B$ do $S 1, R)=\left(\exists k: 0 \leq k \wedge H_{k}(R)\right)$
To tie this general definition to the specific model of iterative control flow in the GOM, recall that $B$ is modeled by the gom-iter-exp attribute and $S 1$ is modeled by the gom-iter-body attribute of the gom-iteration class.

As in the GIM, the task still remains to define $H_{k}(R)$. There is no attempt in this research to define loop invariants [19] for iteration in the GOM. The set of states $H_{k}(R)$ defines the semantics of iteration in the GOM to a level of detail sufficient for this research.

### 4.16 Object-Oriented Subprogram Invocation

Even though the primary means of communication in the OO paradigm consists of objects sending messages to each other, there is still a need to model subprogram invocations in the GOM. Specifically, there will still be calls to utility subprograms such as SIN and COS that must be modeled in the GOM. Since this research involves re-engineering GIM subprograms to the GOM, all user-defined subprograms must be converted to methods and all calls to user-defined subprograms must be converted to messages. This leads to the following restriction of the GOM.

Restriction IV.4. Only calls to non-user-defined subprograms are modeled by subprogram invocations in the GOM.

As in the imperative paradigm, there are two types of subprogram invocation, viz. procedure calls and function calls. If the non-user-defined subprogram being called is a procedure, then the gom-procedure-call class is used to model the procedure call. Figure 72 shows the class definition for the gom-procedure-call class. The attribute gom-

| gom-procedure-call |
| :--- |
| gom-proc-call-identifier : symbol <br> gom-proc-call-actuals : seq(gom-data-construct) |

Figure 72 GOM Procedure Call Class
proc-call-identifier holds a symbol that represents the name of the procedure being called. The gom-proc-call-actuals attribute holds the sequence of actual parameters sent to the procedure.

If the non-user-defined subprogram being called is a function, then the gom-functioncall class is used to model the function call. Figure 73 shows the class description for the

| gom-function-call |
| :--- |
| gom-fun-call-identifier: symbol |
| gom-fun-call-actuals : seq(gom-data-construct) |

Figure 73 GOM Function Call Class
gom-function-call class. The attribute gom-fun-call-identifier models the name of the function being called. The gom-fun-call-actuals attribute models the actual parameters that are passed to the called function.

The formal semantics for calls to non-user-defined procedures in the GOM are similar to the semantics for procedure calls in the GIM (see Section 3.11). As in the GIM, procedures modeled in the GOM are not allowed to have parameters that are both input and output parameters, thus calls to non-user-defined procedures must have no such parameters.

Let $p$ be a non-user-defined procedure and let $S 1$ be the sequence of statements declared in $p$. Let $\bar{a}$ be a vector representing the actual parameters that correspond to input parameters in $p$. Let $\bar{c}$ be a vector representing the actual parameters that correspond
to output parameters in $p$. An invocation of $p$ takes the form:

$$
p(\bar{a}, \bar{c})
$$

To invoke $p$, the actual parameters are copied to the formal parameters and control flow transfers to $p$. Assume the vectors $\bar{x}$ and $\bar{z}$ are the vectors representing the input and output parameters of the non-user-defined procedure, respectively. Executing the procedure call $p$ is equivalent to executing the following sequence

$$
[\bar{x}:=\bar{a}, S 1, \bar{c}:=\bar{z}]
$$

Using weakest precondition notation, the semantics for a call to a non-user-defined procedure are defined below.

Definition IV.10. $w p(p(\bar{a}, \bar{c}), R)=w p([\bar{x}:=\bar{a}, S 1, \bar{c}:=\bar{z}], R)$
The formal semantics for calls to non-user-defined functions in the GOM are similar to the semantics for function calls in the GIM (see Section 3.11). As in the GIM, a restriction is placed on functions as defined below.

Restriction IV.5. Non-user-defined functions must have no output parameters.
Let $f$ be a non-user-defined function and let $S 1$ be the sequence of statements declared in $f$. Let $\bar{a}$ be a vector representing the actual parameters that correspond to the input parameters in $f$. Since every function appears as part of an expression in some statement, let $S$ represent the statement in which $f$ appears. Recall the modified representation of a function presented in Section 3.10 includes one output parameter, $y$, which represents the value returned from the function $f$. Let $f^{\prime}$ be the procedure that represents $f$ with the formal parameters from $f$ and the additional parameter $y$. Let $b$ represent the actual parameter that corresponds to $y$. Using these definitions, an invocation of the function $f$ takes the following form.

An invocation of the procedure, $f^{\prime}$, takes the following form.

$$
f^{\prime}(\bar{a}, b)
$$

The difference between the invocations of $f$ and $f^{\prime}$ is that the invocation of $f$ is part of $S$ and the invocation of $f^{\prime}$ is a single statement. Invoking $f$ is equivalent to invoking $f^{\prime}$ and then substituting $b$ in $S$ for any invocations of $f$. This provides a formal representation of the value returned from $f$. The substitution of $b$ for the call to $f$ in $S$ is represented by the following notation.

$$
S_{b}^{f(\bar{a})}
$$

In this way, the call to $f$ is equivalent to the following sequence of statements.

$$
\left[f^{\prime}(\bar{a}, b), S_{b}^{f(\bar{a})}\right]
$$

As defined in Section 3.11, when the procedure $f^{\prime}$ is invoked, the actual parameters in $\bar{a}$ are copied to the formal parameters in $\bar{x}$ and control flow transfers to $f^{\prime}$. After execution of the function, the value of $y$ is copied to $b$. Hence, executing the call to $f^{\prime}$ is equivalent to executing the following sequence.

$$
[\bar{x}:=\bar{a}, S 1, b:=y]
$$

Using weakest precondition notation, the formal semantics of a call to the non-user-defined function $f$ are defined below.

Definition IV.11. $w p(f(\bar{a}), R)=w p\left(\left[\bar{x}:=\bar{a}, S 1, b:=y, S_{b}^{f(\bar{a})}\right], R\right)$

### 4.17 Recursion

In the object paradigm, it is possible for a method to send a message that invokes itself, which is termed recursion. The GOM does not model recursion, which leads to the following restriction.

Restriction IV.6. Methods modeled in the GOM are not allowed to send messages invoking the method recursively.

Even more restrictively, the call tree consisting of the graph of messages sent between objects is assumed to be a directed acyclic graph.

Restriction IV.7. The call tree of messages sent between objects must be a directed acyclic graph.

### 4.18 Variables

As in the imperative paradigm, variables are used to store pieces of information in the object-oriented paradigm. Variables appear in OO programs as local variables or formal parameters of methods. Local variables are declared in the local scope of a method and the value of the variable is lost when execution of the method ends. Formal parameters of methods are variables that are declared in the method and used to pass data items into the method. This section defines the AST class used to model variables in the GOM and presents an example of modeling a variable.

Both local variables and formal parameters are modeled in the GOM by using the gom-variable class. Figure 74 shows the class description for the gom-variable class.

| gom-variable |
| :--- |
| gom-name : symbol |
| gom-var-scope : symbol |
| gom-var-type : gom-data-type |
| gom-indices : seq(gom-data-construct) |

Figure 74 GOM Variable Class

The gom-name attribute holds a symbol that represents the name of the variable. The gom-var-scope holds a symbol that represents the scope of this variable. See Section 4.21 for a discussion of scoping issues in the GOM. The gom-var-type attribute models the data type of the variable. Section 4.20 describes and lists the data types modeled in the GOM. If a variable is an array with indices, the gom-indices attribute is used to model the access into the array.

> | (gom-variable) |
| :--- |
| gom-name : PDIAM |
| gom-var-scope : PRDIV |
| gom-var-type : gom-real |
| gom-indices : [ ] |

Figure 75 GOM Variable Object Example

For example, Figure 75 shows how the local variable PDIAM, defined in a method PRDIV, is represented in the GOM using an instance of the gom-variable class. The gom-var-scope attribute is set to the symbol PRDIV to indicate this variable is in the scope of the method PRDIV. The data type of PRDIV is modeled using the gom-real object. The gom-indices attribute holds an empty sequence to indicate this variable has no indices.

### 4.19 Attributes

In the object paradigm, data items are also stored as attributes of objects. As described in Section 4.4, classes define a template for objects by declaring attributes that are built into each instance of the class. This forms a collection of variables associated with the object that are accessed as attributes of the object. This section defines the AST class that models object attributes in the GOM and presents an example of how an attribute is modeled.

Attributes are modeled in the GOM using the gom-attribute class. Figure 76 shows

| gom-attribute |
| :--- |
| gom-name : symbol |
| gom-var-scope : symbol |
| gom-var-type : gom-data-type |
| gom-indices : seq(gom-data-construct) |

Figure 76 GOM Attribute Class
the class definition for the gom-attribute class. The gom-name attribute holds a symbol that represents the name of the attribute. The gom-var-scope attribute holds a symbol that represents the scope of this attribute. The gom-var-type attribute models the data type of the attribute. If an attribute is an array, the gom-indices attribute is used to model
the indices used when accessing the array. Note that the classes that model variables and attributes in the GOM are identical. This is because of the similarity between attributes and variables in the OO paradigm.

```
class CLASS-2 attributes ZCOS
    method GET-ZCOS ( C-10 ): real
        begin
            GET-ZCOS := C-10.ZCOS
        end
    method SET-ZCOS ( C-11, VAL-10 )
        begin
            C-11.ZCOS := VAL-10
        end
superclass USER-OBJECT
```

Figure 77 Example of ZCOS Attribute

For example, consider the class shown in Figure 77. The CLASS-2 class has one attribute, ZCOS. The "SET-" and "GET-" methods are also shown in the figure. As described in Section 4.4, the gom-attrs attribute of the class gom-class models the attributes of a class. These attributes are modeled using instances of the gom-attribute class. For
(gom-attribute)
gom-name: ZCOS
gom-var-scope: CLASS-2
gom-var-type : gom-real
gom-indices: []

Figure 78 GOM Attribute Object Example
example, Figure 78 shows the gom-attribute object that models the ZCOS attribute of the CLASS-2 class. The gom-name attribute holds the symbol ZCOS. The gom-var-scope attribute holds the symbol CLASS -2 to represent the fact that ZCOS is declared in the scope of CLASS-2. The gom-var-type attribute models the data type of the attribute, which in this example is real.

### 4.20 Data types

This section describes the data types that are used when modeling variables in the GOM. The data types being modeled are as follows.

## Instance object instances of a class

Integer zero and positive and negative whole numbers
Real rationals and irrationals
Boolean true and false logical values
Character single alpha characters
String sequences of characters
Array homogeneous collections of elements accessed with an index.
The Instance data type is used as the data type of variables in the GOM that are instances of a class. Each of the data types presented here is modeled using a separate class in the GOM. In order to model a specific data type in the GOM, the data type attribute of GOM variables and GOM attributes holds an instance of an AST built from the class that models the data type of the variable or attribute. Examples of this are shown in Figure 75 and Figure 78.

As discussed in Section 3.15, the number of bytes associated with an imperative data type can be specified by the user or derived from pre-defined data types. This is true in the object paradigm as well, so each class that models an object-oriented data type has an attribute gom-type-size that models the number of bytes associated with the data type.

### 4.21 Scoping Issues

As in the imperative paradigm, each variable in the object-oriented paradigm has a scope associated with it that refers to its "visibility". A method that defines a variable has visibility to the variable. A variable that is an instance of a class provides visibility to the attributes of the instance. Attributes in the object-oriented paradigm also have scope. This section describes how the scope of variables and attributes in the GOM is modeled.

In the GOM, each method has a local scope much as subprograms in the GIM have a local scope. If a variable is declared in a method, this local variable is only visible to the declaring method. The formal parameters of a method are also only visible to the declaring
method. In order for statements in a method to access the attributes of an instance, the instance must be a local variable or a formal parameter of the method.

Some object-oriented programming languages allow variables declared outside the local scope to be accessed by a method. This type of access is not allowed in the GOM, which leads to the following restriction.

Restriction IV.8. All variables in a GOM method are either declared locally or are formal parameters of the method.

In the object paradigm, a method passes information to other methods by including variables as parameters of messages. As in the imperative paradigm, parameters are either input or output, i.e. the value of the parameter is either read in the method or written to in the method. If one method passes a variable to another method as an output parameter, the method being called changes the value of the variable in the calling method. As in the imperative paradigm, this is termed pass by reference parameter passing [16]. All parameters are passed by reference in the GOM.

In some object-oriented programming languages, one method can be declared inside another method. This means the method would be visible only to the method in which it was declared. Limiting the visibility of a method in this way is not allowed in the GOM, which leads to the following restriction.

## Restriction IV.9. Methods cannot be declared inside of another method.

Each of the methods declared in the GOM must be part of a class, i.e. in the set of operations for a class. This means a particular class has visibility to each of the methods declared within it. When a message is sent to a target object, the class of the object is used to determine with method to invoke. This mechanism is represented and canonicalized in the GOM by requiring each class to be declared in one overall global scope. This leads to the following restriction on the GOM.

Restriction IV.10. Classes cannot be declared inside of another class.

### 4.22 Homogeneous Data Structures

As in the imperative paradigm, collections of homogeneous elements can be built in the object paradigm using constructs such as lists and arrays. The only such collection modeled in the GOM is the array. Arrays are represented in the GOM by building variables and attributes of the data type gom-array (see Section 4.20).

For example, the following statement includes an access to the array variable GEOM.
RANGE := RANGE + GEOM ( K)

The GOM AST shown in Figure 79 models this access to the Kth element of the GEOM array variable.


Figure 79 GEOM Array Variable Access Object

If an attribute of an object is an array, then the indices used to access the array are passed as parameters to the "GET-" or "SET-" message that accesses the attribute. For example, the following statement accesses the Kth element of the GEOM attribute of the instance variable C-10.
RANGE := RANGE + GET-GEOM ( C-10, K)

The GEOM attribute is an array and the GET-GEDM message is used to access the elements of the array. In the statement, the message GET-GEOM ( $\mathrm{C}-10, \mathrm{~K}$ ) is sent to the target object C-10 requesting element K from the GEOM array attribute. The GOM AST shown in Figure 80 models the message used to access the Kth element of the GEOM array attribute.


## Figure 80 GEOM Attribute Array Access Object

The first parameter of the message is the target object $\mathrm{C}-10$ and the second parameter is the variable K. This variable holds the index of the element to be retrieved from the array.

To set the value of an array attribute, the appropriate "SET-" message is used. For example, the following statement sets the Kth element of the GEOM attribute to the value of the RANGE variable.
SET-GEOM ( C-10, K, RANGE)

In this statement, the message SET-GEOM ( C-10, K, RANGE) is sent to the target object C-10 in order to set the Kth element to the value of the RANGE variable. The GOM AST shown in Figure 81 models the "SET-" message. The first parameter is the target object $\mathrm{C}-10$, the second parameter is the index variable K , and the final parameter is the variable to which the array element will be set, i.e. the RANGE variable.

Figure 82 shows the GET-GEOM and SET-GEOM methods used for accessing and assigning a value to the GEOM array attribute.


Figure 81 GEOM Attribute Array Access Object

### 4.23 Heterogeneous Data Structures

Some object-oriented programming languages allow the programmer to build collections of heterogeneous data items, for example, records in Ada 95 and structs in C++. These heterogeneous data structures are not modeled in the GOM.

Restriction IV.11. The GOM does not model heterogeneous data structures.

### 4.24 Pointers

As in the imperative paradigm, some object-oriented programming languages allow the programmer to store the address of a variable and then access the variable via the address. This mechanism is referred to as using pointers to variables. Pointers are not modeled in the GOM.

Restriction IV.12. The GOM does not model pointers.

```
method GET-GEOM ( C-10, IDX-1 ) : real
    begin
            GET-GEDM := C-10.GEOM ( IDX-1)
    end
method SET-GEOM ( C-11, IDX-2, VAL-10 )
    begin
        C-11.GEOM ( IDX-2) \(:=\) VAL-10
    end
```

Figure 82 "SET-" and "GET-" Methods for GEOM

### 4.25 Object-Oriented Expressions

In the object-oriented paradigm, expressions can be literal values, unary or binary expressions with operators, non-user-defined function calls, functional messages, or variables. This section describes how expressions are modeled in the GOM.

Literal values in the GOM are modeled by the gom-literal-constant class. The

| gom-literal-constant |
| :--- |
| gom-literal-value : any-type |

Figure 83 Modeling Object-Oriented Literal Constants
class description for the gom-literal-constant class is shown in Figure 83. The gom-literal-value attribute models the value of the literal. Figure 84 shows the literal con-

gom-literal-integer<br>gom-literal-real<br>gom-literal-boolean<br>gom-literal-string

Figure 84 Object-Oriented Literal Constants
stants currently modeled in the GOM. Each literal shown is modeled as an AST defined as a subclass of the gom-literal-constant class. As in the GIM, literal constants other than those listed in Figure 84 are not modeled in the GOM, but can be modeled by defining a new subclass under the gom-literal-constant class that describes the new literal constant.

Unary expressions in the GOM are modeled using ASTs built from the gom-unaryexpression class. Figure 85 shows the class description for the gom-unary-expression

| gom-unary-expression |
| :--- |
| gom-unary-operand : gom-expression |

Figure 85 Modeling Object-Oriented Unary Expressions
class. The gom-unary-operand attribute models the single operand of the unary expression. Figure 86 shows all the unary expressions modeled in the GOM. Each unary ex-

$$
\begin{aligned}
& \text { gom-negate } \\
& \text { gom-not }
\end{aligned}
$$

## Figure 86 List of Object-Oriented Unary Expressions

pression shown is modeled as an AST defined as a subclass of the gom-unary-expression class. As with literal constants, any unary expressions other than those listed in Figure 86 are not modeled in the GOM but can be built as a new subclass that defines the new unary expression.

Binary expressions in the GOM are modeled by the gom-binary-expression class. Figure 87 shows the class description for gom-binary-expression class. As in the impera-

| gom-binary-expression |
| :--- |
| gom-bin-exp-operand-1 $:$ gom-expression <br> gom-bin-exp-operand-2 $:$ gom-expression <br> gom-bin-exp-seq $:$ seq(gom-expression) |

Figure 87 GOM Binary Expressions Class
tive paradigm, binary expressions in the object-oriented paradigm have either two operands or multiple operands. For this reason, the gom-binary-expression class has three attributes for modeling the operands of the expression. The gom-bin-exp-operand-1 and gom-bin-exp-operand-2 attributes hold the two operands if the binary expression has only two operands. When the same binary expression is repeated for multiple operands, such as in the expression ZCOS $*$ ZCOS $*$ ZCOS, the multiple operands are modeled using the gom-bin-exp-seq attribute. Certain binary expressions can be repeated in this way,
e.g. addition and multiplication. Others cannot be repeated in this way, e.g. $<,<=$, and $>$.

| gom-division <br> gom-equal <br> gom-exponent |  |
| :--- | :--- |
| gom-greater-than-or-equal |  |
| gom-greater-than | gom-addition |
| gom-less-than-or-equal | gom-and |
| gom-less-than | gom-concat |
| gom-not-equal | gom-multiplication |
| gom-subtraction | gom-or |

(a) Two operand.
(b) Sequence.

Figure 88 Object-Oriented Binary Expressions Modeled in the GOM

Figure 88 lists all the binary expressions that are modeled in the GOM. The binary expressions that cannot be repeated are shown in Figure 4.88(a). These expressions are modeled using the gom-bin-exp-operand-1 and gom-bin-exp-operand-2 attributes. The binary expressions that can be repeated are shown in Figure 4.88(b). These expressions are modeled using the gom-bin-exp-seq attribute.

As with literal constants and unary expressions, any binary expressions not shown in Figure 88 are not modeled in the GOM. New binary expressions are modeled by adding a subclass to the gom-binary-expression class to define the new binary expression.

### 4.26 Object-Oriented Input

This section defines the ASTs that are used to model input statement from objectoriented programming languages. Most object-oriented programming languages allow the programmer to input data items from a file or the console. The GOM models this aspect of object-oriented programming languages using ASTs as defined below.

Input statements are modeled in the GOM by using ASTs built from the gom-input class. Figure 89 shows the class description for the gom-input class. The gom-in-logicalfile attribute is used to model the source of the input. In the GOM, input coming from
gom-in-logical-file : gom-data-construct gom-input-list : seq(gom-format-item)

Figure 89 GOM Input Class
the user is modeled by setting this attribute to the symbol CONSOLE. The gom-input-list attribute models the sequence of items being input.

For example, the following input statement (shown in GOL syntax)
read ( CONSOLE, NLASER)
inputs the value of the variable NLASER from the console, which is indicated by the symbol CONSOLE as the first parameter of the statement. Figure 90 shows how this input statement


Figure 90 GOM Input Object
is modeled using an AST in the GOM. The gom-in-logical-file attribute holds the symbol CONSOLE to indicate the input is coming from the user. The gom-input-list attribute is a singleton list holding the NLASER variable.

As in the imperative paradigm, object-oriented programming languages also allow the programmer to input data from a file. Before inputting data from the file, the file must be "opened" to establish the logical link to the physical file on disk. These files are modeled in the GOM using ASTs built from the gom-file class. Figure 91 shows the class description for the gom-file class. The gom-designator attribute holds the name of the logical file as referenced in the program. The gom-file-name attribute holds the name of the physical file. The gom-access attribute holds the access type for the file, e.g. direct or sequential. The gom-status attribute holds the status of the file, opened either for input or output. ASTs built to model files are stored as attributes of a design in the GOM.

| gom-file |
| :--- |
| gom-designator : gom-data-construct <br> gom-file-name : gom-data-construct <br> gom-access : symbol <br> gom-status : symbol |

Figure 91 GOM File Class
The semantics of the GOM input statement are similar to the semantics of the GOM assignment statement. Both statements define values for variables. In general, an input statement in the GOM is equivalent to a subprogram call where all the parameters are output from the subprogram.

For example, let $I$ represent an input statement and let $\bar{a}$ represent a vector of variables in the gom-input-list attribute of $I$ (as modeled in the GOM). Let $\bar{x}$ represent a vector of variables returned from the execution of the input statement after the values are read from the indicated file (or the console). Execution of the GOM input statement is equivalent to the following sequence:

$$
[I(\bar{a}), \bar{a}:=\bar{x}]
$$

The formal semantics for the GOM input statement $I(\bar{a})$ are defined as follows.
Definition IV.12. $w p(I(\bar{a}), R)=w p([I(\bar{a}), \bar{a}:=\bar{x}], R)$
This indicates each of the variables in the vector $\bar{a}$ is set to the values returned by the input statement $I(\bar{a})$.

### 4.27 Object-Oriented Output

This section defines the ASTs that are used to model output statements from objectoriented programming languages. Most object-oriented programming languages allow the programmer to output data items to a file or to the console. The GOM models this aspect of object-oriented languages as defined below.

Output statements in the GOM using ASTs built from the gom-output class. The class description for the gom-output class is shown in Figure 92. The gom-out-logical-

| gom-output |
| :--- |
| gom-out-logical-file : gom-data-construct <br> gom-output-list : seq(gom-format-item) |

Figure 92 GOM Output Class
file attribute represents whether the output is being sent to the console or to an output file. In the GOM, the symbol STD-OUT is stored in this attribute if the output is going to the console. The gom-output-list models the sequence of data items that are being output.

For example, the following output statement (shown in GOL syntax)

```
write ( STD-OUT, " NLASER =", NLASER);
```

outputs two data items to the console. The string literal " NLASER =" is output followed by the value of the variable NLASER. Figure 93 shows the AST used to model this output


Figure 93 GOM Output Object
statement in the GOM. The gom-out-logical-file attribute holds the symbol STD-OUT to indicate the output is being sent to the standard output port. The gom-output-list attribute holds the sequence of two data items representing the string literal "NLASER $=1$ and the variable NLASER.

In the GOM, an output statement is not allowed to change the state of the program. Let $O$ represent a GOM output statement and let $\bar{a}$ represent a vector of data items appearing in the output statement $O$. The semantics of GOM output statements is defined as follows.

Definition IV.13. $w p(O(\bar{a}), R)=R$
This means the GOM output statements are restricted from changing the state of the program.

The following sections provide formal definitions for object-oriented classes, methods, messages, and designs. Formal transformations of an object-oriented design are provided at the end of this section.

### 4.28 Formalizing GOM Statements and Expressions

This section presents formal representations of GOM statements and expressions. Let $x$ represent a GOM variable and let $e$ represent a GOM expression. A GOM assignment statement is represented formally by the following tuple.

$$
<x,:=, e>
$$

This tuple indicates that $x$ is the variable to be assigned the value of the expression $e$. The $:=$ symbol is a syntactic token that distinguishes this tuple as an assignment statement.

Let $e$ represent a GOM expression and let $S_{1}$ and $S_{2}$ represent sequences of GOM statements. A GOM selection statement is represented formally by the following tuple.

$$
<\text { if, } e \text {, then, } S_{1} \text {, else, } S_{2}>
$$

This tuple indicates that the sequence of statements $S_{1}$ will be executed if the expression $e$ evaluates to true otherwise the $S_{2}$ sequence will be executed. The if, then, and else symbols are syntactic tokens used to identify this tuple.

Similarly, let $e$ represent a GOM expression and let $S_{3}$ represent a sequence of GOM statements. The GOM iteration statement is represented by the following tuple

$$
<\text { while, } e, S_{3}>
$$

This indicates that the sequence of statements $S_{3}$ will be executed while the expression $e$ is true. The while symbol is a syntactic token used to identify this tuple.

Let $E$ represent a sequence of expressions. The GOM input and output statements are represented formally by the following tuples.

$$
\begin{gathered}
\langle\text { input, iport, } E\rangle \\
\langle\text { output, oport, } E\rangle
\end{gathered}
$$

These tuples indicate that the expressions in the sequence $E$ are either input from the input port iport or output to the output port oport.

Expressions in the GOM are represented formally by the following tuples. Let $e_{1}$ and $e_{2}$ be GOM expressions.

$$
\begin{array}{r}
<e_{1},+, e_{2}> \\
<e_{1}, \text { and, } e_{2}> \\
<e_{1}, \text { concat, } e_{2}> \\
<e_{1}, /, e_{2}> \\
<e_{1},=, e_{2}> \\
<e_{1}, * *, e_{2}> \\
<e_{1},>=, e_{2}> \\
<e_{1},>, e_{2}> \\
<e_{1},<=, e_{2}> \\
<e_{1},<, e_{2}> \\
<e_{1}, *, e_{2}> \\
<e_{1},<>, e_{2}> \\
<e_{1}, \text { or }, e_{2}> \\
<e_{1},-, e_{2}> \\
<-, e_{1}> \\
<\text { not }, e_{1}>
\end{array}
$$

These formal representations are used in the formal transformations presented in Chapter VI.

### 4.29 Formalizing Object-Oriented Classes

This section describes how classes modeled by the gom-class AST can be defined more mathematically. The formal definition of an object-oriented class is presented below.
$i d_{C}-$ A symbol storing the name of the class.
$\Phi$ - The set of instance attributes.
$\Omega$ - The set of instance methods.
$\lambda$ - The symbol storing the name of the superclass.
Thus, an object-oriented class, $C$, is defined formally as

$$
\left.C=<i d_{C}, \Phi_{C}, \Omega_{C}, \lambda\right\rangle
$$

```
class CLASS-2 attributes ZCOS, XLAMDA, SIGTRB
    method GET-ZCOS ( C-10 ): real begin GET-ZCOS := C-10.ZCOS
        end
    method SET-ZCOS ( C-11, VAL-10 ) begin C-11.ZCOS := VAL-10
        end
    method GET-XLAMDA ( C-10) : real begin
        GET-XLAMDA := C-10.XLAMDA end
    method SET-XLAMDA ( C-11, VAL-11 ) begin
        C-11.XLAMDA := VAL-11 end
    method GET-SIGTRB ( C-10 ): real begin
        GET-SIGTRB := C-10.SIGTRB end
    method SET-SIGTRB ( C-11, VAL-12 ) begin
        C-11.SIGTRB := VAL-12 end
    superclass USER-OBJECT
```

Figure 94 Class CLASS-2
For example, Figure 94 shows a class (using GOL syntax) that has three attributes ZCOS, XLAMDA, and SIGTRB. The class inherits from the class USER-OBJECT and includes the appropriate methods to set and access its attributes. This class is formally defined by the following tuple.

$$
\begin{aligned}
& C=<\text { CLASS-2 }, \Phi_{C}, \Omega_{C}, \text { USER-OBJECT }>\text { where } \\
& \Phi_{C}=\{\text { ZCOS, XLAMDA, SIGTRB }\} \text { and } \\
& \Omega_{C}=\{\text { SET-ZCOS, SET-XLAMDA, SET-SIGTRB }, \\
&\text { GET-ZCOS, GET-XLAMDA, GET-SIGTRB }\}
\end{aligned}
$$

The name of the class is the symbol CLASS-2. The attributes of the class are represented by the set of data items ZCOS, XLAMDA, and SIGTRB. The operations of the class are represented by the set of methods including SET-ZCOS, SET-XLAMDA, SET-SIGTRB, GET-ZCOS, GET-XLAMDA, GET-SIGTRB. Each operation is represented by a formal definition as presented in the following section.

### 4.30 Formalizing Object-Oriented Methods

This section explains how methods represented using gom-method ASTs can be defined more mathematically. Methods are defined as follows.
$i d_{M}$ - A symbol storing the name of the method.
$\tau$ - The target object.
$Q_{o b j}$ - The sequence of objects passed to the method (other than $\tau$ ).
$Q_{f o r m}$ - The sequence of formal parameters.
$Q_{r e t}$ - The sequence of returned values.
$Q_{l o c}$ - The sequence of local variables.
$\Psi$ - The sequence of object-oriented statements.
Thus, an object-oriented method is defined by the tuple:

$$
<i d_{M}, \tau, Q_{o b j}, Q_{f o r m}, Q_{r e t}, Q_{l o c}, \Psi>
$$

All object-oriented methods are elements of $\Omega$ of some object-oriented class.
For example, Figure 95 shows the method UPLREQ. This method expects an instance of the class CLASS-2 to be passed in as the target object. The method returns a value of type real. This method is defined formally using the following tuple.

```
class CLASS-2 attributes ZCOS, XLAMDA, SIGTRB
    method UPLREQ ( C-12 ): real
        begin
        SIGTO := 3.75d-6;
        TMP1 :=
            GET-XLAMDA ( C-12) * GET-ZCOS ( C-12) * GET-ZCOS ( C-12)
            * GET-ZCOS ( C-12);
        TMP2 := TMP1^0.2;
        SIGUP := SIGT0 / TMP2;
        if GET-SIGTRB ( C-12) > 0.0 then
            UPLREQ := SIGUP / GET-SIGTRB ( C-12)
        else
            UPLREQ := 1000000.0d0
        endif
        end
    superclass USER-OBJECT
```

Figure 95 Method UPLREQ

$$
\begin{aligned}
& <\text { UPLREQ, } \tau, Q_{o b j}, Q_{f o r m}, Q_{r e t}, Q_{l o c}, \Psi>\text { where } \\
& \tau=\mathrm{C}-12 \text { and } \\
& Q_{o b j}=[] \text { and } \\
& Q_{f o r m}=[] \text { and } \\
& Q_{r e t}=[\text { UPLREQ }] \text { and } \\
& Q_{l o c}=[\text { SIGT0, TMP1, TMP2, SIGUP }] \text { and } \\
& \Psi=\text { the statements between the BEGIN and the END }
\end{aligned}
$$

The name of the method is the symbol UPLREQ. The data item C-12 holds the target object passed to the method. UPLREQ expects no other objects or parameters to be passed since the $Q_{o b j}$ and $Q_{f o r m}$ sequences are empty. The data item returned from this method is represented by including the name of the method in the $Q_{r e t}$ sequence. The local variables of UPLREQ are the data items SIGTO, TMP1, TMP2, and SIGUP. The sequence of object-oriented statements between the BEGIN and the END of UPLREQ comprise $\Psi$.

### 4.31 Formalizing Object-Oriented Messages

This section explains how messages represented using the gom-message AST can be defined more mathematically. Messages are defined formally below.
$i d_{o}$ - a symbol that holds the name of the method to be invoked.
$Q_{\text {act }}$ - a sequence of variables that are the actual parameters passed to the method.
$Q_{\text {form }}$ - a sequence of variables that are formal parameters of the method to be invoked.
Let $C_{1}$ be a class that has a method $o_{1}$. Let $C_{2}$ be a class that has a method $o_{2}$ that is invoked in $o_{1}$. Then, the message sent in method $o_{1}$ is represented by the tuple

$$
<i d_{o_{2}}, Q_{a c t}>
$$

In the object-oriented paradigm, the number, order, and type of parameters in $Q_{a c t}$ must match the number, order, and type of the parameters in $Q_{\text {form }}$.

```
class CLASS-5 attributes PHASE, ZENITH, UPLFAC
    method RELAY-PHASE ( C-14, C-22, PHASE )
        begin
        PHASE := UPLREQ ( C-22 )
        end
    superclass USER-OBJECT
```

Figure 96 Class with method RELAY-PHASE

For example, as shown in Figure 96, the message UPLREQ( C-22 ) is being sent by the RELAY-PHASE method. The target object of this message is C-22 and this instance is being asked to execute the method UPLREQ. The object C-22 is an instance of the class CLASS-2. Figure 97 shows the class CLASS-2 and the (partially declared) method UPLREQ. The message UPLREQ ( $\mathrm{C}-22$ ) is defined formally by the tuple

$$
<U P L R E Q,[\mathrm{C}-22]>
$$

Since this message is invoking the UPLREQ method, the UPLREQ identifier is the first item in the tuple. The singleton sequence containing $\mathbf{C - 2 2}$ represents that this call is passing in a single parameter, which is the target object c-22.

```
class CLASS-2 attributes ZCOS, XLAMDA, SIGTRB
    method UPLREQ ( C-12 ): real
        begin
            ...
        end
    superclass USER-OBJECT
```

Figure 97 Class with method UPLREQ

### 4.32 Formalizing the Object Model

Rumbaugh [62] defines an object model in his Object Modeling Technique (OMT). The object model includes definitions for the classes, inheritance, and aggregation associations in an object-oriented design. This section provides a formal definition of an object-oriented design, a discussion of the inheritance and aggregation associations in the design, and formal transformations for the object model.

The formal definition of an object-oriented design is given below. Let $C^{*}$ be a set of object-oriented classes to be included in an object-oriented design. An object-oriented design is represented formally using the following one-tuple.

$$
O O D=C^{*}
$$

An object-oriented design consists of the set of classes to be included in the design. One of these classes in the design is the system class that includes the main method, which is given the flow of control when execution begins.

Inheritance associations are represented in a design by references to the superclass in a class description, i.e. the $\lambda$ item in the tuple representing a class. An aggregation association exists between two classes when object instances of one class are stored as instance attributes of the other class.

Blaha [7] discusses several primitive transformations for object models. The transformations defined to formalize the PBOI method include removing or adding a class and removing or adding an attribute. A mathematical description for these transformations is provided below.

Let $T_{c}^{+}$be a transformation of the object model that adds a new class to an existing design and returns a new object-oriented design. Thus,

$$
\begin{gathered}
T_{c}^{+}(O O D, C)=O O D^{\prime} \text { where } \\
O O D^{\prime}=O O D \cup\{C\}
\end{gathered}
$$

Let $T_{c}^{-}$be a transformation of the object model that removes a class and returns a new object-oriented design. Thus,

$$
\begin{gathered}
T_{c}^{-}(O O D, C)=O O D^{\prime} \text { where } \\
O O D^{\prime}=O O D-\{C\}
\end{gathered}
$$

Let $T_{a}^{+}$be a transformation of the object model that adds a new attribute to a specific class and returns a new object-oriented design. Thus,

$$
\begin{aligned}
& T_{a}^{+}(O O D, C, a)=O O D^{\prime} \text { where } \\
& \quad C=<i d_{C}, \Phi_{C}, \Omega_{C}, \lambda>\text { and } \\
& \quad C^{\prime}=<i d_{C}, \Phi_{C} \cup\{a\}, \Omega_{C}, \lambda>\text { and } \\
& O O D^{\prime}=T_{c}^{+}\left(T_{c}^{-}(O O D, C), C^{\prime}\right)
\end{aligned}
$$

Let $T_{a}^{-}$be a transformation of the object model that removes an attribute of a specific class and returns a new object-oriented design. Thus,

$$
\begin{aligned}
& T_{a}^{-}(O O D, C, a)=O O D^{\prime} \text { where } \\
& \quad C=<i d_{C}, \Phi_{C}, \Omega_{C}, \lambda>\text { and } \\
& C^{\prime}=<i d_{C}, \Phi_{C}-\{a\}, \Omega_{C}, \lambda>\text { and } \\
& O O D^{\prime}=T_{c}^{+}\left(T_{c}^{-}(O O D, C), C^{\prime}\right)
\end{aligned}
$$

Blaha [7] does not address the addition or removal of a method from a class. These additional transformations are introduced below for adding and removing a method, $o$, of a class, $C$.

Let $T_{m}^{+}$be a transformation that adds a new method to a class and returns a new object-oriented design. Thus,

$$
\begin{aligned}
& T_{m}^{+}(O O D, C, o)=O O D^{\prime} \text { where } \\
& \quad C=<i d_{C}, \Phi_{C}, \Omega_{C}, \lambda>\text { and } \\
& \quad C^{\prime}=<i d_{C}, \Phi_{C}, \Omega_{C} \cup\{o\}, \lambda>\text { and } \\
& O O D^{\prime}=T_{c}^{+}\left(T_{c}^{-}(O O D, C), C^{\prime}\right)
\end{aligned}
$$

Let $T_{m}^{-}$be a transformation that removes a method from a class and returns a new object-oriented design. Thus,

$$
\begin{aligned}
& T_{m}^{-}(O O D, C, o)=O O D^{\prime} \text { where } \\
& \quad C=<i d_{C}, \Phi_{C}, \Omega_{C}, \lambda>\text { and } \\
& C^{\prime}=<i d_{C}, \Phi_{C}, \Omega_{C}-\{o\}, \lambda>\text { and } \\
& O O D^{\prime}=T_{c}^{+}\left(T_{c}^{-}(O O D, C), C^{\prime}\right)
\end{aligned}
$$

### 4.33 Summary

This chapter has defined the Generic Object-Oriented Design Model (GOM) which models objects, classes, methods, messages, assignment, variables, expressions, and control flow in object-oriented programming languages. Several restrictions have been placed on the GOM. Some restrictions are intended to canonicalize the representations of objectoriented entities. Other restrictions place limits on which OO entities can possibly be modeled using the GOM.

Specifically, Restriction IV. 1 states that neither functional nor procedural methods can have a parameter that is both and input and an output parameter. This restriction is not unreasonable since any method that does not meet this restriction can be converted using a process similar to the one presented in Appendix D.

Similarly, Restriction IV. 2 does not allow functional methods to return more than one data item. Hypothetically, functional methods of this type can be converted to procedural methods that return multiple values.

Restriction IV. 3 is intended to canonicalize the accessing and assigning of object attributes and does not limit which entities can be model using the GOM.

Restrictions IV. 6 and IV. 7 limit messages to non-recursive invocations. This restriction is reasonable because Knuth [37] argues any recursive algorithm can be represented using an iterative algorithm.

Restriction IV. 8 does not allow global variables to appear in methods. This means methods with variables other than formal parameters or local variables must first be converted into the proper form. It is hypothetically possible to convert global variables into formal parameters throughout the call tree of an object-oriented system, so this restriction is not unreasonable.

Restriction IV. 9 does not allow one method to be declared inside another method. The scoping issues involved in this nesting of methods is more complicated than that of variables, but it is hypothetically possible to convert the nested methods into methods declared at the global level. Similarly, Restriction IV. 10 does not allow one class to be declared inside another class. This is a minor restriction since declaring classes inside of other classes is not a common practice in object-oriented programming.

Restriction IV. 4 requires that only calls to non-user-defined subprograms are modeled by subprogram invocations in the GOM. This means that the GOM represents a "pure" object-oriented design where no mixing of the imperative paradigm and the object-oriented paradigm is allowed. This restriction presents a limitation if the design to be modeled does include such a mixture.

None of the restrictions discussed above preclude the representation of object-oriented entities in the GOM. However, Restriction IV. 11 states that heterogeneous data types are not modeled in the GOM. Restriction IV. 12 states that pointers are not modeled in the GOM. These two restrictions limit the entities that can be modeled by the GOM. If the object-oriented system being modeled includes either heterogeneous data types or pointers, the system cannot be represented using the GOM. This is not unreasonable when re-engineering from the GIM, but is a more serious limitation for representing languages such as C++ or Ada 95.

As presented in this chapter, the GOM provides a canonical form for modeling entities in the object paradigm. Most of the restrictions placed on the GOM are meant to
canonicalize and simplify the representation of these entities. The GOM is used throughout the following chapters to represent object-oriented entities.

## V. Identifying Objects

### 5.1 Introduction

This chapter introduces a novel method, Parameter-Based Object Identification (PBOI), for recovering objects from legacy imperative programs. The method is based on fundamental axioms that relate programs written in imperative languages such as C or Cobol to objects and classes written in object-oriented languages such as Ada 95 or $\mathrm{C}++$. This chapter provides an informal introduction to the object identification methodology. Transformations that formalize the methodology are presented in Chapter VI.

In order to focus the task of transforming imperative subprograms to the objectoriented paradigm, a taxonomy of imperative subprograms has been developed. This chapter describes the taxonomy, which classifies all imperative subprograms into one of six categories and explains how the object identification methodology is used to convert subprograms in each of the categories. Finally, the chapter describes how the process of program slicing [76] is used to focus the transformation task even further.

### 5.2 Taxonomy of Imperative Subprograms

This section describes the taxonomy of imperative subprograms built to focus the task of re-engineering imperative legacy code to the object paradigm. By defining this taxonomy, an imperative subprogram can be converted to the object paradigm by determining the classification of the subprogram and then applying the formal transformation appropriate to that category of subprogram. The taxonomy reduces the re-engineering task to defining the transformations for the six categories of subprograms identified. The following definitions are used to build the taxonomy.

Definition V.1. One subprogram calls another subprogram by using the name of the subprogram to invoke the other subprogram.

Definition V.2. $C_{\text {sub }}$ is the collection of calls a subprogram makes to other subprograms.
Definition V.3. An imperative procedure produces a data item if the data item is an output parameter.

Definition V.4. An imperative function produces the data item returned at the end of its execution.

Definition V.5. $P_{\text {pro }}$ is the collection of data items produced by a subprogram.
The taxonomy of imperative subprograms considers whether or not a subprogram calls another subprogram and whether the subprogram produces zero, one, or multiple data items. The distinction between producing zero or one data item is important for the classification of the main program, i.e. the subprogram that is given the flow of control as the system begins execution. Figure 98 shows the taxonomy of imperative subprograms based on the sizes of $C_{s u b}$ and $P_{p r o}$.

|  | $\left\|P_{\text {pro }}\right\|=0$ | $\left\|P_{\text {pro }}\right\|=1$ | $\left\|P_{\text {pro }}\right\|>1$ |
| :---: | :---: | :---: | :---: |
| $\left\|C_{\text {sub }}\right\|=0$ | 0 | 2 | 4 |
| $\left\|C_{\text {sub }}\right\|>0$ | 1 | 3 | 5 |

Figure 98 Subprogram Taxonomy

Category 0 subprograms produce no data items and call no other subprograms. These subprograms could consist of output statements or busy/wait loops. Category 1 subprograms produce no data items but call other subprograms. These subprograms could be driver programs or the main program. Category 2 subprograms produce a single data item and call no other subprograms. Category 3 subprograms produce a single data item and call other subprograms. Category 4 subprograms produce multiple data items and call no other subprograms. Category 5 subprograms produce multiple data items and call other subprograms.

Since any imperative subprogram can be classified into one of these six categories, the process of transforming the subprogram from the GIM to the GOM is reduced to building transformations for each category.

### 5.3 Parameter-Based Object Identification

Parameter-Based Object Identification (PBOI) is a novel method for identifying objects in imperative legacy code based on the data items passed as parameters in imperative
subprogram calls. The PBOI method provides a rationale for converting imperative subprograms into classes and methods that implement the subprograms. The overall rationale for the PBOI method is based on the following thesis.

Thesis V.1. Object attributes manifest themselves as data items passed from subprogram to subprogram in the imperative paradigm.

Object attributes are extracted from the parameters of imperative subprograms using specific transformations defined by the PBOI methodology. Once the attributes of an object are built into a class, the behavior associated with the attributes is built into the class.

In this methodology, it is assumed that all data items in an imperative subprogram are either passed as parameters to the subprogram or declared locally in the subprogram. In this way, the main program is the origin of all data items being passed to the imperative subprograms.

The PBOI methodology starts the transformation process at the main program. Before the main program is transformed each of the subprograms called by the main program is transformed to the object-oriented paradigm. Similarly, each of the subprograms called by a Category 1,3 , or 5 subprogram is converted before the subprogram itself is converted. This depth-first style of transformation results in incremental designs being merged together at different steps in the transformation.

The following sections define the PBOI method in detail as it applies to the six categories of imperative subprograms. The discussions of the different categories are not presented in numerical order, but in an order that more closely fits the incremental nature of the PBOI method.

### 5.4 Converting Category 2 Subprograms

As a starting point, consider the conversion of Category 2 imperative subprograms. These subprograms produce one data item without calling any other subprograms, so an analogous entity is needed from the object paradigm. In his Object Modeling Technique (OMT), Rumbaugh [62] distinguishes between object-oriented operations that have side
effects and those that merely compute a functional value. He labels the latter form of operation queries. Queries with no arguments except the target object are considered derived attributes [62] of the object. Imperative functions fit this description, so the PBOI method uses the following thesis to convert Category 2 functions to the object-oriented paradigm.

Thesis V.2. Category 2 imperative functions implement derived attribute queries of objects.

Category 2 imperative procedures have a single output parameter, so the PBOI method uses the following thesis to convert Category 2 procedures to the object-oriented paradigm.

Thesis V.3. Category 2 imperative procedures implement side-effecting operations of objects.

Based on these two theses, the PBOI methodology converts each formal parameter of a Category 2 subprogram into an attribute of a class and transforms the subprogram into a method of the class.

```
real function CAPTURE
    ( BEAMRA, AXMAJ, AXMIN, ANGFAC )
    begin
    TILTFA := DSIN ( ANGFAC);
    RMAREA := AXMAJ * AXMIN;
    EFAREA := RMAREA * TILTFA;
    BMAREA := BEAMRA * BEAMRA;
    if EFAREA <= BMAREA * 6.0dO
        then USED \(:=\) EFAREA / BMAREA;
            CAPTURE := 1.OdO - DEXP ( -USED)
        else CAPTURE := 1.OdO endif
    end
```

Figure 99 Imperative function CAPTURE

For example, consider the imperative function CAPTURE shown in Figure 99. This function is shown using the GIL surface syntax. The CAPTURE function is a Category 2 subprogram that returns a single value of type real. The parameters BEAMRA, AXMAJ, AXMIN,
and ANGFAC are passed into the function and used by the statements of the function to calculate the value returned. The CAPTURE function is converted to the object-oriented paradigm using the PBOI method. Figure 100 shows the class and method built for CAPTURE.

```
class CLASS-2 attributes
    AXMIN, AXMAJ, BEAMRA, ANGFAC
    method CAPTURE ( C-2 ): real
        begin
        TILTFA := DSIN ( GET-ANGFAC ( C-2));
        RMAREA := GET-AXMAJ ( C-2)
            * GET-AXMIN ( C-2);
        EFAREA := RMAREA * TILTFA;
        BMAREA := GET-BEAMRA ( C-2)
            * GET-BEAMRA ( C-2);
    if EFAREA <= BMAREA * 6.0dO
            then USED := EFAREA / BMAREA;
                CAPTURE := 1.0dO - DEXP ( -USED)
            else CAPTURE := 1.OdO endif
        end
    superclass USER-OBJECT
```

Figure 100 Class and method built for CAPTURE

This class and method are represented in the figure using the GOL surface syntax. Note that each of the parameters BEAMRA, AXMAJ, AXMIN, and ANGFAC are attributes of the class CLASS-2. The method that implements the CAPTURE subprogram has been built as the single method of CLASS-2. This method expects one parameter, C-2, which is an instance of CLASS-2, and the method returns a value of type real. The statements of the method now access the BEAMRA, AXMAJ, AXMIN, and ANGFAC data items from the object C-2 in order to calculate the value returned from the method.

Because of Restriction IV.3, the BEAMRA, AXMAJ, AXMIN, and ANGFAC attributes are accessed using the "GET-" and "SET-" methods. Each "GET-" and "SET-" method is generated for each attribute and added to the set of operations for the class. Similarly, a method to instantiate instances of the class is needed, so a "CREATE-" method is generated for each class. These methods are not generated until the main program is
transformed, which explains why they do not appear in Figure 100. The transformation that formalizes the generation of these methods is presented in Section 6.11.2.

The conversion of Category 2 procedures is identical to the conversion of Category 2 functions except that the single output parameter is converted to an attribute of the new class built for the subprogram. By using this part of the PBOI method, all Category 2 subprograms can be converted to the object-oriented paradigm.

### 5.5 Converting Category 0 Subprograms

A Category 0 subprogram produces no data items and calls no other subprograms. These subprograms may be routines that include imperative-output statements, or they may be needed in the system for timing considerations. Category 0 subprograms are treated as a special case of Category 2 subprograms. All Category 0 subprograms are transformed to the object-oriented paradigm by using the transformation defined for Category 2 subprograms.

### 5.6 Converting Category 3 Subprograms

Category 3 subprograms produce a single data item and call other subprograms. Since Category 3 subprograms call other subprograms, the attributes of objects that have already been built can be analyzed using the PBOI methodology. This section describes the depth-first analysis that is done to the object-oriented design while transforming Category 3 subprograms to the object-oriented paradigm.

The first step in converting a Category 3 subprogram is to transform all of the subprograms that it calls. Then a new class with a method that implements the Category 3 subprogram is built. Initially this class has an attribute built from each of the parameters of the subprogram, as was done for Category 2 subprograms. As Category 3 subprograms are converted to the object-oriented paradigm, the attributes of this class, as well as the attributes of the other classes in the design, are filtered to determine which should remain attributes and which should be converted from attributes to parameters of the methods.

Specifically, consider the case where a Category 3 subprogram $S_{1}$ makes one call to another subprogram $S_{2}$ ( $S_{2}$ may be from any category in the taxonomy). Each of the actual parameters from $S_{1}$ in the call to $S_{2}$ is linked to a corresponding formal parameter in the declaration of $S_{2}$. Let $C_{1}$ represent the class built for $S_{1}$ and let $C_{2}$ represent the class built for $S_{2}$.

An actual parameter from $S_{1}$ may or may not also be a formal parameter of $S_{1}$, and the corresponding formal parameter in $S_{2}$ may be an attribute of $C_{2}$ or a parameter of the method in $C_{2}$. Figure 101 shows the four cases considered for each actual parameter when converting a Category 3 subprogram. In each PBOI case, a specific change is made to the

|  | Actual is Formal | Actual is not Formal |
| :---: | :---: | :---: |
| Formal is Attribute | PBOI Case 1 | PBOI Case 3 |
| Formal is Parameter | PBOI Case 2 | PBOI Case 4 |

Figure 101 PBOI Cases
design developed so far as explained below.

PBOI Case 1 The data item remains an attribute of $C_{2}$ and is removed as an attribute of $C_{1}$.

PBOI Case 2 The data item is converted from an attribute of $C_{1}$ to a parameter of $C_{1}$ 's method.

PBOI Case 3 The data item is converted from an attribute of $C_{2}$ to a parameter of $C_{2}$ 's method.

PBOI Case 4 No change is required for the design.
In Case 1, the data item has already been made an attribute of $C_{2}$, so it should not be built as an attribute of another class, i.e. $C_{1}$. In Case 2, it has been determined through some earlier filtering that the data item should not be an attribute, so it is not built as an attribute of $C_{1}$. In Case 3, the data item is not being passed down from the main program, so it is not part of an object. Thus, it is filtered out of $C_{2}$ and moved to a parameter. In Case 4, the data item is not being passed down and it is already a parameter instead of an attribute. These four cases are used to evaluate the entire design built so far when each

Category 3 subprogram is converted to the object-oriented paradigm. For clarification, an example is given in the following sections for each of the PBOI cases.
5.6.1 PBOI Case 1. As an example of PBOI Case 1, consider the Category 3 subprogram BOUNCE shown in Figure 102 (using the GIL surface syntax). The BOUNCE

```
real function BOUNCE ( ANGFAC, RNGFAC, PROJRA,
    SIGB, RANGE, AXMAJ, AXMIN, XLAMDA)
    begin
    RHOSTD := 0.95 * PROJRA * ANGFAC;
    BEAMRA :=
        RADIUS ( PROJRA, SIGB, RNGFAC, RANGE);
    BOUNCE :=
        CAPTURE ( BEAMRA, AXMAJ, AXMIN, ANGFAC)
        * RHO ( RHOSTD, XLAMDA, ANGFAC)
    end
```

Figure 102 Subprogram BOUNCE
subprogram calls the Category 2 subprogram RADIUS shown in Figure 103, as well as the CAPTURE and RHO subprograms (not shown in the figures). Figure 104 shows the class built for the RADIUS subprogram (using the GOL surface syntax) by using the rule for Category 2 subprograms. Note that the formal parameters of RADIUS have been built as attributes of the class CLASS-1.

```
real function RADIUS
    ( PROJRA, SIGB, RNGFAC, RANGE )
    begin
    if RNGFAC > 0.0 and RANGE > 0.0
        then SIGABS := DABS ( SIGB);
            SLOPE := SIGABS - PROJRA / RANGE;
            SPOT := SIGABS * RANGE;
            RAD := DABS ( PROJRA + SLOPE * RNGFAC);
            RADIUS := DMAX1 ( SPOT, RAD)
        else
            RADIUS := PROJRA endif
    end
```

Figure 103 Subprogram RADIUS

In the call to RADIUS, the actual parameters PROJRA, SIGB, RNGFAC and RANGE are also formal parameters of the BOUNCE subprogram. These four data items provide an example

```
class CLASS-1 attributes
    RANGE, RNGFAC, SIGB, PROJRA
    method RADIUS ( C-1 ): real
        begin
        if GET-RNGFAC ( C-1) > 0.0 and
                GET-RANGE ( C-1) > 0.0
            then SIGABS := DABS ( GET-SIGB ( C-1));
                SLOPE := SIGABS - GET-PROJRA ( C-1)
                    / GET-RANGE ( C-1);
                SPOT := SIGABS * GET-RANGE ( C-1);
                RAD := DABS ( GET-PROJRA ( C-1) +
                        SLOPE * GET-RNGFAC ( C-1));
            RADIUS := DMAX1 ( SPOT, RAD)
            else RADIUS := GET-PROJRA ( C-1) endif
        end
    superclass USER-OBJECT
```

Figure 104 Class and method built for RADIUS
of PBOI Case 1 and remain attributes of CLASS-1.
Figure 105 shows the initial class and method built for the Category 3 subprogram BOUNCE. Note that the subprogram calls to RADIUS, CAPTURE, and RHO have not yet been transformed to messages, i.e. the transformation is incomplete. Also note that each of the parameters from the subprogram BOUNCE has been built as an attribute of the class CLASS-4 including the formal parameters PROJRA, SIGB, RNGFAC and RANGE. Since these data items are passed to the RADIUS subprogram and have been built as attributes of the class built for RADIUS, they are examples of PBOI Case 1 and are converted from attributes of CLASS-4 to attributes of CLASS-1.

Figure 106 shows CLASS-4 after the attributes PROJRA, SIGB, RNGFAC and RANGE have been converted from CLASS-4 to CLASS-1. As shown in the figure, these data items are no longer attributes of CLASS-4. The message GET-PROJRA is now sent to the object C-5, an instance of CLASS-1, instead of being sent to the object C-4, an instance of CLASS-4.

```
class CLASS-4 attributes ANGFAC, RNGFAC, PROJRA,
            SIGB, RANGE, AXMAJ, AXMIN, XLAMDA
    method BOUNCE ( C-4 ): real
        begin
        RHOSTD := 0.95 * GET-PROJRA ( C-4)
            * GET-ANGFAC ( C-4);
        BEAMRA :=
            RADIUS ( PROJRA, SIGB, RNGFAC, RANGE);
        BOUNCE :=
            CAPTURE ( BEAMRA, AXMAJ, AXMIN, ANGFAC)
            * RHO ( RHOSTD, XLAMDA, ANGFAC)
        end
    superclass USER-OBJECT
```

Figure 105 Initial class built for BOUNCE

The call to the subprogram RADIUS has been transformed into a message sent to C-5 that invokes the RADIUS method of CLASS-1.
5.6.2 PBOI Case 2. As an example of PBOI Case 2, consider another version of CLASS-1 shown in Figure 107 that might have been built for the RADIUS subprogram. The class shown in the figure includes the PROJRA data item as a parameter to the RADIUS method instead of as an attribute of the class CLASS-1. This data item provides an example of PBOI Case 2. Figure 105 shows the initial class built for the BOUNCE subprogram. Each of the formal parameters of the subprogram BOUNCE has been built as an attribute of CLASS-4, including the PROJRA data item. In this example, the PROJRA data item is passed to the RADIUS subprogram but was not built as a parameter of the class built for RADIUS. This is an example of PBOI Case 2, so the data item PROJRA is transformed from an attribute of CLASS-4 into a parameter of the methods of CLASS-4.

Figure 108 shows the updated class after converting the data item PROJRA from an attribute of CLASS-4 into a parameter of the BOUNCE method. The PROJRA data item is passed as an actual parameter to the RADIUS method. The message GET-PROJRA ( C-4) that was used to access PROJRA as an attribute of the class has been converted to an access of the parameter PROJRA. The class in the figure has also been updated to convert the data items SIGB, RNGFAC and RANGE from attributes of CLASS-4 to attributes of CLASS-1 since

```
class CLASS-4 attributes
    ANGFAC, AXMAJ, AXMIN, XLAMDA
    method BOUNCE ( C-4, C-5 ): real
        begin
        RHOSTD := 0.95 * GET-PROJRA ( C-5)
            * GET-ANGFAC ( C-4);
        BEAMRA := RADIUS ( C-5);
        BOUNCE :=
            CAPTURE ( BEAMRA, AXMAJ, AXMIN, ANGFAC)
                * RHO ( RHOSTD, XLAMDA, ANGFAC)
    end
    superclass USER-OBJECT
```

Figure 106 Updated class built for BOUNCE
these data items are examples of PBOI Case 1. The message RADIUS ( C-5, PROJRA) now matches the signature of the method RADIUS and is passed an instance of CLASS-1 and the parameter PROJRA.
5.6.3 PBOI Case 3. As an example of PBOI Case 3, Figure 109 shows the Category 3 imperative subprogram BOUNCE and the Category 2 subprogram CAPTURE, which is called by BOUNCE. The class originally built for the CAPTURE subprogram is shown in Figure 110. Since CAPTURE is a Category 2 subprogram, each of the parameters, including the data item BEAMRA, is converted into an attribute of the class CLASS-2. The BEAMRA data item is not a formal parameter of the subprogram BOUNCE, so it does not remain an attribute of CLASS-2. This data item is an example of PBOI Case 3, so it must be converted from an attribute of CLASS-2 into a parameter of the CAPTURE method. Figure 111 shows CLASS-2 after this change has been made. Figure 105 shows the initial class built for the BOUNCE subprogram. The data item BEAMRA is not a formal parameter of the BOUNCE subprogram, so it has not been built as a parameter of the class CLASS-4. There is no change required by PBOI Case 3 to the attributes of CLASS-4 or the parameters of the BOUNCE method. The only change made to the statements of the method is to ensure the BEAMRA data item is passed as a parameter of the CAPTURE message. To illustrate, note

```
class CLASS-1 attributes RANGE, RNGFAC, SIGB
    method RADIUS ( C-1, PROJRA ): real
        begin
        if GET-RNGFAC ( C-1) > 0.0 and
                GET-RANGE ( C-1) > 0.0
            then SIGABS := DABS ( GET-SIGB ( C-1));
                SLOPE := SIGABS - PROJRA
                        / GET-RANGE ( C-1);
                SPOT := SIGABS * GET-RANGE ( C-1);
                RAD := DABS ( PRDJRA + SLOPE *
                    GET-RNGFAC ( C-1));
                RADIUS := DMAX1 ( SPOT, RAD)
            else RADIUS := PROJRA endif
        end
    superclass USER-OBJECT
```

Figure 107 Class built for RADIUS
that the AXMAJ, AXMIN, and ANGFAC data items are examples of PBOI Case 1, so they are converted from attributes of CLASS-4 to attributes of CLASS-2.

Figure 112 shows CLASS-4 after this transformation has been done and the call to CAPTURE has been converted into a message. As shown in the figure, the message CAPTURE ( $\mathrm{C}-6$, BEAMRA) invokes the CAPTURE method in CLASS-2. The instance variable C-6 now includes the AXMAJ, AXMIN, and ANGFAC data items as attributes (because of PBOI Case 1), and the BEAMRA data item is passed as an actual parameter of the message (because of PBOI Case 3).
5.6.4 PBOI Case 4. As an example of PBOI Case 4, consider the version of CLASS-2 shown in Figure 111. This class has been built for the subprogram CAPTURE and, for this example, the data items AXMAJ, AXMIN, and ANGFAC have been built as attributes of the class. For this example, the data item BEAMRA has been built as a parameter of the CAPTURE method. As shown in Figure 109, the BEAMRA data item is a formal parameter of the CAPTURE subprogram.

Figure 113 shows the initial class built for the BOUNCE subprogram, as originally presented in Figure 105. The BEAMRA data item is not a formal parameter of the BOUNCE

```
class CLASS-4 attributes
    ANGFAC, AXMAJ, AXMIN, XLAMDA
    method BOUNCE ( C-4, C-5, PROJRA ): real
        begin
        RHOSTD := 0.95 * PROJRA
            * GET-ANGFAC ( C-4);
        BEAMRA := RADIUS (C-5, PROJRA);
        BOUNCE :=
            CAPTURE ( BEAMRA, AXMAJ, AXMIN, ANGFAC)
                * RHO ( RHOSTD, XLAMDA, ANGFAC)
        end
    superclass USER-OBJECT
```

Figure 108 Updated class built for BOUNCE
subprogram (as shown in Figure 109). Since BEAMRA is not a formal parameter and it has not been built as an attribute of CLASS-2, it is an example of PBOI Case 4 . In this case, there are no changes required to either CLASS-2 or CLASS-4. The only change made to the statements of the method BOUNCE is to ensure that BEAMRA is included as an actual parameter of the CAPTURE message.

Figure 114 shows CLASS-4 after the data items AXMAJ, AXMIN, and ANGFAC are converted from attributes of CLASS-4 to attributes of CLASS-2 (because of PBOI Case 1) and the call to CAPTURE has been converted to a message. As shown in the figure, the message CAPTURE ( C-6, BEAMRA) includes C-6, an instance of CLASS-2, and the data item BEAMRA passed as actual parameters.
5.6.5 Eliminating Duplicate Classes. In the call tree of an imperative design, it is possible that one subprogram will have multiple calls to another subprogram. Since the first step in converting a Category 3 subprogram is to transform all of the subprograms that it calls, the subprogram that is called multiple times will be converted multiple times. This introduces duplicate classes into the design returned to the Category 3 subprogram. Furthermore, there may be different combinations of PBOI cases for each of the calls to a subprogram. This means the signature of the method built for one call to a subprogram may not match the signature of the method built for another call to the subprogram.

```
real function CAPTURE
            ( BEAMRA, AXMAJ, AXMIN, ANGFAC )
    begin
    TILTFA := DSIN ( ANGFAC);
    RMAREA := AXMAJ * AXMIN;
    EFAREA := RMAREA * TILTFA;
    BMAREA := BEAMRA * BEAMRA;
    if EFAREA <= BMAREA * 6.0d0
        then USED := EFAREA / BMAREA;
            CAPTURE := 1.OdO - DEXP ( -USED)
        else CAPTURE := 1.OdO endif
    end
real function BOUNCE
    ( ANGFAC, RNGFAC, PROJRA, SIGB,
            RANGE, AXMAJ, AXMIN, XLAMDA)
    begin
    RHOSTD := 0.95 * PROJRA * ANGFAC;
    BEAMRA :=
        RADIUS ( PROJRA, SIGB, RNGFAC, RANGE);
    BOUNCE :=
        CAPTURE ( BEAMRA, AXMAJ, AXMIN, ANGFAC)
            * RHO ( RHOSTD, XLAMDA, ANGFAC)
    end
```

Figure 109 Subprograms CAPTURE and BOUNCE

To resolve this possible mismatch between the signature of methods in duplicate classes, the duplicate classes are identified and eliminated as part of the conversion of Category 3 subprograms. Two duplicate classes are replaced in the design with a new class that is built as follows. The sets of attributes from the two duplicate classes are compared and any attributes not in the intersection are moved from attributes to parameters. The attributes in the intersection become attributes of the new class. Since both duplicate classes include a method that implements a single subprogram, the set of operations for the new class includes the method built for this subprogram. The resulting class replaces the two duplicate classes in the design. If any other methods include calls to the methods from the duplicate classes, these calls are updated to match the signature of the method in the new class.

```
class CLASS-2 attributes
    AXMAJ, AXMIN, ANGFAC, BEAMRA
    method CAPTURE ( C-2 ): real
        begin
        TILTFA := DSIN ( GET-ANGFAC ( C-2));
        RMAREA := GET-AXMAJ ( C-2) *
            GET-AXMIN ( C-2);
        EFAREA := RMAREA * TILTFA;
    BMAREA := GET-BEAMRA ( C-2) *
        GET-BEAMRA ( C-2);
    if EFAREA <= BMAREA * 6.0d0
        then USED := EFAREA / BMAREA;
                            CAPTURE := 1.0dO - DEXP ( -USED)
        else CAPTURE := 1.OdO endif
        end
    superclass USER-OBJECT
```

Figure 110 Original class built for CAPTURE

For example, Figure 115 shows two classes built for the subprogram CAPTURE. The signatures of the two methods do not match since BEAMRA is passed as a parameter in one version and ANGFAC is passed as a parameter in the other. The classes are easily identified as duplicates because the names of the two classes are the same. These duplicate classes are replaced in the design by the class shown in Figure 116. The new version of CLASS-2 includes attributes from the intersection of the sets of attributes from the duplicate classes. The single operation in the new class is the method from the duplicate classes updated to properly access the attributes and parameters of the new class.

Duplicate classes may also be introduced into the design when different subprograms call the same subprogram. These duplicates will be identified and eliminated when the call subtrees that include the two calling subprograms join. This is guaranteed to happen at least at the main program.

### 5.7 Converting Category 1 Subprograms

Category 1 subprograms produce no data items but call other subprograms. These subprograms provide another opportunity to extract objects using the PBOI methodology.

```
class CLASS-2 attributes AXMAJ, AXMIN, ANGFAC
    method CAPTURE ( C-2, BEAMRA ): real
        begin
        TILTFA := DSIN ( GET-ANGFAC ( C-2));
        RMAREA := GET-AXMAJ ( C-2) *
            GET-AXMIN ( C-2);
        EFAREA := RMAREA * TILTFA;
        BMAREA := BEAMRA * BEAMRA;
        if EFAREA <= BMAREA * 6.0dO
            then USED := EFAREA / BMAREA;
                            CAPTURE := 1.0dO - DEXP ( -USED)
            else CAPTURE := 1.OdO endif
        end
    superclass USER-OBJECT
```

Figure 111 Updated class built for CAPTURE

Typically, the main program is classified as a Category 1 subprogram. All Category 1 subprograms other than the main program are transformed using the PBOI methodology defined for Category 3 subprograms. Because of the unique handling of the main program, the following assumption applies.

Assumption V.1. The main program of the system that is being converted to the object paradigm is uniquely identified.

There is no attempt in this research to identify the main program automatically. Since it is at the top of the call tree of imperative subprograms, such automatic identification of the main program should be straightforward to implement. The main program is transformed to the object-oriented paradigm as described in the following section.

### 5.8 Converting the Main Program

The main program of a system provides a unique opportunity to update the objectoriented design developed so far and is transformed using the rationale defined in this section. Before the main program is transformed, each of the subprograms called by the main program is transformed to the object-oriented paradigm. This depth-first transformation results in an object-oriented design that includes classes built for all of the sub-

```
class CLASS-4 attributes RNGFAC, PROJRA, SIGB,
    RANGE, XLAMDA
    method BOUNCE ( C-4, C-6 ): real
        begin
        RHOSTD := 0.95 * GET-PROJRA ( C-4)
            * GET-ANGFAC ( C-6);
        BEAMRA :=
            RADIUS ( PROJRA, SIGB, RNGFAC, RANGE);
        BOUNCE :=
            CAPTURE ( C-6, BEAMRA)
            * RHO ( RHOSTD, XLAMDA, ANGFAC)
        end
    superclass USER-OBJECT
```

Figure 112 Updated class built for BOUNCE
programs in the system (except the main program). A special class is built for the main program to represent the overall system class in the object-oriented design. This class has no attributes and only one method which implements the main program. The name of this class is CLASS-SYSTEM. In this way, the entire imperative design is converted into an object-oriented design using depth-first transformations.

In this research, an assumption is made that all data items in the imperative subprograms are either passed as parameters to the subprogram or declared locally in the subprogram; thus the main program is the origin of all data items being passed to the imperative subprograms. For this reason, the data items in the main program are used to create all of the instances required throughout the entire object-oriented design. Creating every object instance from data items in the main program leads to the following thesis.

Thesis V.4. Attributes that can be linked to the same main program data item are part of the same object.

This is used as the rationale for two design updates done at this stage of the main program transformation, as discussed in Section 5.8.1 and Section 5.8.2.

Since each of the subprograms called by the main program has been converted to a method, the subprogram calls in the main program must be transformed into messages that invoke these methods. Each method includes one or more object instances in its

```
class CLASS-4 attributes ANGFAC, RNGFAC, PROJRA,
            SIGB, RANGE, AXMAJ, AXMIN, XLAMDA
    method BOUNCE ( C-4 ): real
        begin
        RHOSTD := 0.95 * GET-PROJRA ( C-4)
            * GET-ANGFAC ( C-4);
        BEAMRA :=
            RADIUS ( PROJRA, SIGB, RNGFAC, RANGE);
        BOUNCE :=
            CAPTURE ( BEAMRA, AXMAJ, AXMIN, ANGFAC)
                * RHO ( RHOSTD, XLAMDA, ANGFAC)
        end
    superclass USER-OBJECT
```

Figure 113 Initial class built for BOUNCE
signature, so these instances must be created by the main program. Data items which appear as parameters in the imperative paradigm must be built as attributes of object instances in the object paradigm. The assignment statements that create these instances are inserted in the sequence of main program statements just before the message.

For example, Figure 117 shows the partial declaration of the subprogram LNKCALDWELLT. Assume this subprogram is called by the main program BMDSIM1. The formal parameters BETA, XLAMDA, and DIAM are shown in the figure as part of the signature of this subprogram. Figure 118 shows the partial declaration of the class CLASS-20 which has the method LNKCAL-DWELLT that implements the subprogram LNKCAL-DWELLT from Figure 117. The signature of the method is radically different from the signature of the subprogram because the data items BETA, XLAMDA, and DIAM are now attributes of a class and do not appear as parameters in the signature of the method. Instead, an instance of the class that has BETA, XLAMDA, and DIAM as attributes is included in the method LNKCAL-DWELLT, viz. C-22. The instance object C-22 is a formal parameter of the method LNKCAL-DWELLT and a corresponding object must be passed as an actual parameter of any messages that invoke the method LNKCAL-DWELLT. Since, in this example, the LNKCAL-DWELLT method is invoked by the main program BMDSIM1, the main program must build an instance to send in the message that invokes LNKCAL-DWELLT.

```
class CLASS-4 attributes RNGFAC, PROJRA, SIGB,
    RANGE, XLAMDA
    method BOUNCE ( C-4, C-6 ): real
        begin
        RHOSTD := 0.95 * GET-PROJRA ( C-4)
            * GET-ANGFAC ( C-6);
        BEAMRA :=
            RADIUS ( PROJRA, SIGB, RNGFAC, RANGE);
        BOUNCE :=
            CAPTURE ( C-6, BEAMRA)
                * RHO ( RHOSTD, XLAMDA, ANGFAC)
        end
    superclass USER-OBJECT
```

Figure 114 Updated class built for BOUNCE

Figure 119 shows the class CLASS-5 that, in this example, is the class of which C-22 is an instance. Note the method CREATE-CLASS-5 shown in the figure. This method is used to build new instances of CLASS-5 and is generated automatically when using the PBOI methodology. Section 6.11 .2 presents the transformation that formalizes the generation of this method. Figure 120 shows the class CLASS-SYSTEM built for the main program BMDSIM1. Some of the statements from the main program are shown in the figure. Specifically, the message that invokes the LNKCAL-DWELLT method is shown, and the message that creates C-22 is shown. Since C-22 must be passed to the LNKCAL-DWELLT method, it must first be created and inserted into the statements of the BMDSIM1 method. The variable C-22 (a computer-generated name) is assigned the object instance returned from the CREATE-CLASS-5 method shown in Figure 119. Note that the signature of the LNKCAL-DWELLT message is incomplete. If there are any other objects in the signature of the LNKCAL-DWELLT method, assignment statements are built and inserted into the statements of BMDSIM1.
5.8.1 Removing Duplicate Object Instances. Following the creation of object instances and the transformation of subprogram calls into messages, the next design update is to replace duplicate object instances. Duplicate object instances are defined as follows.

```
class CLASS-2 attributes AXMAJ, AXMIN, ANGFAC
    method CAPTURE ( C-2, BEAMRA ): real
        begin
        TILTFA := DSIN ( GET-ANGFAC ( C-2));
        RMAREA := GET-AXMAJ ( C-2) *
            GET-AXMIN ( C-2);
        EFAREA := RMAREA * TILTFA;
        BMAREA := BEAMRA * BEAMRA;
        if EFAREA <= BMAREA * 6.0d0
            then USED := EFAREA / BMAREA;
                    CAPTURE := 1.OdO - DEXP ( -USED)
            else CAPTURE := 1.0dO endif
        end
    superclass USER-OBJECT
class CLASS-2 attributes AXMAJ, AXMIN, BEAMRA
    method CAPTURE ( C-2, ANGFAC ): real
        begin
        TILTFA := DSIN ( ANGFAC);
        RMAREA := GET-AXMAJ ( C-2) *
            GET-AXMIN ( C-2);
        EFAREA := RMAREA * TILTFA;
        BMAREA := GET-BEAMRA ( C-2) * GET-BEAMRA ( C-2);
        if EFAREA <= BMAREA * 6.0dO
            then USED := EFAREA / BMAREA;
                    CAPTURE := 1.OdO - DEXP ( -USED)
        else CAPTURE := 1.0d0 endif
        end
    superclass USER-OBJECT
```

Figure 115 Duplicate classes built for CAPTURE

```
class CLASS-2 attributes AXMAJ, AXMIN
    method CAPTURE ( C-2, BEAMRA ): real
        begin
        TILTFA := DSIN ( ANGFAC);
        RMAREA := GET-AXMAJ ( C-2) *
                GET-AXMIN ( C-2);
        EFAREA := RMAREA * TILTFA;
        BMAREA := BEAMRA * BEAMRA;
        if EFAREA <= BMAREA * 6.0dO
            then USED := EFAREA / BMAREA;
                    CAPTURE := 1.0dO - DEXP ( -USED)
            else CAPTURE := 1.0dO endif
        end
    superclass USER-OBJECT
```

Figure 116 Class to replace the duplicates

```
procedure LNKCAL-DWELLTT ( ..., BETA, XLAMDA, DIAM, ... )
    begin
    ...
    end
```

Figure 117 Partial Signature of Subprogram LNKCAL-DWELLT

Definition V.6. Separate object instances that are built from the same class using the same data items are duplicate object instances.

Duplicate object instances which are identified in the main program are replaced with a single object instance built from the class. This is done to avoid duplicate copies of the instance attributes which may be updated by different methods. Having duplicate object instances would result in an inconsistent system state, so they are eliminated.

For example, Figure 121 shows the overall system class CLASS-SYSTEM built for the main program BMDSIM1. As shown in the figure, the three variables $\mathrm{C}-22, \mathrm{C}-32$, and $\mathrm{C}-36$ are created before being passed as actual parameters in the LNKCAL-DWELLT message. These three variables are all instances of CLASS-5, which is also shown in the figure. These instances are built by using the class's creation method, CREATE-CLASS-5. All three objects are built from the data items BETA, XLAMDA, and DIAM, thus all three objects are duplicate

```
class CLASS-20 attributes MIRR, MIRF, SLACT, IENG, SLANG
    method LNKCAL-DWELLT ( . . ., C-22, ..., )
        begin
        end
```

Figure 118 Partial Signature of Method LNKCAL-DWELLT

```
class CLASS-5 attributes BETA, XLAMDA, DIAM
    method CREATE-CLASS-5 ( A-BETA, A-XLAMDA, A-DIAM ) : a CLASS-5
        begin
        INST-CLASS-5 := new ( CLASS-5);
        SET-BETA ( INST-CLASS-5, A-BETA);
        SET-XLAMDA ( INST-CLASS-5, A-XLAMDA);
        SET-DIAM ( INST-CLASS-5, A-DIAM);
        CREATE-CLASS-5 := INST-CLASS-5
        end
```

Figure 119 Partial Class Containing Attributes BETA, XLAMDA, and DIAM
object instances. Figure 122 shows how method BMDSIM1 is updated by replacing the duplicate object instances $\mathrm{C}-22, \mathrm{C}-32$, and $\mathrm{C}-36$ with the single instance $\mathrm{C}-60$. The $\mathrm{C}-60$ instance is created using the CREATE-CLASS-5 and passed to the LNKCAL-DWELLT method in place of $\mathrm{C}-22, \mathrm{C}-32$, and $\mathrm{C}-36$. The instance $\mathrm{C}-60$ is passed as three actual parameters in order to properly replace the object instances C-22, C-32, and C-36 in the invocation of LNKCAL-DWELLT. This allows the LNKCAL-DWELLT method to remain unchanged and properly refer to the instance $\mathbf{C - 6 0}$ using its three formal parameters. Future research should explore the changes required to methods such as LNKCAL-DWELLT in order to refer to a single formal parameter when eliminating duplicate objects.
5.8.2 Merging Overlapping Classes. The second update done to the design when transforming the main program is to merge classes that overlap.

Definition V.7. A class overlaps another class when an instance of each class is built using at least one common data item.

```
class CLASS-SYSTEM attributes
    method BMDSIM1 ( )
        begin
        C-22 := CREATE-CLASS-5 ( BETA, XLAMDA, DIAM);
        LNKCAL-DWELLT ( C-46, ..., C-22, ... );
        *
        end
```

Figure 120 Creating an Instance of CLASS-5

In this case, the overlapping classes are merged into a new class. The merging of two classes unions the attributes and operations of the classes into a new class. A single new instance that is built from this new class replaces the two separate instances. Throughout the entire design, any instances from the overlapping classes are replaced by an instance of the new class. As with duplicate object instances, overlapping classes are merged in order to avoid duplicating instance attributes which may be changed by different methods. Building object instances from overlapping classes would result in an inconsistent system state, so the classes are merged.

For example, Figure 123 shows overlapping classes CLASS-19 and CLASS-17 as well as the overall system class CLASS-SYSTEM built for the main program BMDSIM1. As shown in the figure, the instance C-56 is built as an instance of CLASS-19 using the data items DWELLT and NMIRL. The instance C-57 is built as an instance of CLASS-17 using the data item DWELLT. By Definition V.7, classes CLASS-19 and CLASS-17 overlap and are merged into one new class.

Figure 124 shows the result of merging CLASS-19 and CLASS-17 into the new class CLASS-20. The attributes of CLASS-20 are the union of the attributes of CLASS-19 and CLASS-17. The names of the attributes in CLASS-20 match the names of the data items in the main program. The attribute names are changed because the names of the attributes in overlapping classes do not always match. ${ }^{1}$ As shown in the figure, CLASS-20 includes

[^4]```
class CLASS-5 attributes BETA, XLAMDA, DIAM
    method CREATE-CLASS-5
            ( A-BETA, A-XLAMDA, A-DIAM ) : a CLASS-5
        begin
        INST-CLASS-5 := new ( CLASS-5);
        SET-BETA ( INST-CLASS-5, A-BETA);
        SET-XLAMDA ( INST-CLASS-5, A-XLAMDA);
        SET-DIAM ( INST-CLASS-5, A-DIAM);
        CREATE-CLASS-5 := INST-CLASS-5
        end
class CLASS-SYSTEM attributes
    method BMDSIM1 ( )
        begin
        C-22 := CREATE-CLASS-5 ( BETA, XLAMDA, DIAM);
        C-32 := CREATE-CLASS-5 ( BETA, XLAMDA, DIAM);
        C-36 := CREATE-CLASS-5 ( BETA, XLAMDA, DIAM);
        LNKCAL-DWELLT ( C-46, C-22, C-32, C-36, ... );
        end
```

Figure 121 Duplicate object instances
all of the methods from CLASS-19 and CLASS-17. The new instance C-61 is created using the creation method from CLASS-20 and passed to the LNKCAL-DWELLT message in place of C-56 and C-57. The design is updated by removing CLASS-19 and CLASS-17 and adding CLASS-20. All instances of the classes CLASS-19 and CLASS-17 are converted to instances of the class CLASS-20 throughout the entire design. In this way, the design is updated to remove the overlapping classes when transforming the main program.

### 5.9 Converting Category \& Subprograms

Category 4 subprograms produce multiple outputs without calling other subprograms. By using program slicing [22,76], a multiple-output subprogram is converted into independent program slices that each produce a single output. These program slices are used to build new imperative subprograms, so a Category 4 subprogram is sliced into

```
class CLASS-SYSTEM attributes
    method BMDSIM1 ( )
        begin
    ...
    C-60 := CREATE-CLASS-5 ( BETA, XLAMDA, DIAM);
    LNKCAL-DWELLT ( C-46, C-60, C-60, C-60, ... );
    end
```

Figure 122 Duplicates removed
multiple Category 2 subprograms. This program slicing process is discussed in detail in Section 5.11.

### 5.10 Converting Category 5 Subprograms

A Category 5 subprogram produces multiple outputs and calls other subprograms. As with Category 4 subprograms, program slicing is used to convert a Category 5 subprogram into multiple slices. These slices are used to build new imperative subprograms that may or may not call other subprograms, so the Category 5 subprograms are sliced into new subprograms that are classified as either Category 3 or Category 2 subprograms. This process of program slicing is discussed in detail in Section 5.11.

### 5.11 Program Slicing

This section defines how program slicing is used in the PBOI method to simplify the transformation task. Program slicing has already been defined in the literature [22,30,76]. In this research, a new imperative subprogram is built by providing an appropriate name and using a program slice as the sequence of statements in the subprogram. The sequence of formal parameters for this new subprogram consists of all of the original formal parameters that are referenced in the program slice.

By using program slicing in this way, the behavior of a Category 4 or Category 5 subprogram is projected into multiple subprograms. Each of the subprograms produces one

```
class CLASS-19 attributes A, NVAL
    method MAXA-NMAX ( C-45, NMAX, AMAX )
        begin
        end
class CLASS-17 attributes DWELLT
    method DASET-DWELLT ( C-43, DT, TLASE)
        begin
        ..
        end
class CLASS-SYSTEM attributes
    method BMDSIM1 ( )
        begin
        C-56 := CREATE-CLASS-19 ( DWELLT, NMIRL);
        C-57 := CREATE-CLASS-17 ( DWELLT );
        LNKCAL-DWELLT ( C-46, ..., C-56, C-57);
        end
```

Figure 123 Overlapping object classes

```
class CLASS-20 attributes NMIRL, DWELLT
    method CREATE-CLASS-20
            ( A-NMIRL, A-DWELLT ) : a CLASS-20
        begin
        INST-CLASS-20 := new (CLASS-20);
        SET-NMIRL ( INST-CLASS-20, A-NMIRL);
        SET-DWELLT ( INST-CLASS-20, A-DWELLT);
        CREATE-CLASS-20 := INST-CLASS-20
        end
    method MAXA-NMAX ( C-45, NMAX, AMAX )
        begin
        end
    method DASET-DWELLT ( C-43, DT, TLASE)
        begin
    end
class CLASS-SYSTEM attributes
    method BMDSIM1 ( )
        begin
        C-61 := CREATE-CLASS-20 ( NMIRL, DWELLT);
        LNKCAL-DWELLT ( C-46, ..., C-61, C-61);
        end
```

Figure 124 Overlapping classes merged
of the data items originally produced by the Category 4 or Category 5 subprogram. Since the PBOI method relies on the parameters being passed from subprogram to subprogram and the new subprograms include only those parameters required to produce the data item, program slicing has grouped these parameters according to how they are used instead how they are organized. This new grouping helps the PBOI method move away from the structured analysis and design that was done on the original system (if any) and move towards an object-oriented design.

For example, Figure 125 shows the imperative subprogram BOOSTR using GIL syntax. This is a Category 4 subprogram because it produces the two data items $R$ and $V$ and calls no other (user-defined) subprograms. Figure 126 shows the new imperative subprogram

```
procedure BOOSTR ( ITYPE, T, R, V )
    begin
    RE := 6378.16d0;
    PI2 := 1.570796327d0;
    ITYP := ABS ( ITYPE);
    if ITYP = 1
        then ALT := -5.71d0 + (0.24d0 + 0.00363d0 * T) * T;
            RANG := -0.172d0 + (-0.419d0 + 0.0112dO * T) * T;
            VEL := -0.196d0 + (0.0236d0 + 1.6d-6 * T) * T;
            GAMA := 1.12dO + (-0.00751d0 + 1.82d-5 * T) * T
        else endif;
    if ALT < 0 then ALT := 0 else endif;
    if RANG < O then RANG := 0 else endif;
    if GAMA > PI2 then GAMA := PI2 else endif;
    RM := RE + ALT;
    THETA := RANG / RE;
    R ( 1) := RM * DCOS ( THETA);
    R ( 2) := RM * DSIN ( THETA);
    R (3) := 0;
    BETA := THETA + PI2 - GAMA;
    V ( 1) := VEL * DCOS ( BETA);
    V ( 2) := VEL * DSIN ( BETA);
    V ( 3) := 0
    end
```

Figure 125 Imperative subprogram BOOSTR
built from the program slice for the data item $R$. The name of the subprogram is built by appending the data item identifier onto the name of the original subprogram, hence the name BOOSTR-R. The formal parameters of the new subprogram are the data items ITYPE, T, and R. The new subprogram is classified as a Category 2 subprogram because it produces the single data item $R$ and calls no other subprograms. Figure 127 shows the new imperative subprogram built from the program slice for the data item V. This new subprogram is classified as a Category 2 subprogram because it produces the single data item $V$ and calls no other subprograms.

```
procedure BOOSTR-R ( ITYPE, T, R ) begin
    RE := 6378.16d0;
    ITYP := ABS ( ITYPE);
    if ITYP = 1
        then ALT := -5.71d0 + (0.24d0 + 0.00363d0 * T) * T;
            RANG := -0.172d0 + (-0.419d0 + 0.0112d0 * T) * T
        else endif;
    if ALT < 0 then ALT := 0 else endif;
    if RANG < 0 then RANG := 0 else endif;
    RM := RE + ALT;
    THETA := RANG / RE;
    R ( 1) := RM * DCOS ( THETA);
    R ( 2) := RM * DSIN ( THETA);
    R ( 3) := 0
    end
```


## Figure 126 Imperative subprogram BOOSTR-R

### 5.12 Inter-Procedural Slicing

When a subprogram being sliced includes calls to other subprograms, a more robust program slicing process is required. Inter-procedural slicing [30] is a special form of program slicing that builds a program slice from a subprogram taking into consideration the calls being made to other subprograms. Only the calls to subprograms that are producing data items required for the slice are included in the program slice.

This research uses inter-procedural slicing when slicing Category 5 subprograms. A new subprogram is built from the program slice generated, as explained in Section 5.11. For example, Figure 128 shows the Category 5 subprogram TRAJ, which includes calls to the BOOSTR-R subprogram and BOOSTR-V subprogram. Originally, the TRAJ subprogram called the BOOSTR subprogram. Because BOOSTR is a Category 4 subprogram it has already been sliced and the calls to the new subprograms have replaced the call to BOOSTR. In this research, any calls in a Category 5 subprogram that sliced using inter-procedural slicing must produce a single data item, i.e. calls to Category 2 or Category 3 subprograms. An overall process that meets this constraint is used to convert Category 4 and Category 5 subprograms as explained in Section 5.15. Since the TRAJ subprogram includes the calls to BOOSTR-R and BOOSTR-V, inter-procedural slicing must be used. For example, Figure 129

```
procedure BOOSTR-V ( ITYPE, T, V ) begin
    RE := 6378.16d0;
    PI2 := 1.570796327d0;
    ITYP := ABS ( ITYPE);
    if ITYP = 1
        then RANG := -0.172d0 + (-0.419d0 + 0.0112d0 * T) * T;
            VEL := -0.196d0 + (0.0236d0 + 1.6d-6 * T) * T;
            GAMA := 1.12d0 + (-0.00751d0 + 1.82d-5 * T) * T
        else endif;
    if RANG < 0 then RANG := 0 else endif;
    if GAMA > PI2 then GAMA := PI2 else endif;
    THETA := RANG / RE;
    BETA := THETA + PI2 - GAMA;
    V ( 1) := VEL * DCOS ( BETA);
    V ( 2) := VEL * DSIN ( BETA);
    V ( 3) := 0
    end
```

Figure 127 Imperative subprogram BOOSTR-V
shows the result of inter-procedural slicing the TRAJ subprogram for the data item $R$. The TRAJ-R subprogram includes a call to the BOOSTR-R subprogram, but not a call to the BOOSTR-V subprogram. This is because the data item produced by the BOOSTR-V subprogram is not needed for the slice on data item $R$ in TRAJ-R. Figure 130 shows the result of inter-procedural slicing the TRAJ subprogram for the data item $V$. The TRAJ-V subprogram includes a call to the BOOSTR-V subprogram but not the BOOSTR-R subprogram.

### 5.13 Masking Output Parameters

Program slicing does not always convert a Category 4 or Category 5 subprogram into a Category 2 or Category 3 subprogram. In some cases, one output data item must be defined in order to produce a second output data item. Thus, if both data items were originally produced by the subprogram, then the new subprogram built from a program slice on the second output data item will still include the first output data item and will be classified as either a Category 4 or Category 5 subprogram.

For example, Figure 131 shows the Category 4 subprogram LASP. This subprogram

```
procedure TRAJ
    ( ITYPE, T, VI, RBO, VBO, TBO, TFBOT, AR, R, V )
    begin
    XMU := 398601.2d0;
    WDOT := 7.292115600000001d-5;
    if T < TBO
        then BOOSTR-R ( ITYPE, T, RP);
            BOOSTR-V ( ITYPE, T, VP);
            MTRT ( RP, AR, RO);
            MTRT ( VP, AR, VT);
            TEMP-40 := 1;
            VADD ( TEMP-40, VT, VI, VO)
        else endif;
    DLONT := -WDOT * T;
    TEMP-41 := 1;
    MAT ( TEMP-41, DLONT, ARL);
    MTRT ( RO, ARL, R);
    MTRT ( VO, ARL, V)
    end
```

Figure 128 Original imperative subprogram TRAJ
produces the data items BFLUX, BFLU, and TD. Figure 132 shows the new subprograms LASP-BFLUX, LASP-BFLU, and LASP-TD, built from the program slices for each of these three data items. Note that the LASP-TD subprogram still includes definitions of the BFLUX and BFLU output data items. In fact, there is no difference between the LASP subprogram and the LASP-TD subprogram. The new subprogram is still classified as a Category 4 subprogram and has not been converted to a Category 2 subprogram.

To solve this problem, certain data items in the new subprogram can be masked so that they are not produced by the subprogram. To mask a data item, a new local variable is created and any references to the data item are replaced with references to the local variable. Because of Restriction III.1, the data items being masked cannot be both input and output parameters. This means there is no need to initialize the local variable used to mask a data item. The definition of the data item becomes a definition of the local variable, thus the data item is no longer produced by the new subprogram. Figure 133 shows the LASP-TD subprogram after the BFLUX and BFLU data items have been masked.

```
procedure TRAJ-R
    ( ITYPE, T, RBO, VBO, TBO, TFBOT, AR, R )
    begin
    XMU := 398601.2d0;
    WDOT := 7.292115600000001d-5;
    if T < TBO
        then BOOSTR-R ( ITYPE, T, RP);
            MTRT ( RP, AR, RO)
        else endif;
    DLONT := -WDOT * T;
    TEMP-41 := 1;
    MAT ( TEMP-41, DLONT, ARL);
    MTRT ( RO, ARL, R)
    end
```

Figure 129 Imperative subprogram TRAJ-R

These data items still appear in the signature of the LASP-TD subprogram in order to localize the masking process. The identifiers that are used for the formal parameters must not match the identifiers used for the local variables in order to created the disconnect that masks the output parameters.

### 5.14 Conservative Slicing

By including all statements from a subprogram that are needed to produce a data item, statements that appear to be unnecessary appear in the program slice. For example, Figure 134 shows the program slice built for the data item $R$ from the subprogram KEP. The second while loop in the subprogram is unnecessary code, but appears in the program slice because definitions of the data item I are required to produce R. Extra statements such as this while loop are included in the slice because this research is using a conservative approach to program slicing. There is no attempt in the research to optimize the slices or filter unnecessary statements.

```
procedure TRAJ-V
    ( ITYPE, T, VI, RBO, VBO, TBD, TFBOT, AR, V )
    begin
    XMU := 398601.2dO;
    WDOT := 7.292115600000001d-5;
    if T < TBO
        then BOOSTR-V ( ITYPE, T, VP);
            MTRT ( VP, AR, VT);
            TEMP-40 := 1;
            VADD ( TEMP-40, VT, VI, VO)
        else endif;
    DLONT := -WDOT * T;
    TEMP-41 := 1;
    MAT ( TEMP-41, DLONT, ARL);
    MTRT ( VO, ARL, V)
    end
```

Figure 130 Imperative subprogram TRAJ-V

```
procedure LASP
    ( POWL, EFFNCY, AREA, CFLUM, CFLUS, XKF, BFLUX, BFLU, TD )
    begin
    BFLUX := EFFNCY * POWL / AREA;
    BFLU := CFLUM + XKF * CFLUS;
    TD := BFLU / BFLUX
    end
```

Figure 131 Imperative subprogram LASP

### 5.15 Slicing for the Main Program

This section defines the overall process used to ensure all Category 4 and Category 5 subprograms are converted to Category 2 or Category 3 subprograms. The overall collection of imperative subprograms to be converted to the object-oriented paradigm is stored in a structured design, as described in Section 3.7.

The first step in this process is to build a program slice for each of the data items produced by a Category 4 or Category 5 subprogram. Inter-procedural slicing is not used at this point. A new subprogram is built for each program slice and added to the structured design. Next, the original subprogram calls in all Category 3, Category 5, and Category 1

```
procedure LASP-BFLUX ( POWL, EFFNCY, AREA, BFLUX ) begin
    BFLUX := EFFNCY * POWL / AREA end
procedure LASP-BFLU ( CFLUM, CFLUS, XKF, BFLU ) begin
    BFLU := CFLUM + XKF * CFLUS end
procedure LASP-TD
    ( POWL, EFFNCY, AREA, CFLUM, CFLUS, XKF, BFLUX, BFLU, TD )
    begin
    BFLUX := EFFNCY * POWL / AREA;
    BFLU := CFLUM + XKF * CFLUS;
    TD := BFLU / BFLUX
    end
```

Figure 132 Imperative subprogram LASP-TD

```
procedure LASP-TD
    ( POWL, EFFNCY, AREA, CFLUM, CFLUS, XKF, BFLUX, BFLU, TD )
    begin
    LOCAL-1 := EFFNCY * POWL / AREA;
    LOCAL-2 := CFLUM + XKF * CFLUS;
    TD := LOCAL-2 / LOCAL-1
    end
```

Figure 133 BFLUX and BFLU masked
subprograms other than the main program are replaced by calls to the new subprograms just created. This means all the original Category 4 and Category 5 subprograms are still in the structured design, but the only calls to such subprograms are in the main program.

At this point, program slices are built using inter-procedural slicing for each data item produced by a subprogram that is called from the main program. New subprograms are built from these program slices. Each call to the original subprogram is replaced by calls to the new subprograms. Note that calls to Category 0 and Category 1 subprograms are not changed. The calls to Category 2 and Category 3 subprograms are also not changed because a subprogram built from the program slice of a Category 2 or Category 3 subprogram is identical to the original subprogram.

```
procedure KEP-R ( RO, VO, T, XMU, R )
    begin
    I := 1;
    while I <= 3 do
        begin
        R (I) := RO (I) + VO (I);
        I := I + 1
        end;
    I := 1;
    while I <= 3 do
        begin
        I := I + 1
        end
    end
```

Figure 134 Imperative subprogram KEP-R

However, it is important to use inter-procedural slicing for Category 3 subprograms because they are allowed to call subprograms from any other category. The first two steps presented above ensure the calls to Category 4 or Category 5 subprograms have already been converted into calls to new subprograms. Inter-procedural slicing ensures that only the calls to subprograms that produce data items required for the program slice are included in the program slice.

For example, Figure 135 shows an alternate version of the TRAJ subprogram that is classified as a Category 3 subprogram producing the single data item $R$. By using interprocedural slicing on this Category 3 subprogram, the subprogram calls not producing data items for the program slice are eliminated. Figure 136 shows the updated TRAJ subprogram. Note the call to BOOSTR-R and two calls to MTRT have been eliminated. Subprogram calls such as these constitute dead code [22] and are eliminated by inter-procedural slicing.

In the main program, the calls to Category 4 and Category 5 subprograms are replaced with calls to the new subprograms built from program slices. In this way, none of the Category 4 or Category 5 subprograms are called by any of the subprograms in the design. Furthermore, by using inter-procedural slicing, all of the subprograms called

```
procedure TRAJ
    ( ITYPE, T, VI, RBO, VBO, TBO, TFBOT, AR, R )
    begin
    XMU := 398601.2d0;
    WDOT := 7.292115600000001d-5;
    if T < TBO
        then BOOSTR-R ( ITYPE, T, RP);
                BOOSTR-V ( ITYPE, T, VP);
            MTRT ( RP, AR, RO);
            MTRT ( VP, AR, VT);
            TEMP-40 := 1;
            VADD ( TEMP-40, VT, VI, VO)
        else endif;
    DLONT := -WDOT * T;
    TEMP-41 := 1;
    MAT ( TEMP-41, DLONT, ARL);
    MTRT ( RO, ARL, R);
    MTRT ( VO, ARL, V)
    end
```

Figure 135 TRAJ as a Category 3 subprogram
by the main program produce exactly one data item. Several transformations have been defined that formalize this conversion process. Chapter VI presents these transformations.

### 5.16 Discussion

The conversion of imperative subprograms to the object paradigm using the PBOI methodology results in a rudimentary object-oriented design. The inheritance associations are rudimentary because the inheritance built during the conversion is a flat hierarchy where every class built in the object model inherits from the overall super class USER-OBJECT. The aggregation associations are also rudimentary because, while allowed, aggregation associations are not identified by the PBOI methodology. This discussion leads to the following limitations on the conversions from the imperative subprograms to the object paradigm.

Limitation V.1. Every class in the design extracted using the PBOI methodology inherits from the overall super-class USER-OBJECT.

```
procedure TRAJ
    ( ITYPE, T, RBO, VBO, TBO, TFBOT, AR, R )
    begin
    XMU := 398601.2d0;
    WDOT := 7.292115600000001d-5;
    if T < TBO
        then BOOSTR-R ( ITYPE, T, RP);
            MTRT ( RP, AR, RO)
        else endif;
    DLONT := -WDOT * T;
    TEMP-41 := 1;
    MAT ( TEMP-41, DLONT, ARL);
    MTRT ( RO, ARI, R)
    end
```

Figure 136 Updated Category 3 subprogram TRAJ

Limitation V.2. Aggregation associations are not identified by the PBOI methodology.
These two limitations are not unreasonable. Future research could possibly remove these limitations by expanding the PBOI methodology.

### 5.17 Summary

This chapter has provided an informal description of how imperative subprograms are transformed to the object-oriented paradigm along with some rationale for the approach. The Parameter-Based Object Identification (PBOI) method for extracting objects from imperative subprograms was presented. The taxonomy of imperative subprograms used to simplify the development of the PBOI method was also presented. Each category of imperative subprogram was explained and the PBOI transformation for converting that category of subprogram to the object-oriented paradigm was defined. The process of converting Category 4 and Category 5 subprograms to Category 2 and Category 3 subprograms by using program slicing and inter-procedural slicing was also explained. Transformations that formalize the PBOI method have been defined and are presented in the following chapter.

## VI. Formal Transformations

### 6.1 Introduction

This chapter presents transformations that formalize the PBOI methodology, which was presented in Chapter V. Proof that these transformations maintain functional equivalence is presented in Chapter VII. Table 1 shows the order in which the transformations are presented in the chapter. The transformations that convert statements and expressions are presented first, since the transformations for subprograms build on them. Transformations for each category of subprogram are presented followed by the formalized transformations for program slicing. Finally, the transformations for the main program are presented along with the transformation for the entire imperative design.

In order to formally classify a subprogram $S$, let $C a t_{i}(S)$ indicate the category to which a subprogram belongs, such that $\operatorname{Cat}_{i}(S)$ is true when $S$ is a Category $i$ subprogram. Let $C a t_{M}(S)$ indicate that the subprogram $S$ is a Category 1 subprogram that is also the main program.

### 6.2 Transforming Statements ( $\theta$ )

This section defines the formal transformation for converting GIM statements to GOM statements. This transformation is used when converting a subprogram to the object paradigm. Recall from Section 3.24 that a GIM subprogram is defined by the following tuple.

$$
<i d_{S}, P_{i n}, P_{o u t}, P_{r e t}, P_{l o c}, \Sigma>
$$

The GOM entity that is analogous to a GIM subprogram is the method. In order to convert the GIM subprogram to the object paradigm, the GIM subprogram is converted to a GOM method, which is represented as a tuple of the following form.

$$
<i d_{S}, Q_{t a r}, Q_{o b j}, Q_{f o r m}, Q_{r e t}, Q_{l o c}, \Psi>
$$

| Transformation | Constructs |
| :---: | :---: |
| $\theta_{v} \theta_{V}$ | variables |
| $\theta_{e} \theta_{E}$ | expressions |
| $\theta$ | statements |
| $\delta_{e}$ | accesses in expressions |
| $\delta$ | variable accesses |
| $\delta_{e}^{-1}$ | accesses in expressions |
| $\delta^{-1}$ | attribute accesses |
| $\sigma$ | subprograms |
| $\sigma_{2}$ | Category 2 subprograms |
| $T_{A}^{\delta}$ | parameters to attributes |
| $T_{A}^{\delta^{-1}}$ | attributes to parameters |
| $\gamma$ | attribute accesses |
| $T_{A}^{\gamma}$ | attributes |
| $T_{A}^{\gamma}$ | known attributes |
| $T_{p b o i}^{1} T_{p b o i}^{2} T_{p b o i}^{3} T_{p b o i}^{4}$ | subprograms |
| $T_{\text {pboi }}$ | subprograms |
| $v_{m}$ | subprogram calls |
| $v_{e}$ | calls in expressions |
| $v$ | calls in statements |
| $T_{l}^{v}$ | calls in subprograms |
| $T_{o}^{v}$ | messages in methods |
| $T_{\text {class }}^{\text {dup }}$ | duplicate classes |
| $\sigma_{3}$ | Category 3 subprograms |
| $\sigma_{1}$ | Category 1 subprograms |
| $T_{o b j}^{d u p}$ | duplicate objects |
| $\kappa \kappa^{-1}$ | "GET-", "SET-", and "CREATE-" methods |
| $T_{\text {class }}^{\text {merge }}$ | classes |
| $T_{m}^{a s} v_{e}^{a s}$ | assignment statements |
| $v_{M}$ | main program messages |
| $T_{M}^{v}$ | main program messages |
| $\sigma_{M}$ | main program |
| $T_{p s}^{\text {build }} T_{p s}^{\text {calls }}$ | program slices |
| $T_{p s}^{\text {main }}$ | main program slice calls |
| $\sigma_{F}$ | imperative design |

Table 1 Formal transformations

As an imperative subprogram is converted to an object-oriented method, each imperative statement in $\Sigma$ must be converted to an object-oriented statement in $\Psi$. One difference between an imperative statement and an object-oriented statement is the way these statements access data items. In an imperative statement, data items are stored in variables including local variables and parameters. These variables are accessed by using the name of the variable. In object-oriented statements, data items may be stored in attributes of objects. These data items must be accessed either directly by using the name of the attribute and the name of the object or indirectly by sending a message to the appropriate object. Because of this difference, each GIM statement must be converted to a GOM statement.

Part of the conversion process is to transform each GIM variable into a GOM variable. Let $\theta_{v}$ be the transformation that converts a GIM variable into a GOM variable. Let $v$ be a GIM variable and let $v^{\prime}$ be a GOM variable. In order to formalize the conversion, a notation is presented here that denotes the class of AST that represents the GIM entity and the GOM entity. For example, GOM variables are represented using the gom-variable class. This is formalized by using a boolean function named gom-variable that returns true if the entity is a gom-variable and false otherwise. The $\theta_{v}$ transformation is defined below.

$$
\begin{aligned}
& \theta_{v}(v)=v^{\prime} \text { where } \\
& \quad \text { gim-variable }(v) \text { and gom-variable }\left(v^{\prime}\right) \text { and } \\
& \forall v_{1}, v_{2} \\
& \quad \theta_{v}\left(v_{1}\right)=\theta_{v}\left(v_{2}\right) \Rightarrow v_{1}=v_{2}
\end{aligned}
$$

Since GIM input statements include sequences of variables, the following transformation of sequences of variables is presented. Let $V$ represent a sequence of GIM variables and let $V^{\prime}$ represent a sequence of GOM variables. Let $\operatorname{pos}(v, V)$ represent the ordinal position of the variable $v$ in the sequence $V$. The $\theta_{V}$ transformation converts $V$ to $V^{\prime}$ as defined below.

$$
\begin{aligned}
& \theta_{V}(V)=V^{\prime} \text { where } \\
& \forall v \in V \\
& v^{\prime}=\theta_{v}(v) \text { and } \\
& v^{\prime} \in V^{\prime} \text { and } \\
& \quad \operatorname{pos}(v, V)=\operatorname{pos}\left(v^{\prime}, V^{\prime}\right)
\end{aligned}
$$

Another part of the conversion process is to transform each of the GIM expressions into GOM expressions. Let $\theta_{e}$ be the transformation that converts a GIM expression into a GOM expression. Let $e$ be a GIM expression and let $e^{\prime}$ be a GOM expression. The $\theta_{e}$ transformation is defined as shown in Figure 137.

GIM output statements include a sequence of expressions. In order to transform such sequences, let $\theta_{E}$ represent a formal transformation of a sequence of expressions. Let $E$ be a sequence of expressions. Let $\operatorname{pos}(e, E)$ represent the ordinal position of a expression $e$ in the sequence of expressions $E$. The transformation is defined below.

$$
\begin{aligned}
& \theta_{E}(E)=E^{\prime} \text { where } \\
& \forall e \in E \\
& \quad e^{\prime}=\theta_{e}(e) \text { and } \\
& \quad e^{\prime} \in E^{\prime} \text { and } \\
& \quad \operatorname{pos}(e, E)=\operatorname{pos}\left(e^{\prime}, E^{\prime}\right)
\end{aligned}
$$

Let $\theta$ be a transformation that converts a sequence of imperative statements into a sequence of object-oriented and imperative statements. Let $\Sigma$ represent a sequence of imperative statements and let $\Psi^{\prime}$ represent a sequence of object-oriented and imperative statements. Let $\operatorname{pos}(s, \Sigma)$ represent the ordinal position of a statement $s$ in the sequence $\Sigma$. The $\theta$ transformation uses the $\theta_{v}, \theta_{e}, \theta_{V}$, and $\theta_{E}$ transformations and is defined as shown in Figure 138. GIM procedure call statements are not transformed to objectoriented statements by the $\theta$ transformation. GIM function calls in expressions are not transformed by the $\theta_{e}$ transformation. This explains why $\Psi^{\prime}$ is a mixture of imperative statements and object-oriented statements.
$\theta_{e}(e)=e^{\prime}$ where
imperative-variable $(e) \Rightarrow e^{\prime}=\theta_{v}(e)$ and
imperative-function-call $(e) \Rightarrow e^{\prime}=e$ and
imperative-literal-boolean $(e) \Rightarrow$ gom-literal-boolean $\left(e^{\prime}\right)$ and $e=e^{\prime}$ and imperative-literal-integer $(e) \Rightarrow$ gom-literal-integer $\left(e^{\prime}\right)$ and $e=e^{\prime}$ and imperative-literal-real $(e) \Rightarrow$ gom-literal-real $\left(e^{\prime}\right)$ and $e=e^{\prime}$ and imperative-literal-string $(e) \Rightarrow$ gom-literal-string $\left(e^{\prime}\right)$ and $e=e^{\prime}$ and imperative-addition $(e)$ and $e=\left\langle e_{1},+, e_{2}\right\rangle \Rightarrow$
gom-addition $\left(e^{\prime}\right)$ and $e^{\prime}=<\theta_{e}\left(e_{1}\right),+, \theta_{e}\left(e_{2}\right)>$ and
imperative-and $(e)$ and $e=<e_{1}$, and, $\left.e_{2}\right\rangle \Rightarrow$
$\operatorname{gom}-\operatorname{and}\left(e^{\prime}\right)$ and $e^{\prime}=<\theta_{e}\left(e_{1}\right)$, and, $\theta_{e}\left(e_{2}\right)>$ and
imperative-concat $(e)$ and $e=\left\langle e_{1}, \&, e_{2}\right\rangle \Rightarrow$
gom-concat $\left(e^{\prime}\right)$ and $e^{\prime}=<\theta_{e}\left(e_{1}\right), \&, \theta_{e}\left(e_{2}\right)>$ and
imperative-division(e) and $e=\left\langle e_{1}, /, e_{2}\right\rangle \Rightarrow$
gom-division $\left(e^{\prime}\right)$ and $\boldsymbol{e}^{\prime}=<\theta_{e}\left(e_{1}\right), /, \theta_{e}\left(e_{2}\right)>$ and
imperative-equal $(e)$ and $e=<e_{1},=, e_{2}>\Rightarrow$
gom-equal $\left(e^{\prime}\right)$ and $e^{\prime}=<\theta_{e}\left(e_{1}\right),=, \theta_{e}\left(e_{2}\right)>$ and
imperative-exponent(e) and $e=\left\langle e_{1},{ }^{* *}, e_{2}\right\rangle \Rightarrow$
gom-exponent $\left(e^{\prime}\right)$ and $e^{\prime}=<\theta_{e}\left(e_{1}\right),{ }^{* *}, \theta_{e}\left(e_{2}\right)>$ and
imperative-greater-than-or-equal $(e)$ and $\left.e=\left\langle e_{1},\right\rangle=, e_{2}\right\rangle \Rightarrow$
gom-greater-than-or-equal $\left(e^{\prime}\right)$ and $\left.e^{\prime}=\left\langle\theta_{e}\left(e_{1}\right),\right\rangle=, \theta_{e}\left(e_{2}\right)\right\rangle$ and
imperative-greater-than $(e)$ and $\left.e=\left\langle e_{1},\right\rangle, e_{2}\right\rangle \Rightarrow$
gom-greater-than $\left(e^{\prime}\right)$ and $e^{\prime}=<\theta_{e}\left(e_{1}\right),>, \theta_{e}\left(e_{2}\right)>$ and
imperative-less-than-or-equal $(e)$ and $e=\left\langle e_{1},<=, e_{2}\right\rangle \Rightarrow$
gom-less-than-or-equal $\left(e^{\prime}\right)$ and $e^{\prime}=<\theta_{e}\left(e_{1}\right),<=, \theta_{e}\left(e_{2}\right)>$ and
imperative-less-than $(e)$ and $e=\left\langle e_{1},<, e_{2}\right\rangle \Rightarrow$
gom-less-than $\left(e^{\prime}\right)$ and $e^{\prime}=<\theta_{e}\left(e_{1}\right),<, \theta_{e}\left(e_{2}\right)>$ and
imperative-multiplication(e) and $e=\left\langle e_{1},{ }^{*}, e_{2}\right\rangle \Rightarrow$
gom-multiplication $\left(e^{\prime}\right)$ and $e^{\prime}=<\theta_{e}\left(e_{1}\right),{ }^{*}, \theta_{e}\left(e_{2}\right)>$ and
imperative-not-equal $(e)$ and $e=\left\langle e_{1},\langle \rangle, e_{2}\right\rangle \Rightarrow$
gom-not-equal $\left(e^{\prime}\right)$ and $e^{\prime}=\left\langle\theta_{e}\left(e_{1}\right),<>, \theta_{e}\left(e_{2}\right)>\right.$ and
imperative-or $(e)$ and $e=\left\langle e_{1}\right.$, or, $\left.e_{2}\right\rangle \Rightarrow$
$\operatorname{gom}-\operatorname{or}\left(e^{\prime}\right)$ and $e^{\prime}=<\theta_{e}\left(e_{1}\right)$, or, $\theta_{e}\left(e_{2}\right)>$ and
imperative-subtraction(e) and $e=\left\langle e_{1},-, e_{2}\right\rangle \Rightarrow$
gom-subtraction $\left(e^{\prime}\right)$ and $e^{\prime}=<\theta_{e}\left(e_{1}\right),-, \theta_{e}\left(e_{2}\right)>$ and
imperative-negate( $e$ ) and $\left.e=<-, e_{1}\right\rangle \Rightarrow$
gom-negate $\left(e^{\prime}\right)$ and $e^{\prime}=<-, \theta_{e}\left(e_{1}\right)>$ and
imperative-not $(e)$ and $\left.e=<\operatorname{not}, e_{1}\right\rangle \Rightarrow$
$\operatorname{gom}-\operatorname{not}\left(e^{\prime}\right)$ and $e^{\prime}=<\operatorname{not}, \theta_{e}\left(e_{1}\right)>$

Figure 137 The $\theta_{e}$ transformation

```
\(\theta(\Sigma)=\bar{\Psi}\) where
    For each \(s \in \Sigma\)
    imperative-procedure-call \((s) \Rightarrow s^{\prime}=s\) and
    \(s=\langle x,:=, e\rangle \Rightarrow\)
        \(s^{\prime}=<\theta_{v}(x),:=, \theta_{e}(e)>a n d\)
    \(s=\left\langle\mathrm{if}, e\right.\), then, \(S_{1}\), else, \(\left.S_{2}\right\rangle \Rightarrow\)
    \(s^{\prime}=<\) if, \(\theta_{e}(e)\), then, \(\theta\left(S_{1}\right)\), else, \(\theta\left(S_{2}\right)>\) and
    \(s=<\) while, \(\left.e, S_{3}\right\rangle \Rightarrow s^{\prime}=<\) while, \(\theta_{e}(e), \theta\left(S_{3}\right)>\) and
    \(s=\langle\) input, iport, \(V\rangle \Rightarrow s^{\prime}=\left\langle\right.\) input, iport, \(\left.\theta_{V}(V)\right\rangle\) and
    \(s=<\) output, oport, \(E>\Rightarrow s^{\prime}=<\) output, oport, \(\left.\theta_{E}(E)\right\rangle\) and
    \(s^{\prime} \in \Psi^{\prime}\) and
    \(\operatorname{pos}(s, \Sigma)=\operatorname{pos}\left(s^{\prime}, \bar{\Psi}\right)\)
```

Figure 138 The $\theta$ transformation

### 6.3 Transforming Accesses ( $\delta$ )

As discussed in Section 5.3, when converting a GIM subprogram to the GOM using the PBOI method, certain variables in the subprogram are converted to attributes of the class built for the subprogram. After a variable has been converted to an attribute, the statements in the GOM method must be updated to access the data item from the object attribute instead of the variable. The formal transformation for converting variable accesses in GOM statements to attribute accesses of an object is defined in this section.
6.3.1 Transforming Expressions ( $\delta_{e}$ ). As part of the conversion of variable accesses to attribute accesses, it is necessary to transform the GOM expressions found in GOM statements. This section discusses a formal transformation of GOM expressions that replaces references to variables with accesses of attributes.

Let $\delta_{e}$ be a transformation that converts an expression with variable accesses into an expression with object attribute accesses. Let $e$ be a GOM expression, let $c$ be an instance of a class $C$, and let $A$ be a set of GOM variables. Let GET-a represent the symbol resulting from prepending "GET-" to the GOM variable $a$. The $\delta_{e}$ transformation is defined as shown in Figure 139. This transformation replaces all occurrences of $a$ in the

$$
\delta_{e}(e, c, A)=e^{\prime} \text { where }
$$

gom-variable(e) and $e=a$ and $a \in A \Rightarrow e^{\prime}=\langle$ GET-a, $[c]\rangle$
gom-variable(e) and $e=a$ and $a \notin A \Rightarrow e^{\prime}=e$ and
$e=\langle$ GET-a, $[c]\rangle \Rightarrow e^{\prime}=e$ and
gom-function-call $(e) \Rightarrow e^{\prime}=e$ and
gom-literal-boolean $(e) \Rightarrow e^{\prime}=e$ and
gom-literal-integer $(e) \Rightarrow e^{\prime}=e$ and
gom-literal-real $(e) \Rightarrow e^{\prime}=e$ and
gom-literal-string $(e) \Rightarrow e^{t}=e$ and
$e=\left\langle e_{1},+, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle\delta_{e}\left(e_{1}\right),+, \delta_{e}\left(e_{2}\right)\right\rangle$ and
$e=\left\langle e_{1}\right.$, and, $\left.e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle\delta_{e}\left(e_{1}\right)\right.$, and, $\left.\delta_{e}\left(e_{2}\right)\right\rangle$ and
$e=\left\langle e_{1}, \&, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle\delta_{e}\left(e_{1}\right), \&, \delta_{e}\left(e_{2}\right)\right\rangle$ and
$e=\left\langle e_{1}, /, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle\delta_{e}\left(e_{1}\right), /, \delta_{e}\left(e_{2}\right)\right\rangle$ and
$e=\left\langle e_{1},=, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle\delta_{e}\left(e_{1}\right),=, \delta_{e}\left(e_{2}\right)\right\rangle$ and
$e=\left\langle e_{1},{ }^{* *}, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle\delta_{e}\left(e_{1}\right),{ }^{* *}, \delta_{e}\left(e_{2}\right)\right\rangle$ and
$\left.\left.e=\left\langle e_{1},\right\rangle=, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle\delta_{e}\left(e_{1}\right),\right\rangle=, \delta_{e}\left(e_{2}\right)\right\rangle$ and
$\left.\left.e=\left\langle e_{1},\right\rangle, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle\delta_{e}\left(e_{1}\right),\right\rangle, \delta_{e}\left(e_{2}\right)\right\rangle$ and
$e=\left\langle e_{1},<=, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle\delta_{e}\left(e_{1}\right),\left\langle=, \delta_{e}\left(e_{2}\right)\right\rangle\right.$ and
$e=\left\langle e_{1},\left\langle, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle\delta_{e}\left(e_{1}\right),\left\langle, \delta_{e}\left(e_{2}\right)\right\rangle\right.\right.$ and
$e=\left\langle e_{1},{ }^{*}, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle\delta_{e}\left(e_{1}\right),{ }^{*}, \delta_{e}\left(e_{2}\right)\right\rangle$ and
$e=\left\langle e_{1},\langle \rangle, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle\delta_{e}\left(e_{1}\right),\langle \rangle, \delta_{e}\left(e_{2}\right)\right\rangle$ and
$e=\left\langle e_{1}\right.$, or, $\left.e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle\delta_{e}\left(e_{1}\right)\right.$, or, $\left.\delta_{e}\left(e_{2}\right)\right\rangle$ and
$e=\left\langle e_{1},-, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle\delta_{e}\left(e_{1}\right),-, \delta_{e}\left(e_{2}\right)\right\rangle$ and
$e=\left\langle-, e_{1}\right\rangle \Rightarrow e^{\prime}=\left\langle-, \delta_{e}\left(e_{1}\right)\right\rangle$ and
$e=\left\langle\operatorname{not}, e_{1}\right\rangle \Rightarrow e^{\prime}=\left\langle\operatorname{not}, \delta_{e}\left(e_{1}\right)\right\rangle$

Figure 139 The $\delta_{e}$ transformation
expression $e$ with GOM "GET-" messages that accesses the attribute $a$ from the object instance $c$. If none of the GOM variables in the set $A$ are part of the expression $e$ or the set $A$ is empty, then the original expression $e$ is returned. For example, the following expression (shown in GOL syntax) shows a GOM multiplication expression with the XLAMDA and ZCOS variables.

```
XLAMDA * ZCOS * ZCOS * ZCOS
```

Let $e$ represent this expression and let C-18 be an object instance variable. Consider the following transformation.

$$
\delta_{e}(e, \mathrm{C}-18,\{\mathrm{ZCOS}\})
$$

This transformation replaces all variable accesses of $2 \operatorname{COS}$ in $e$ with messages that access the attribute ZCOS from C-18. The following expression shows the result of this transformation.

$$
\text { XLAMDA } * \operatorname{GET}-\mathrm{ZCOS}(\mathrm{C}-18) * \operatorname{GET}-\mathrm{ZCOS}(\mathrm{C}-18) * \operatorname{GET}-\mathrm{ZCOS}(\mathrm{C}-18)
$$

All accesses to the data item ZCOS are now "GET-" messages that access the attribute from C-18.
6.3.2 Transforming GOM Variable Accesses. Now that accesses to variables in expressions can be transformed, a formal transformation is presented that converts a sequence of statements that include variable accesses into a sequence of statements that include attribute accesses. Let $\delta$ be a transformation that converts GOM variable accesses in sequences of statements to GOM object attribute accesses. Let $\Psi$ be a sequence of GOM statements, let $c$ be an instance of a class $C$ and $A$ be a set of GOM variables. Let SET-a represent the symbol resulting from prepending "SET-" to the name of the GOM variable $a$. Recall that the tuple $<$ SET-a, $[c, e]>$ represents a message named SET-a with two parameters, $c$ and $e$. Let instance $(c, C)$ represent whether or not an object $c$ is an instance of class $C$. The $\delta$ transformation is defined as shown in Figure 140. This is a recursive transformation that uses the structure of the GOM statements to transform each access to the GOM variables $a$ in $A$ to GOM messages that access $a$ as an attribute. If the statement in $\Psi$ is an assignment to $a$, then the entire statement is transformed to a

$$
\begin{aligned}
& \delta(\Psi, c, A)=\Psi^{\prime} \text { where } \\
& \text { instance }(c, C) \text { and } \\
& C=<d_{C}, \Phi_{C}, \Omega_{C}, \lambda> \\
& \forall a \in A \text { and } \forall s \in \Psi \\
& a \in \Phi_{C} \text { and } \\
& s=<a,:=, e>\Rightarrow \\
& s^{\prime}=<\text { SET-a, }\left[c, \delta_{e}(e, c, A)\right]>\text { and } \\
& s=<x,:=, e>\text { and } x \notin A \Rightarrow \\
& s^{\prime}=<x,:=, \delta_{e}(e, c, A)>\text { and } \\
& s=<\text { if, } e, \text { then, } S_{1}, \text { else, } S_{2}>\Rightarrow \\
& s^{\prime}=<\text { if, } \delta_{e}(e, c, A), \text { then, } \delta\left(S_{1}, c, A\right), \text { else, } \delta\left(S_{2}, c, A\right)>\text { and } \\
& s=<\text { while, e, } S_{3}>\Rightarrow \\
& s^{\prime}=<\text { while, } \delta_{e}(e, c, A), \delta\left(S_{3}, c, A\right)>\text { and } \\
& s=<\text { output, oport, } E>\Rightarrow \\
& s^{\prime}=<\text { output, oport }, E^{\prime}>\text { and } \\
& E^{\prime}=\left[\delta_{e}(e, c, A) \mid e \in E\right] \text { and } \\
& s^{\prime} \in \Psi^{\prime}
\end{aligned}
$$

Figure 140 The $\delta$ transformation

GOM "SET-" message. The expression $e$ in the assignment statement is transformed using the $\delta_{e}$ transformation in order to transform any accesses to $a$ in $e$. If the statement is an assignment to a variable other than $a$, the expression $e$ must be transformed using the $\delta_{e}$ transformation. If the statement is a selection statement, the expression $e$ is transformed by the $\delta_{e}$ transformation and the sequences of statements $S_{1}$ and $S_{2}$ are transformed by using the $\delta$ transformation recursively. If the statement is an iteration statement, the expression $e$ is transformed using the $\delta_{e}$ transformation and the sequence of statements $S_{3}$ is transformed by using the $\delta$ transformation recursively. If the statement is an input or an output statement, then the sequence of expressions, $E$, in the statement is transformed by using the $\delta_{e}$ transformation on each of the expressions in the sequence. If none of the GOM variables in the set $A$ are accessed in the statements in $\Psi$ or the set $A$ is empty, then the original statements in $\Psi$ are returned unchanged.
6.3.3 Transforming Attribute Accesses $\left(\delta^{-1}\right)$. As part of the PBOI method, in some cases it is necessary to convert an attribute of a class back into a formal parameter of a method. In this case, the attribute accesses must be converted back to accesses of a variable since a formal parameter in a method is a variable defined by the method. This means the $\delta$ and $\delta_{e}$ transformations must be reversible.

Let $\delta_{e}^{-1}$ be a transformation that converts object attribute accesses in GOM expressions to variable accesses. Let $e$ be a GOM expression, let $c$ be an instance of a class $C$, and let $A$ be a set of GOM variables. Let GET-a represent the symbol resulting from prepending "GET-" to the GOM variable $a$. The $\delta_{e}^{-1}$ transformation is defined as shown in Figure 141. This transformation replaces all occurrences of GOM "GET-" messages that access the attribute $a$ in the expression with a reference to $a$. If none of the GOM variables in the set $A$ are part of the expression $e$ or the set $A$ is empty, then the original expression $e$ is returned.

The inverse of the transformation of statements is defined below. Let $\delta^{-1}$ be a transformation that converts GOM object attribute accesses in statements to GOM variable accesses. Let $\Psi$ be a sequence of GOM statements, let $c$ be an instance of a class $C$ and $A$ be a set of GOM variables. Let SET-a represent the symbol resulting from prepending "SET-" to the name of the GOM variable $a$. The $\delta^{-1}$ transformation is defined as shown in Figure 142. This is a recursive transformation that uses the structure of the GOM statements to transform each attribute access of $a$ into a reference to the variable $a$. If the statement in $\Psi$ is a "SET-" message that sets the value of $a$, then the "SET-" message is replaced with an assignment statement that sets the value of the variable $a$. The expression $e$ from the message is transformed using the $\delta_{e}^{-1}$ transformation in order to transform any accesses of $a$. If the statement is an assignment to a variable other than $a$, the expression $e$ must be transformed using the $\delta_{e}^{-1}$ transformation. If the statement is a selection statement, the expression $e$ is transformed by the $\delta_{e}^{-1}$ transformation and the sequences of statements $S_{1}$ and $S_{2}$ are transformed by using the $\delta^{-1}$ transformation recursively. If the statement is an iteration statement, the expression $e$ is transformed using the $\delta_{e}^{-1}$ transformation and the sequence of statements $S_{3}$ is transformed by using the $\delta^{-1}$ transformation recursively. If the statement is an input or an output statement,

$$
\begin{aligned}
& \delta_{e}^{-1}(e, c, A)=e^{\prime} \text { where } \\
& e=<\text { GET-a, }[c]>\text { and } a \in A \Rightarrow \operatorname{gom}-\operatorname{variable}\left(e^{\prime}\right) \text { and } e^{\prime}=a \text { and } \\
& e=<\text { GET-a, }[c]>\text { and } a \notin A \Rightarrow e^{\prime}=e \text { and } \\
& \text { gom-variable }(e) \Rightarrow e^{\prime}=e \text { and } \\
& \text { gom-function-call }(e) \Rightarrow e^{\prime}=e \text { and } \\
& \text { gom-literal-boolean }(e) \Rightarrow e^{\prime}=e \text { and } \\
& \text { gom-literal-integer }(e) \Rightarrow e^{\prime}=e \text { and } \\
& \text { gom-literal-real }(e) \Rightarrow e^{\prime}=e \text { and } \\
& \text { gom-literal-string }(e) \Rightarrow e^{\prime}=e \text { and } \\
& e=<e_{1},+, e_{2}>\Rightarrow e^{\prime}=<\delta_{e}^{-1}\left(e_{1}\right),+, \delta_{e}^{-1}\left(e_{2}\right)>\text { and } \\
& e=<e_{1} \text {, and, } e_{2}>\Rightarrow e^{\prime}=<\delta_{e}^{-1}\left(e_{1}\right) \text {, and, } \delta_{e}^{-1}\left(e_{2}\right)>\text { and } \\
& e=<e_{1}, \&, e_{2}>\Rightarrow e^{\prime}=<\delta_{e}^{-1}\left(e_{1}\right), \&, \delta_{e}^{-1}\left(e_{2}\right)>\text { and } \\
& e=<e_{1}, /, e_{2}>\Rightarrow e^{\prime}=<\delta_{e}^{-1}\left(e_{1}\right), /, \delta_{e}^{-1}\left(e_{2}\right)>\text { and } \\
& e=<e_{1},=, e_{2}>\Rightarrow e^{\prime}=<\delta_{e}^{-1}\left(e_{1}\right),=, \delta_{e}^{-1}\left(e_{2}\right)>\text { and } \\
& e=<e_{1},{ }^{* *}, e_{2}>\Rightarrow e^{\prime}=<\delta_{e}^{-1}\left(e_{1}\right),{ }^{* *}, \delta_{e}^{-1}\left(e_{2}\right)>\text { and } \\
& e=<e_{1},>=, e_{2}>\Rightarrow e^{\prime}=<\delta_{e}^{-1}\left(e_{1}\right),>=, \delta_{e}^{-1}\left(e_{2}\right)>\text { and } \\
& e=<e_{1},>, e_{2}>\Rightarrow e^{\prime}=<\delta_{e}^{-1}\left(e_{1}\right),>, \delta_{e}^{-1}\left(e_{2}\right)>\text { and } \\
& e=<e_{1},<=, e_{2}>\Rightarrow e^{\prime}=<\delta_{e}^{-1}\left(e_{1}\right),<=, \delta_{e}^{-1}\left(e_{2}\right)>\text { and } \\
& e=<e_{1},<, e_{2}>\Rightarrow e^{\prime}=<\delta_{e}^{-1}\left(e_{1}\right),<, \delta_{e}^{-1}\left(e_{2}\right)>\text { and } \\
& e=<e_{1},{ }^{*}, e_{2}>\Rightarrow e^{\prime}=<\delta_{e}^{-1}\left(e_{1}\right),^{*}, \delta_{e}^{-1}\left(e_{2}\right)>\text { and } \\
& e=<e_{1},<>, e_{2}>\Rightarrow e^{\prime}=<\delta_{e}^{-1}\left(e_{1}\right),<>, \delta_{e}^{-1}\left(e_{2}\right)>\text { and } \\
& e=<e_{1} \text {, or, } e_{2}>\Rightarrow e^{\prime}=<\delta_{e}^{-1}\left(e_{1}\right) \text {, or, } \delta_{e}^{-1}\left(e_{2}\right)>\text { and } \\
& e=<e_{1},-, e_{2}>\Rightarrow e^{\prime}=<\delta_{e}^{-1}\left(e_{1}\right),-, \delta_{e}^{-1}\left(e_{2}\right)>\text { and } \\
& e=<-, e_{1}>\Rightarrow e^{\prime}=<-, \delta_{e}^{-1}\left(e_{1}\right)>\text { and } \\
& e=<\operatorname{not}, e_{1}>\Rightarrow e^{\prime}=<\operatorname{not}, \delta_{e}^{-1}\left(e_{1}\right)>
\end{aligned}
$$

Figure 141 The $\delta_{e}^{-1}$ transformation

$$
\begin{aligned}
& \delta^{-1}(\Psi, c, A)=\Psi^{\prime} \text { where } \\
& \forall a \in A \text { and } \forall s \in \Psi \\
& s=<\text { SET-a, }[c, e]>\Rightarrow \\
& s^{\prime}=<a,:=, \delta_{e}^{-1}(e, c, A)>\text { and } \\
& s=<x,:=, e>\Rightarrow \\
& s^{\prime}=<x,:=, \delta_{e}^{-1}(e, c, A)>\text { and } \\
& s=<\text { if, } e, \text { then, } S_{1}, \text { else }, S_{2}>\Rightarrow \\
& s^{\prime}=<\text { if, } \delta_{e}^{-1}(e, c, A), \text { then, } \delta^{-1}\left(S_{1}, c, A\right), \text { else, } \delta^{-1}\left(S_{2}, c, A\right)>\text { and } \\
& s=<\text { while } e, S_{3}>\Rightarrow \\
& s^{\prime}=<\text { while, } \delta_{e}^{-1}(e, c, A), \delta^{-1}\left(S_{3}, c, A\right)>\text { and } \\
& s=<\text { input, iport, } E>\Rightarrow \\
& s^{\prime}=<\text { input, iport, } E^{\prime}>\text { and } \\
& \\
& E^{\prime}=\left[\delta_{e}^{-1}(e, c, A) \mid e \in E\right] \text { and } \\
& s=<\text { output, oport, } E>\Rightarrow \\
& s^{\prime}=<\text { output, oport, } E^{\prime}>\text { and } \\
& E^{\prime}=\left[\delta_{e}^{-1}(e, c, A) \mid e \in E\right] \text { and } \\
& s^{\prime} \in \Psi^{\prime}
\end{aligned}
$$

Figure 142 The $\delta^{-1}$ transformation
then the sequence of expressions, $E$, in the statement is transformed by using the $\delta_{e}^{-1}$ transformation on each of the expressions in the sequence. If none of the GOM variables in the set $A$ are accessed in the statements in $\Psi$ or the set $A$ is empty, then the statements in $\Psi$ are returned unchanged.

### 6.4 Transforming Subprograms ( $\sigma$ )

This section defines the high-level transformation, $\sigma$, that applies lower-level transformations to formalize the PBOI method defined in Section 5.3. The $\sigma$ transformation uses the taxonomy of GIM subprograms defined in Section 5.2 to classify the input subprogram and apply the transformation appropriate for that category of subprograms.

Let $S$ represent an imperative subprogram and $O O D$ represent the object-oriented design developed so far. The $\sigma$ transformation is defined as follows.

$$
\begin{aligned}
\sigma(O O D, S) & =O O D^{\prime} \text { where } \\
C a t_{0}(S) & \Rightarrow O O D^{\prime}=\sigma_{2}(O O D, S) \text { and } \\
C a t_{1}(S) & \Rightarrow O O D^{\prime}=\sigma_{1}(O O D, S) \text { and } \\
C a t_{2}(S) & \Rightarrow O O D^{\prime}=\sigma_{2}(O O D, S) \text { and } \\
C a t_{3}(S) & \Rightarrow O O D^{\prime}=\sigma_{3}(O O D, S)
\end{aligned}
$$

Note that Category 0 subprograms are treated as a special case of Category 2 subprograms, so the $\sigma_{2}$ transformation, defined in the following section, is used to transform both Category 0 and Category 2 subprograms. The transformations for Category 1 and Category 3 subprograms are defined in Section 6.8. The transformations that formalize the program slicing process used to convert Category 4 and Category 5 subprograms are defined in Section 6.12.

### 6.5 Transforming Category 2 Subprograms ( $\sigma_{2}$ )

This section describes the formal transformation of Category 2 subprograms to the object paradigm. As defined in Section 5.3, the PBOI method assumes Category 2 subprograms implement derived attribute queries. According to Proposition V.2, each Category 2 subprogram is converted to a method in a class where the attributes of the class are built from the formal parameters of the Category 2 subprogram (see Section 5.3).

The formal transformation $\sigma_{2}$ transforms an imperative subprogram, $S$, into a class, $C$, which includes a method that implements $S$. The new class, $C$, is added to the objectoriented design developed so far, $O O D$, and an updated design $O O D^{\prime}$ is returned from the transformation.

Let $i d_{C}$ be a symbol representing the name of the newly formed class $C$. This symbol can be a computer generated one or can be provided by the user. Let $\lambda_{0}$ be a symbol that represents the name of the overall super-class ( 'USER-OBJECT ) from which every object inherits in the design being developed. Let $c$ represent a parameter that holds an instance of the class $C$. Then, the transformation $\sigma_{2}$ for a Category 2 subprogram $S$ is defined as follows.

$$
\begin{aligned}
& \sigma_{2}(O O D, S)=O O D^{\prime} \text { where } \\
& \quad S=<i d_{S}, P_{\text {in }}, P_{\text {out }}, P_{\text {ret }}, P_{\text {loc }}, \Sigma_{S}>\text { and } \\
& \quad P_{\text {form }}=P_{\text {in }} \oplus P_{\text {out }} \text { and } \\
& C=<i d_{C}, \Phi_{C}, \Omega_{C}, \lambda_{0}>\text { and } \\
& \quad \Phi_{C}=\operatorname{range}\left(\theta_{V}\left(P_{\text {form }}\right)\right) \text { and } \\
& Q_{r e t}=\theta_{V}\left(P_{r e t}\right) \text { and } \\
& Q_{l o c}=\theta_{V}\left(P_{l o c}\right) \text { and } \\
& \Psi_{C}=\delta\left(\theta\left(\Sigma_{S}\right), c, \Phi_{C}\right) \text { and } \\
& \Omega_{C}=\left\{<i d_{S}, c, \emptyset, \emptyset, Q_{r e t}, Q_{l o c}, \Psi_{C}>\right\} \text { and } \\
& O O D^{\prime}=T_{c}^{+}(O O D, C)
\end{aligned}
$$

This transformation builds an object-oriented class $C$ from the imperative subprogram $S$ by first transforming all the input parameters in $P_{\text {in }}$ and output parameters in $P_{\text {out }}$ into instance attributes, $\Phi_{C}$, of $C$ by using the $\theta_{V}$ transformation. The return value and the local variables are also converted to GOM variables using $\theta_{V}$. The newly formed class $C$ is a subclass of $\lambda_{0}$ and includes one method in $\Omega_{C}$ that implements the functionality of the imperative subprogram. The newly formed method is named using the name of the imperative subprogram $i d_{S}$. The method must be passed an instance of the class $C$ as the target object and is not passed any other objects or input or output parameters. Each statement of the subprogram in $\Sigma_{S}$ is transformed into an object-oriented statement using the $\theta$ transformation. In these statements, all accesses to the variables that are now included in $\Phi_{C}$ are changed to accesses of these attributes by using the $\delta$ transformation. The new class $C$ is added to the object design by using the $T_{c}^{+}(O O D, C)$ transformation.

### 6.6 Transforming Parameters

This section presents formal transformations for manipulating parameters of methods and attributes of classes. As presented in Section 5.3, in some cases when using the PBOI method it is necessary to add or remove attributes of classes. In order to maintain the visibility to the data item, an attribute removed from a class is added as a parameter to each method of the class. Conversely, when an attribute is added to a class the corresponding parameter is removed from each of the methods of the class.

For these reasons, formal transformations that convert parameters of the methods of a class into attributes of that class (and vice versa) have been developed. The transformations presented in Section 4.32 for adding and removing an attribute of a class ( $T_{a}^{+}$and $T_{a}^{-}$) are sufficient for developing a design, but are not rigorous enough for re-engineering. They do not consider the effects on the methods of a class that adding or removing an attribute will have. The transformations $T_{a}^{+}$and $T_{a}^{-}$will not be used when converting GIM subprograms to the GOM. Instead, the following transformations will be used to add and delete attributes of a class.
6.6.1 Moving Parameters to Attributes ( $T_{A}^{\delta}$ ). Let $T_{A}^{\delta}$ be a transformation that moves parameters of the methods of a class to attributes of the class. Let $O O D$ represent an object-oriented design, let $C$ represent a class, and let $A$ represent a set of GOM variables to be converted from parameters to attributes. Let $\operatorname{seq}(A)$ represent the conversion of the set $A$ to a sequence of arbitrary order. The $T_{A}^{\delta}$ transformation is defined as shown below.

$$
\begin{aligned}
& T_{A}^{\delta}(O O D, C, A)=C^{\prime} \text { where } \\
& C=<i d_{C}, \Phi_{C}, \Omega_{C}, \lambda>\text { and } \\
& \Phi_{C}^{\prime}=\Phi_{C} \cup A \text { and } \\
& \text { For each } o \in \Omega_{C} \\
& \quad o=<i d_{o}, c, Q_{o b j}, Q_{\text {form }}, Q_{r e t}, Q_{l o c}, \Psi>\text { and } \\
& \quad o^{\prime}=<i d_{o}, c, Q_{o b j}, Q_{f o r m} \ominus \operatorname{seq}(A), Q_{r e t}, Q_{l o c}, \delta(\Psi, c, A)>\text { and } \\
& o^{\prime} \in \Omega_{C}^{\prime} \text { and } \\
& C^{\prime}=<i d_{C}, \Phi_{C}^{\prime}, \Omega_{C}^{\prime}, \lambda>
\end{aligned}
$$

In this transformation, the attributes in $A$ are added to the set of attributes for the class, i.e. $\Phi_{C}$. Each method $o$ of the class $C$ is transformed by using the sequence subtraction operation $\Theta$ to ensure the data items in $A$ are not in the sequence of formal parameters. All accesses to the data items in $A$ in the statements of $o^{\prime}$ are transformed to attribute accesses of $c$ using the $\delta$ transformation (see Section 6.3). If the set $A$ is empty, then the $T_{A}^{\delta}$ transformation adds no attributes and changes no statements, so the class $C$ is not changed by the transformation. The transformed class $C^{\prime}$ is returned as the result of the transformation.
6.6.2 Moving Attributes to Parameters ( $T_{A}^{\delta^{-1}}$ ). When using the PBOI method, it is also necessary to transform attributes of classes into parameters of its methods. For this reason, the "inverse" of the $T_{A}^{\delta}$ transformation is defined. Let $T_{A}^{\delta^{-1}}$ be a transformation that converts a set of attributes of a class into parameters of the methods of the class. Let $O O D$ represent an object-oriented design, let $C$ represent a class, and let $A$ represent a set of data items to be converted from attributes to parameters. Let $\operatorname{seq}(A)$ represent the conversion of the set $A$ to a sequence of arbitrary order. The $T_{A}^{\delta^{-1}}$ transformation is defined as follows.

$$
\begin{aligned}
& T_{A}^{\delta^{-1}}(O O D, C, A)=C^{\prime} \text { where } \\
& C=<i d_{C}, \Phi_{C}, \Omega_{C}, \lambda>\text { and } \\
& \quad \Phi_{C}^{\prime}=\Phi_{C}-A \text { and } \\
& \quad \text { For each } o \in \Omega_{C} \\
& \quad o=<i d_{o}, c, Q_{o b j}, Q_{f o r m}, Q_{r e t}, Q_{l o c}, \Psi>\text { and } \\
& \quad o^{\prime}=<i d_{o}, c, Q_{o b j}, Q_{f o r m} \oplus \operatorname{seq}(A), Q_{r e t}, Q_{l o c}, \delta^{-1}(\Psi, c, A)>\text { and } \\
& o^{\prime} \in \Omega_{C}^{\prime} \text { and } \\
& C^{\prime}=<i d_{C}, \Phi_{C}^{\prime}, \Omega_{C}^{\prime}, \lambda>
\end{aligned}
$$

In this transformation, the attributes in $A$ are removed from the set of attributes for the class, $\Phi_{C}$. Each of the methods in the class are transformed by adding the data items in $A$ as parameters of $o^{\prime}$. Finally, the $\delta^{-1}$ transformation is used to convert all attribute accesses in the statements of $o^{\prime}$ to variable accesses. If the set $A$ is empty, then the $T_{A}^{\delta^{-1}}$ transformation removes no attributes and changes no statements, so the class $C$ is not changed by the transformation. The updated class $C^{\prime}$ is returned as the result of the transformation. Note that the $T_{A}^{\delta}$ transformation and the $T_{A}^{\delta^{-1}}$ transformation are not true inverses because $T_{A}^{\delta-1}\left(T_{A}^{\delta}(O O D, C, A), C, A\right) \neq C$. This is because the order of some parameters may have changed.

### 6.7 Transforming Attributes ( $\gamma$ )

This section defines a formal transformation for moving an attribute from one class to another. In some cases when using the PBOI method (see Section 5.3), it may be determined that a parameter should be built as an attribute of one class instead of another.

To formalize this case, a transformation is needed that moves an attribute from one class to another. This section presents this formal transformation and a simplified version of the transformation.

To define the transformation that moves an attribute from one class, $C_{1}$, to another class, $C_{2}$, a transformation is needed that replaces instances of $C_{1}$ with instances of $C_{2}$ in any messages where $C_{1}$ is the target object. Let $\gamma$ be a transformation that changes all attribute accesses of specific attributes from an instance of one class to an instance of a second class. Let $A$ be a subset of attributes from a class $C_{1}$. Let $c_{1}$ be an instance of $C_{1}$ and $c_{2}$ be an instance of the class $C_{2}$. Let $\Psi$ be a sequence of object-oriented statements. Then,

$$
\Psi^{\prime}=\gamma\left(\Psi, c_{1}, c_{2}, A\right)
$$

where $\Psi^{\prime}$ is a sequence of object-oriented statements where all attribute accesses of the attributes in $A$ from the instance $c_{1}$ are now attribute accesses of the attributes in $A$ for the instance $\boldsymbol{c}_{2}$.

For example, consider the following statement

$$
\mathrm{X}:=\mathrm{GET}-\mathrm{BETA}(\mathrm{C}-8)
$$

The following transformation

$$
\gamma([\mathrm{x}:=\mathrm{GET}-\mathrm{BETA}(\mathrm{C}-8)], \mathrm{C}-8, \mathrm{C}-9,\{\mathrm{BETA}\})
$$

results in the following updated statement

$$
\mathrm{X}:=\text { GET-BETA ( C-9 ) }
$$

The access of BETA now comes from C-9 instead of C-8.
Using the $\gamma$ transformation, it is now possible to define the $T_{A}^{\gamma}$ transformation that moves attributes from one class to another. Let $O O D$ represent an object-oriented design, let $C_{1}$ be a class, let $c_{1}$ be an instance of $C_{1}$, let $C_{2}$ be a class, and let $c_{2}$ be an instance of $C_{2}$. Let $A$ be a set of attributes of $C_{1}$ that will be moved from $C_{1}$ to $C_{2}$. The $T_{A}^{\gamma}$
transformation assumes that the instance $c_{1}$ is passed as a parameter to the methods of the class $C_{2}$ in order to have visibility to the attributes in $A$. The $T_{A}^{\gamma}$ transformation is defined as follows.

$$
\begin{aligned}
& T_{A}^{\gamma}\left(O O D, C_{1}, C_{2}, A\right)=O O D^{\prime} \text { where } \\
& C_{1}=<i d_{C_{1}}, \Phi_{C_{1}}, \Omega_{C_{1}}, \lambda_{C_{1}}>\text { and } \\
& \text { instance }\left(c_{1}, C_{1}\right) \text { and } \\
& C_{2}=<i d_{C_{2}}, \Phi_{C_{2}}, \Omega_{C_{2}}, \lambda_{C_{2}}>\text { and } \\
& \text { instance }\left(c_{2}, C_{2}\right) \text { and } \\
& \Phi_{C_{1}}^{\prime}=\Phi_{C_{1}}-A \text { and } \\
& \Phi_{C_{2}}^{\prime}=\Phi_{C_{2}} \cup A \text { and } \\
& \text { For each } o \in \Omega_{C_{1}} \\
& \quad o=<i d_{o}, c_{1}, Q_{o b j}, Q_{f o r m}, Q_{r e t}, Q_{l o c}, \Psi>\text { and } \\
& \quad o^{\prime}=<i d_{o}, c_{1}, Q_{o b j} \oplus\left[c_{2}\right], Q_{\text {form }}, Q_{r e t}, Q_{l o c}, \gamma\left(\Psi, c_{1}, c_{2}, A\right)>\text { and } \\
& o^{\prime} \in \Omega_{C_{1}}^{\prime} \text { and }
\end{aligned}
$$

For each $o \in \Omega_{C_{2}}$

$$
\begin{aligned}
& \quad o=<i d_{o}, c_{2}, Q_{o b j}, Q_{f o r m}, Q_{r e t}, Q_{l o c}, \Psi>\text { and } \\
& \quad o^{\prime}=<i d_{o}, c_{2}, Q_{o b j}, Q_{\text {form }}, Q_{r e t}, Q_{l o c}, \gamma\left(\Psi, c_{1}, c_{2}, A\right)>\text { and } \\
& \quad o^{\prime} \in \Omega_{C_{2}}^{\prime} \text { and } \\
& C_{1}^{\prime}=<i d_{C_{1}}, \Phi_{C_{1}}^{\prime}, \Omega_{C_{1}}^{\prime}, \lambda_{C_{1}}>\text { and } \\
& C_{2}^{\prime}=<i d_{C_{2}}, \Phi_{C_{2}}^{\prime}, \Omega_{C_{2}}^{\prime}, \lambda_{C_{2}}>\text { and } \\
& O O D^{\prime}=T_{c}^{+}\left(T_{c}^{-}\left(T_{c}^{+}\left(T_{c}^{-}\left(O O D, C_{1}\right), C_{1}^{\prime}\right), C_{2}\right), C_{2}^{\prime}\right)
\end{aligned}
$$

This transformation removes the attributes in $A$ from $C_{1}$ and adds them to the set of attributes in $C_{2}$. Each method in $C_{1}$ is updated by adding the instance $c_{2}$ to the sequence of objects passed into the method. The statements of each method are changed by the $\gamma$ transformation to send any accessing and assigning messages to $c_{2}$ instead of $c_{1}$. The statements of each method are changed by the $\gamma$ transformation to also send any accessing and assigning messages to $c_{2}$ instead of $c_{1}$. The methods in both $C_{1}$ and $C_{2}$ now access the attributes in $A$ from the instance $c_{2}$. If the set $A$ is empty, then the $T_{A}^{\gamma}$ transformation moves no attributes and neither $C_{1}$ nor $C_{2}$ are changed by the transformation. The updated classes are added to the design $O O D$ in order to produce the updated design $O O D^{\prime}$.

As an example, Figure 143 shows an object-oriented design with two classes where the GOM variable DIAM is an attribute of CLASS-4. The PRDIV method is passed C-9, an
instance of CLASS-4, in order to access the DIAM attribute. Figure 144 shows the result of the transformation

$T_{A}^{\gamma}(O O D$, CLASS-4, CLASS-5, DIAM)

The DIAM attribute has been moved from CLASS-4 to CLASS-5 and the methods have been updated to reflect this change.

The $T_{A}^{\gamma}$ transformation can be simplified if it is known that the attributes in $A$ are already attributes of $C_{2}$ and the desired processing is to make appropriate updates to $C_{1}$. Given this precondition, define $T_{A}^{\gamma^{\prime}}$ as follows.

$$
\begin{aligned}
& T_{A}^{\gamma^{\prime}}\left(O O D, C_{1}, C_{2}, A\right)=C_{1}^{\prime} \text { where } \\
& C_{1}=<i d_{C_{1}}, \Phi_{C_{1}}, \Omega_{C_{1}}, \lambda_{C_{1}}>\text { and } \\
& \text { instance }\left(c_{1}, C_{1}\right) \text { and } \\
& \text { instance }\left(c_{2}, C_{2}\right) \text { and } \\
& \Phi_{C_{1}}^{\prime}=\Phi_{C_{1}}-A \text { and }
\end{aligned}
$$

For each $o \in \Omega_{C_{1}}$

$$
\begin{aligned}
& \quad o=<i d_{o}, c_{1}, Q_{o b j}, Q_{f o r m}, Q_{r e t}, Q_{l o c}, \Psi>\text { and } \\
& \quad o^{\prime}=<i d_{o}, c_{1}, Q_{o b j} \oplus\left[c_{2}\right], Q_{f o r m}, Q_{r e t}, Q_{l o c}, \gamma\left(\Psi, c_{1}, c_{2}, A\right)>\text { and } \\
& o^{\prime} \in \Omega_{C_{1}}^{\prime} \text { and } \\
& C_{1}^{\prime}=<i d_{C_{1}}, \Phi_{C_{1}}^{\prime}, \Omega_{C_{1}}^{\prime}, \lambda_{C_{1}}>
\end{aligned}
$$

Since it is assumed that the attributes in $A$ have already been built as attributes of $C_{2}$, this transformation removes the attributes from the set of attributes for $C_{1}$ but makes no change to the set of attributes for $C_{2}$. Similarly, only the methods of $C_{1}$ are updated to access the attributes in $A$ from $c_{2}$ instead of $c_{1}$. The methods in $C_{2}$ are assumed to already access the attributes from $c_{2}$. If the set of attributes $A$ is empty, then the $T_{A}^{\gamma^{\prime}}$ transformation removes no attributes, so $C_{1}$ is not changed by the transformation. The updated class $C_{1}$ is returned as the result of this transformation.

For example, Figure 145 shows the DIAM attribute as an attribute of both CLASS-4 and CLASS-5. This attribute is being accessed correctly in CLASS-5 and moving the attribute from CLASS-4 to CLASS-5 is a matter of making the correct updates to CLASS-4. Figure 146

```
class CLASS-4 attributes DIAM, UPLFAC, ZENITH
    method RELAY-PHASE ( C-8, PHASE )
        begin
        PHASE := 1;
        SIGPR := GET-DIAM ( C-8 );
        PHASE := UPLREQ ( SIGPR )
        end
    superclass USER-OBJECT
class CLASS-5 attributes BETA, XLAMDA
    method PRDIV ( C-3, C-9 ): real begin
        FAC := 1.22d0;
        QUAL := MAX ( GET-BETA ( C-3), 1.0dO);
        PDIAM := MAX ( GET-DIAM ( C-9), 0.1d0);
        WAVELN := GET-XLAMDA ( C-3) * 9.9d-7;
        PRDIV := QUAL * WAVELN * FAC / PDIAM
        end
    superclass USER-OBJECT
```

Figure 143 DIAM as Attribute of CLASS-4

```
class CLASS-4 attributes UPLFAC, ZENITH
    method RELAY-PHASE ( C-8, C-9, PHASE )
        begin
        PHASE := 1;
        SIGPR := GET-DIAM ( C-9 );
        PHASE := UPLREQ ( SIGPR )
        end
    superclass USER-OBJECT
class CLASS-5 attributes BETA, XLAMDA, DIAM
    method PRDIV ( C-3 ): real begin
        FAC := 1.22dO;
        QUAL := MAX ( GET-BETA ( C-3), 1.0d0);
        PDIAM := MAX ( GET-DIAM ( C-3), 0.1d0);
        WAVELN := GET-XLAMDA ( C-3) * 9.9d-7;
        PRDIV := QUAL * WAVELN * FAC / PDIAM
        end
    superclass USER-OBJECT
```

Figure 144 DIAM as Attribute of CLASS-5

```
class CLASS-4 attributes DIAM, UPLFAC, ZENITH
    method RELAY-PHASE ( C-8, PHASE )
        begin
        PHASE := 1;
        SIGPR := GET-DIAM ( C-8 );
        PHASE := UPLREQ ( SIGPR )
        end
    superclass USER-OBJECT
class CLASS-5 attributes BETA, XLAMDA, DIAM
    method PRDIV ( C-3 ): real begin
        FAC := 1.22d0;
        QUAL := MAX ( GET-BETA ( C-3), 1.0dO);
        PDIAM := MAX ( GET-DIAM ( C-3), 0.1d0);
        WAVELN := GET-XLAMDA ( C-3) * 9.9d-7;
        PRDIV := QUAL * WAVELN * FAC / PDIAM
        end
    superclass USER-OBJECT
```

Figure 145 DIAM as Attribute of CLASS-4 and CLASS-5.
shows the result of applying the following transformation

$$
T_{A}^{\gamma^{\prime}}(O O D, \mathrm{CLASS}-4, \mathrm{CLASS}-5, \mathrm{DIAM})
$$

The DIAM attribute is removed from CLASS-4 and C-9, the instance of CLASS-5, is added as an object parameter to the RELAY-PHASE method. The GET-DIAM message is updated to access DIAM from C-9 instead of C-8.

```
class CLASS-4 attributes UPLFAC, ZENITH
    method RELAY-PHASE ( C-8, C-9, PHASE )
        begin
        PHASE := 1;
        SIGPR := GET-DIAM ( C-9 );
        PHASE := UPLREQ ( SIGPR )
        end
    superclass USER-OBJECT
class CLASS-5 attributes BETA, XLAMDA, DIAM
    method PRDIV ( C-3 ): real begin
        FAC := 1.22dO;
        QUAL := MAX ( GET-BETA ( C-3), 1.0dO);
        PDIAM := MAX ( GET-DIAM ( C-3), 0.1d0);
        WAVELN := GET-XLAMDA ( C-3) * 9.9d-7;
        PRDIV := QUAL * WAVELN * FAC / PDIAM
        end
    superclass USER-OBJECT
```

Figure 146 DIAM as Attribute of CLASS-5

### 6.8 Transforming Category 3 Subprograms

This section discusses the transformations that formalize the conversion of Category 3 subprograms. As defined in Section 5.3, there are four cases to consider when extracting object attributes from the imperative subprogram calls found in Category 3 subprograms. Some fundamental mappings required for all the PBOI transformations are presented first followed by the transformations that formalize each of the four cases. The overall highlevel transformation that determines which of these transformations to apply is presented at the end of this section.
6.8.1 Linking Classes and Subprograms ( $\rho$ ). In order to build the PBOI transformations, a mapping is needed that links an imperative subprogram to the class that was built for the subprogram. Let $\rho$ represent this mapping as defined below.

$$
\begin{aligned}
& \rho(S)=C \text { such that } \\
& \quad S=<i d_{S}, P_{\text {in }}, P_{o u t}, P_{r e t}, P_{l o c}, \Sigma_{S}>\text { and } \\
& C=<i d_{C}, \Phi_{C}, \Omega_{C}, \lambda_{0}>\text { and } \\
& o=<i d_{S}, c, Q_{o b j}, Q_{\text {form }}, Q_{r e t}, Q_{l o c}, \Psi>\text { and } \\
& o \in \Omega_{C}
\end{aligned}
$$

This mapping links the imperative subprogram $S$ to a class that has a method $o$ with the same name as $S$, i.e. $i d_{S}$.

Since, at this point in the development of the design, there is only one class built for each subprogram and each class is built from only one subprogram, the $\rho$ mapping is invertible. Let the $\rho^{-1}$ mapping map a class $C$ to the subprogram from which $C$ was built. The $\rho^{-1}$ mapping is defined as follows.

$$
\begin{aligned}
& \rho^{-1}(C)=S \text { such that } \\
& \quad C=<i d_{C}, \Phi_{C}, \Omega_{C}, \lambda_{0}>\text { and } \\
& \quad o=<i d_{S}, c, Q_{o b j}, Q_{f o r m}, Q_{r e t}, Q_{l o c}, \Psi>\text { and } \\
& o \in \Omega_{C} \text { and } \\
& \quad S=<i d_{S}, P_{i n}, P_{o u t}, P_{r e t}, P_{l o c}, \Sigma_{S}>
\end{aligned}
$$

This mapping links the class $C$ that was built for the subprogram $S$ to this subprogram $S$.

$$
\begin{aligned}
& T_{p b o i}^{1}(O O D, S, P)=O O D^{\prime} \text { where } \\
& C_{1}=\rho(S) \text { and } \\
& \forall p \in P \\
& P^{\prime}=\left\{p \mid \exists C_{2} \in \rho^{*}(S)\right. \text { such that } \\
& \left.a^{\prime}=\theta_{v}\left(\mu^{*}(p)\right) \text { and } a^{\prime} \in \Phi_{C_{2}}\right\} \text { and } \\
& C_{1}^{\prime}=T_{A}^{\gamma^{\prime}}\left(O O D, C_{1}, C_{2}, \theta_{v}\left(P^{\prime}\right)\right) \text { and } \\
& C_{1}^{\prime} \in O O D^{\prime} \wedge C_{1} \notin O O D^{\prime}
\end{aligned}
$$

Figure 147 The $T_{p b o i}^{1}$ transformation
6.8.2 Formalizing PBOI Case 1. The first case for converting Category 3 subprograms is that the formal parameter is an attribute and the actual parameter is also a formal. This section develops the transformation that formalizes the PBOI Case 1 as presented in Section 5.3. When transforming a subprogram $S$, the transformation required for PBOI Case 1 is to move attributes from the class built for $S$ to another class in the design. This transformation is done on any of the formal parameters in $S$ that can be linked to the attributes of another class through the $\theta_{v}$ transformation and the $\mu^{*}$ relation. An example is given at the end of the section.

Let $T_{p b o i}^{1}$ be the transformation that formalizes the first PBOI case. Recall from Section 3.25 , the $\mu$ relation maps an actual parameter of a subprogram call to a formal parameter in the called subprogram. Let $\mu^{*}$ represent the transitive closure of the $\mu$ relation. Let $\operatorname{call}\left(S_{1}, S_{2}\right)$ be a relation from subprogram $S_{1}$ to subprogram $S_{2}$ that indicates $S_{1}$ includes an imperative subprogram call to $S_{2}$. Let call ${ }^{*}(S)$ represent the transitive closure of the call relation for an imperative subprogram $S$. This is also termed the call tree of $S$. Let $\rho^{*}(S)$ represent the set of classes built for the subprograms in the call tree of $S$. Let $O O D$ represent an object-oriented design, let $S$ represent an imperative subprogram, and let $P$ represent the set of actual parameters from $S$ that are also formal parameters of $S$. The $T_{p b o i}^{1}$ transformation is defined as shown in Figure 147. This transformation moves each attribute in $C_{1}$ that has already been built as an attribute in $C_{2}$ from $C_{1}$ to $C_{2}$ using the $T_{A}^{\gamma^{\prime}}$ transformation. The transitive closure of the links between actual parameters and formal parameters, $\mu^{*}$, is used to find formal parameters of subprograms that are in the
call tree of $S$ that have been converted to attributes (using $\theta_{v}$ ) of the corresponding class. The set $P^{\prime}$ collects each of the parameters from $P$ that is linked to an attribute $a^{\prime}$ through $\mu^{*}(p)$ and $\theta_{v}$. Each $a^{\prime}$ is an attribute of a class in the set of classes built for the call tree of $S$. The set $P^{\prime}$ is used in the $T_{A}^{\gamma^{\prime}}$ transformation to indicate which attributes of $C_{1}$ to move from class $C_{1}$ to class $C_{2}$. The $T_{A}^{\gamma^{\prime}}$ transformation is used because the attribute $a^{\prime}$ is already an attribute of class $C_{2}$. The original class $C_{1}$ is removed from the design and the updated class $C_{1}^{\prime}$ is added to the design in order to produce the updated design $O O D^{\prime}$.

For example, Figure 148 shows the subprograms, RADIUS, CAPTURE, and BOUNCE. The BOUNCE subprogram calls the RADIUS and CAPTURE subprograms, so the mappings $\operatorname{call(BOUNCE,RADIUS)}$ and $\operatorname{call(BOUNCE,CAPTURE)}$ are in the transitive closure call $^{*}($ BOUNCE $)$. Figure 149 shows the classes built for RADIUS, CAPTURE, and BOUNCE before applying the $T_{p b o i}^{1}$ transformation. Since RADIUS and CAPTURE are called by BOUNCE, the transformations from a subprogram to a class have already been done. The RADIUS subprogram has been transformed into CLASS-1, which is shown in the figure. The CAPTURE subprogram has been transformed into CLASS-2, which is also shown in Figure 149. The transformation of the subprogram BOUNCE is incomplete in this example, but the class being built for BOUNCE is shown as CLASS-4 in the figure. Notice at this point in the transformation of BOUNCE, none of the subprogram calls to other subprograms have been converted to messages. Also note that CLASS-4 currently has the attributes AXMAJ, AXMIN, ANGFAC, PROJRA, SIGB, RNGFAC, RANGE, and other attributes.

Let $O O D$ represent the design that includes these classes built for BOUNCE, CAPTURE, and RADIUS. Consider the following transformation.
$T_{p b o i}^{1}(O O D, B O U N C E,\{$ AXMAJ, AXMIN, ANGFAC, RANGE, RNGFAC, SIGB, PROJRA \} )

This transformation is used to update $O O D$ based on whether the parameters AXMAJ, AXMIN, ANGFAC, PROJRA, SIGB, RNGFAC, and RANGE have been built as attributes of any of the classes built for the call tree of BOUNCE. Note in Figure 148, BOUNCE calls RADIUS passing the PROJRA, SIGB, RNGFAC, and RANGE data items as actual parameters. The formal parameters that correspond to these actual parameters are the parameters PROJRA, SIGB, RNGFAC, and

```
real function RADIUS ( PROJRA, SIGB, RNGFAC, RANGE )
    begin
    if RNGFAC > 0.0 and RANGE > 0.0
        then SIGABS := DABS ( SIGB);
            SLOPE := SIGABS - PROJRA / RANGE;
            SPOT := SIGABS * RANGE;
            RAD := DABS ( PROJRA + SLOPE * RNGFAC);
            RADIUS := DMAX1 ( SPOT, RAD)
        else
            RADIUS := PROJRA
        endif
    end
real function CAPTURE ( BEAMRA, AXMAJ, AXMIN, ANGFAC )
    begin
    TILTFA := DSIN ( ANGFAC);
    RMAREA := AXMAJ * AXMIN;
    EFAREA := RMAREA * TILTFA;
    BMAREA := BEAMRA * BEAMRA;
    if EFAREA <= BMAREA * 6.0dO
        then USED := EFAREA / BMAREA;
            CAPTURE := 1.OdO - DEXP ( -USED)
        else CAPTURE := 1.0d0
        endif
    end
real function BOUNCE ( ANGFAC, RNGFAC, PROJRA, SIGB,
            RANGE, AXMAJ, AXMIN, XLAMDA)
    begin
    RHOSTD := 0.95 * PROJRA * ANGFAC;
    BEAMRA := RADIUS ( PROJRA, SIGB, RNGFAC, RANGE);
    BOUNCE :=
        CAPTURE ( BEAMRA, AXMAJ, AXMIN, ANGFAC)
            * RHO ( RHOSTD, XLAMDA)
    end
```

Figure 148 Subprograms RADIUS, CAPTURE, and BOUNCE

```
class CLASS-1 attributes RANGE, RNGFAC, SIGB, PROJRA
    method RADIUS ( C-1 ): real begin
        if GET-RNGFAC ( C-1) > 0.0 and GET-RANGE (C-1) > 0.0
        then SIGABS := DABS ( GET-SIGB ( C-1));
            SLOPE := SIGABS - GET-PROJRA ( C-1) / GET-RANGE ( C-1);
            SPOT := SIGABS * GET-RANGE ( C-1);
            RAD := DABS ( GET-PROJRA ( C-1) + SLOPE * GET-RNGFAC ( C-1));
            RADIUS := DMAX1 ( SPOT, RAD)
            else RADIUS := GET-PROJRA ( C-1) endif
        end
    superclass USER-OBJECT
class CLASS-2 attributes BEAMRA, AXMAJ, AXMIN, ANGFAC
    method CAPTURE ( C-2 ): real
        begin
        TILTFA := DSIN ( GET-ANGFAC ( C-2));
        RMAREA := GET-AXMAJ ( C-2) * GET-AXMIN ( C-2);
        EFAREA := RMAREA * TILTFA;
        BMAREA := GET-BEAMRA (C-2) * GET-BEAMRA ( C-2);
        if EFAREA <= BMAREA * 6.0d0
            then USED := EFAREA / BMAREA;
                CAPTURE := 1.OdO - DEXP ( -USED)
            else CAPTURE := 1.0d0
            endif
        end
    superclass USER-OBJECT
class CLASS-4 attributes ANGFAC, RNGFAC, PROJRA, SIGB,
            RANGE, AXMAJ, AXMIN, XLAMDA
    method BOUNCE ( C-4 ): real
        begin
        RHOSTD := 0.95 * GET-PROJRA ( C-4)
            * GET-ANGFAC ( C-4);
        BEAMRA := RADIUS ( PROJRA, SIGB, RNGFAC, RANGE);
        BOUNCE :=
            CAPTURE ( BEAMRA, AXMAJ, AXMIN, ANGFAC)
                * RHO ( RHOSTD, XLAMDA)
        end
    superclass USER-OBJECT
```

Figure 149 Classes Built for RADIUS, CAPTURE, and BOUNCE

RANGE declared in RADIUS. This means the following mappings are in the transitive closure $\mu^{*}$.

$$
\begin{aligned}
\text { BOUNCE :: PROJRA } & \xrightarrow{\mu^{*}} \text { RADIUS :: PROJRA } \\
\text { BOUNCE ::SIGB } & \xrightarrow{\mu^{*}} \text { RADIUS ::SIGB } \\
\text { BOUNCE :: RNGFAC } & \xrightarrow{\mu^{*}} \text { RADIUS :: RNGFAC } \\
\text { BOUNCE :: RANGE } & \xrightarrow{\mu^{*}} \text { RADIUS :: RANGE }
\end{aligned}
$$

Also note that each of these formal parameters have been built as an attribute of the class built for RADIUS, i.e. CLASS-1 shown in Figure 149. All of this information means the $T_{p b o i}^{1}$ transformation is performed and the PROJRA, SIGB, RNGFAC, and RANGE attributes are moved from CLASS-4 to CLASS-1 using the $T_{A}^{\gamma^{\prime}}$ transformation. All accesses to PROJRA, SIGB, RNGFAC, and RANGE in CLASS-4 are changed from accessing instances of CLASS-4 to accessing instances of CLASS-1.

Similarly, BOUNCE calls CAPTURE passing the BEAMRA, AXMAJ, AXMIN, and ANGFAC data items as actual parameters. The formal parameters that correspond to these actual parameters are the BEAMRA, AXMAJ, AXMIN, and ANGFAC parameters. The following mappings are in the transitive closure $\mu^{*}$.

$$
\begin{aligned}
\text { BOUNCE }:: ~ B E A M R A & \xrightarrow{\mu^{*}} \\
\text { BOUNCE }:: A X M A J & \xrightarrow{\mu^{*}} \text { CAPTURE }:: ~ B E A M R A \\
\text { BOUNCE }:: A X M I N & \xrightarrow{\mu^{*}} \text { CAPTURE :: AXMIN } \\
\text { BOUNCE }:: A N G F A C & \xrightarrow{\mu^{*}} \text { CAPTURE }:: A N G F A C
\end{aligned}
$$

Note that only the AXMAJ, AXMIN, and ANGFAC parameters are both formal and actual parameters in BOUNCE. In adherence with PBOI Case 1, these data items are sent to the $T_{p b o i}^{1}$ transformation, but the BEAMRA data item is not. Also note that the AXMAJ, AXMIN, and ANGFAC parameters have been built as attributes of the class CLASS-2. This means the AXMAJ, AXMIN, and ANGFAC attributes of CLASS-4 are moved from CLASS-4 to CLASS-2 using the $T_{A}^{\gamma^{\prime}}$ transformation. All accesses of AXMAJ, AXMIN, and ANGFAC attributes are converted from accessing instances of CLASS-4 to accessing instances of CLASS-1.

Figure 150 shows the result of the $T_{p b o i}^{1}$ transformation. Note in the GET-PROJRA message, the PROJRA attribute is now obtained from C-5, an instance of CLASS-1, instead

```
class CLASS-4 attributes XLAMDA
    method BOUNCE ( C-4, C-5, C-6): real
        begin
        RHOSTD := 0.95 * GET-PROJRA ( C-5)
            * GET-ANGFAC ( C-6);
        BEAMRA := RADIUS ( PROJRA, SIGB, RNGFAC, RANGE);
        BOUNCE :=
            CAPTURE ( BEAMRA, AXMAJ, AXMIN, ANGFAC)
            * RHO ( RHOSTD, XLAMDA)
        end
    superclass USER-OBJECT
```

Figure 150 Updated CLASS-4 for BOUNCE
of C-4, an instance of CLASS-4. In the GET-ANGFAC message, the ANGFAC attribute is now obtained from C-6, an instance of CLASS-2, instead of C-4, an instance of CLASS-4. The $T_{A}^{\gamma^{\prime}}$ transformation has added C-5 and C-6 as formal parameters of the method BOUNCE.
6.8.3 Formalizing PBOI Case 2. The second case for converting Category 3 subprograms is that the formal parameter is a parameter of the method and the actual parameter is a formal. This section defines the transformation that formalizes the second PBOI case as presented in Section 5.3. The transformation needed in this case is to identify parameters of a called subprogram that are not attributes and move them to parameters of the method built for the calling subprogram. The transformation is defined and an example is given.

Recall from Section 6.8.2, $\mu^{*}$ represents the transitive closure of the $\mu$ relation, call ${ }^{*}(S)$ (the call tree of $S$ ) represents the transitive closure of the call relation, and $\rho^{*}(S)$ represents the set of classes built for the subprograms in the call tree of $S$. Let $O O D$ represent an object-oriented design, let $S$ represent the subprogram to be transformed, and let $P$ be the set of data items to consider in the transformation.

Using these definitions, the $T_{p b o i}^{2}$ transformation that formalizes PBOI Case 2 is defined as shown in Figure 151. This transformation checks each of the classes built for the subprograms in the call tree of $S$. For any data item $p$, if there are no classes that have built an attribute that is linked to the parameter $p$ through the $\mu^{*}$ mapping, then

$$
\begin{aligned}
& T_{p b o i}^{2}(O O D, S, P)=O O D^{\prime} \text { where } \\
& \qquad \begin{array}{l}
P^{\prime}=\left\{p \mid p \in P \text { and } \neg \exists C_{2} \in \rho^{*}(S)\right. \text { such that } \\
\left.a^{\prime}=\theta_{v}\left(\mu^{*}(p)\right) \text { and } a^{\prime} \in \Phi_{C_{2}}\right\} \text { and } \\
C_{1}=\rho(S) \text { and } \\
C_{1}^{\prime}=T_{A}^{\delta^{-1}}\left(O O D, C_{1}, \theta_{v}\left(P^{\prime}\right)\right) \text { and } \\
O O D^{\prime}=T_{c}^{+}\left(T_{c}^{-}\left(O O D, C_{1}\right), C_{1}^{\prime}\right)
\end{array}
\end{aligned}
$$

Figure 151 The $T_{p b o i}^{2}$ transformation
an instance of PBOI Case 2 has been found. The set $P^{\prime}$ collects these data items so they can be changed from attributes to parameters. All of the data items in $P^{\prime}$ are changed from attributes of the class built for $S$, i.e. $C_{1}$, into parameters of the methods of $C_{1}$ using the $T_{A}^{\delta^{-1}}$ transformation. Once $p$ is converted to a GOM variable using the $\theta_{v}$ transformation, it is appropriate to move the variable from an attribute to a parameter because both attributes and parameters are variables in the object paradigm. For example, Figure 152 shows the subprograms CAPTURE and BOUNCE (whose statements do not match those shown in Figure 148). The BOUNCE subprogram calls the CAPTURE subprogram, so the mapping call $(B O U N C E, C A P T U R E)$ is one of the mappings in the transitive closure call* ${ }^{*}(B O U N C E)$. Figure 153 shows the classes built for CAPTURE and BOUNCE. The subprogram CAPTURE has been completely transformed since it is called by BOUNCE. CLASS-4 in the figure is the incomplete class being built for BOUNCE. At this point in the transformation, none of the subprogram calls have been converted to messages in CLASS-4.

Let $O O D$ represent the design that includes the classes built for BOUNCE and CAPTURE. Consider the transformation

$$
T_{p b o i}^{2}(O O D, \text { BOUNCE, }\{\text { AXMAJ, AXMIN, ANGFAC \} ) }
$$

This transformation is used to update $O O D$ based on whether the parameters AXMAJ, AXMIN, or ANGFAC have been built as attributes of any of the classes in the call tree of BOUNCE. Note in the figure that even though BOUNCE calls the other subprograms RADIUS and RHO, the AXMAJ, AXMIN, and ANGFAC data items are only passed to the CAPTURE subprogram.

```
real function CAPTURE ( BEAMRA, AXMAJ, AXMIN, ANGFAC )
    begin
    TILTFA := DSIN ( ANGFAC);
    RMAREA := AXMAJ * AXMIN;
    EFAREA := RMAREA * TILTFA;
    BMAREA := BEAMRA * BEAMRA;
    if EFAREA <= BMAREA * 6.0dO
        then USED := EFAREA / BMAREA;
            CAPTURE := 1.0dO - DEXP ( -USED)
        else CAPTURE := 1.OdO
        endif
    end
real function BOUNCE ( ANGFAC, RNGFAC, PROJRA, SIGB,
            RANGE, AXMAJ, AXMIN, XLAMDA)
    begin
    RHOSTD := 0.95 * ANGFAC;
    BEAMRA := RADIUS ( PROJRA, SIGB, RNGFAC, RANGE);
    BOUNCE :=
        CAPTURE ( BEAMRA, AXMAJ, AXMIN, ANGFAC)
            * RHO ( RHOSTD, XLAMDA)
    end
```

Figure 152 Subprograms CAPTURE and BOUNCE

This means, for this example, the only possible class that could have attributes that are linked to these data items through the $\mu^{*}$ relation is the class built for CAPTURE. As is seen in Figure 152, the actual parameters AXMAJ, AXMIN, and ANGFAC are linked to the formal parameters AXMAJ, AXMIN, and ANGFAC, respectively, declared in the CAPTURE subprogram. This means the following mappings are in the transitive closure $\mu^{*}$.

$$
\begin{array}{rll}
B O U N C E:: A X M A J & \xrightarrow{\mu^{*}} & C A P T U R E:: A X M A J \\
B O U N C E:: A X M I N & \xrightarrow{\mu^{*}} & C A P T U R E:: A X M I N \\
B O U N C E:: A N G F A C & \xrightarrow{\mu^{*}} & C A P T U R E:: A N G F A C
\end{array}
$$

As shown in Figure 153, the AXMAJ and AXMIN data items have been built as attributes of the class CLASS-2, but the ANGFAC data item has not been built as an attribute. The ANGFAC data item is a parameter of the method that implements CAPTURE. This means in the $T_{p b o i}^{2}$ transformation, the only data item in the set $P^{\prime}$ is ANGFAC. This attribute is

```
class CLASS-2 attributes BEAMRA, AXMAJ, AXMIN
    method CAPTURE ( C-2, ANGFAC ): real
        begin
        TILTFA := DSIN ( ANGFAC);
        RMAREA := GET-AXMAJ ( C-2) * GET-AXMIN ( C-2);
        EFAREA := RMAREA * TILTFA;
        BMAREA := GET-BEAMRA ( C-2) * GET-BEAMRA ( C-2);
        if EFAREA <= BMAREA * 6.0d0
            then USED := EFAREA / BMAREA;
                CAPTURE := 1.0dO - DEXP ( -USED)
            else CAPTURE := 1.0d0
            endif
        end
    superclass USER-OBJECT
class CLASS-4 attributes ANGFAC, RNGFAC, PROJRA, SIGB,
        RANGE, AXMAJ, AXMIN, XLAMDA
    method BOUNCE ( C-4 ): real
        begin
        RHOSTD := 0.95 * GET-ANGFAC ( C-4);
        BEAMRA := RADIUS ( PROJRA, SIGB, RNGFAC, RANGE);
        BOUNCE :=
            CAPTURE ( BEAMRA, AXMAJ, AXMIN, ANGFAC)
                * RHO ( RHOSTD, XLAMDA)
        end
    superclass USER-OBJECT
```

Figure 153 Classes Built for CAPTURE and BOUNCE
removed from CLASS-4 and added as a parameter to the method that implements BOUNCE by using the $T_{A}^{\delta^{-1}}$ transformation. Figure 154 shows the result of the $T_{p b o i}^{2}$ transformation. The ANGFAC data item has been changed from an attribute of CLASS-4 to a parameter of the BOUNCE method. The message GET-ANGFAC ( C-6), which accesses the attribute angFac, has been changed to access the parameter angFac. This updated class replaces the original class in the design $O O D$ to produce the updated design $O O D^{\prime}$.
6.8.4 Formalizing PBOI Case 3. The third case for converting Category 3 subprograms is that the formal parameter is an attribute and the actual parameter is not a formal. This section defines the transformation that formalizes the third case for PBOI

```
class CLASS-4 attributes RNGFAC, PROJRA, SIGB,
            RANGE, AXMAJ, AXMIN, XLAMDA
    method BOUNCE ( C-4, ANGFAC ): real
        begin
        RHOSTD := 0.95 * ANGFAC;
        BEAMRA := RADIUS ( PROJRA, SIGB, RNGFAC, RANGE);
        BOUNCE :=
            CAPTURE ( BEAMRA, AXMAJ, AXMIN, ANGFAC)
            * RHO ( RHOSTD, XLAMDA)
        end
    superclass USER-OBJECT
```

Figure 154 Updated Class for BOUNCE
as described in Section 5.3. This case identifies parameters from the called subprogram that have been built as attributes of a class but are not formal parameters of the calling subprogram. The transformation needed for this case is to move the identified data items from attributes to parameters. The transformation for case three is defined below followed by an example.

As defined in Section 6.8.2, $\mu^{*}$ represents the transitive closure of the $\mu$ relation, call ${ }^{*}\left(S_{1}\right)$ (the call tree of $S_{1}$ ) represents the transitive closure of the call relation, and $\rho^{*}\left(S_{1}\right)$ represents the set of classes built for the subprograms in the call tree of $S_{1}$. Let $O O D$ represent an object-oriented design, let $S_{1}$ represent the subprogram being transformed, and let $P$ be the set of data items being considered in the transformation. Let $\mu^{*}\left(P^{\prime}\right)$ represent the set of data items resulting from the application of $\mu^{*}$ to the data items in $P^{\prime}$.

Using these definitions, the transformation that formalizes PBOI Case 3 is defined as shown in Figure 155. This transformation is fundamentally different from the transformations that formalize PBOI Case 1 and PBOI Case 2. The changes that were made to the design $O O D$ in Case 1 and Case 2 were made only to the class that was built for the subprogram $S$. In the transformation for Case 3 , no change is required for the class built for $S_{1}$, but other classes in the call tree of $S_{1}$ may be changed. If a class is found in $\rho^{*}\left(S_{1}\right)$ that has an attribute that can be linked to a parameter $p$ through $\theta_{v}$ and the $\mu^{*}$ relation, then each of the classes built for the subprograms that include $p$ as a formal parameter is

$$
\begin{aligned}
& T_{p b o i}^{3}\left(O O D, S_{1}, P\right)=O O D^{\prime} \text { where } \\
& C_{1}=\rho\left(S_{1}\right) \text { and } \\
& P^{\prime}=\left\{p \mid p \in P \text { and } \exists C_{3} \in \rho^{*}\left(S_{1}\right)\right. \text { such that } \\
& \left.b^{\prime}=\theta_{v}\left(\mu^{*}(p)\right) \text { and } b^{\prime} \in \Phi_{C_{3}}\right\} \text { and } \\
& \forall C_{2} \in O O D \\
& S_{2}=\rho^{-1}\left(C_{2}\right) \text { and } \\
& S_{2}=<i d_{S_{2}}, P_{\text {in }_{S_{2}}}, P_{\text {out }_{S_{2}}}, P_{\text {ret }}, P_{\text {loc }}, \Sigma_{S_{2}}>\text { and } \\
& P_{\text {form }_{S_{2}}}=P_{\text {in }_{S_{2}}} \oplus P_{\text {out }_{S_{2}}} \text { and } \\
& A=\left\{a^{\prime} \mid p^{\prime} \in P^{\prime} \text { and } a^{\prime}=\mu^{*}\left(p^{\prime}\right) \text { and } a^{\prime} \in P_{\text {forms }_{2}}\right\} \\
& C_{2}^{\prime}=T_{A}^{\delta^{-1}}\left(O O D, C_{2}, \theta_{v}(A)\right) \text { and } \\
& C_{2}^{\prime} \in O O D^{\prime} \wedge C_{2} \notin O O D^{\prime}
\end{aligned}
$$

Figure 155 The $T_{p b o i}^{3}$ transformation
transformed. The set $P^{\prime}$ collects any such parameters from $P$. All the classes in the design $O O D$ are examined to see if they require an update. The subprogram that is associated with each class $C_{2}$ is determined (by using $\rho^{-1}$ ) and the formal parameters of this subprogram $S_{2}$ are examined. If $S_{2}$ has a formal parameter that can be linked to $p^{\prime} \in P^{\prime}$ by the $\mu^{*}\left(p^{\prime}\right)$ transitive closure, then the $T_{p b o i}^{3}$ transformation must ensure this parameter has not been built as an attribute. The $T_{A}^{\delta^{-1}}$ transformation is used to transform the attributes associated with such parameters into parameters of the methods of $C_{2}$.

Let the term call path represent a mapping in the call ${ }^{*}\left(S_{1}\right)$ transitive closure such that $\operatorname{call}\left(S_{1}, S_{2}\right)$ indicates a call path from $S_{1}$ to $S_{2}$. Since the $\operatorname{call}^{*}\left(S_{1}\right)$ relation is a transitive closure, there may be multiple call mappings in call ${ }^{*}\left(S_{1}\right)$ of the form $\operatorname{call}\left(S_{1}, S_{n}\right)$ and $\operatorname{call}\left(S_{n}, S_{m}\right)$ and $\operatorname{call}\left(S_{m}, S_{2}\right)$ that were used to build the mapping $\operatorname{call}\left(S_{1}, S_{2}\right)$. All these subprograms, $S_{1}, S_{n}, S_{m}$, and $S_{2}$, are considered to be in the call path from $S_{1}$ to $S_{2}$. Let $S_{3}=\rho^{-1}\left(C_{1}\right)$, where $C_{1}$ is the class that has built the parameter $p$ as an attribute. The $T_{p b o i}^{3}$ transformation is built specifically to update all the classes that have been built for the subprograms in the call path from $S_{1}$ and $S_{3}$. This is a critical part of the transformation because each method that implements the subprograms in the call path from $S_{1}$ to $S_{3}$ must now access $p$ as a parameter instead of as an attribute of some object.

This is why each of the classes in the design $O O D$ is checked and the $\mu^{*}\left(p^{\prime}\right)$ transitive closure is used to identify such parameters.

In some classes, the data item $a^{\prime}$ will not be an attribute of the class $C_{2}$. The $T_{A}^{\delta^{-1}}$ transformation will make the correct transformation because set difference is used by the $T_{A}^{\delta^{-1}}$ transformation to remove $a^{\prime}$ from the set of attributes for the class. Whether or not $a^{\prime}$ is in the set of attributes, the result is a set of attributes that does not contain $a^{\prime}$. In some cases, the data item $a^{\prime}$ will already be a parameter of the methods of a class. In this case the $T_{A}^{\delta^{-1}}$ transformation will make the correct transformation because the $\oplus$ operation (defined in Section 3.24) is used to add $a^{\prime}$ to the sequence of formal parameters of each method. The $\oplus$ operation adds a parameter to a sequence of parameters without allowing duplicates. Whether or not $a^{\prime}$ is originally in the sequence of parameters for the method, the result is a sequence of parameters that now includes $a^{\prime}$.

As an example, Figure 156 shows the subprograms RHO, BOUNCE, and RELAY-PHASE. These subprograms have been altered from previous examples. The RELAY-PHASE subpro-
 included in the transitive closure call* $(R E L A Y-P H A S E)$. The BOUNCE subprogram calls the RHO subprogram, so the $\operatorname{call}(B O U N C E, R H O)$ and the $\operatorname{call(RELAY-PHASE,RHO)}$ mappings are also included in the transitive closure call ${ }^{*}(R E L A Y-P H A S E)$. In this example, the call path from RELAY-PHASE to RHO includes the subprograms RHO, BOUNCE, and RELAY-PHASE.

The classes built for these subprograms are shown in Figure 157. The subprograms BOUNCE and RHO have already been transformed since they are called by RELAY-PHASE. The incomplete class being built for RELAY-PHASE is shown in the figure as CLASS-5. The subprogram call to BOUNCE has not yet been converted to a message.

Let $O O D$ represent the design that includes these three classes. Consider the transformation

$$
T_{p b o i}^{3}(O O D, \text { RELAY-PHASE, }\{\text { RHOSTD, XLAMDA \} ) }
$$

```
real function RHO ( RHOSTD, LAMBDA)
    begin
        RHO := RHOSTD * LAMBDA
    end
real function BOUNCE ( ANGFAC, RNGFAC, PROJRA, SIGB,
            RANGE, AXMAJ, AXMIN, RHOSTD, XLAMDA)
    begin
    BEAMRA := RADIUS ( PROJRA, SIGB, RNGFAC, RANGE);
    BOUNCE :=
        CAPTURE ( BEAMRA, AXMAJ, AXMIN, ANGFAC)
            * RHO ( RHOSTD, XLAMDA)
    end
procedure RELAY-PHASE ( DIAM, UPLFAC, RNGFAC, ZENITH, SIGB,
        AXMAJ, AXMIN, PHASE )
        begin
        PHASE := 1;
        RHOSTD := 0.95;
        ANGFAC := UPLFAC * 3.14159;
        PROJRA := DIAM / 2;
        RANGE := ZENITH * PROJRA * ANGFAC;
        XLAMDA := 0.625;
        PHASE := BOUNCE ( ANGFAC, RNGFAC, PROJRA, SIGB,
        RANGE, AXMAJ, AXMIN, RHOSTD, XLAMDA)
    end
```

Figure 156 Subprograms RHO, BOUNCE, and RELAY-PHASE

```
class CLASS-3 attributes RHOSTD, LAMBDA
    method RHO ( C-3 ): real
        begin
            RHO := GET-RHOSTD ( C-3) * GET-LAMBDA ( C-3)
        end
    superclass USER-OBJECT
class CLASS-4 attributes
    method BOUNCE ( C-4, C-5, C-6, C-7 ): real
        begin
        BEAMRA := RADIUS ( C-5);
        BOUNCE := CAPTURE ( C-6, BEAMRA) * RHO ( C-7)
        end
    superclass USER-OBJECT
class CLASS-5 attributes DIAM, UPLFAC, RNGFAC, ZENITH, SIGB,
            AXMAJ, AXMIN, PHASE
    method RELAY-PHASE ( C-8 )
        begin
        SET-PHASE ( C-8, 1);
        RHOSTD := 0.95;
        ANGFAC := GET-UPLFAC ( C-8) * 3.14159;
        PROJRA := GET-DIAM ( C-8) / 2;
        RANGE := GET-ZENITH ( C-8) * PROJRA * ANGFAC;
        XLAMDA := 0.625;
        SET-PHASE ( C-8, BOUNCE ( ANGFAC, RNGFAC, PROJRA, SIGB,
            RANGE, AXMAJ, AXMIN, RHOSTD, XLAMDA))
        end
    superclass USER-OBJECT
```

Figure 157 Classes Built for RHO, BOUNCE, and RELAY-PHASE

This transformation updates the $O O D$ to comply with PBOI Case 3 since the data items RHOSTD and XLAMDA are actual parameters but are not formal parameters in the subprogram RELAY-PHASE. As shown in the figure, the RELAY-PHASE subprogram calls the BOUNCE subprogram passing RHOSTD and XLAMDA as actual parameters. The corresponding formal parameters in BOUNCE are the RHOSTD and XLAMDA formal parameters. This means the following mappings are in the transitive closure $\mu^{*}$.

$$
\begin{array}{rlll}
\text { RELAY-PHASE :: RHOSTD } & \xrightarrow{\mu^{*}} & \text { BOUNCE :: RHOSTD } \\
\text { RELAY-PHASE }:: X L A M D A & \xrightarrow{\mu^{*}} & \text { BOUNCE :: XLAMDA }
\end{array}
$$

The subprogram BOUNCE calls the subprogram RHO passing the RHOSTD and XLAMDA parameters as actual parameters. The formal parameters that correspond to these actual parameters are the RHOSTD and LAMBDA formal parameters declared in RHO. This means the following mappings are in the transitive closure $\mu^{*}$.

$$
\begin{array}{rll}
\text { BOUNCE :: RHOSTD } & \xrightarrow{\mu^{*}} & \text { RHO :: RHOSTD } \\
\text { BOUNCE }:: \text { XLAMDA } & \xrightarrow{\mu^{*}} & \text { RHO :: LAMBDA } \\
\text { RELAY-PHASE :: RHOSTD } & \xrightarrow{\mu^{*}} & \text { RHO :: RHOSTD } \\
\text { RELAY-PHASE :: XLAMDA } & \xrightarrow{\mu^{*}} & \text { RHO :: LAMBDA }
\end{array}
$$

Notice in Figure 157, the class CLASS-3 built for RHO has built the RHOSTD and LAMBDA data items as attributes of the class. This means a class exists in the call tree for RELAY-PHASE that includes an attribute that can be linked to the RHOSTD and XLAMDA data items in RELAY-PHASE. Both RHOSTD and XLAMDA are collected into the set $P^{\prime}$ for further evaluation. The $T_{p b o i}^{3}$ transformation continues by examining each class in the design $O O D$ to see if it needs to be transformed.

For this example, $O O D$ includes the three classes CLASS-3, CLASS-4, and CLASS-5. For CLASS-5, the $T_{p b o i}^{3}$ transformation uses the $\rho^{-1}$ mapping to find the subprogram RELAY-PHASE. The set $A$ for this subprogram is empty since, in this example, each $p^{\prime}$ is a variable defined in RELAY-PHASE and there are no mappings in $\mu^{*}$ that map data items in RELAY-PHASE to other data items in RELAY-PHASE. The only way for those mappings to be in $\mu^{*}$ would be from a recursive call to RELAY-PHASE, and recursion is not allowed in the

GOM. This means the empty set $A$ is passed to the $T_{A}^{\delta-1}$ transformation and no change is made to CLASS-5, as required.

For CLASS-4, the subprogram associated with this class is the BOUNCE subprogram. The set $A$ for BOUNCE is \{ RHOSTD, XLAMDA \} since these data items have mappings in $\mu^{*}$ to the RHOSTD and XLAMDA data items, respectively, in RELAY-PHASE. The $T_{A}^{\delta^{-1}}$ transformation is passed these data items which ensures RHOSTD and XLAMDA aren't attributes of CLASS-4, but are parameters of the methods of CLASS-4.

```
class CLASS-4 attributes
    method BOUNCE ( C-4, C-5, C-6, C-7, RHOSTD, XLAMDA ): real
        begin
        BEAMRA := RADIUS ( C-5);
        BOUNCE := CAPTURE ( C-6, BEAMRA)
            * RHO ( C-7, RHOSTD, XLAMDA)
        end
    superclass USER-OBJECT
```

Figure 158 Updated Class for BOUNCE

Figure 158 shows the result of this transformation. The RHOSTD and XLAMDA data items are not attributes of CLASS-4, but are formal parameters of the method BOUNCE and actual parameters in the message RHO.

For CLASS-3, the subprogram associated with this class is the RHO subprogram. The set $A$ for RHO is \{ RHOSTD, LAMBDA \} since these data items have mappings in $\mu^{*}$ to the RHOSTD and XLAMDA data items, respectively, in RELAY-PHASE. The $T_{A}^{\delta^{-1}}$ transformation is passed \{ RHOSTD, LAMBDA \} which ensures RHOSTD and LAMBDA are not attributes of CLASS-3, but are parameters of the methods of CLASS-3.

Figure 159 shows the result of the $T_{A}^{\delta^{-1}}$ transformation on CLASS-3. The RHOSTD and LAMBDA data items are now parameters of the RHO method instead of attributes of the CLASS-3 class. The three updated classes are included in the updated design returned from the $T_{p b o i}^{3}$ transformation.

```
class CLASS-3 attributes
    method RHO ( C-3, RHOSTD, LAMBDA ): real
        begin
            RHO := RHOSTD * LAMBDA
        end
    superclass USER-OBJECT
```

Figure 159 Updated Class for RHO

$$
\begin{aligned}
& T_{p b o i}^{4}(O O D, S, P)=O O D^{\prime} \text { where } \\
& P^{\prime}=\left\{p \mid p \in P \text { and } \neg \exists C_{1} \in \rho^{*}(S)\right. \text { such that } \\
& \left.a^{\prime}=\theta_{v}\left(\mu^{*}(p)\right) \text { and } a^{\prime} \in \Phi_{C_{1}}\right\} \text { and } \\
& O O D^{\prime}=O O D
\end{aligned}
$$

Figure 160 The $T_{p b o i}^{4}$ transformation
6.8.5 Formalizing PBOI Case 4. The fourth case for converting Category 3 subprograms is that the formal parameter is a parameter of the method and the actual parameter is not a formal. This section defines the formal transformation that formalizes the fourth case of the PBOI method, as defined in Section 5.3. This case identifies parameters of the called subprogram that are not attributes of a class and are not formal parameters of the calling subprogram. There is no change to the object-oriented design required for this case. The transformation is defined below and an example is given at the end of this section.

As defined in Section 6.8.2, $\mu^{*}$ represents the transitive closure of the $\mu$ relation, call $^{*}(S)$ (the call tree of $S$ ) represents the transitive closure of the call relation, and $\rho^{*}(S)$ represents the set of classes built for the subprograms in the call tree of $S$. The $T_{p b o i}^{4}$ formal transformation is defined in Figure 160. This transformation checks each class in the set of classes built for the subprograms in the call tree of $S$. If none of the classes have an attribute that can be linked through the $\mu^{*}$ relation to the parameter $p$, then there is no change required in the design.

For example, consider the RADIUS and BOUNCE subprograms shown in Figure 161. These subprograms have been changed slightly to illustrate PBOI Case 4. The PROJRA

```
real function RADIUS ( PROJRA, SIGB, RNGFAC, RANGE )
    begin
    if RNGFAC > 0.0 and RANGE > 0.0
        then SIGABS := DABS ( SIGB);
            SLOPE := SIGABS - PROJRA / RANGE;
            SPOT := SIGABS * RANGE;
            RAD := DABS ( PROJRA + SLOPE * RNGFAC);
            RADIUS := DMAX1 ( SPOT, RAD)
        else
                RADIUS := PROJRA
        endif
    end
real function BOUNCE ( ANGFAC, RNGFAC, SIGB,
            RANGE, AXMAJ, AXMIN, XLAMDA)
    begin
    PROJRA := XLAMDA * 3.14159;
    RHOSTD := 0.95;
    BEAMRA := RADIUS ( PROJRA, SIGB, RNGFAC, RANGE);
    BOUNCE :=
        CAPTURE ( BEAMRA, AXMAJ, AXMIN, ANGFAC)
        * RHO ( RHOSTD, XLAMDA)
    end
```

Figure 161 Subprograms RADIUS and BOUNCE

```
class CLASS-1 attributes RANGE, RNGFAC, SIGB
    method RADIUS ( C-1, PROJRA ): real begin
        if GET-RNGFAC ( C-1) > 0.0 and GET-RANGE ( C-1) > 0.0
            then SIGABS := DABS ( GET-SIGB ( C-1));
                SLOPE := SIGABS - PROJRA / GET-RANGE ( C-1);
                SPOT := SIGABS * GET-RANGE ( C-1);
                RAD := DABS ( PROJRA + SLOPE * GET-RNGFAC ( C-1));
                RADIUS := DMAX1 ( SPOT, RAD)
            else
                RADIUS := PROJRA
            endif
        end
    superclass USER-DBJECT
class CLASS-4 attributes ANGFAC, RNGFAC, SIGB,
            RANGE, AXMAJ, AXMIN, XLAMDA
    method BOUNCE ( C-4 ): real
        begin
        PROJRA := XLAMDA * 3.14159;
        RHOSTD := 0.95;
        BEAMRA := RADIUS ( PROJRA, SIGB, RNGFAC, RANGE);
        BOUNCE :=
            CAPTURE ( BEAMRA, AXMAJ, AXMIN, ANGFAC)
                * RHO ( RHOSTD, XLAMDA)
        end
    superclass USER-OBJECT
```

Figure 162 Classes Built for RADIUS and BOUNCE
data item is now a local variable of the BOUNCE subprogram. In the example, the BOUNCE subprogram calls the RADIUS subprogram, so the $\operatorname{call(BOUNCE,RADIUS)~mapping~is~}$ included in the transitive closure call ${ }^{*}(B O U N C E)$. Figure 162 shows the classes that are built for the RADIUS and BOUNCE subprograms. The RADIUS subprogram has been completely transformed into class CLASS-1 since it is called by BOUNCE. This class is different than the other example transformation of RADIUS (see Figure 149). In Figure 162, the PROJRA data item has not been built as an attribute of the class built for RADIUS. The incomplete class being built for BOUNCE is CLASS-4 as shown in the figure.

Let $O O D$ represent the design developed so far that includes these two classes. Consider the transformation

$$
T_{p b o i}^{4}(O O D, \text { BOUNCE, PROJRA })
$$

This transformation checks all the classes built for the subprograms in the call tree of BOUNCE to see if any of the classes have an attribute that can be linked through the $\mu^{*}$ relation to the PROJRA data item. The BOUNCE subprogram calls the RADIUS subprogram passing in PROJRA as an actual parameter. The formal parameter that corresponds to this actual parameter is the PROJRA formal parameter declared in RADIUS. This means the following mapping is in the transitive closure $\mu^{*}(B O U N C E:: ~ P R O J R A)$.

$$
\text { BOUNCE :: PROJRA } \xrightarrow{\mu^{*}} \text { RADIUS :: PROJRA }
$$

The PROJRA data item in RADIUS has not been built as an attribute. Since in the call tree of BOUNCE the PROJRA data item is passed only to the RADIUS subprogram, it can be concluded that there is not a class in the call tree of BOUNCE that has built PROJRA as an attribute.
6.8.6 Formalizing PBOI. This section defines the high-level transformation combining the four transformations that formalize the four PBOI cases. The formal transformation is defined and an example is given.

Let $O O D$ represent an object-oriented design, let $S$ represent the subprogram being transformed. Let $C_{s u b_{S}}$ be the set of imperative subprogram calls in $S$. Let $P_{a c t}^{*}$ be the sequence collecting all the actual parameters from all the subprogram calls in $S$, such that duplicate parameters are not included and the order of the sequence is arbitrary. Let $T_{p b o i}$ be the high-level transformation that determines which of the four PBOI transformations to apply. The $T_{p b o i}$ transformation is defined as shown in Figure 163. This transformation is used to update the object-oriented design developed so far by applying the four PBOI transformations defined in the previous sections. This high-level transformation examines the subprogram calls that the input subprogram $S$ makes and determines which PBOI formal transformation to apply for each actual parameter $p$ of the call. The reduce operation

$$
\begin{aligned}
& T_{p b o i}(O O D, S)=O O D_{4} \text { where } \\
& S=<i d_{S}, P_{\text {in }}, P_{\text {out }}, P_{r e t}, P_{l o c}, \Sigma_{S}>\text { and } \\
& P_{\text {form }}=P_{\text {in }} \oplus P_{\text {out }} \text { and } \\
& P_{a c t}^{*}=\operatorname{reduce}\left(\oplus,\left[P_{\text {act }}^{n}|~| l \in C_{s u b_{S}}\right. \text { and }\right. \\
& \left.\left.l=<i d_{S_{n}}, P_{a c t_{n}}>\right]\right) \text { and } \\
& O O D_{1}=T_{p b o i}^{1}\left(O O D, S, P_{f o r m} \cap P_{a c t}^{*}\right) \text { and } \\
& O O D_{2}=T_{p b o i}^{2}\left(O O D_{1}, S, P_{\text {form }} \cap P_{a c t}^{*}\right) \text { and } \\
& O O D_{3}=T_{p b o i}^{3}\left(O O D_{2}, S, P_{\text {form }}-P_{\text {act }}^{*}\right) \text { and } \\
& O O D_{4}=T_{p b o i}^{4}\left(O O D_{3}, S, P_{\text {form }}-P_{a c t}^{*}\right)
\end{aligned}
$$

Figure 163 The $T_{p b o i}$ transformation
is used to apply the $\oplus$ operator in order to combine the sequences of actual parameters. Each actual parameter is checked to see if it is also a formal parameter of the calling subprogram $S$. If an actual parameter is also a formal parameter, then either PBOI Case 1 or PBOI Case 2 applies. If the actual parameter is not a formal parameter then either PBOI Case 3 or PBOI Case 4 applies.

### 6.9 Transforming Subprogram Calls

This section defines the formal transformations used to convert imperative subprogram calls to object-oriented messages. These transformations are used when converting Category 3 subprograms. Let $v_{m}$ be the transformation that converts subprogram calls to messages. In order to transform a subprogram call $l$ into a message $m$, it is assumed that the method to be invoked by $m$ already exists in some class $C$. Let instance $(c, C)$ represent whether or not an object $c$ is an instance of class $C$. The $v_{m}$ transformation is defined as shown in Figure 164. This transformation ensures that each of the actual parameters in the subprogram call $l$ is either an attribute of the target object $c$, an attribute of one of the other objects passed to the method, or a formal parameter of the method. The target object, each of the other objects required, and the formal parameters are built as actual parameters of the message $m$.

```
\(v_{m}(S, l)=m\) where
    \(l=\left\langle i d_{S_{n}}, P_{\text {act }}>\right.\) and
    ( \(\exists C_{2} \in \rho^{*}(S)\) such that
        \(C_{2}=\left\langle i d_{C_{2}}, \Phi_{C_{2}}, \Omega_{C_{2}}, \lambda_{0}\right\rangle\) and
        \(\left.\Omega_{C_{2}}=\left\{<i d_{S_{n}}, \tau_{2}, Q_{o b j_{2}}, Q_{\text {form }_{2}}, Q_{\text {ret }}^{2},\left(Q_{l o c_{2}}, \Psi_{C_{2}}\right\rangle\right\}\right)\) and
    instance (c, \(C_{2}\) ) and
    \(\forall p \in P_{\text {act }}\)
        \(\left[a^{\prime}=\theta_{v}\left(\mu^{*}(p)\right)\right.\) and \(\left.a^{\prime} \in \Phi_{C_{2}}\right]\) or
        \(\left[\exists C_{3} \in \rho^{*}(S)\right.\) such that
            \(C_{3}=\left\langle i d_{C_{3}}, \Phi_{C_{3}}, \Omega_{C_{3}}, \lambda_{0}\right\rangle\) and
            \(b^{\prime}=\theta_{v}\left(\mu^{*}(p)\right)\) and \(b^{\prime} \in \Phi_{C_{3}}\) and
            instance \(\left(q, C_{3}\right)\) and
            \(q \in Q_{o b j_{2}}\) and \(q \in Q_{o b j}\) and \(\left.q \in Q_{o b j}^{\prime}\right]\) or
            \(\left[d^{\prime}=\theta_{v}\left(\mu^{*}(p)\right)\right.\) and \(d^{\prime} \in Q_{\text {form }_{2}}\) and
            \(p^{\prime}=\theta_{v}(p)\) and \(\left.p^{\prime} \in Q_{\text {form }}^{\prime}\right]\) and
    \(Q_{a c t}^{\prime}=[c] \oplus Q_{o b j}^{\prime} \oplus Q_{f o r m}^{\prime}\) and
    \(m=\left\langle i d_{S_{n}}, Q_{a c t}^{\prime}\right\rangle\)
```

Figure 164 The $v_{m}$ transformation

The $v_{e}$ transformation, shown in Figure 165, is defined to use the $v_{m}$ transformation in order to transform each function call in an imperative expression. This transformation uses the formal definition for each imperative expression in its definition.

In order to transform subprogram calls in each statement and expression, the $v$ transformation has been defined as shown in Figure 166. This transformation uses the formal definitions of imperative statements in its definition. Recall that pos $(s, \Psi)$ indicates the ordinal position of a statement, $s$, in the sequence, $\Psi$. This transformation takes as input a sequence of imperative statements and object-oriented statements. The resulting sequence is a sequence of object-oriented statements where the subprogram calls from the imperative statements and expressions have been replaced by messages.

Let $T_{l}^{v}$ be the formal transformation that takes an object-oriented design, a subprogram, and a class and replaces all the subprogram calls in the statements of the class' method with messages. This transformation is defined as follows.

$$
\begin{aligned}
& T_{l}^{v}(O O D, S, C)=O O D^{\prime} \text { where } \\
& \quad C=<i d_{C}, \Phi_{C}, \Omega_{C}, \lambda_{0}>\text { and } \\
& \Omega_{C}=\left\{<i d_{o}, \tau, Q_{o b j}, Q_{f o r m}, Q_{r e t}, Q_{l o c}, \Psi_{o}>\right\} \text { and } \\
& \Psi_{o}^{\prime}=v\left(S, \Psi_{o}\right) \text { and } \\
& \Omega_{C}^{\prime}=\left\{<i d_{o}, \tau, Q_{o b j}, Q_{f o r m}, Q_{r e t}, Q_{l o c}, \Psi_{o}^{\prime}>\right\} \text { and } \\
& C^{\prime}=<i d_{C}, \Phi_{C}, \Omega_{C}^{\prime}, \lambda_{0}>\text { and } \\
& O O D^{\prime}=T_{c}^{+}\left(T_{c}^{-}(O O D, C), C^{\prime}\right)
\end{aligned}
$$

Each subprogram call in $\Psi_{o}$ is replaced by corresponding message by using the $v$ transformation. These updated statements are used to update the method in $\Omega_{C}^{\prime}$ in the updated class $C^{\prime}$. This class replaces the class $C$ in the design and the updated design is returned as the result of the transformation.

For example, Figure 167 shows the class CLASS-3 containing the method RHO. The original imperative subprogram call to the subprogram RHO is given below.

RHO ( RHOSTD, XLAMDA, ANGFAC)
Let C-7 be an instance of CLASS-3. The subprogram call is transformed into the following message.

$$
\begin{aligned}
& v_{e}(S, e)=e^{\prime} \text { where } \\
& \left.e=<i d_{S_{n}}, P_{\text {act }}\right\rangle \Rightarrow e^{\prime}=v_{m}(S, e) \text { and } \\
& \text { gom-variable }(e) \Rightarrow e^{t}=e \text { and } \\
& e=<\text { GET-a, }[c]>\Rightarrow e^{\prime}=e \text { and } \\
& \text { gom-literal-boolean }(e) \Rightarrow e^{\prime}=e \text { and } \\
& \text { gom-literal-integer }(e) \Rightarrow e^{\prime}=e \text { and } \\
& \text { gom-literal-real }(e) \Rightarrow e^{\prime}=e \text { and } \\
& \text { gom-literal-string }(e) \Rightarrow e^{\prime}=e \text { and } \\
& e=\left\langle e_{1},+, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle v_{e}\left(S, e_{1}\right),+, v_{e}\left(S, e_{2}\right)\right\rangle \text { and } \\
& e=\left\langle e_{1} \text {, and, } e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle v_{e}\left(S, e_{1}\right) \text {, and, } v_{e}\left(S, e_{2}\right)\right\rangle \text { and } \\
& e=\left\langle e_{1}, \&, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle v_{e}\left(S, e_{1}\right), \&, v_{e}\left(S, e_{2}\right)\right\rangle \text { and } \\
& e=\left\langle e_{1}, /, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle v_{e}\left(S, e_{1}\right), /, v_{e}\left(S, e_{2}\right)\right\rangle \text { and } \\
& e=\left\langle e_{1},=, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle v_{e}\left(S, e_{1}\right),=, v_{e}\left(S, e_{2}\right)\right\rangle \text { and } \\
& e=\left\langle e_{1},{ }^{* *}, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle v_{e}\left(S, e_{1}\right),{ }^{* *}, v_{e}\left(S, e_{2}\right)\right\rangle \text { and } \\
& \left.\left.e=\left\langle e_{1},\right\rangle=, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle v_{e}\left(S, e_{1}\right),\right\rangle=v_{e}\left(S, e_{2}\right)\right\rangle \text { and } \\
& \left.\left.e=\left\langle e_{1},\right\rangle, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle v_{e}\left(S, e_{1}\right),\right\rangle, v_{e}\left(S, e_{2}\right)\right\rangle \text { and } \\
& e=\left\langle e_{1},<=, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle v_{e}\left(S, e_{1}\right),<=, v_{e}\left(S, e_{2}\right)\right\rangle \text { and } \\
& e=\left\langle e_{1},\left\langle, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle v_{e}\left(S, e_{1}\right),\left\langle, v_{e}\left(S, e_{2}\right)\right\rangle\right. \text { and }\right. \\
& e=\left\langle e_{1},{ }^{*}, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle v_{e}\left(S, e_{1}\right),{ }^{*}, v_{e}\left(S, e_{2}\right)\right\rangle \text { and } \\
& e=\left\langle e_{1},\langle \rangle, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle v_{e}\left(S, e_{1}\right),\langle \rangle, v_{e}\left(S, e_{2}\right)\right\rangle \text { and } \\
& e=\left\langle e_{1} \text {, or, } e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle v_{e}\left(S, e_{1}\right) \text {, or, } v_{e}\left(S, e_{2}\right)\right\rangle \text { and } \\
& e=\left\langle e_{1},-, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle v_{e}\left(S, e_{1}\right),-, v_{e}\left(S, e_{2}\right)\right\rangle \text { and } \\
& e=\left\langle-, e_{1}\right\rangle \Rightarrow e^{\prime}=\left\langle-, v_{e}\left(S, e_{1}\right)\right\rangle \text { and } \\
& e=\left\langle\operatorname{not}, e_{1}\right\rangle \Rightarrow e^{\prime}=\left\langle\operatorname{not}, v_{e}\left(S, e_{1}\right)\right\rangle
\end{aligned}
$$

Figure 165 The $v_{e}$ transformation

$$
\begin{aligned}
& v(S, \Psi)=\Psi^{\prime} \text { where } \\
& \forall s \in \Psi \\
& s=<i d_{S_{n}}, P_{\text {act }}>\Rightarrow \\
& s^{\prime}=v_{m}(S, s) \text { and } \\
& s=<x,:=, e>\Rightarrow \\
& s^{\prime}=<x,:=, v_{e}(S, e)>\text { and } \\
& s=<\text { if }, e, \text { then, } S_{1}, \text { else }, S_{2}>\Rightarrow \\
& s^{\prime}=<\text { if, } v_{e}(S, e), \text { then }, v\left(S, S_{1}\right), \text { else }, v\left(S, S_{2}\right)>\text { and } \\
& s=<\text { while }, e, S_{3}>\Rightarrow \\
& s^{\prime}=<\text { while, } v_{e}(S, e), v\left(S, S_{3}\right)>\text { and } \\
& s=<\text { output, oport }, E>\Rightarrow \\
& s^{\prime}=<\text { output, oport, } E^{\prime}>\text { and } \\
& E^{\prime}=\left[v_{e}(S, e) \mid e \in E\right] \text { and } \\
& s^{\prime} \in \Psi^{\prime} \text { and } \\
& \operatorname{pos}(s, \Psi)=\operatorname{pos}\left(s^{\prime}, \Psi^{\prime}\right)
\end{aligned}
$$

Figure 166 The $v$ transformation
RHO ( C-7, RHOSTD)
using the $T_{l}^{v}$ transformation. The RHOSTD actual parameter is passed as a parameter of the message, the XLAMDA actual parameter has become the LAMBDA attribute of CLASS-3, and the actual parameter ANGFAC has also become an attribute of CLASS-3.

When duplicate classes are eliminated from the object-oriented design using the PBOI methodology, the signature of methods in the resulting class may be changed. The signatures of any messages that call these methods must be updated to match. Let the

```
class CLASS-3 attributes LAMBDA, ANGFAC
    method RHO ( C-3, RHOSTD ): real
        begin
            RHO := RHOSTD
        end
    superclass USER-OBJECT
```

Figure 167 Class CLASS-3 with Method RHO

$$
\begin{aligned}
& T_{o}^{v}(O O D, C)=O O D^{\prime} \text { where } \\
& C=<i d_{C}, \Phi_{C}, \Omega_{C}, \lambda_{0}>\text { where } \\
& \Omega_{C}=\left\{<i d_{S_{n}}, \tau, Q_{o b j}, Q_{f o r m}, Q_{r e t}, Q_{l o c}, \Psi_{o}>\right\} \text { and } \\
& \forall C_{1} \in O O D \\
& \quad C_{1}=<i d_{C_{1}}, \Phi_{C_{1}}, \Omega_{C_{1}}, \lambda_{0}>\text { and } \\
& \Omega_{C_{1}}=\left\{<i d_{o}, \tau, Q_{o b j_{1}}, Q_{\text {form }_{1}}, Q_{r e t_{1}}, Q_{l o c_{1}}, \Psi_{o_{1}}>\right\} \text { and } \\
& \text { For each } m \in C_{s u b_{o}} \\
& m=<i d_{S_{n}}, Q_{a c t}>\Rightarrow \\
& \quad v_{s}\left(Q_{a c t}^{\prime}, Q_{f o r m}\right) \text { and } \\
& m^{\prime}=<i d_{S_{n}}, Q_{a c t}^{\prime}>\text { and } m^{\prime} \in C_{s u b_{o}}^{\prime} \text { and } \\
& m \neq<i d_{S_{n}}, Q_{a c t}>\Rightarrow \\
& \quad m^{\prime}=m \text { and } m^{\prime} \in C_{s u b_{o}}^{\prime} \text { and } \\
& \Psi_{o_{1}}^{\prime}=\Psi_{o_{1}} \text { with } m \in C_{s u b_{o}} \text { replaced by } m^{\prime} \in C_{s u b_{o}}^{\prime} \text { and } \\
& \Omega_{C_{1}}^{\prime}=\left\{<i d_{o}, \tau, Q_{o b j_{1}}, Q_{f o r m_{1}}, Q_{r e t_{1}}, Q_{l o c_{1}}, \Psi_{o_{1}}^{\prime}>\right\} \text { and } \\
& C_{1}^{\prime}=<i d_{C_{1}}, \Phi_{C_{1}}, \Omega_{C_{1}}^{\prime}, \lambda_{0}>\text { and } \\
& C_{1} \notin O O D^{\prime} \text { and } C_{1}^{\prime} \in O O D^{\prime}
\end{aligned}
$$

Figure 168 The $T_{o}^{v}$ transformation
$T_{o}^{v}$ transformation represent this updating of message signatures as defined below. Let $O O D$ represent an object-oriented design, let $C_{m s g_{o}}$ represent the collection of messages in a method. Let $v_{s}\left(Q_{a c t}^{\prime}, Q_{f o r m}\right)$ represent whether or not the number, order, and type of the actual parameters in $Q_{a c t}^{\prime}$ match the number, order, and type of the formal parameters in $Q_{f o r m}$. The $T_{o}^{v}$ transformation is shown in Figure 168. This transformation checks each method in each class in $O O D$ for messages being sent to the method in $C$. If the call identifier of the message $m$ matches the name of the method in $C$, then the $v_{s}$ condition is used to ensure the signature of $m$ matches the formals in $Q_{\text {form }}$. The updated message, $m^{\prime}$, replaces $m$ in $C_{1}^{\prime}$ and this updated class replaces $C_{1}$ in $O O D$. If the call identifier of $m$ is not referring to the method in $C$, then no update is required and the original message, $m$, is stored in $C_{1}^{\prime}$.

### 6.10 Category 3 Subprogram Transformation ( $\sigma_{3}$ )

This section defines the formal transformation used to convert Category 3 subprograms to the object paradigm. As discussed in Section 5.3, the transformation of Category 3 subprograms to the object paradigm uses the PBOI method to update the object design developed so far.

At this point of the transformation, there may be several classes in the design that have been built for the same subprogram. This is because a subprogram may be called multiple times and a new class is built for the subprogram for each of the calls. These classes built for the same subprogram are termed duplicate classes because they have the same name. Duplicate classes are eliminated from the design by merging any duplicate classes into a new class that replaces the duplicates throughout the entire design. The set of attributes for the new class is the intersection of the sets of attributes from the duplicate classes. The set of operations for the new class is the union of the sets of operations from the duplicate classes. The formal transformation that defines the removal of duplicate classes is defined below.

Let $T_{\text {class }}^{d u p}$ represent the formal transformation that removes duplicate classes from the object-oriented design. Let $O O D$ represent an object-oriented design. The $T_{\text {class }}^{d u p}$ transformation is defined as shown in Figure 169.

This transformation examines all pairs of classes in $O O D$. Duplicate classes are identified by comparing the name of the class. This implies the generated name of a class must be the same each time a class is built for a subprogram. If the names of two classes $C_{1}$ and $C_{2}$ are the same, then a new class $C_{3}$ is built to represent the intersection of these two classes. The attributes of $C_{3}$ are built by initially unioning $\Phi_{C_{1}}$ and $\Phi_{C_{2}}$ and then using the $T_{A}^{\delta^{-1}}$ transformation to remove those attributes not in the intersection of $\Phi_{C_{1}}$ and $\Phi_{C_{2}}$. The operations of $C_{3}$ are built by unioning $\Omega_{C_{1}}$ and $\Omega_{C_{2}}$. The union is based on the fact that two operations are equal if they have the same name. The $C_{3}$ class replaces the two duplicate classes in $O O D^{\prime}$. The final step of the transformation is to ensure any calls to the updated method in $C_{3}$ match the signature of the method.

$$
\begin{aligned}
& T_{c l a s s}^{d u p}(O O D)=O O D^{\prime \prime} \text { where } \\
& \forall C_{1} \text { and } C_{2} \in O O D \\
& C_{1} \neq C_{2} \text { and } \\
& C_{1}=<i d_{C}, \Phi_{C_{1}}, \Omega_{C_{1}}, \lambda_{0}>\text { and } \\
& C_{2}=<i d_{C}, \Phi_{C_{2}}, \Omega_{C_{2}}, \lambda_{0}>\Rightarrow \\
& C_{3}=<i d_{C}, \Phi_{C_{3}}, \Omega_{C_{3}}, \lambda_{0}>\text { and } \\
& \Phi_{C_{3}}=\Phi_{C_{1}} \cup \Phi_{C_{2}} \text { and } \\
& \Omega_{C_{3}}=\Omega_{C_{1}} \cup \Omega_{C_{2}} \text { and } \\
& A=\left(\Phi_{C_{1}} \cup \Phi_{C_{2}}\right)-\left(\Phi_{C_{1}} \cap \Phi_{C_{2}}\right) \text { and } \\
& C_{3}^{\prime}=T_{A}^{\delta^{-1}}\left(O O D, C_{3}, A\right) \text { and } \\
& C_{1} \notin O O D^{\prime} \text { and } C_{2} \notin O O D^{\prime} \text { and } \\
& C_{3}^{\prime} \in O O D^{\prime} \text { and } \\
& O O D^{\prime \prime}=T_{o}^{v}\left(O O D^{\prime}, C_{3}^{\prime}\right)
\end{aligned}
$$

Figure 169 The $T_{\text {class }}^{d u p}$ transformation

Now that the transformation for eliminating duplicate classes has been defined, the transformation of Category 3 subprograms is defined as follows. Let $O O D$ represent an object-oriented design, let $S$ represent the Category 3 subprogram to be transformed. Let $O O D^{\prime}$ represent the updated object-oriented design that results from converting $S$ to the object paradigm. Let $P_{\text {form }}$ be the sequence of formal parameters from the subprogram $S$. Let $C_{s u b_{s}}$ be the set of imperative subprogram calls in $S$. The formal transformation for Category 3 subprograms is defined as shown in Figure 170. This transformation converts each of the subprograms called by $S$ and unions all the designs that are returned. This is done by applying the $\sigma$ transformation to each of the subprograms in $C_{s u b_{S}}^{*}$. Initially, a class is built to represent the Category 3 subprogram by building all the formal parameters of the subprogram as attributes of the class. The subprogram $S$ is initially transformed into a method included in $\Omega_{C}$. Each statement in the subprogram is transformed into an objectoriented statement by $\theta$ and accesses to parameters of the subprogram are transformed by $\delta$ into attribute accesses of the instance object $\tau$. This initial class is added to the design built so far, viz. $O O D^{\prime}$, and the resulting design is passed to the PBOI transformation along with $S$. Since the PBOI transformation has the potential to change the class $C$,

$$
\begin{aligned}
& \sigma_{3}(O O D, S)=O O D^{(4)} \text { where } \\
& S=\left\langle i d_{S}, P_{\text {in }}, P_{\text {out }}, P_{\text {ret }}, P_{\text {loc }}, \Sigma_{S}\right\rangle \text { and } \\
& P_{\text {form }}=P_{\text {in }} \oplus P_{\text {out }} \text { and } \\
& \forall l \in C_{s u b_{s}} \\
& l=\left\langle i d_{S_{n}}, P_{a c t_{n}}>\right.\text { and } \\
& S_{n} \in C_{s u b_{S}}^{*} \\
& O O D^{\prime}=\bigcup_{i=1}^{n}\left(\sigma\left(O O D, S_{i}\right)\right) \text { where } S_{i} \in C_{s u b_{s}}^{*} \text { and } \\
& C=<i d_{C}, \Phi_{C}, \Omega_{C}, \lambda_{0}>\text { and } \\
& \Phi_{C}=\operatorname{range}\left(\theta_{v}\left(P_{\text {form }}\right)\right) \text { and } \\
& \tau \text { is an instance of } C \text { and } \\
& \Psi=\delta\left(\theta\left(\Sigma_{S}\right), \tau, \Phi_{C}\right) \text { and } \\
& Q_{r e t}=\theta_{v}\left(P_{r e t}\right) \text { and } \\
& Q_{l o c}=\theta_{v}\left(P_{l o c}\right) \text { and } \\
& \Omega_{C}=\left\{<i d_{S}, \tau, \emptyset, \emptyset, Q_{r e t}, Q_{l o c}, \Psi>\right\} \text { and } \\
& O O D^{\prime \prime}=T_{p b o i}\left(T_{c}^{+}\left(O O D^{\prime}, C\right), S\right) \text { and } \\
& C^{\prime}=\text { updated } C \in O O D^{\prime \prime} \text { and } \\
& O O D^{\prime \prime \prime}=T_{l}^{v}\left(O O D^{\prime \prime}, S, C^{\prime}\right) \text { and } \\
& O O D^{(4)}=T_{\text {class }}^{\text {dup }}\left(O O D^{\prime \prime \prime}\right)
\end{aligned}
$$

Figure 170 The $\sigma_{3}$ transformation

```
procedure RELAY-EFFNCY
    ( BETA, XLAMDA, DIAM, UPLFAC, SIGJ, ZENITH, MIRRORS, GEOM,
        AXMAJ, AXMIN, SIGJR, EFFNCY )
begin
    ...
    if MIRRORS > 0
        then RNGFAC := 0.0;
            K := 1;
            while K <= MIRRORS do begin
                ANGFAC := GEOM ( 1, K);
                RNGFAC := RNGFAC + GEOM ( 2, K);
                TRANS := TRANS
                    * BOUNCE ( ANGFAC, RNGFAC, PROJRA, SIGB,
                        RANGE, AXMAJ, AXMIN, XLAMDA);
                K := K + 1
                end
        else endif;
    ANGFAC := GEOM ( 1, MIRRORS + 1);
    EFFNCY := TRANS
    end
```

Figure 171 Partial declaration of RELAY-EFFNCY
the class $C^{\prime}$ represents this updated class after the PBOI transformation. This updated class $C^{\prime}$ is passed to the $T_{l}^{v}$ transformation in order to transform the subprogram calls in $C^{\prime}$ to messages. The class returned from the $T_{l}^{v}$ transformation replaces $C^{\prime}$ in the design. The $T_{\text {class }}^{\text {dup }}$ transformation is used to remove duplicate classes and the resulting design is returned as the result of the transformation.

For example, Figure 171 shows the partial declaration of the imperative subprogram RELAY-EFFNCY (using GIL syntax). This subprogram is a Category 3 subprogram that calls the BOUNCE subprogram. The first step in converting RELAY-EFFNCY to the object paradigm is to transform the BOUNCE subprogram and store the resulting design in $O O D^{\prime}$. Next, an initial class $C$ is built for the RELAY-EFFNCY subprogram and added to this design. The partial declaration of this class is shown in Figure 172. This class has an attribute built from each of the formal parameters of the RELAY-EFFNCY subprogram. The operations for this class include one method built to implement RELAY-EFFNCY. For brevity,

```
class CLASS-9 attributes BETA, XLAMDA, DIAM, UPLFAC,
        SIGJ, ZENITH, MIRRORS, GEDM, AXMAJ, AXMIN, SIGJR, EFFNCY
    method RELAY-EFFNCY ( C-39)
        begin
        end
    superclass USER-OBJECT
```

Figure 172 Classes after PBOI Transformation
the statements of this method are not shown. The class CLASS-9 is added to the design $O O D^{\prime}$ and the resulting design is passed to the $T_{p b o i}$ transformation.

### 6.11 Transforming Category 1 Subprograms ( $\sigma_{1}$ )

This section defines the formal transformation for converting Category 1 subprograms to the object-oriented paradigm. As explained in Section 5.8, the main program is typically classified as a Category 1 subprogram. Category 1 subprograms other than the main program are transformed to the object paradigm using the Category 3 transformation, $\sigma_{3}$.

More formally, let $O O D$ represent an object-oriented design, let $S$ be an imperative subprogram, and let $M$ be the main program. Recall from Section 5.8 that the main program for a system of imperative subprograms is identified by the user. The transformation of any Category 1 subprogram is defined as follows.

$$
\begin{aligned}
& \sigma_{1}(O O D, S)=O O D^{\prime} \text { where } \\
& \quad S \neq M \Rightarrow O O D^{\prime}=\sigma_{3}(O O D, S) \text { and } \\
& S=M \Rightarrow O O D^{\prime}=\sigma_{M}(O O D, S)
\end{aligned}
$$

This transformation determines whether the input subprogram is the main program or not. If it is not the main program, then the $\sigma_{3}$ transformation is used. If it is the main program, then the $\sigma_{M}$ transformation specifically defined for the main program is used.

The rationale for the transformation of the main program is given in Section 5.8. Specifically, there are three updates done on the design while transforming the main program. First, duplicate object instances are replaced in the main program with a single
object instance. Second, the "GET-", "SET-", and "CREATE-" methods are added to each class in the design. Finally, overlapping classes in the design are merged into a new class and instances of the overlapping classes are replaced with a single instance of the new class. The transformations that formalize these updates are defined below and an example of each transformation is provided.
6.11.1 Removing duplicate objects. This section defines the transformations needed to remove duplicate objects from the main program. Two duplicate objects are instances of the same class that are built using the same data items. Duplicate objects are eliminated by replacing the two assignment statements that are used to build the instances with a new assignment statement that builds just one instance. In order to replace the two variables that hold the duplicate objects, the $v_{e}^{d u p}$ and $v^{d u p}$ transformations are defined in this section.

The $v_{e}^{d u p}$ transformation replaces any occurrences of the $v_{1}$ variable in an expression $e$ with the $v_{2}$ variable, which results in a new expression $e$. Let the $\operatorname{pos}(v, P)$ predicate indicate the ordinal position of a variable, $v$, in the sequence, $P$. The $v_{e}^{d u p}$ transformation is defined as shown in Figure 173.

The $v^{d u p}$ transformation has been defined as shown in Figure 174. This transformation uses the formal definitions of imperative statements in its definition.

Let $T_{o b j}^{d u p}$ represent the transformation that removes duplicate object instances from the object-oriented design. Let $O O D$ represent the object-oriented design, let $C$ represent the overall system class built for the main program, and let $i d_{\text {main }}$ be the name of the main program. The $T_{o b j}^{d u p}$ transformation is defined as shown in Figure 175. This transformation compares the assignment statements in the method built for the main program in order to find "CREATE-" messages that are instantiating duplicate objects. The transformation replaces the $i_{1}$ and $i_{2}$ assignment statements with the $i_{3}$ assignment statement and the variables $v_{1}$ and $v_{2}$ with the variable $v_{3}$.

For example, Figure 176 shows the overall system class CLASS-SYSTEM. The variables C-5 and C-6 hold object instances that are passed to the BOUNCE method. Both object instances are built from CLASS-2 using the data items ANGFAC and BEAMRA and are duplicate

$$
\begin{aligned}
& v_{e}^{d u p}\left(e, v_{1}, v_{2}\right)=e^{\prime} \text { where } \\
& e=<i d_{S_{n}}, P_{a c t}>\text { and } v_{1} \in P_{a c t} \Rightarrow \\
& e^{\prime}=<i d_{S_{n}}, P_{a c t}^{\prime}>\text { and } \\
& v_{1} \notin P_{a c t}^{\prime} \text { and } v_{2} \in P_{a c t}^{\prime} \text { and } \\
& \operatorname{pos}\left(v_{1}, P_{a c t}^{\prime}\right)=\operatorname{pos}\left(v_{2}, P_{a c t}^{\prime}\right) \text { and } \\
& \text { gom-variable }(e) \text { ande }=v_{1} \Rightarrow \\
& e^{\prime}=v_{2} \text { and } \\
& \text { gom-literal-boolean }(e) \Rightarrow e^{\prime}=e \text { and } \\
& \text { gom-literal-integer }(e) \Rightarrow e^{\prime}=e \text { and } \\
& \text { gom-literal-real }(e) \Rightarrow e^{\prime}=e \text { and } \\
& \text { gom-literal-string }(e) \Rightarrow e^{\prime}=e \text { and } \\
& e=<e_{1},+, e_{2}>\Rightarrow e^{\prime}=<v_{e}^{d u p}\left(e_{1}, v_{1}, v_{2}\right),+, v_{e}^{d u p}\left(e_{2}, v_{1}, v_{2}\right)>\text { and } \\
& e=<e_{1} \text {, and, } e_{2}>\Rightarrow e^{\prime}=<v_{e}^{d u p}\left(e_{1}, v_{1}, v_{2}\right) \text {, and, } v_{e}^{d u p}\left(e_{2}, v_{1}, v_{2}\right)>\text { and } \\
& e=<e_{1}, \&, e_{2}>\Rightarrow e^{\prime}=<v_{e}^{d u p}\left(e_{1}, v_{1}, v_{2}\right), \&, v_{e}^{d u p}\left(e_{2}, v_{1}, v_{2}\right)>\text { and } \\
& e=<e_{1}, /, e_{2}>\Rightarrow e^{\prime}=<v_{e}^{d u p}\left(e_{1}, v_{1}, v_{2}\right), /, v_{e}^{d u p}\left(e_{2}, v_{1}, v_{2}\right)>\text { and } \\
& e=<e_{1},=, e_{2}>\Rightarrow e^{\prime}=\left\langle v_{e}^{d u p}\left(e_{1}, v_{1}, v_{2}\right),=, v_{e}^{d u p}\left(e_{2}, v_{1}, v_{2}\right)\right\rangle \text { and } \\
& e=<e_{1},{ }^{* *}, e_{2}>\Rightarrow e^{\prime}=<v_{e}^{d u p}\left(e_{1}, v_{1}, v_{2}\right),^{* *}, v_{e}^{d u p}\left(e_{2}, v_{1}, v_{2}\right)>\text { and } \\
& \left.\left.e=\left\langle e_{1},\right\rangle=, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle v_{e}^{d u p}\left(e_{1}, v_{1}, v_{2}\right),\right\rangle=, v_{e}^{d u p}\left(e_{2}, v_{1}, v_{2}\right)\right\rangle \text { and } \\
& \left.e=<e_{1},>, e_{2}>\Rightarrow e^{\prime}=<v_{e}^{d u p}\left(e_{1}, v_{1}, v_{2}\right),>, v_{e}^{d u p}\left(e_{2}, v_{1}, v_{2}\right)\right\rangle \text { and } \\
& e=<e_{1},<=, e_{2}>\Rightarrow e^{\prime}=<v_{e}^{d u p}\left(e_{1}, v_{1}, v_{2}\right),<=, v_{e}^{d u p}\left(e_{2}, v_{1}, v_{2}\right)>\text { and } \\
& e=<e_{1},<, e_{2}>\Rightarrow e^{\prime}=<v_{e}^{d u p}\left(e_{1}, v_{1}, v_{2}\right),<, v_{e}^{d u p}\left(e_{2}, v_{1}, v_{2}\right)>\text { and } \\
& e=\left\langle e_{1},{ }^{*}, e_{2}>\Rightarrow e^{\prime}=\left\langle v_{e}^{d u p}\left(e_{1}, v_{1}, v_{2}\right),^{*}, v_{e}^{d u p}\left(e_{2}, v_{1}, v_{2}\right)\right\rangle\right. \text { and } \\
& \left.\left.e=\left\langle e_{1},<\right\rangle, e_{2}\right\rangle \Rightarrow e^{\prime}=\left\langle v_{e}^{d u p}\left(e_{1}, v_{1}, v_{2}\right),<\right\rangle, v_{e}^{d u p}\left(e_{2}, v_{1}, v_{2}\right)\right\rangle \text { and } \\
& e=<e_{1} \text {, or, } e_{2}>\Rightarrow e^{\prime}=<v_{e}^{d u p}\left(e_{1}, v_{1}, v_{2}\right) \text {, or, } v_{e}^{d u p}\left(e_{2}, v_{1}, v_{2}\right)>\text { and } \\
& e=<e_{1},-, e_{2}>\Rightarrow e^{\prime}=<v_{e}^{d u p}\left(e_{1}, v_{1}, v_{2}\right),-, v_{e}^{d u p}\left(e_{2}, v_{1}, v_{2}\right)>\text { and } \\
& e=<-, e_{1}>\Rightarrow e^{\prime}=<-, v_{e}^{d u p}\left(e_{1}, v_{1}, v_{2}\right)>\text { and } \\
& e=<\operatorname{not}, e_{1}>\Rightarrow e^{\prime}=<\operatorname{not}, v_{e}^{d u p}\left(e_{1}, v_{1}, v_{2}\right)>
\end{aligned}
$$

Figure 173 The $v_{e}^{d u p}$ transformation

```
\(v^{d u p}\left(\Psi, v_{1}, v_{2}\right)=\Psi^{\prime}\) where
    \(\forall s \in \Psi\)
        \(s=<i d_{S_{n}}, P_{a c t}>\) and \(v_{1} \in P_{a c t} \Rightarrow\)
        \(s^{\prime}=\left\langle i d_{S_{n}}, P_{\text {act }}^{\prime}>\right.\) and
        \(v_{1} \notin P_{a c t}^{\prime}\) and \(v_{2} \in P_{a c t}^{\prime}\) and
        \(\operatorname{pos}\left(v_{1}, P_{a c t}^{\prime}\right)=\operatorname{pos}\left(v_{2}, P_{a c t}^{\prime}\right)\) and
        \(s=\langle x,:=, e\rangle \Rightarrow\)
            \(s^{\prime}=\left\langle x,:=, v_{e}^{d u p}\left(e, v_{1}, v_{2}\right)\right\rangle\) and
        \(s=\left\langle\right.\) if, \(e\), then, \(S_{1}\), else, \(\left.S_{2}\right\rangle \Rightarrow\)
            \(s^{\prime}=<\) if, \(v_{e}^{d u p}\left(e, v_{1}, v_{2}\right)\), then, \(v^{d u p}\left(S_{1}, v_{1}, v_{2}\right)\), else, \(v^{d u p}\left(S_{2}, v_{1}, v_{2}\right)>\) and
\(s=<\) while, \(\left.e, S_{3}\right\rangle \Rightarrow\)
            \(s^{\prime}=\left\langle\right.\) while, \(\left.v_{e}^{d u p}\left(e, v_{1}, v_{2}\right), v^{d u p}\left(S_{3}, v_{1}, v_{2}\right)\right\rangle\) and
        \(s=<\) output, oport, \(E>\Rightarrow\)
            \(s^{\prime}=\left\langle\right.\) output, oport, \(\left.E^{\prime}\right\rangle\) and
                \(E^{\prime}=\left[v_{e}^{d u p}\left(e, v_{1}, v_{2}\right) \mid e \in E\right]\) and
\(s^{\prime} \in \Psi^{\prime}\) and
\(\operatorname{pos}(s, \Psi)=\operatorname{pos}\left(s^{\prime}, \Psi^{\prime}\right)\)
```

Figure 174 The $v^{d u p}$ transformation

$$
\begin{aligned}
& T_{o b j}^{d u p}(O O D, C)=O O D^{\prime} \text { where } \\
& \quad C=<\text { CLASS-SYSTEM, } \Phi_{C}, \Omega_{C}, \lambda_{0}>\text { and } \\
& \Omega_{C}=\left\{<i d_{m a i n}, \tau, \emptyset, \emptyset, Q_{r e t}, Q_{l o c}, \Psi>\right\} \text { and }
\end{aligned}
$$

$\forall i_{1}$ and $i_{2} \in \Psi$ and

$$
\left[i_{1}=\left\langle v_{1},:=,<C R E A T E-C_{1},\left[d_{1}, d_{2}, \ldots, d_{n}\right] \gg\right. \text { and }\right.
$$

$$
\left.i_{2}=<v_{2},:=,<C R E A T E-C_{1},\left[d_{1}, d_{2}, \ldots, d_{n}\right] \gg\right] \Rightarrow
$$

$$
i_{3}=\left\langle v_{3},:=,<C R E A T E-C_{1},\left[d_{1}, d_{2}, \ldots, d_{n}\right] \gg\right. \text { and }
$$

$$
i_{1} \notin \Psi^{\prime} \text { and } i_{2} \notin \Psi^{\prime} \text { and } i_{3} \in \Psi \text { and }
$$

$$
\operatorname{pos}\left(i_{1}, \Psi\right)=\operatorname{pos}\left(i_{3}, \Psi^{\prime}\right) \text { and }
$$

$$
\Psi^{\prime \prime}=v^{d u p}\left(\Psi, v_{1}, v_{3}\right) \text { and }
$$

$$
\Psi^{\prime \prime \prime}=v^{d u p}\left(\Psi, v_{2}, v_{3}\right) \text { and }
$$

$$
\Omega_{C}^{\prime}=\left\{<i d_{m a i n}, \tau, \emptyset, \emptyset, Q_{r e t}, Q_{l o c}, \Psi^{\prime \prime \prime}>\right\} \text { and }
$$

$$
C^{\prime}=<\operatorname{CLASS}-S Y S T E M, ~ \Phi_{C}, \Omega_{C}^{\prime}, \lambda_{0}>\text { and }
$$

$$
O O D^{\prime}=T_{c}^{+}\left(T_{c}^{-}(O O D, C), C^{\prime}\right)
$$

Figure 175 The $T_{o b j}^{d u p}$ transformation

```
class CLASS-2 attributes ANGFAC, BEAMRA
    method CREATE-CLASS-2
            ( A-ANGFAC, A-BEAMRA ): a CLASS-2
        begin
        end
class CLASS-SYSTEM attributes
    method BMDSIM1 ( C-9 )
        begin
        C-5 := CREATE-CLASS-2 ( ANGFAC, BEAMRA );
        C-6 := CREATE-CLASS-2 ( ANGFAC, BEAMRA );
        BOUNCE ( C-4, C-5, C-6 );
        end
```


## Figure 176 Duplicate object instances

object instances. Figure 177 shows that the method BMDSIM1 has been updated by replacing

```
class CLASS-SYSTEM attributes
    method BMDSIM1 ( C-9 )
        begin
        C-7 := CREATE-CLASS-2 ( ANGFAC, BEAMRA );
        BOUNCE ( C-4, C-7, C-7 );
        end
```

Figure 177 Duplicates removed
the duplicate object instances. The variables C-5 and C-6 have been replaced with the variable C-7. The new instance is instantiated using the CREATE-CLASS-2 method, stored in C-7, and passed to the BOUNCE method in place of C-5 and C-6.
6.11.2 Generating Methods ( $\kappa$ ). As part of the transformation to merge overlapping classes, the "SET-", "GET-", and "CREATE-" methods are added to each class

```
class CLASS-17 attributes DWELLT
    method CREATE-CLASS-17 ( A-DWELLT ) : a CLASS-17
        begin
        INST-CLASS-17 := new ( CLASS-17 );
        SET-DWELLT ( INST-CLASS-17, A-DWELLT );
        CREATE-CLASS-17 := INST-CLASS-17
        end
    method GET-DWELLT ( C-10 ): real
        begin
        GET-DWELLT := C-10.DWELLT
        end
    method SET-DWELLT ( C-11, VAL-10 )
        begin
        C-11.DWELLT := VAL-10
        end
```

Figure 178 Instantiation Method for CLASS-17
in the object-oriented design. This section defines a formal transformation that generates these methods.

Figure 178 shows the "SET-", "GET-", and "CREATE-" methods for the class CLASS-17. The mechanics of "SET-" and "GET-" methods were covered in Section 6.3, so they won't be covered further here. The "CREATE-" method for this class is explained in detail.

The CLASS-17 class has only one attribute DWELLT, so the "CREATE-" method for this class expects one parameter that is used to initialize the attribute. The "CREATE-" method includes a call to the intrinsic functional method named new, which returns a new instance of the class. The INST-CLASS-17 local variable holds the newly instantiated object and is sent the "SET-" method to assign the input value of the DWELLT attribute. Finally, the new initialized instance is returned as the result of this method call.

Let $\kappa$ be a transformation that adds the "SET-", "GET-", and "CREATE-" methods to a class. Let $C$ be a class, and let $\tau$ be the target object, which is an instance of $C$. Let $\beta$ be an expression that will be passed as the new value in "SET-" methods. Let SET-a represent a symbol that results from prepending "SET-" to the name of the attribute

$$
\begin{aligned}
& \kappa(C)=C^{\prime} \text { where } \\
& C=<i d_{C}, \Phi_{C}, \Omega_{C}, \lambda>\text { and } \\
& \text { For each } a \in \Phi_{C} \\
& s=\langle\text { SET-a }, \tau,[],[\beta],[],[],[], \Pi\rangle \text { and } \\
& \Pi=[<\tau . a,:=, \beta>] \text { and } \\
& g=\langle\text { GET-a }, \tau,[],[],[],[a],[], \Psi\rangle \text { and } \\
& \Psi=[\langle\text { GET-a, }:=, \tau . a\rangle] \text { and } \\
& s \in \Omega_{C}^{\prime} \text { and } g \in \Omega_{C}^{\prime} \\
& \text { For each } a \in \Phi_{C} \\
& \text { A-a } \in \chi \text { and } \\
& \left.<\text { SET-a, [ INST- } i d_{C}, \text { A-a }\right]>\in \psi \text { and } \\
& \Xi=\left[\left\langle\operatorname{INST} \mathrm{id} d_{C},:=, \operatorname{new}\left(i d_{C}\right)\right\rangle\right. \text {, } \\
& \psi \text {, } \\
& <{\text { CREATE- } i d_{C}},:=,{\text { INST- } i d_{C}}^{>} \text {] and } \\
& \left.r=<\text { CREATE-i }_{C}, \tau,[], \chi,\left[\text { INST- } i d_{C}\right],\left[\text { INST- } i d_{C}\right], \Xi\right\rangle \text { and } \\
& r \in \Omega_{C}^{\prime} \text { and } \\
& C^{\prime}=\left\langle i d_{C}, \Phi_{C}, \Omega_{C}^{\prime}, \lambda\right\rangle
\end{aligned}
$$

Figure 179 The $\kappa$ transformation
$a \in \Phi_{C}$. Similarly, let GET-a represent the symbol resulting from the prepending of "GET" to the name of $a$. Let CREATE- $i d_{C}$ represent the symbol that results from prepending "CREATE-" to the name of the class $C$. Let A-a represent the symbol resulting from prepending "A-" to the name of $a$. This symbol is used for formal parameters in the "CREATE-" method. Let $\chi$ represent the sequence of these formal parameters. Let INST-\$id_\{C\}\$ represent the symbol resulting from prepending "INST-" to the name of the class $C$. Let $\psi$ represent the sequence of "SET-" messages used to assign the input values to the object attributes. The $\kappa$ transformation is defined as shown in Figure 179.

All the methods shown in Figure 178 are generated by the transformation

$$
\begin{aligned}
& \kappa^{-1}(C)=C^{\prime} \text { where } \\
& \quad C=<i d_{C}, \Phi_{C}, \Omega_{C}, \lambda>\text { and } \\
& \text { For each } a \in \Phi_{C} \\
& \quad s=<\text { SET-a }, \tau,[],[\beta],[],[],[], \Pi>\text { and } \\
& \quad g=<\text { GET-a, } \tau,[],[],[],[a],[], \Psi>\text { and } \\
& \quad s \notin \Omega_{C}^{\prime} \text { and } g \notin \Omega_{C}^{\prime} \text { and } \\
& r=<\text { CREATE-id },[],[], \chi,\left[\text { INST- } i d_{C}\right],[\text { INST-id } C], \Xi>\text { and } \\
& r \notin \Omega_{C}^{\prime} \text { and } \\
& C^{\prime}=<i d_{C}, \Phi_{C}, \Omega_{C}^{\prime}, \lambda>
\end{aligned}
$$

Figure 180 The $\kappa^{-1}$ transformation

The $\kappa$ transformation returns a new class $C^{\prime}$ that must replace the old class $C$ in the object design. This updating of the object design is not shown here.

As part of the PBOI method, it is in some cases required to remove the "SET-", "GET-", and "CREATE-" methods from a class. For example, when merging two classes. For this reason, the $\kappa$ transformation is reversible. The "inverse" transformation for $\kappa$ is defined below.

Let $\kappa^{-1}$ be the inverse of $\kappa$. The $\kappa$ transformation is defined as shown in Figure 180. All the methods shown in Figure 178 are removed by the transformation

As with $\kappa$, the $\kappa^{-1}$ transformation returns a new class $C^{\prime}$ that must replace the old class $C$ in the object design. This updating of the object design is not shown.
6.11.3 Merging overlapping classes. The $T_{\text {class }}^{\text {merge }}$ transformation is used in the main program to merge overlapping classes. Let $O O D$ represent the design built so far and let $C$ represent the class built for the main program. Let $\operatorname{set}(Q)$ represent the conversion of the sequence $Q$ to a set. Let $Q_{a c t}$ represent a sequence of actual parameters and let $Q_{f o r m}$ represent a sequence of formal parameters. Let $v_{s}\left(Q_{a c t}, Q_{f o r m}\right)$ indicate the number, order, and type of the parameters in $Q_{\text {act }}$ matches the number, order, and type
of the parameters in $Q_{\text {form }}$. The $T_{\text {class }}^{\text {merge }}$ transformation is defined as shown in Figure 181. This transformation examines each assignment statement in the method built for the main program in order to find "CREATE-" messages that are building instances of overlapping classes. The classes from the "CREATE-" messages in the $i_{1}$ and $i_{2}$ assignment statements are merged to form the new class $C_{3}$. The overlapping classes $C_{1}$ and $C_{2}$ do not appear in the updated design, but the new class does. The actual parameters for the "CREATE$"$ message in the new assignment statement $i_{3}$ are built from the actual parameters in the "CREATE-" messages in $i_{1}$ and $i_{2}$. The $v$ predicate ensures that this new sequence of actual parameters matches the number, order, and type of the formal parameters in the "CREATE-" method of the new class $C_{3}$. The $i_{1}$ and $i_{2}$ assignment statements are replaced by the new assignment statement $i_{3}$ and the variables $v_{1}$ and $v_{2}$ are replaced by the variable $v_{3}$. The updated system class replaces the original system class in the $O O D^{\prime}$ design and the resulting design is returned as the result of the transformation.

For example, Figure 182 shows the classes CLASS-2, CLASS-3, and CLASS-SYSTEM. In the BMDSIM1 method, the variable C-5 holds an instance of CLASS-2 that is built using the data items ANGFAC and BEAMRA. The variable C-8 holds an instance of CLASS-3 that is built using the data items ANGFAC and RHOSTD. Since the sequence of actuals have the ANGFAC data item in common, the classes CLASS-2 and CLASS-3 overlap and are merged into one new class.

Figure 183 shows the result of merging CLASS-2 and CLASS-3 into the new class CLASS-10. In the BMDSIM1 method, the variable C-11 holds the new instance instantiated from CLASS-10. The C-11 variable is passed to the BOUNCE message in place of the variables C-5 and C-8. The design is updated by removing CLASS-2 and CLASS-3 and adding CLASS-10.
6.11.4 Converting the Main Program. This section defines the formal transformation required to convert the main program. In the main program, as the calls to other subprograms are transformed into messages, the object instances required by the signature of the method being invoked must be built. For each message being built, it is assumed that the corresponding method exists in some class in the design.

$$
\begin{aligned}
& T_{\text {class }}^{\text {merge }}(O O D, C)=O O D^{\prime \prime} \text { where } \\
& C=<\text { CLASS-SYSTEM, } \emptyset, \Omega_{C}, \lambda_{0}>\text { and } \\
& \left.\Omega_{C}=\left\{<i d_{\text {main }}, \tau, \emptyset, \emptyset, \emptyset, Q_{l o c}, \Psi\right\rangle\right\} \text { and } \\
& i_{1} \text { and } i_{2} \in \Psi \text { and } \\
& \operatorname{pos}\left(i_{1}, \Psi\right)<p o s\left(i_{2}, \Psi\right) \text { and } \\
& i_{1}=\left\langle v_{1},:=,<C R E A T E-C_{1}, Q_{a c t_{1}} \gg\right. \text { and } \\
& i_{2}=<v_{2},:=,<C R E A T E-C_{2}, Q_{a c t_{2}} \gg \text { and } \\
& \operatorname{set}\left(Q_{a c t_{1}}\right) \cap \operatorname{set}\left(Q_{a c t_{2}}\right) \neq \emptyset \Rightarrow \\
& C_{1} \in O O D \text { and } C_{1}^{\prime}=\kappa^{-1}\left(C_{1}\right) \text { and } \\
& C_{2} \in O O D \text { and } C_{2}^{\prime}=\kappa^{-1}\left(C_{2}\right) \text { and } \\
& C_{1}^{\prime}=\left\langle i d_{C_{1}^{\prime}}, \Phi_{C_{1}^{\prime}}, \Omega_{C_{1}^{\prime}}, \lambda_{0}\right\rangle \text { and } \\
& C_{2}^{\prime}=\left\langle i d_{C_{2}^{\prime}}, \Phi_{C_{2}^{\prime}}, \Omega_{C_{2}^{\prime}}, \lambda_{0}\right\rangle \text { and } \\
& C_{3}=\left\langle i d_{C_{3}}, \Phi_{C_{3}}, \Omega_{C_{3}}, \lambda_{0}\right\rangle \text { and } \\
& \Phi_{C_{3}}=\Phi_{C_{1}^{\prime}} \cup \Phi_{C_{2}^{\prime}} \text { and } \\
& \Omega_{C_{3}}=\Omega_{C_{1}^{\prime}} \cup \Omega_{C_{2}^{\prime}} \text { and } \\
& C_{1} \notin O O D^{\prime} \text { and } C_{2} \notin O O D^{\prime} \text { and } \kappa\left(C_{3}\right) \in O O D^{\prime} \text { and } \\
& i_{3}=\left\langle v_{3},:=,<C R E A T E-C_{3}, Q_{a c t} \gg\right. \text { where } \\
& Q_{a c t_{3}}=Q_{a c t_{1}} \oplus Q_{a c t_{2}} \text { and } \\
& v_{s}\left(Q_{a_{c t}^{3}}, Q_{\text {form }_{C R E A T E-C_{3}}}\right) \text { and } \\
& i_{1} \notin \Psi^{\prime} \text { and } i_{2} \notin \Psi^{\prime} \text { and } i_{3} \in \Psi \text { and } \\
& \operatorname{pos}\left(i_{1}, \Psi\right)=\operatorname{pos}\left(i_{3}, \Psi^{\prime}\right) \text { and } \\
& \Psi^{\prime \prime}=v^{d u p}\left(\Psi, v_{1}, v_{3}\right) \text { and } \\
& \Psi^{\prime \prime \prime}=v^{d u p}\left(\Psi, v_{2}, v_{3}\right) \text { and } \\
& \Omega_{C}^{\prime}=\left\{<i d_{\text {main }}, \tau, \emptyset, \emptyset, Q_{r e t}, Q_{l o c}, \Psi^{\prime \prime \prime}>\right\} \text { and } \\
& C^{\prime}=<\text { CLASS-SYSTEM, } \Phi_{C}, \Omega_{C}^{\prime}, \lambda_{0}>\text { and } \\
& O O D^{\prime \prime}=T_{c}^{+}\left(T_{c}^{-}\left(O O D^{\prime}, C\right), C^{\prime}\right)
\end{aligned}
$$

Figure 181 The $T_{\text {class }}^{\text {merge }}$ transformation

```
class CLASS-2 attributes ANGFAC, BEAMRA
    method CREATE-CLASS-2
            ( A-ANGFAC, A-BEAMRA ): a CLASS-2
        begin
        end
    method CAPTURE ( C-2 ) : real
        begin
        ...
        end
class CLASS-3 attributes ANGFAC, RHOSTD
    method CREATE-CLASS-3
            ( A-ANGFAC, A-RHOSTD ) : a CLASS-3
        begin
        ...
        end
    method RHO ( C-3 ) : real
        begin
        end
class CLASS-SYSTEM attributes
    method BMDSIM1 ( C-9 )
        begin
        ...
        C-5 := CREATE-CLASS-2 ( ANGFAC, BEAMRA );
        C-8 := CREATE-CLASS-3 ( ANGFAC, RHOSTD );
        BOUNCE ( C-4, C-5, C-8 );
        ...
        end
```

Figure 182 Overlapping object classes

```
class CLASS-10 attributes ANGFAC, BEAMRA, RHOSTD
    method CREATE-CLASS-10
        ( A-ANGFAC, A-BEAMRA, A-RHOSTD ) : a CLASS-10
        begin
        end
    method CAPTURE ( C-10 ) : real
        begin
        ...
        end
    method RHO ( C-10 ) : real
        begin
        ..
        end
class CLASS-SYSTEM attributes
    method BMDSIM1 ( C-9 )
        begin
        ..
        C-11 :=
            CREATE-CLASS-10 ( ANGFAC, BEAMRA, RHOSTD );
        BOUNCE ( C-4, C-11, C-11 );
        ..
        end
```

Figure 183 Overlapping classes merged

The $T_{m}^{a s}$ transformation generates a sequence of assignment statements that instantiate the object instances required and stores them in variables used as actual parameters in the message. Let $v_{c}$ represent a GOM variable generated to hold the target object instance. Let $v_{i}$ represent one of the variables generated to hold the other object instances required for a specific message, $m$. Let the $v_{s}\left(Q_{a c t_{1}}, Q_{f o r m_{o}}\right)$ predicate indicate whether or not the number, order, and type of the actual parameters in $Q_{\text {act }}$ match the number, order, and type of the parameters in $Q_{f o r m_{o}}$. The $T_{m}^{a s}$ transformation is shown in Figure 184. This transformation builds an assignment statement, $a s_{1}$, that includes a call to the "CREATE-" message from class $C_{2}$ in order to instantiate the target object $c$. The actual parameters to be sent to the "CREATE-" message are determined by using the $\mu^{*}$ transitive closure and the actual parameters from the subprogram call $l$. The assignment statement $a s_{1}$ stores the instance into the variable $v_{c}$. For each object instance $q$ in $Q_{o b j}$, an assignment statement is built that includes a call to the "CREATE-" message from $q$ 's class $C_{i}$. The assignment statement stores the instance in the variable $v_{i}$.

For example, consider the following message from the main program.

```
BOUNCE ( C-4, C-5, C-8 );
```

This message is passing the $\mathrm{C}-4, \mathrm{C}-5$, and $\mathrm{C}-8$ object instances to the BOUNCE method. The target object of the message is C-4, an instance of CLASS-4, which includes the BOUNCE method. For this example, assume that C-5 and C-8 are instances of CLASS-2 and CLASS-3, respectively. The original call to the subprogram BOUNCE is shown below.

```
BOUNCE ( XLAMDA, ANGFAC, BEAMRA, RHOSTD );
```

The sequence of statements generated to build the instances required for this message are shown below.

```
C-4 := CREATE-CLASS-4 ( XLAMDA );
C-5 := CREATE-CLASS-2 ( ANGFAC, BEAMRA);
C-8 := CREATE-CLASS-3 ( ANGFAC, RHOSTD );
```

The sequence of assignment statements returned from the $T_{m}^{a s}$ transformation must be spliced into the sequence of statements in the main program right before the message

```
\(T_{m}^{a s}(O O D, l, m)=A S\) where
    \(l=\left\langle i d_{S}, P_{\text {act }}\right\rangle\) and
    \(m=<i d_{S}, Q_{\text {act }}>\) and
    \(Q_{a c t}=[c] \oplus Q_{o b j} \oplus Q_{f o r m}\) and
    instance \(\left(c, C_{2}\right)\) and
    \(C_{2} \in O O D\) and
    \(C_{2}=\left\langle i d_{C_{2}}, \Phi_{C_{2}}, \Omega_{C_{2}}, \lambda_{0}\right\rangle\) and
    \(o \in \Omega_{C_{2}}\) and
    \(o=\left\{\left\langle\right.\right.\) CREATE-i \(\left.\left._{C_{2}}, \tau,[], Q_{\text {form }_{o}}, Q_{r e t_{o}}, Q_{l o c_{o}}, \Psi_{o}\right\rangle\right\}\) and
    \(Q_{a c t_{1}}=\left[\theta_{v}\left(d_{k}\right) \mid d_{k} \in P_{\text {act }}\right.\) and
        \(\left.\theta_{v}\left(\mu^{*}\left(d_{k}\right)\right) \in Q_{\text {formo }}\right]\) and
    \(v_{s}\left(Q_{a c t_{1}}, Q_{f o r m_{o}}\right)\) and
    \(A S_{1}=\left[\left\langle v_{c},:=,<\right.\right.\) CREATE-C \(\left.C_{2}, Q_{a c t_{1}} \gg\right]\) and
    \(\forall q \in Q_{o b j}\)
        instance \(\left(q, C_{i}\right)\) and
        \(C_{i} \in O O D\) and
        \(C_{i}=\left\langle i d_{C_{i}}, \Phi_{C_{i}}, \Omega_{C_{i}}, \lambda_{0}\right\rangle\) and
        \(o_{i} \in \Omega_{C_{i}}\) and
        \(o_{i}=\left\{<\right.\) CREATE-C \(\left.\mathcal{C}_{i}, \tau_{o_{i}},[], Q_{\text {form }_{o_{i}}}, Q_{\text {ret }_{o_{i}}}, Q_{l o o_{o_{i}}}, \Psi_{o_{i}}>\right\}\) and
        \(Q_{a c t_{i}}=\left[d_{j} \mid \mu^{*}\left(d_{j}\right) \in Q_{\text {form }_{o_{i}}}\right]\) and
        \(v_{s}\left(Q_{a c t_{i}}, Q_{\text {form }_{o_{i}}}\right)\) and
        \(a s_{i}=\left\langle v_{i},:=,<\right.\) CREATE- \(i d_{C_{i}}, Q_{a c t_{i}} \gg\) and
        \(a s_{i} \in A S_{2}\) and
    \(A S=A S_{1} \oplus A S_{2}\)
```

Figure 184 The $T_{m}^{a s}$ transformation

$$
\begin{aligned}
& v_{e}^{a s}(O O D, S, e)=A S \text { where } \\
& e=<i d_{S_{n}}, Q_{a c t}>\Rightarrow \\
& A S=T_{m}^{a s}\left(O O D, e, v_{m}(O O D, S, e)\right) \text { and } \\
& e=<e_{1},+, e_{2}>\Rightarrow A S=v_{e}^{a s}\left(O O D, S, e_{1}\right) \oplus v_{e}^{a s}\left(O O D, S, e_{2}\right) \text { and } \\
& e=<e_{1}, \text { and }, e_{2}>\Rightarrow A S=v_{e}^{a s}\left(O O D, S, e_{1}\right) \oplus v_{e}^{a s}\left(O O D, S, e_{2}\right) \text { and } \\
& e=<e_{1}, \&, e_{2}>\Rightarrow A S=v_{e}^{a s}\left(O O D, S, e_{1}\right) \oplus v_{e}^{a s}\left(O O D, S, e_{2}\right) \text { and } \\
& e=<e_{1}, /, e_{2}>\Rightarrow A S=v_{e}^{a s}\left(O O D, S, e_{1}\right) \oplus v_{e}^{a s}\left(O O D, S, e_{2}\right) \text { and } \\
& e=<e_{1},=, e_{2}>\Rightarrow A S=v_{e}^{a s}\left(O O D, S, e_{1}\right) \oplus v_{e}^{a s}\left(O O D, S, e_{2}\right) \text { and } \\
& e=<e_{1}, * *, e_{2}>\Rightarrow A S=v_{e}^{a s}\left(O O D, S, e_{1}\right) \oplus v_{e}^{a s}\left(O O D, S, e_{2}\right) \text { and } \\
& e=<e_{1},>=, e_{2}>\Rightarrow A S=v_{e}^{a s}\left(O O D, S, e_{1}\right) \oplus v_{e}^{a s}\left(O O D, S, e_{2}\right) \text { and } \\
& e=<e_{1},>, e_{2}>\Rightarrow A S=v_{e}^{a s}\left(O O D, S, e_{1}\right) \oplus v_{e}^{a s}\left(O O D, S, e_{2}\right) \text { and } \\
& e=<e_{1},<=, e_{2}>\Rightarrow A S=v_{e}^{a s}\left(O O D, S, e_{1}\right) \oplus v_{e}^{a s}\left(O O D, S, e_{2}\right) \text { and } \\
& e=<e_{1},<, e_{2}>\Rightarrow A S=v_{e}^{a s}\left(O O D, S, e_{1}\right) \oplus v_{e}^{a s}\left(O O D, S, e_{2}\right) \text { and } \\
& e=<e_{1}, *, e_{2}>\Rightarrow A S=v_{e}^{a s}\left(O O D, S, e_{1}\right) \oplus v_{e}^{a a}\left(O O D, S, e_{2}\right) \text { and } \\
& e=<e_{1},<>, e_{2}>\Rightarrow A S=v_{e}^{a s}\left(O O D, S, e_{1}\right) \oplus v_{e}^{a s}\left(O O D, S, e_{2}\right) \text { and } \\
& e=<e_{1}, \text { or, } e_{2}>\Rightarrow A S=v_{e}^{a s}\left(O O D, S, e_{1}\right) \oplus v_{e}^{a s}\left(O O D, S, e_{2}\right) \text { and } \\
& e=<e_{1},-, e_{2}>\Rightarrow A S=v_{e}^{a s}\left(O O D, S, e_{1}\right) \oplus v_{e}^{a s}\left(O O D, S, e_{2}\right) \text { and } \\
& e=<-, e_{1}>\Rightarrow A S=v_{e}^{a s}\left(O O D, S, e_{1}\right) a n d \\
& e=<\operatorname{not}, e_{1}>\Rightarrow A S=v_{e}^{a s}\left(O O D, S, e_{1}\right)
\end{aligned}
$$

Figure 185 The $v_{e}^{a s}$ transformation
$m$. The $v_{e}^{a s}$ transformation, defined in Figure 185, is used to build the sequence of assignment statements for functional messages that appear in expressions. Let $O O D$ represent the object-oriented design, and let $S$ represent the original subprogram. The $v_{M}$ transformation is defined to update the statements from the main program, $\Psi$, by replacing all subprogram calls with messages. For procedural messages, the sequence of assignment statements generated by the $T_{m}^{a s}$ transformation is spliced into $\Psi$ before the message. For functional messages, the sequence of assignment statements is spliced into $\Psi$ right before the statement that includes the message. The $v_{M}$ transformation is defined in Figure 186. This transformation uses the $v_{m}$ and $v_{e}$ transformations to transform subprogram calls into messages. The reduce operation is used for output statements in order to combine the results of the $v_{e}^{a s}$ transformation of the expressions using the $\oplus$ operator.

$$
\begin{aligned}
& v_{M}(S, \Psi)=\Psi^{\prime} \text { where } \\
& \forall s \in \Psi \\
& \Psi=\Psi_{1} \oplus[s] \oplus \Psi_{2} \text { and } \\
& s=<i d_{S_{n}}, P_{a c t}>\Rightarrow \\
& s^{\prime}=v_{m}(S, s) \text { and } \\
& A S=T_{m}^{a s}\left(O O D, s, s^{\prime}\right) \text { and } \\
& s=<x,:=, e>\Rightarrow \\
& s^{\prime}=<x,:=, v_{e}(S, e)>\text { and } \\
& A S=v_{e}^{a s}(O O D, S, e) \text { and } \\
& s=<\text { if, } e \text { then, } S_{1}, \text { else, } S_{2}>\Rightarrow \\
& s^{\prime}=<\text { if, } v_{e}(S, e), \text { then }, v_{M}\left(S, S_{1}\right), \text { else }, v_{M}\left(S, S_{2}\right)>\text { and } \\
& A S=v_{e}^{a s}(O O D, S, e) \text { and } \\
& s=<\text { while,e, } S_{3}>\Rightarrow \\
& s^{\prime}=<\text { while, } v_{e}(S, e), v_{M}\left(S, S_{3}\right)>\text { and } \\
& A S=v_{e}^{a s}(O O D, S, e) \text { and } \\
& s=<\text { input, iport }, E>\Rightarrow \\
& s^{\prime}=s \text { and } \\
& s=<\text { output, oport, } E>\Rightarrow \\
& s^{\prime}=<\text { output, oport }, E^{\prime}>\text { and } \\
& E^{\prime}=\left[v_{e}(S, e) \mid e \in E\right] \text { and } \\
& A S=\text { reduce }\left(\oplus,\left[v_{e}^{a s}(O O D, S, e) \mid e \in E\right]\right) \text { and } \\
& \Psi^{\prime}=\Psi_{1} \oplus A S \oplus\left[s^{\prime}\right] \oplus \Psi_{2} \text { and }
\end{aligned}
$$

Figure 186 The $v_{M}$ transformation

```
class CLASS-SYSTEM attributes
    method BMDSIM1 ( C-9 )
        begin
        BOUNCE ( XLAMDA, ANGFAC, BEAMRA, RHOSTD );
    end
```

Figure 187 BMDSIM1 before BOUNCE converted

The formal transformation $T_{M}^{v}$ applies the $v_{M}$ transformation to the main method of the overall system class and is defined as follows. Let $O O D$ represent an object-oriented design, let $M$ represent the main program from the imperative system, let $C$ represent the CLASS-SYSTEM built for the main program.

$$
\begin{aligned}
& T_{M}^{v}(O O D, M, C)=O O D^{\prime} \text { where } \\
& C=<\text { CLASS-SYSTEM, } \emptyset, \Omega_{C}, \lambda_{0}>\text { and } \\
& \Omega_{C}=\left\{<i d_{\text {main }}, \tau, \emptyset, \emptyset, \emptyset, Q_{l o c}, \Psi_{\text {main }}>\right\} \text { and } \\
& \Psi_{\text {main }}^{\prime}=v_{M}(M, \Psi \text { main }) \text { and } \\
& \Omega_{C}^{\prime}=\left\{<i d_{\text {main }}, \tau, \emptyset, \emptyset, \emptyset, Q_{l o c}, \Psi_{\text {main }}^{\prime}>\right\} \text { and } \\
& C^{\prime}=<{\text { CLASS-SYSTEM, } \emptyset, \Omega_{C}^{\prime}, \lambda_{0}>\text { and }}_{O O D^{\prime}=T_{c}^{+}\left(T_{c}^{-}(O O D, C), C^{\prime}\right)} .
\end{aligned}
$$

For example, Figure 187 shows the CLASS-SYSTEM main program method BMDSIM1 before the $T_{\text {main }}^{v}$ transformation. The BOUNCE subprogram call has not yet been converted to a message. Figure 188 shows the BMDSIM1 method after the BOUNCE subprogram call has been converted to a message and the object instances required for the message have been inserted.

In order to convert the main program, the $\sigma_{M}$ transformation is defined in Figure 189. Let $O O D$ represent an object-oriented design and let $M$ represent the main program. This transformation takes the main program, $M$, as input and unions the designs built from applying $\sigma$ to each of the subprograms called by $M$. Each subprogram called by the main program will also apply $\sigma$ to any subprograms it calls. This formalizes the depth-first approach used in the PBOI methodology. The resulting designs are unioned

```
class CLASS-SYSTEM attributes
    method BMDSIM1 ( C-9 )
        begin
        C-4 := CREATE-CLASS-4 ( XLAMDA );
        C-5 := CREATE-CLASS-2 ( ANGFAC, BEAMRA );
        C-8 := CREATE-CLASS-3 ( ANGFAC, RHOSTD );
        BOUNCE ( C-4, C-5, C-8);
        nd
```

Figure 188 BMDSIM1 after BOUNCE converted
together to build $O O D^{\prime}$. The CLASS-SYSTEM class is built with no attributes and the main program statements are converted using the $\theta$ transformation. There is no need to use the $\delta$ transformation because CLASS-SYSTEM has no attributes. Each of the subprogram calls is converted to a message using the $T_{M}^{v}$ transformation. Duplicate classes are removed by the $T_{\text {class }}^{\text {dup }}$ transformation, duplicate objects are removed by the $T_{o b j}^{d u p}$ transformation, and overlapping classes are merged using the $T_{\text {class }}^{\text {merge }}$ transformation.

### 6.12 Eliminating Category 4 and Category 5 Subprograms

This section defines the formal transformations used to convert Category 4 and Category 5 subprograms (in the GIM) into collections of Category 2 or Category 3 subprograms (in the GIM). These transformations are built using the formal definitions of a subprogram and subprogram call provided in Chapter III. Since only procedures are allowed to have multiple outputs, the transformations are restricted to this form of subprogram.

As presented in Chapter III, a procedure, $S$, is represented by the following tuple.

$$
<i d_{S}, P_{\text {in }}, P_{\text {out }},[], P_{\text {loc }}, \Sigma>
$$

Let $\Sigma_{p}$ represent the program slice on the statements in $\Sigma$ required to produce $p$. Weiser [76] defines a program slice as a projection of the behavior of the original procedure. Let $\pi$ represent a function that indicates a sequence of statements, $\Sigma_{p}$, projects the behavior

$$
\begin{aligned}
& \sigma_{M}(O O D, M)=O O D^{(5)} \text { where } \\
& M=<i d_{\text {main }}, \emptyset, \emptyset, \emptyset, P_{l o c}, \Sigma_{M}>\text { and } \\
& \forall l \in C_{s u b_{M}} \\
& \quad l=<i d_{S_{n}}, P_{\text {act }}> \\
& \quad S_{n} \in C_{s u b_{M}}^{*} \\
& O O D^{\prime}=\bigcup_{i=1}^{n}\left(\sigma\left(O O D, S_{i}\right)\right) \text { where } S_{i} \in C_{s u b_{M}}^{*} \text { and } \\
& O O D^{\prime \prime}=T_{\text {class }}^{\text {dup }}\left(O O D^{\prime}\right) \text { and } \\
& \forall C_{i} \in O O D^{\prime \prime} \\
& \quad \kappa\left(C_{i}\right) \in O O D^{\prime \prime \prime} \text { and } \\
& C=<\operatorname{CLASS-SYSTEM,~} \emptyset, \Omega_{C}, \lambda_{0}>\text { and } \\
& \text { instance }(\tau, C) \text { and } \\
& \Psi=\theta\left(\Sigma_{M}\right) \text { and } \\
& Q_{l o c}=\theta_{v}\left(P_{l o c}\right) \text { and } \\
& \Omega_{C}=\left\{<i d_{\text {main }}, \tau, \emptyset, \emptyset, \emptyset, Q_{l o c}, \Psi>\right\} \text { and } \\
& O O D^{(4)}=T_{M}^{u}\left(O O D^{\prime \prime \prime}, M, C\right) \text { and } \\
& C^{\prime}=\text { updated } C \in O O D^{(4)} \text { and } \\
& O O D^{(5)}=T_{o b j}^{\text {dup }}\left(O O D^{(4)}, C^{\prime}\right) \text { and } \\
& C^{\prime \prime}=\text { updated } C^{\prime} \in O O D^{(5)} \text { and } \\
& O O D^{(6)}=T_{c l a s s}^{\text {merge }}\left(O O D^{(5)}, C^{\prime \prime}\right)
\end{aligned}
$$

Figure 189 The $\sigma_{M}$ transformation
of another sequence of statements, $\Sigma$, in order to produce a variable $p$. Formally,

$$
\Sigma_{p}=\pi(\Sigma, p)
$$

Any sequence of statements that meets the condition, $\pi$, is a program slice of the original procedure and produces the single data item $p$. A new subprogram built from this program slice is represented as a tuple that is defined as follows. Let $i d_{S_{p}}$ represent the name of the new subprogram. Let $P_{i n_{p}}$ represent the input parameters that are referenced in the program slice, such that $P_{i n_{p}} \subseteq P_{i n}$. Let $P_{l o c_{p}}$ represent any local variables referenced in the program slice, such that $P_{\text {loc } p} \subseteq P_{l o c_{p}}$. Given a data item, $p \in P_{o u t}$ that is produced
by $\Sigma$, the program slice, $\Sigma_{p}$, is defined as shown below.

$$
<i d_{S_{p}}, P_{i n_{p}},[p],[], P_{l o c_{p}}, \pi\left(\Sigma_{p}\right)>
$$

As presented in Chapter III, a call to procedure $S$ is defined by the following tuple.

$$
<i d_{S}, P_{a c t}>
$$

In order to differentiate between the actual parameters that are input parameters and the actual parameters that are output parameters, the representation of a procedure call is extended as follows. Let $P_{\text {act } t_{i n}}$ represent the input actual parameters and let $P_{\text {actout }}$ represent the output actual parameters, such that $P_{\text {actin }} \oplus P_{a c t o u t}=P_{a c t}$. The procedure call is represented by the following tuple.

$$
<i d_{S}, P_{a c t_{i n}}, P_{\text {actout }}>
$$

A call to the new subprogram $S_{p}$ is represented as follows. Let $P_{\text {actin }{ }^{\text {r }}}$ represent the input actual parameters that match formal parameters from the new subprogram. Let $a$ represent the output actual parameter that corresponds to the output formal parameter $p$ from the new subprogram, i.e. $\mu(a)=p$. The procedure call to $S_{p}$ is represented by the following tuple.

$$
<i d_{S_{p}}, P_{a c t_{i n_{p}}},[a]>
$$

6.12.1 Build Slices Transformation. As defined in Section 5.15, the first step in converting Category 4 and Category 5 subprograms is to build one new subprogram for each of the data items produced by the Category 4 or Category 5 subprograms. This section defines the formal transformation used to generate the program slices for all Category 4 and Category 5 subprograms. Once the slices are built, the transformation for inter-procedural slicing is used to call each program slice as needed. That transformation is also defined in this section.

$$
\begin{aligned}
& T_{p s}^{\text {build }}(S D)=S D^{\prime} \text { where } \\
& \quad \text { For each } S \in S D \\
& \quad S=<i d_{S}, P_{\text {in }}, P_{o u t},[], P_{l o c}, \Sigma_{S}>\text { and } \\
& S \in S D^{\prime} \text { and } \\
& \quad \operatorname{Cat}_{4}(S) \text { or } \text { Cat }_{5}(S) \Rightarrow \\
& \quad\left[\forall p \in P_{\text {out }}\right. \\
& \quad S_{p}=<i d_{S_{p}}, P_{i n_{p}},[p],[], P_{l o c_{p}}, \pi\left(\Sigma_{S}\right)>\text { and } \\
& \left.\quad S_{p} \in S D^{\prime}\right]
\end{aligned}
$$

Figure 190 The $T_{p s}^{\text {build }}$ transformation
Let $T_{p s}^{\text {build }}$ represent the transformation that builds each of the program slices for Category 4 and Category 5 subprograms. Let $S D$ represent a collection of imperative subprograms. Let $\Sigma_{S_{p}}$ represent the program slice generated for the data item $p$ from the subprogram $S$. The $T_{p s}^{\text {build }}$ transformation is defined as shown in Figure 190.

This transformation generates a program slice for each output parameter, $p$, of Category 4 and Category 5 subprograms. Each of the original subprograms is included in $S D^{\prime}$, and the subprograms built from the program slices are also included. The next step is to change each subprogram call to a Category 4 or Category 5 subprogram into a sequence of calls to the new subprograms built from its program slices.

Let $T_{p s}^{c a l l s}$ represent the transformation that builds the sequence of calls to the new subprograms. Let $S D$ represent a collection of imperative subprograms, and let $S$ represent one of these subprograms. The $T_{p s}^{c a l l s}$ transformation is defined in Figure 191. This transformation creates a sequence of subprogram calls, $L_{S_{n}}$ for each call in $S$ that is invoking a Category 4 or Category 5 subprogram. The $L_{S_{n}}$ sequence collects the calls to the new subprograms that are built from the program slices. Each data item in $S$ that is produced from the Category 4 or Category 5 subprogram is now produced in $S^{\prime}$ by calling the new subprogram. The updated subprogram, $S^{\prime}$, is added to the updated design, $S D^{\prime}$, which is returned as the result of the transformation.

$$
\begin{aligned}
& T_{p s}^{\text {calls }}(S D, S)=S D^{\prime} \text { where } \\
& S=<i d_{S}, P_{\text {in }}, P_{\text {out }}, P_{\text {ret }}, P_{\text {loc }}, \Sigma_{S}>\text { and } \\
& \forall s \in \Sigma_{S} \\
& \Sigma_{S}=\Sigma_{S_{1}} \oplus[s] \oplus \Sigma_{S_{2}} \text { and }
\end{aligned}
$$

$$
\begin{aligned}
& S_{n} \in S D \text { and } \\
& S_{n}=<i d_{S_{n}}, P_{\text {in }}, P_{o u t},[], P_{\text {loc }}, \Sigma_{S_{n}}>\text { and } \\
& \left|P_{\text {out }}\right|>1 \Rightarrow \\
& \forall p \in P_{\text {out }} \\
& S_{p}=<i d_{S_{p}}, P_{i n_{p}},[p],[], P_{l o c_{p}}, \Sigma_{S_{p}}>\text { and } \\
& S_{p} \in S D^{\prime} \text { and } \\
& L_{S_{n}}=\left[l_{p} \mid l_{p}=\left\langle i d_{S_{p}}, P_{a_{\text {act }}^{\text {in }}},[p]\right\rangle\right. \text { and } \\
& P_{a c t_{i_{p}}}=\left[a \mid a \in P_{a_{c t} t_{i n}}\right. \text { and } \\
& \left.b \in P_{i n_{p}} \text { and } \mu(a)=b\right] \text { and } \\
& \left|P_{\text {out }}\right|<=1 \Rightarrow \\
& L_{S_{n}}=[s] \text { and } \\
& \Sigma_{S}^{\prime}=\Sigma_{S_{1}} \oplus L_{S_{n}} \oplus \Sigma_{S_{2}} \text { and } \\
& s \neq<i d_{S_{n}}, P_{\text {act }}^{\text {in }}, ~ P_{\text {actout }}>\Rightarrow \\
& \Sigma_{S}^{\prime}=\Sigma_{S_{1}} \oplus[s] \oplus \Sigma_{S_{2}} \text { and } \\
& S^{\prime}=<i d_{S}, P_{\text {in }}, P_{\text {out }}, P_{\text {ret }}, P_{\text {loc }}, \Sigma_{S}^{\prime}>\text { and } \\
& S D^{\prime}=T_{s}^{+}\left(T_{s}^{-}(S D, S), S^{\prime}\right)
\end{aligned}
$$

Figure 191 The $T_{p s}^{\text {calls }}$ transformation

Let $T_{p s}^{\text {main }}$ represent the transformation that builds all the program slices and updates the imperative subprograms to call the program slices instead of the Category 4 and Category 5 subprograms. Let $S D$ represent a collection of imperative subprograms.

$$
\begin{gathered}
T_{p s}^{\text {main }}(S D)=S D^{\prime \prime} \text { where } \\
S D^{\prime}=T_{p s}^{\text {build }}(S D) \text { and } \\
S D^{\prime \prime}=T_{p s}^{\text {calls }}\left(S D^{\prime}, S\right)
\end{gathered}
$$

The result of this transformation is an imperative design where each Category 4 and Category 5 subprogram has been replaced by the procedures built from program slicing the subprogram. Each subprogram in this design is updated to call the program slices instead of the Category 4 or Category 5 subprogram.

### 6.13 Converting an Imperative Design

The PBOI method, as a whole, converts a collection of imperative subprograms into a collection of classes and methods. The collection of imperative subprograms is represented as an imperative design, and the collection of classes and methods is represented as an object-oriented design. The conversion of the imperative design begins with the main program, which is identified by the user (see Assumption V. 1 in Chapter V). The transformation that formalizes the conversion of the imperative design to an object-oriented design is the $\sigma_{F}$ transformation. This transformation is defined below.

Let $S D$ represent an imperative design, and let $O O D$ represent an object-oriented design. The $\sigma_{F}$ transformation is defined as follows.

$$
\begin{aligned}
& \sigma_{F}(S D)=O O D \text { where } \\
& \quad S D^{\prime}=T_{p s}^{\text {main }}(S D) \text { and } \\
& M^{\prime} \in S D^{\prime} \text { and } C a t_{M}\left(M^{\prime}\right) \text { and } \\
& O O D=\sigma\left(\emptyset, M^{\prime}\right)
\end{aligned}
$$

This transformation uses the $T_{p s}^{\text {main }}$ transformation to build a new imperative design without Category 4 or Category 5 subprograms. The $\sigma_{M}$ transformation is used to convert the main program of this imperative design. The result of this transformation is the objectoriented design OOD.

### 6.14 Summary

This chapter has presented transformations that formalize the conversion of imperative statements, expressions, parameters, attributes, and subprograms using the PBOI method. Section 6.1 provides a table listing the transformations that are presented in this chapter. The transformations that formalize the program slicing process have also been presented in this chapter. Using these transformations, a collection of imperative subprograms can be converted to a functionally equivalent collection of classes and methods that implement the subprograms. This claim is discussed and proven in the following chapter.

## VII. Proving Functional Equivalence

### 7.1 Introduction

In order to evaluate the Object-Oriented (OO) design that is extracted using the PBOI methodology, a proof is developed in this chapter that shows the extracted OO code (represented in the GOM) is a consistent implementation of the original imperative code (represented in the GIM). Given the same input, a consistent implementation produces the same output as the original code. To build this proof, the behavior of both the original imperative code and the extracted OO code are defined.

Let $F$ be the original imperative code as represented in the GIM. Let $F^{\prime}$ be the extracted OO code as represented in the GOM. One undesirable approach for showing $F^{\prime}$ is a consistent implementation of $F$ is to delineate all the inputs and outputs for $F$ and show that an input given to $F^{\prime}$ produces the same output as $F$. This is an impractical approach and is not discussed further. At the highest level, $F$ represents the main program from the imperative design and $F^{\prime}$ represents the main method from the object-oriented design. The transformation that converts $F$ to $F^{\prime}$ is the $\sigma_{F}$ transformation, i.e. $F^{\prime}=$ $\sigma_{F}(F)$. Since the semantics of both $F$ and $F^{\prime}$ are expressed using the weakest precondition notation, the proof of functional equivalence is based on their weakest preconditions. Given a postcondition $R$, the semantics of $F$ are defined as follows.

$$
\{w p(F, R)\} F\{R\}
$$

Since the postcondition and weakest precondition are predicates (expressions) defined in terms of imperative variables, they must be transformed as well. The $\theta_{e}$ transformation built to convert expressions is used to convert these predicates. Hence, the semantics of $F^{\prime}$ are defined as follows.

$$
\left\{w p\left(F^{\prime}, \theta_{e}(R)\right\} F^{\prime}\left\{\theta_{e}(R)\right\}\right.
$$

If it can be proven that the transformation of imperative subprograms is a morphism [46], it can be proven that the weakest precondition predicate is "preserved" by the
transformation. If this can be proven, then the semantics for the imperative subprograms are equivalent to the semantics of the object-oriented methods, i.e. they are functionally equivalent. The $\sigma_{F}$ transformation is a morphism if the following equality can be proven correct.

$$
\theta_{e}(w p(F, R)) \Leftrightarrow w p\left(\sigma_{F}(F), \theta_{e}(R)\right)
$$

The rest of this chapter provides lemmas, theorems, and proofs which are used to prove that this equality holds. Since the $\sigma_{F}$ transformation uses several other transformations, proofs are presented that prove these transformations maintain functional equivalence. The proofs for the $\theta$ and $\delta$ transformations are presented first, followed by proofs for the $v$ and $\sigma$ transformations. The proof for the $\sigma_{F}$ transformation is presented in Section 7.6, at the end of the chapter.

### 7.2 Proof for Statement Transformations ( $\theta$ )

The $\theta$ transformation is used to convert a sequence of GIM statements to a sequence of GOM statements. Theorems are presented and proven below showing that the $\theta$ transformation of assignment statements, input statements, output statements, sequential control flow, selective control flow, and iterative control flow maintains functional equivalence. Imperative subprograms and subprogram calls are not transformed by $\theta$, but by $\sigma$ and $v$, respectively. The proofs for $\sigma$ and $v$ appear in Section 7.4.

Since the textual substitution [19] notation, $R_{e}^{x}$, is used for defining the semantics of assignment statements, the following lemma is provided for $\theta_{e}\left(R_{e}^{x}\right)$.

Lemma VII.1. $\theta_{e}\left(R_{e}^{x}\right) \Leftrightarrow \theta_{e}(R)_{\theta_{e}(e)}^{\theta_{v}(x)}$

Proof. 1. The notation $R_{e}^{x}$ symbolizes the simultaneous replacement of all free occurrences of $x$ with $e$ in the predicate $R$.
2. When $\theta_{e}$ is applied to $R_{e}^{x}$ on the left-hand-side of the equality, the textual substitution is done first and the $\theta_{e}$ transformation is applied to $R$ as well as the newly substituted occurrences of $e$, i.e. $\theta_{e}(e)$.
3. When the $\theta_{e}$ transformation is applied to $R$ on the right-hand-side of the equality, the occurrences of $x$ are converted using $\theta_{v}(x)$ so the substitution in $\theta_{e}(R)$ works properly.
4. The $\theta_{e}$ transformation is applied to $e$ before it replaces $\theta_{v}(x)$ in $\theta_{e}(R)$ so that the $\theta_{e}$ transformation of $R$ is complete after the substitution.
5. Thus, on both sides of the equality, $\theta_{e}$ is applied to all newly substituted occurrences of $e$ in $R$.
7.2.1 Proof for Assignment Statements. The following theorem is proven in order to shown that the transformation of assignment statements preserves functional equivalence.

Theorem VII.1. The $\theta$ transformation of assignment statements in the GIM preserves functional equivalence.

Proof.

$$
\begin{aligned}
& w p\left(\theta(<\mathrm{x},:=, \mathrm{e}>), \theta_{e}(R)\right) \Leftrightarrow \\
& w p\left(<\theta_{v}(x),:=, \theta_{e}(e)>, \theta_{e}(R)\right) \Leftrightarrow \\
& \theta_{e}(R)_{\theta_{e}(x)}^{\theta_{v}(x)} \Leftrightarrow \\
& \theta_{e}\left(R_{e}^{x}\right) \Leftrightarrow \text { by Lemma VII. } 1 \\
& \theta_{e}(w p(<\mathrm{x},:=, \mathrm{e}>, R))
\end{aligned}
$$

7.2.2 Proof for Input and Output Statements. The following theorem is proven in order to shown that the transformation of input statements preserves functional equivalence. The proof is given for a sequence of 2 inputs. The extension to arbitrary length sequences is obvious.

Theorem VII.2. The $\theta$ transformation of input statements in the GIM preserves functional equivalence.

Proof.

$$
\begin{aligned}
& w p\left(\theta\left(<\text { input, iport, }\left[v_{1}, v_{2}\right]>\right), \theta_{e}(R)\right) \Leftrightarrow \\
& w p\left(<\text { input, iport, } \theta_{V}\left(\left[v_{1}, v_{2}\right]\right)>, \theta_{e}(R)\right) \Leftrightarrow \\
& w p\left(<\text { input, iport, }\left[\theta_{v}\left(v_{1}\right), \theta_{v}\left(v_{2}\right)\right]>, \theta_{e}(R)\right) \Leftrightarrow \\
& w p\left(\left[\left\langle\theta_{v}\left(v_{1}\right),:=, \theta_{e}\left(x_{1}\right)\right\rangle,\left\langle\theta_{v}\left(v_{2}\right),:=, \theta_{e}\left(x_{2}\right)\right\rangle\right], \theta_{e}(R)\right) \Leftrightarrow \\
& w p\left(\left[<\theta_{v}\left(v_{1}\right),:=, \theta_{e}\left(x_{1}\right)>, \theta_{e}(R)_{\theta_{e}\left(x_{2}\right)}^{\theta_{v}\left(v_{2}\right)} \Leftrightarrow\right.\right. \\
& \left(\theta_{e}(R)_{\theta_{e}\left(x_{1}\right)}^{\theta_{v}\left(v_{1}\right)}\right)_{\theta_{e}\left(x_{2}\right)}^{\theta_{v}\left(v_{2}\right)} \Leftrightarrow \\
& \theta_{e}\left(\left(R_{x_{1}}^{v_{1}}\right)_{x_{2}}^{v_{2}}\right) \Leftrightarrow \text { by Lemma VII. } 1 \\
& \left.\theta_{e}\left(w p\left(<v_{1},:=, x_{1}\right\rangle, R_{x_{2}}^{v_{2}}\right)\right) \Leftrightarrow \\
& \left.\theta_{e}\left(w p\left(<v_{1},:=, x_{1}\right\rangle, w p\left(<v_{2},:=, x_{2}>, R\right)\right)\right) \Leftrightarrow \\
& \theta_{e}\left(w p\left(\left[<v_{1},:=, x_{1}\right\rangle,\left\langle v_{2},:=, x_{2}>\right], R\right)\right) \Leftrightarrow \\
& \theta_{e}\left(w p\left(<\text { input, iport, }\left[v_{1}, v_{2}\right]>, R\right)\right)
\end{aligned}
$$

The following theorem is proven in order to shown that the transformation of output statements preserves functional equivalence.

Theorem VII.3. The $\theta$ transformation of output statements in the GIM preserves functional equivalence.

Proof.

$$
\begin{aligned}
& w p\left(\theta(<\text { output, oport, } E>), \theta_{e}(R)\right) \Leftrightarrow \\
& \quad w p\left(<\text { output, oport, } \theta_{E}(E)>, \theta_{e}(R)\right) \Leftrightarrow \\
& \theta_{e}(R) \Leftrightarrow \\
& \theta_{e}(w p(<\text { output, oport, } E>, R))
\end{aligned}
$$

7.2.3 Proof for Skip Statement. The following theorem is proven in order to shown that the transformation of skip statements preserves functional equivalence.

Theorem VII.4. The $\theta$ transformation of skip statements in the GIM preserves functional equivalence. ${ }^{1}$

[^5]Proof.

$$
\begin{gathered}
w p\left(\theta(s k i p), \theta_{e}(R)\right) \Leftrightarrow \\
w p\left(s k i p, \theta_{e}(R)\right) \Leftrightarrow \\
\theta_{e}(R) \Leftrightarrow \\
\theta_{e}(w p(s k i p, R))
\end{gathered}
$$

7.2.4 Proof for Sequential Control Flow. The following theorem is proven in order to shown that the transformation of sequences of statements preserves functional equivalence.

Theorem VII.5. The $\theta$ transformation of sequential control flow for sequences of assignment, input, output, and skip statements preserves functional equivalence.

Proof. Since it has already been proven that the $\theta$ transformation maintains functional equivalence for the assignment, input, output, and skip statements, the following equality holds.

$$
\begin{aligned}
& w p\left(\theta\left(\left[s_{1}, s_{2}\right]\right), \theta_{e}(R)\right) \Leftrightarrow \\
& \quad w p\left(\left[\theta\left(s_{1}\right), \theta\left(s_{2}\right)\right], \theta_{e}(R)\right) \Leftrightarrow \\
& w p\left(\theta\left(s_{1}\right), w p\left(\theta\left(s_{2}\right), \theta_{e}(R)\right)\right) \Leftrightarrow \\
& w p\left(\theta\left(s_{1}\right), \theta_{e}\left(w p\left(s_{2}, R\right)\right)\right) \Leftrightarrow \\
& \theta_{e}\left(w p\left(s_{1}, w p\left(s_{2}, R\right)\right)\right) \Leftrightarrow \\
& \theta_{e}\left(w p\left(\left[s_{1}, s_{2}\right], R\right)\right)
\end{aligned}
$$

7.2.5 Proof for Selective Control Flow. This section provides a proof showing that the $\theta$ transformation of selective control flow statements maintains functional equivalence. The following theorem is proven below.

Theorem VII.6. The $\theta$ transformation of selective control flow in the GIM preserves functional equivalence.

Proof by Induction. 1. Base Case: The proof of Theorem VII. 5 shows that $\theta$ maintains functional equivalence for sequences of assignment, input, output, or skip statements. Thus, the following equality holds.

$$
\begin{aligned}
& w p\left(\theta\left(<\text { if, } e, \text { then, } S_{1}, \text { else, } S_{2}>\right), \theta_{e}(R)\right) \Leftrightarrow \\
& \left.\quad w p\left(<\text { if, } \theta_{e}(e), \text { then, } \theta\left(S_{1}\right), \text { else }, \theta\left(S_{2}\right)>\right), \theta_{e}(R)\right) \Leftrightarrow \\
& \quad\left(\left(\theta_{e}(e) \Rightarrow w p\left(\theta\left(S_{1}\right), \theta_{e}(R)\right)\right) \wedge\left(\neg \theta_{e}(e) \Rightarrow w p\left(\theta\left(S_{2}\right), \theta_{e}(R)\right)\right)\right) \Leftrightarrow \\
& \quad\left(\left(\theta_{e}(e) \Rightarrow \theta_{e}\left(w p\left(S_{1}, R\right)\right)\right) \wedge\left(\neg \theta_{e}(e) \Rightarrow \theta_{e}\left(w p\left(S_{2}, R\right)\right)\right)\right) \Leftrightarrow \\
& \left(\theta_{e}\left(\left(e \Rightarrow w p\left(S_{1}, R\right)\right)\right) \wedge \theta_{e}\left(\left(\neg e \Rightarrow w p\left(S_{2}, R\right)\right)\right)\right) \Leftrightarrow \\
& \theta_{e}\left(\left(\left(e \Rightarrow w p\left(S_{1}, R\right)\right) \wedge\left(\neg e \Rightarrow w p\left(S_{2}, R\right)\right)\right)\right) \Leftrightarrow \\
& \theta_{e}\left(w p\left(<\text { if, } e, \text { then, } S_{1}, \text { else, } S_{2}>, R\right)\right)
\end{aligned}
$$

2. Inductive Hypothesis: Assume the sequences of statements $S_{1}$ and $S_{2}$ also include if-then-else statements, and the $\theta$ transformation correctly transforms an if-then-else statement with an arbitrary number, $n$, of nested if-then-else statements.
3. Inductive Step: Prove that adding the $n+1$ nested if-then-else statement maintains the functional equivalence. Since this new nested if-then-else is the final nested statement, it includes only assignment, input, output, or skip statements in the sequences $S_{1}$ and $S_{2}$. The functional equivalence is maintained for this statement as shown for the base case.

The corollary shown below follows from the proof that the $\theta$ transformation of the selective statement maintains functional equivalence.

Corollary VII.1. The $\theta$ transformation of sequential control flow for sequences of assignment, input, output, skip, and selective statements preserves functional equivalence.
7.2.6 Proof for Iterative Control Flow. This section provides a proof showing that the $\theta$ transformation of iterative control flow statements maintains functional equivalence.

Theorem VII.7. The $\theta$ transformation for iterative control flow in the GIM preserves functional equivalence.

Proof by Induction. 1. Base Case: The proof of Theorem VII. 5 shows that $\theta$ maintains functional equivalence for sequences of assignment, input, output, or skip statements. Thus, the equality shown in Figure 192 holds.
2. Inductive Hypothesis: Assume the sequence of statements $S_{3}$ also includes while statements, and the $\theta$ transformation correctly transforms a while statement with an arbitrary number, $n$, of nested while statements.
3. Inductive Step: Prove that adding the $n+1$ nested while statement maintains functional equivalence. Since this new nested while is the final nested statement, it includes only assignment, input, output, or skip statements in the sequence $S_{3}$. The functional equivalence is maintained for this statement as shown for the base case.

The corollary shown below follows from the proof that the transformation of iterative statements maintains functional equivalence.

Corollary VII.2. The $\theta$ transformation of sequential control flow for sequences of assignment, input, output, skip, selective, and iterative statements preserves functional equivalence.

Given that proofs have been provided for selective and iterative control flow, the following two corollaries follow from these proofs.

Corollary VII.3. The $\theta$ transformation of selective control flow for sequences of assignment, input, output, skip, selective, and iterative statements preserves functional equivalence.

Corollary VII.4. The $\theta$ transformation of iterative control flow for sequences of assignment, input, output, skip, selective, and iterative statements preserves functional equivalence.

Finally, the corollary shown below follows from all of the proofs and corollaries provided for the $\theta$ transformation to this point. This corollary provides an assertion that the $\theta$ transformation preserves functional equivalence for each of the statements defined in the GIM, except subprogram definitions and calls.

```
wp(0(< while, e, S S > ), 在(R))\Leftrightarrow
    wp(< while, ,
    wp([< if, 抆(e), then, 0(S3), else, }0([skip])>, iteration 1
        < if, }\mp@subsup{0}{e}{}(e),\mathrm{ then, }0(\mp@subsup{S}{3}{}), else, 0([skip])>, iteration 2
                        ...
```




```
        wp(< if, 紋(e), then, 0(S3), else, }0([skip])>, iteration 2
            wp(< if, 紋(e), then, 0(S S ), else, }0([skip])>,\mp@subsup{0}{e}{}(R))))\Leftrightarrow\mathrm{ iteration k
        wp(< if, 蛴(e), then, 0(S3), else, }0([skip])>, iteration 1
        wp(< if, 访(e), then, 0(S3), else, }0([skip])>, iteration 2
                            ...
            \mp@subsup{0}{e}{}(wp(< if, e, then, S3, else, [skip]>, R)))) \Leftrightarrow iteration k
    wp}(< if, 埌(e), then, 0(S3), else, 0([skip])>, iteration 1
        0e}(wp(< if, e, then, S3, else, [skip]>, iteration 2
            ...
            wp(< if, e, then, S S , else, [skip]>, R)))) \Leftrightarrow iteration k
    0e}(wp(< if, e, then, S3, else, [skip] >, iteration 1
        wp(< if, e, then, S3, else, [skip]>, iteration 2
            wp(< if, e, then, S S , else, [skip]>, R)))) \Leftrightarrow iteration k
    0
            < if, e, then, S}\mp@subsup{S}{3}{}\mathrm{ , else, [skip]>, iteration 2
            < if, e, then, S S , else, [skip] >],R)) \Leftrightarrow iteration k
    0e}(wp(< while, e, S S >, R))
```

Figure 192 Proof for iterative base case．

Corollary VII.5. The $\theta$ transformation of sequences of assignment, input, output, skip, selective, and iterative statements preserves functional equivalence.

### 7.3 Proof for Parameters and Attributes ( $\delta$ )

This section presents theorems and proofs that the $\delta$ transformation preserves functional equivalence. This transformation is used by the $\sigma$ transformation when GOM parameters are moved to attributes and vice versa. Since the $\delta$ transformation converts accesses of GOM variables into "GET-" and "SET-" messages, it must proven that an access to variable is functionally equivalent to a "GET-" message. It must also be proven that the assignment of a GOM variable is functionally equivalent to a "SET-" message. These transformations involve only GOM entities, so there is no need to convert the postcondition using $\theta_{e}$. Let $R^{\prime}$ represent a postcondition represented using GOM variables.

The following theorem is proven below in order to show that the $\delta$ transformation of an access of a variable into a "GET-" message preserves functional equivalence.

Theorem VII.8. The access of a variable, a, in a statement is functionally equivalent to a call to the "GET-a" message.

As an example of this theorem, consider the expression that appears on the right-hand-side of an assignment statement. The following proof shows that if this expression is the variable $a$, then the semantics of the assignment statement are preserved.

Proof.

$$
\begin{aligned}
& w p\left(\delta\left(\left[<x^{\prime},:=, a>\right], c,\{a\}\right), R^{\prime}\right) \Leftrightarrow \\
& w p\left(<x^{\prime},:=, \delta_{e}(a)>, R^{\prime}\right) \Leftrightarrow \\
& w p\left(<x^{\prime},:=,<\text { GET- } a,[c] \gg, R^{\prime}\right) \Leftrightarrow \\
& w p\left(\left[y:=c . a, b:=y,<x^{\prime},:=, b>\right], R^{\prime}\right) \Leftrightarrow \\
& w p\left(y:=c . a, w p\left(b:=y, w p\left(<x^{\prime},:=, b>, R^{\prime}\right)\right)\right) \Leftrightarrow \\
& w p\left(y:=c . a, w p\left(b:=y, R_{b}^{\prime x^{\prime}}\right)\right) \Leftrightarrow \\
& w p\left(y:=c . a,\left(R_{b}^{\prime x^{\prime}}\right)_{y}^{b}\right) \Leftrightarrow \\
& \left(\left(R_{b}^{\prime x^{\prime}}\right)_{y}^{b}\right)_{c . a}^{y} \Leftrightarrow \\
& R_{c . a}^{x^{\prime}} \Leftrightarrow \\
& R_{a}^{\prime x^{\prime}} \Leftrightarrow \\
& w p\left(<x^{\prime},:=, a>, R^{\prime}\right)
\end{aligned}
$$

Other examples of these expressions are limited to the expressions that appear in the boolean tests of selective and iterative statements. Similar arguments are made for these expressions, so in general, Theorem VII. 8 holds for any statement in the GOM.

The following theorem is proven below in order to show that the $\delta$ transformation of an assignment of a variable into a "SET-" message preserves functional equivalence. Let $R^{\prime}$ represent a postcondition represented using GOM variables.

Theorem VII.9. The assignment of a variable, $a$, is functionally equivalent to a call to the "SET-a" message.

Proof.

$$
\begin{aligned}
& w p\left(\delta([<a,:=, e>], c,\{a\}), R^{\prime}\right) \Leftrightarrow \\
& w p\left(<\operatorname{SET}-\mathrm{a},\left[c, \delta_{e}(e)\right]>, R^{\prime}\right) \Leftrightarrow \\
& R_{\delta_{e}(e)}^{c \cdot a} \Leftrightarrow \\
& \left.R_{\delta_{e}(e)}^{\prime \prime}\right) \Leftrightarrow \\
& w p\left(<a,:=, \delta_{e}(e)>, R^{\prime}\right) \Leftrightarrow \\
& w p\left(<a,:=, e>, R^{\prime}\right) \text { from Theorem VII.8. }
\end{aligned}
$$

Similarly, the $\delta^{-1}$ transformation converts an access of a specific object attribute into an access of a GOM variable. Specifically, each "GET-a" message is converted into a reference to the variable $a$, and each "SET-a" message is converted into an assignment to $a$. The following theorem is proven in order to show that replacing the "GET-a" message with an access of a variable, $a$, in an expression maintains functional equivalence. Let $R^{\prime}$ represent a postcondition represented using GOM variables.

Theorem VII.10. The call to a "GET-a" message in a statement is functionally equivalent to the access of the variable, a.

As an example of this theorem, consider the expression that appears on the right-hand-side of an assignment statement. The following proof shows that replacing the "GET-a" message with an access to the variable $a$ maintains the functionality of the assignment statement.

Proof.

$$
\begin{aligned}
& w p\left(\delta^{-1}([<x,:=,<\text { GET- } a,[c] \gg], c,\{a\}), R^{\prime}\right) \Leftrightarrow \\
& w p\left(<x,:=, a>, R^{\prime}\right) \Leftrightarrow \\
& R_{a}^{\prime x} \Leftrightarrow \\
& R_{c . a}^{\prime x} \Leftrightarrow \\
& \left(\left(R_{b}^{\prime x} b_{y}^{b}\right)_{c . a}^{y} \Leftrightarrow\right. \\
& w p\left(y:=c . a,\left(R_{b}^{\prime x}\right)_{y}^{b}\right) \Leftrightarrow \\
& w p\left(y:=c . a, w p\left(b:=y, R_{b}^{\prime x}\right)\right) \Leftrightarrow \\
& w p\left(y:=c . a, w p\left(b:=y, w p\left(<x,:=, b>, R^{\prime}\right)\right)\right) \Leftrightarrow \\
& w p\left([y:=c . a, b:=y,<x,:=, b>], R^{\prime}\right) \Leftrightarrow \\
& w p\left(<x,:=,<\text { GET- } a,[c] \gg, R^{\prime}\right)
\end{aligned}
$$

Other examples of these expressions are limited to the expressions that appear in the boolean tests of selective and iterative statements. Similar arguments are made for these expressions, so in general, Theorem VII. 10 holds for any statement in the GIM.

The following theorem is proven in order to show that replacing the "SET-a" message with an assignment of a variable, $a$, maintains functional equivalence. Let $R^{\prime}$ represent a postcondition represented using GOM variables.

Theorem VII.11. The "SET-a" message is functionally equivalent to an assignment of the variable $a$.

Proof.

$$
\begin{gathered}
w p\left(\delta^{-1}(<\mathrm{SET}-\mathrm{a},[c, e]>), R^{\prime}\right) \Leftrightarrow \\
w p\left(<a,:=, e>, R^{\prime}\right) \Leftrightarrow \\
R_{e}^{\prime a} \Leftrightarrow \\
R_{e}^{\prime c . a} \Leftrightarrow \\
w p\left(<\mathrm{SET}-\mathrm{a},[c, e]>, R^{\prime}\right)
\end{gathered}
$$

## 7. 4 Proof for Subprogram Conversion ( $\sigma$ )

The following inductive proof shows that the $\sigma$ transformation of imperative subprograms whose statements also include procedure calls maintains functional equivalence.

Theorem VII.12. The $\sigma$ transformation of imperative subprograms where $\Sigma_{S}$ is comprised of assignment, input, output, skip, selective, and iterative statements, as well as procedure calls, preserves functional equivalence.

Proof by Induction. 1. Base Case: The $\sigma$ transformation of a subprogram that calls a procedure whose $\Sigma_{S}$ is comprised of assignment, input, output, skip, selective, and iterative statements preserves functional equivalence. Using Corollary VII.5, the base case is proven by showing the $v_{m}$ transformation of such a call preserves functional equivalence.

$$
\begin{aligned}
& w p\left(v_{m}\left(S,<i d_{S}, P_{\text {in }}, P_{\text {out }}>\right), \theta_{e}(R)\right) \Leftrightarrow \\
& w p\left(<i d_{S}, \theta_{V}\left(P_{\text {in }}\right), \theta_{V}\left(P_{\text {out }}\right),>, \theta_{e}(R)\right) \Leftrightarrow \\
& w p\left(\left[\bar{x}:=\theta_{V}\left(P_{\text {in }}\right), \theta(S 1), \theta_{V}\left(P_{\text {out }}\right):=\bar{z}\right], \theta_{e}(R)\right) \Leftrightarrow \\
& w p\left(\bar{x}:=\theta_{V}\left(P_{\text {in }}\right), w p\left(\theta(S 1), w p\left(\theta_{V}\left(P_{\text {out }}\right):=\bar{z}, \theta_{e}(R)\right)\right)\right) \Leftrightarrow \\
& \left.w p\left(\bar{x}:=\theta_{V}\left(P_{\text {in }}\right), w p\left(\theta(S 1), \theta_{e}(R)_{\bar{z}}^{\theta_{V}\left(P_{o u t}\right)}\right)\right)\right) \Leftrightarrow \\
& w p\left(\bar{x}:=\theta_{V}\left(P_{i n}\right), w p\left(\theta(S 1), \theta_{e}\left(R_{\bar{z}}^{P o u t}\right)\right)\right) \Leftrightarrow \\
& w p\left(\bar{x}:=\theta_{V}\left(P_{i n}\right), \theta_{e}\left(w p\left(S 1, R_{\bar{z}}^{P_{o u t}}\right)\right)\right) \Leftrightarrow \\
& \theta_{e}\left(w p\left(S 1, R_{\bar{z}}^{P_{o u t}}\right)\right)_{\theta_{V}\left(P_{\text {in }}\right)}^{\bar{x}} \Leftrightarrow \\
& \theta_{e}\left(w p\left(S 1, R_{\bar{z}}^{P_{o u t}}\right)_{P_{i n}}^{\bar{x}}\right) \Leftrightarrow \\
& \theta_{e}\left(w p\left(\bar{x}:=P_{\text {in }}, w p\left(S 1, R_{\bar{z}}^{P_{o u t}}\right)\right)\right) \Leftrightarrow \\
& \theta_{e}\left(w p\left(\bar{x}:=P_{\text {in }}, w p\left(S 1, w p\left(P_{\text {out }}:=\bar{z}, R\right)\right)\right)\right) \Leftrightarrow \\
& \theta_{e}\left(w p\left(\left[\bar{x}:=P_{\text {in }}, S 1, P_{\text {out }}:=\bar{z}\right], R\right)\right) \Leftrightarrow \\
& \left.\theta_{e}\left(w p\left(<i d_{S}, P_{\text {in }}, P_{\text {out }}\right\rangle, R\right)\right)
\end{aligned}
$$

2. Inductive Hypothesis: Assume $\Sigma_{S}$ of the called procedure also includes calls to other procedures. Since $\sigma$ is used to transform these called procedures, assume that the $\sigma$ transformation correctly transforms an arbitrary number, $n$, of nested procedure calls.
3. Inductive Step: Prove that adding the $n+1$ nested procedure call maintains functional equivalence. Since this new nested procedure call is the final nested call, it includes only assignment, input, output, skip, selective, or iterative statements in $\Sigma_{S}$. The functional equivalence is maintained for this call as shown for the base case.

Corollary VII.6. The $\sigma$ transformation of imperative subprograms where $\Sigma_{S}$ includes statements with expressions that include function calls preserves functional equivalence.

### 7.5 Proof for Program Slicing ( $T_{p s}^{\text {main }}$ )

This section presents a proof that the $T_{p s}^{\text {main }}$ transformation preserves functional equivalence. This transformation builds a program slice for each data item being produced from a Category 4 or Category 5 subprogram using the projection function $\pi$. The col-
lection of program slices produced duplicate the functionality of the original subprogram. This is expressed formally as shown below. Let $\Sigma$ represent the sequence of statement from the original subprogram which produces data items $a$ and $b$.

$$
w p(\Sigma, R) \Leftrightarrow w p([\pi(\Sigma, a), \pi(\Sigma, b)])
$$

The $T_{p s}^{m a i n}$ transformation uses the $T_{p s}^{\text {calls }}$ transformation to replace each call to a Category 4 or Category 5 subprogram with a sequence of calls to the program slices built for the subprogram. The proof of the following theorem shows that these transformations preserve functional equivalence.

Theorem VII.13. The transformation of GIM subprograms and subprogram calls using the $T_{p s}^{\text {calls }}$ transformation preserves functional equivalence.

Proof.

$$
\begin{aligned}
& w p\left(T_{p s}^{c a l l s}\left(\left[<i d_{S}, P_{i n},[a, b]>\right]\right), R\right) \Leftrightarrow \\
& \quad w p\left(\left[<i d_{S_{a}}, P_{i n_{a}},[a]>,<i d_{S_{b}}, P_{i n_{b}},[b]>\right], R\right) \Leftrightarrow \\
& \quad w p\left(<i d_{S_{a}}, P_{i n_{a}},[a]>, w p\left(<i d_{S_{b}}, P_{i n_{b}},[b]>, R\right)\right) \Leftrightarrow \\
& \quad w p\left(\left[\bar{x}_{a}:=P_{i n_{a}}, \pi(\Sigma, a), a:=z\right], w p\left(\left[\bar{x}_{b}:=P_{i n_{b}}, \pi(\Sigma, b), b:=v\right], R\right)\right) \Leftrightarrow \\
& \quad w p\left(\left[\bar{x}_{a}:=P_{i n_{a}}, \pi(\Sigma, a), a:=z, \bar{x}_{b}:=P_{i n_{b}}, \pi(\Sigma, b), b:=v\right], R\right) \Leftrightarrow \\
& \quad w p\left(\left[\bar{x}_{a}:=P_{i n_{a}}, \bar{x}_{b}:=P_{i n_{b}}, \pi(\Sigma, a), a:=z, \pi(\Sigma, b), b:=v\right], R\right) \Leftrightarrow \\
& \quad w p\left(\left[\bar{x}_{a}:=P_{i n_{a}}, \bar{x}_{b}:=P_{i n_{b}}, \pi(\Sigma, a), \pi(\Sigma, b), a:=z, b:=v\right], R\right) \Leftrightarrow \\
& \quad w p\left(\left[\bar{x}_{a}:=P_{i n_{a}} \bar{x}_{b}:=P_{i n_{b}}\right], w p([\pi(\Sigma, a), \pi(\Sigma, b)], w p([a:=z, b:=v], R))\right) \Leftrightarrow \\
& \quad w p\left(\left[\bar{x}:=P_{i n}\right], w p\left(\Sigma_{S}, w p([a:=z, b:=v], R)\right)\right) \Leftrightarrow \\
& \quad w p\left(\left[\bar{x}:=P_{i n}, \Sigma_{S}, a:=z, b:=v\right], R\right) \Leftrightarrow \\
& \quad w p\left(\left\langle i d_{S}, P_{i n},[a, b]>, R\right)\right.
\end{aligned}
$$

### 7.6 Proof for Imperative Design ( $\sigma_{F}$ )

This section provides a proof that the object-oriented code extracted using the PBOI methodology is a consistent implementation of the legacy imperative code. As discussed in Section 7.1, the functionality of the imperative code is embodied at the highest level in
the main program, $F$. The functionality of the object-oriented code is embodied at the highest level in the main method, $F^{\prime}$. The $\sigma_{F}$ transformation is used to transform $F$ to $F^{\prime}$, and this section provides a proof that the $\sigma_{F}$ transformation produces an $F^{\prime}$ that is functionally equivalent to $F$.

Theorem VII.14. The $\sigma_{F}$ transformation preserves functional equivalence.

Proof.

$$
\begin{aligned}
& w p\left(\sigma_{F}(F), \theta_{e}(R)\right) \Leftrightarrow \\
& \quad w p\left(\sigma\left(T_{p s}^{\operatorname{main}}(F)\right), \theta_{e}(R)\right) \Leftrightarrow \\
& \theta_{e}\left(w p\left(T_{p s}^{\text {main }}(F), R\right)\right) \Leftrightarrow \\
& \theta_{e}(w p(F, R))
\end{aligned}
$$

Since this proof shows that the $\sigma_{F}$ transformation preserves functional equivalence, the object-oriented design extracted using the PBOI methodology is functionally equivalent to the legacy imperative design.

### 7.7 Summary

This chapter presented several definitions, theorems, and proofs in order to show that using the PBOI methodology results in an object-oriented design which is a consistent implementation of the original imperative design. The overall functionality of the imperative design was proven to be maintained in the extracted object-oriented design. This was done by proving that the $\sigma_{F}$ transformation preserves functional equivalence. The following chapter presents a feasibility demonstration of the PBOI methodology.

### 8.1 Introduction

This chapter discusses the proof-of-concept prototype developed to implement the PBOI methodology. As a feasibility demonstration, a legacy imperative system consisting of over 3000 lines of FORTRAN- 77 code was converted to the object-oriented paradigm. This chapter describes the transformations used to convert FORTRAN-77 code to the GIM and presents a summary of the legacy system that was converted. A description of the extracted object-oriented design is presented at the end of the chapter.

### 8.2 Converting FORTRAN to the GIM

The GIM has been developed in order to provide a canonical form for representing imperative programming languages. As a proof-of-concept, automatic transformations from FORTRAN-77 to the GIM have been developed. These transformations were built using the Software Refinery ${ }^{T M}$ development environment. The Refine/FORTRAN ${ }^{T M}$ reverse engineering tool, part of Software Refinery ${ }^{T M}$, includes a domain model and grammar for FORTRAN-77. Refine/FORTRAN parses in FORTRAN source code and builds Abstract Syntax Trees (ASTs) that store information about the source code. The transformations build new GIM ASTs based on these FORTRAN-77 ASTs.

The first step in re-engineering a legacy system is to parse each subprogram using Refine/FORTRAN ${ }^{T M}$. Refine/FORTRAN ${ }^{T M}$ has a constraint that each of the subprograms must reside in its own file, and the files to be parsed are collected into a system. An AST is built for the system and each AST for a subprogram is part of this AST. After parsing, the system AST is stored in one file called the analysis file. The analysis file is loaded into the re-engineering prototype, so the transformations can be applied to each AST.

A typical transformation has the following form.

$$
<\text { FORTRAN AST }>\longrightarrow<\text { GIM AST }>
$$

The approach taken in the prototype is to generate a new GIM AST using the transformations, as opposed to transforming the existing FORTRAN ASTs. This means as FORTRAN ASTs are found that match the left-hand-side of the transformation, the GIM AST from the right-hand-side of the transformation is built. The identifiers on the left-hand-side indicate ASTs from the FORTRAN-77 domain model provided in the Refine/FORTRAN ${ }^{T M}$ User's Guide [55] and the identifiers on the right-hand-side indicate ASTs from the GIM domain model as defined in Chapter III. All of the transformations are applied to each node in the FORTRAN AST by using the REFINE pre-order traversal command. The traversal begins with the FORTRAN AST representing the overall system.

A Refine/FORTRAN ${ }^{T M}$ workbench has been built that displays a subprogram from the legacy system (using the FORTRAN-77 surface syntax) and the corresponding subprogram as represented in the GIM (using the GIL surface syntax). Figure 193 shows this workbench. The upper-left subwindow of the workbench shows the original FORTRAN legacy subroutine. The upper-right subwindow shows the GIM representation of this subroutine. The two subwindows are hyper-linked to show the connection between the legacy FORTRAN code and the representation of the code in the GIM. When the cursor is over an AST entity in the GIL Window, the surface syntax of the corresponding FORTRAN AST is highlighted in the Legacy System Window. In the figure, the cursor is over the GIM addition expression R 1 (2) $+\mathrm{F} * \mathrm{R} 2$ ( 2 ), as indicated by the rubber-band box. The FORTRAN AST entity from which this expression was transformed is highlighted in the Legacy System Window using reverse video. Note that the entire FORTRAN subroutine is not visible in the upper-left subwindow, but the scroll bar can be used to view the rest of the subroutine. The workbench also shows the control flow graph and structure chart for the VADD subprogram. The structure chart is provided as part of the Refine/FORTRAN ${ }^{T M}$ workbench; the control flow graph is not.

Because of the limited documentation provided with Refine/FORTRAN ${ }^{T M}$, it was difficult to determine the attributes of an AST node included in the FORTRAN-77 domain model without parsing a statement that builds the node. For this reason, only the transformations that were needed to convert the selected proof-of-concept legacy system were developed in the prototype.


Figure 193 FORTRAN Workbench

| Program level objects |
| :--- |
| EXECUTABLE-PROGRAM |
| PROGRAM-UNIT |
| END-STATEMENT |
| HEADER-STATEMENT |
| PROGRAM-STATEMENT |
| SUBROUTINE-STATEMENT |
| PARAMETER |
| Variables object |
| INDEXABLE-NAME |
| IDENTIFIER |
| IMPLICIT-STATEMENT |
| TYPE-STATEMENT |
| TYPE-BYTE |
| TYPE-CHARACTER |
| TYPE-DOUBLE-PRECISION |
| TYPE-INTEGER |
| TYPE-LOGICAL |
| TYPE-REAL |
| NAME-DECLARATOR |
| POSSIBLY-INITIALIZED-NAME |
| DIMENSION-DECLARATOR |
| SIMPLE-LENGTH-DECL |
| COMMON-DECCLARATOR |
| COMMONN-STATEMENT |
| EQUIVALENCEEDECLLARATOR |
| EQUIVALENCE-STATEMENT |
| DIMENSION-STATEMENT |
| RECORD-VAR-DECL |
| STRUCT-REFERENCE |
| STRUCT-ACCESS |

Table 2 Fortran ASTs Used

| Expressions objects |
| :--- |
| ADD-EXPRESSION |
| AND-EXPRESSION |
| CONCATENATE-EXPRESSION |
| DIVIDE-EXPRESSION |
| EQ-EXPRESSION |
| EQV-EXPRESSION |
| EXPONENTIATE-EXPRESSION |
| GE-EXPRESSION |
| GT-EXPRESSION |
| LE-EXPRESSION |
| LT-EXPRESSION |
| MULTIPLY-EXPRESSION |
| NE-EXPRESSION |
| NEQV-EXPRESSION |
| OR-EXPRESSION |
| SUBTRACT-EXPRESSION |
| LITERAL-FALSE |
| LITERAL-TRUE |
| LITERAL-INTEGER |
| LITERAL-RIGHT-TEXT |
| LITERAL-DP |
| LITERAL-REAL |
| LITERAL-CHARSTRING |
| LITERAL-HOLLERITH |
| IDENTITY-EXPRESSION |
| NEGATE-EXPRESSION |
| NOT-EXPRESSION |
| NULL-EX |
| Assignment objects |
| ASSIGNMENTT-STATEMENT |
| PARAMETER-STATEMENT |
| Control flow statement objects |
| CONTTINUE-STATEMENT |
| RETURN-STATEMENT |
| INCLUDE-STATEMENT |
| LABEL-DEFINITION |
| LABEL-USE |
| UNCONDITIONAL-GOTO |
| CALL-STATEMENT |
| BLOCK-IF-STATEMENT |
| LOGICAL-IF-STATEMENT |
| IF-THEN-ELSE-STATEMENT |
| DO-STATEMENT |

Table 3 Fortran ASTs Used (cont)

| I/O statements |
| :--- |
| OPEN-STATEMENT |
| READ-STATEMENT |
| WRITE-STATEMENT |
| PRINT-STATEMENT |
| FORMAT-STATEMENT |
| FORMAT-COMMA-SEPARATOR |
| FORMAT-CLOSER |
| UNIT-IDENTIFIER |
| FORMAT-IDENTIFIER |
| IO-FULL-SPECIFIER |
| LIT-STRING-FORMAT |
| FORMAT-SLASH-SEPARATOR |
| FORMAT-DOUBLE-SLASH |
| X-FORMAT |
| I-FORMAT |
| F-FORMAT |
| E-FORMAT |
| O-FORMAT |
| Z-FORMAT |
| R-FORMAT |
| A-FORMAT |

Table 4 Fortran ASTs Used (cont)

Table 2, Table 3, and Table 4 show the domain model ASTs that are included in the prototype. The other ASTs shown in Chapter 14 of the Refine/FORTRAN ${ }^{T M}$ User's $^{T}$ Guide [54] were not used. A separate process was used to evaluate the completeness of the transformation after the pre-order traversal process completed execution. This process is described in detail in Section 8.3.

The following sections describe the transformations that have been developed and included in the prototype. These transformations convert FORTRAN statements, variables, and expressions to the GIM.
8.2.1 FORTRAN Assignment Statement Transformation. Variables in FORTRAN are assigned values using the assignment statement and the data statement. The transformation of these two statements is shown in Figure 194.


Figure 194 Assignment Transformations

The following FORTRAN assignment statement is represented using the FORTRAN assignment-statement AST.

$$
\mathrm{RM}=\operatorname{VMAG}(\mathrm{R})
$$

The variable on the left-hand-side of this assignment is converted to a GIM variable and stored in the left-hand-side attribute of the GIM imperative-assignment AST. The expression on the right-hand-side of this assignment is converted to a GIM expression and stored in the right-hand-side attribute of the GIM AST. The GIM AST built from this transformation is shown below using GIL syntax.

```
RM := VMAG ( R);
```

The FORTRAN AST that represents the following DATA statement is transformed to the GIM by building two imperative-assignment ASTs.

DATA TWOPI/6.2831853072DO/ ,XMU/398601.2DO/

Each data item being initialized is converted to a GIM variable and stored in the left-hand-side attribute of a GIM imperative-assignment AST. The corresponding value for each data item is converted to a GIM expression and stored as the right-hand-side of the assignment statement. The two GIM assignment statement ASTs generated for this data statement are shown below using GIL syntax.

TWOPI := 6.2831853072DO;
XMU := 398601.2DO;
Because of the syntactic complexity allowed for data statements, a restriction is placed on the data statements that are transformed using the prototype. Specifically, the number of the data elements being assigned values must match the number of the data constants provided.
8.2.2 FORTRAN Sequential Control Flow Transformation. Sequential control flow in the FORTRAN AST is represented by storing statements in sequences. Sequences are also used in the GIM to store the ASTs built from the transformation of these statements. Specifically, statements in a sequence are transformed one after another (sequentially), and the corresponding GIM AST built from the transformation is appended to the corresponding sequence in the GIM. Sequences are used in FORTRAN ASTs to represent the statements in subprograms, if-then-else statements, and looping statements.
8.2.3 FORTRAN Selective Control Flow Transformation. Selective control flow in the FORTRAN programming language is implemented by the if-then-else statement, the if-then statement, and the logical-if statement. Since an if-then statement is an if-then-else statement without an else part, both statements are represented using the REFINE/FORTRAN block-if-statement AST.

Figure 195 shows the transformations that have been built in the prototype to convert FORTRAN selective control flow.

For example, consider the following FORTRAN if-then-else statement.

$$
\begin{aligned}
\text { block-if-statement } & \longrightarrow \text { imperative-selection } \\
\text { logical-if-statement } & \longrightarrow \text { imperative-selection }
\end{aligned}
$$

Figure 195 Selective Control Flow Transformations

```
IF(ITYPE.LT.0) THEN
CALL CSP(RT,RBO,TFBOT,XMU,VBOPP)
ELSE
CALL CSP(RBO,RT,TFBOT,XMU,VBO)
END IF
```

The attributes of the FORTRAN block-if-statement AST that represents this statement are used to build the GIM imperative-selection AST shown below (using GIL syntax).

```
if ITYPE < 0 then
    CSP ( RT, RBO, TFBOT, XMU, VBOPP);
else
    CSP ( RBO, RT, TFBOT, XMU, VBO)
endif
```

The FORTRAN expression is converted to a GIM expression and stored as the expression of the imperative-selection AST. Each statement in the then part is converted and stored in the then part of the imperative-selection AST. Each statement in the else part is also converted and stored in the else part of the imperative-selection AST.

The transformation of the FORTRAN if-then statement is only slightly different. The following statement is represented using the FORTRAN block-if-statement AST.

```
IF(THETA.GT.PI) THEN
ITYPE = -ABS(ITYPE)
THETA = 2.DO*PI - THETA
END IF
```

In this case, the else part of statement is empty, which is represented in the FORTRAN AST as an empty sequence. The expression from this AST is converted and stored as before, and the statements from the then part are converted and stored. The empty else part of this statement is also represented as an empty sequence in the GIM AST. The GIM AST is shown below using GIL syntax.

```
if THETA > PI then
    ITYPE := -ABS ( ITYPE);
    THETA := 2.OdO * PI - THETA
```

```
else
endif;
```

The GIL syntax for the if-then statement is not like the FORTRAN syntax because the GIL syntax includes the else keyword even if there are no statements in the else part of an if-then-else statement.

The FORTRAN logical-if statement provides a short-hand syntax for the if-then statement. The following statement is represented using the FORTRAN logical-if AST.

$$
I F(A L T . L T .0 .) \text { ALT }=0
$$

As with the if-then statement, the expression is converted to a GIM expression and stored. The logical-if always includes a single statement, so this statement is converted and stored in a sequence in the then part of the GIM AST. An empty sequence is stored in the else part of the AST as was done with the if-then statement. The GIM AST is shown below using GIL syntax.

```
if ALT < 0 then
    ALT := 0
else
endif;
```

The FORTRAN programming language also provides a way to nest if-then-else statements together into one statement. For example, the following FORTRAN statement is represented using a REFINE/FORTRAN block-if-statement AST.

```
IF(ITYP.EQ.1) THEN
ALT = -5.71DO
ELSE IF(ITYP.EQ.2) THEN
ALT = -1.57DO
ELSE IF(ITYP.EQ.3) THEN
ALT = -9.46DO
END IF
```

Note that there is only one END IF keyword even though there are multiple IF clauses in this statement. In the AST, the ELSE IF branches of the statement are collected into a sequence of block-if-statements. Since not all imperative languages provide such a way to nest if-then-else statements, this statement is transformed into an equivalent collection of embedded if-then-else statements. The transformed statement is shown below.

```
if ITYP = 1 then
    ALT := -5.71d0;
else
    if ITYP = 2 then
        ALT := -1.57d0;
    else
        if ITYP = 3 then
            ALT := -9.46d0;
        else
        endif
    endif
endif;
```

At the top level, this is a single if-then-else statement with a second if-then-else statement embedded in the else clause. A third if-then-else statement is embedded in the else clause of the second if-then-else statement.

Appendix C shows a proof that the transformation of nested if-then-else statements to embedded if-then-else statements maintains the functionality of the original FORTRAN statement.

### 8.2.4 FORTRAN Iterative Control Flow Transformation. Structured iterative

 control flow in the FORTRAN language is implemented using the do statement and the proper combination of the if-then statement and the goto statement. Let $\xrightarrow{\text { goto }}$ represent the transformation that only builds a GIM imperative-iteration AST if this proper combination of statements is used. The transformations built in the prototype for converting FORTRAN iterative control flow constructs are shown in Figure 196.$$
\begin{array}{rll}
\text { do-statement } & \longrightarrow \text { imperative-iteration } \\
\text { block-if-statement } & \xrightarrow{\text { goto }} & \text { imperative-iteration }
\end{array}
$$

## Figure 196 Iterative Control Flow Transformations

The following do statement is represented using the FORTRAN do-statement AST.
DO $100 \mathrm{I}=1,3$
$U(I)=R(I) / R M$
100 CONTINUE

The attributes of this AST are used to build the GIM imperative-iteration AST as follows. The label 100 is used to indicate syntactically which statements will be iterated. The FORTRAN AST has an attribute that stores this sequence of statements, so each statement is converted to the GIM and stored in the imperative-iteration AST. The loop variable syntax $\mathrm{I}=1,3$ is represented by an attribute for the loop variable, the initial value and the final value. A GIM imperative-assignment AST is generated to assign the initial value to the loop variable and is returned as part of the imperative-iteration AST transformation. This, in effect, inserts an assignment statement into the sequence of GIM statements right before the imperative-iteration statement. The expression built for the GIM imperative-iteration AST is always a less-than-or-equal expression that tests whether the loop variable is less than or equal to the final value. The transformed GIM imperative-iteration AST is shown below using the GIL syntax.

```
I := 1;
while I <= 3 do
        begin
        U ( I) := R ( I) / RM;
        I := I + 1
    end
```

The only if-then statements transformed into imperative-iteration ASTs have a single goto statement as the last statement of the then clause. This goto statement must cause program control to return to the line that includes the if-then statement. For example, the following if-then statement has such a goto statement.

```
150 IF(C2.GT.1.ODO) THEN
    E=E - DE
    DE = DE/2.
    E = E + DE
    C1 = 1.ODO - E**2
    C2 = C1/C2GAM
    GO TO 150
    END IF
```

This if-then statement is transformed into the following GIM iterative statement.

```
while C2 > 1.0dO do begin
    E := E - DE;
    DE := DE / 2;
    E := E + DE;
    C1 := 1.0d0 - E^2;
    C2 := C1 / C2GAM
    end;
```

8.2.5 FORTRAN Subprogram Transformation. Imperative subprograms are implemented in the FORTRAN language by the subroutine statement and the function statement. Subroutines are invoked using the FORTRAN call statement. Functions are invoked by using the function's identifier in an expression. Let $\xrightarrow{\text { call }}$ represent the transformation that only builds a GIM imp-function-call AST if the identifier refers to a function begin invoked. The transformations that have been built in the prototype to convert these entities are shown in Figure 197.

| subroutine-statement | $\longrightarrow$ | imperative-procedure |
| ---: | :--- | :--- |
| function-statement | $\longrightarrow$ | imperative-function |
| call-statement | $\longrightarrow$ imp-procedure-call |  |
| identifier | $\xrightarrow{\text { call }}$ imp-function-call |  |

Figure 197 Subprogram Transformations

```
SUBROUTINE VADD(IOP,R1,R2,R3)
    INCLUDE 'bdincl.f'
C
C THIS ROUTINE PERFORMS VECTOR ADDITION FOR THREE DIMENSIONAL
C VECTORS
C
    DIMENSION R1(3),R2(3),R3(3)
    F = FLOAT(IOP)
R3(1) = R1(1) + F*R2(1)
R3(2) = R1(2) + F*R2(2)
R3(3) = R1(3) + F*R2(3)
RETURN
END
```

Figure 198 FORTRAN Subroutine VADD

For example, Figure 198 shows the FORTRAN subroutine VADD. The attributes of the FORTRAN AST built for VADD are used to build an imperative-procedure AST in the GIM as follows. The subroutine identifier is converted into a GIM variable and stored as the identifier of the imperative-procedure AST. The formal parameters IOP, R1, R2, R3 are converted into GIM variables and stored (in the same order) in the sequence of formal parameters for the GIM AST. Each statement from the subroutine is converted into a GIM statement and stored (in the same order) in the sequence of statements for the imperative-procedure AST. The GIM AST built from this transformation is shown below using GIL syntax.

```
procedure VADD ( IOP, R1, R2, R3 )
    begin
    F := FLOAT ( IOP);
    R3 ( 1) := R1 ( 1) + F * R2 ( 1);
    R3 ( 2) := R1 ( 2) + F * R2 ( 2);
    R3 ( 3) := R1 ( 3) + F * R2 ( 3)
    end
```

Note that the comments from the FORTRAN subroutine are not modeled in the GIM AST. There is a attribute in the REFINE/FORTRAN AST that stores the comments from a subroutine, but no attempt is made in this research to carry the comments forward. The FORTRAN call statement that invokes VADD is shown below.

```
CALL VADD(-1,RB,R(1,J,K),RRB)
```

The call identifier from the FORTRAN AST is converted to a GIM variable and stored as the identifier of the imp-procedure-call AST. The sequence of actual parameters is converted to a sequence of GIM variables (with the same order) and stored as the actual parameters in the GIM AST. Because of Restriction III.3, an extra step is performed in this transformation to ensure all actual parameters are variables. Specifically, the -1 parameter in the call to VADD is replaced by a temporary variable and an assignment statement is inserted before the call. The two GIM ASTs built from this transformation are shown below using GIL syntax.

```
TEMP-29 := -1;
VADD ( TEMP-29, RB, R ( 1, J, K), RRB);
```

8.2.6 FORTRAN Variable Transformation. Each FORTRAN variable in the legacy system must be converted to a GIM variable. The representation of variables in the GIM consists of two parts. An imperative-variable AST is built for each variable and stored in the Imperative Symbol Table (IST). All of the information needed to build an imperative-variable is taken from the FORTRAN symbol table generated during compilation. The symbol table includes the variable's identifier, scope, data type, and a constant value (if any). If the FORTRAN variable is an array, the indices are not stored with the variable since the entire array is considered one variable.

References to a FORTRAN variable are modeled using the imperative-name AST shown in Figure 199.

| imperative-name |
| :--- |
| imp-scope : symbol |
| imp-identifier $:$ symbol |
| imp-indices : seq(imperative-data-construct) |

Figure 199 Imperative Name Class

A new instance of this class is built for each reference to a FORTRAN variable. If the variable is an array, the array indices are converted to GIM expressions and stored in the imperative-name object. When a FORTRAN variable is transformed, the IST is checked to see if an imperative-variable AST has already been built to represent the FORTRAN variable. If not, an imperative-variable is built and stored in the IST.

Using the imperative-name AST was meant to simplify the process of representing variables. However, in hind-sight, it is just as easy to represent each FORTRAN variable using an instance of the imperative-variable class. The only difference between an imperative-name AST and an imperative-variable AST is that an imperative-name doesn't store the data type and an imperative-variable doesn't store the indices of an array variable. These two ASTs should be combined into one.

Because of the information provided by the FORTRAN symbol table, declarations of variables are not transformed or modeled in the GIM. This means FORTRAN common statements and dimension statements have no corresponding construct in the GIM. Since
there is no modeling of variable declarations in the GIM, a system built to convert GIM ASTs into an imperative language such as $C$ would have to automatically generate the appropriate declarations for each variable as required by the language. There is no surface syntax provided for a GIM imperative-variable ASTs, only imperative-name ASTs.
8.2.7 FORTRAN Data Types Transformation. The transformations built to convert FORTRAN data types (except implicit types and built-in types) are shown in Figure 200.

| type-integer | $\longrightarrow$ imperative-integer |
| ---: | :--- |
| type-byte | $\longrightarrow$ imperative-integer |
| type-real | $\longrightarrow$ imperative-real |
| type-double-precision | $\longrightarrow$ imperative-real |
| type-logical | $\longrightarrow$ imperative-boolean |
| type-character | $\longrightarrow$ imperative-string |
| array-type | $\longrightarrow$ imperative-array |

Figure 200 Data Type Transformations

For implicit types, if the type-expr-name attribute of the type-expr-type attribute of an implicit-type FORTRAN AST equals the symbol real, then an imperative-real GIM AST is built. If the attribute equals the symbol integer, an imperative-integer GIM AST is built. Similarly, if the type-expr-name attribute of a built-in-type FORTRAN AST equals the symbol real, then an imperative-real GIM AST is built. If the attribute equals the symbol integer, an imperative-integer GIM AST is built.
8.2.8 FORTRAN Expression Transformation. A FORTRAN expression can be either a variable, a binary expression, a unary expression, or a literal constant value. This section describes each of the transformations for these kinds of expressions. The transformation of a variable is described in Section 8.2.6. The transformations included in the prototype for FORTRAN binary expressions are shown in Figure 201. The transformations built to convert FORTRAN unary expressions are shown in Figure 202. The transformations built to convert FORTRAN literal expressions are shown in Figure 203.

$$
\begin{aligned}
\text { add-expression } & \longrightarrow \text { imperative-addition } \\
\text { and-expression } & \longrightarrow \text { imperative-and } \\
\text { concatenate-expression } & \longrightarrow \text { imperative-concat } \\
\text { divide-expression } & \longrightarrow \text { imperative-division } \\
\text { eq-expression } & \longrightarrow \text { imperative-equal } \\
\text { eqv-expression } & \longrightarrow \text { imperative-equal } \\
\text { exponentiate-expression } & \longrightarrow \text { imperative-exponent } \\
\text { ge-expression } & \longrightarrow \text { imperative-greater-than-or-equal } \\
\text { gt-expression } & \longrightarrow \text { imperative-greater-than } \\
\text { le-expression } & \longrightarrow \text { imperative-less-than-or-equal } \\
\text { lt-expression } & \longrightarrow \text { imperative-less-than } \\
\text { multiply-expression } & \longrightarrow \text { imperative-multiplication } \\
\text { ne-expression } & \longrightarrow \text { imperative-not-equal } \\
\text { neqv-expression } & \longrightarrow \text { imperative-not-equal } \\
\text { or-expression } & \longrightarrow \text { imperative-or } \\
\text { subtract-expression } & \longrightarrow \text { imperative-subtraction }
\end{aligned}
$$

Figure 201 Binary Expression Transformations

$$
\begin{aligned}
\text { negate-expression } & \longrightarrow \text { imperative-negate } \\
\text { not-expression } & \longrightarrow \text { imperative-not } \\
\text { null-ex } & \longrightarrow \text { imperative-null }
\end{aligned}
$$

Figure 202 Unary Expression Transformations
8.2.9 FORTRAN Input/Output Transformation. Input is implemented in the FORTRAN language by the read statement. Output is implemented by the write and format statements. The transformations built in the prototype to convert FORTRAN input and output statements are shown in Figure 204.

### 8.3 Completeness of the Transformation

Since transformations were not built for the entire domain model provided by REFINE/FORTRAN, a process was defined to assess the completeness of the transformation. Specifically, as the FORTRAN AST was being transformed, each transformation set a flag

$$
\begin{aligned}
\text { literal-false } & \longrightarrow \text { imperative-literal-boolean } \\
\text { literal-true } & \longrightarrow \text { imperative-literal-boolean } \\
\text { literal-hex } & \longrightarrow \text { imperative-literal-integer } \\
\text { literal-integer } & \longrightarrow \text { imperative-literal-integer } \\
\text { literal-octal } & \longrightarrow \text { imperative-literal-integer } \\
\text { literal-real } & \longrightarrow \text { imperative-literal-real } \\
\text { literal-quad } & \longrightarrow \text { imperative-literal-real } \\
\text { literal-double } & \longrightarrow \text { imperative-literal-real } \\
\text { literal-charstring } & \longrightarrow \text { imperative-literal-charstring } \\
\text { literal-hollerith } & \longrightarrow \text { imperative-literal-charstring } \\
\text { literal-right-text } & \longrightarrow \text { imperative-literal-charstring }
\end{aligned}
$$

Figure 203 Literal Expression Transformations

$$
\begin{array}{rll}
\text { read-statement } & \longrightarrow \text { imperative-input } \\
\text { write-statement } & \longrightarrow \text { imperative-output } \\
\text { format-statement } & \longrightarrow \text { imperative-output }
\end{array}
$$

Figure 204 Input and Output Transformations
on the AST node being transformed. A separate traversal of the FORTRAN AST examines each node and report any nodes not transformed. This process proved to be quite effective and uncovered problems with the overall transformation that were easily corrected. An excerpt from the transcript generated by the conversion of the ANG subprogram is shown below.

```
Converting "ANG" to the GIM...
No side effects in this function.
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 60
Number of untransformed nodes: 0
Percentage of AST transformed: 100.0\%
Saving to pob...
Saving AST and Symbol Table in file...
Saving surface syntax in file...
```

This transcript reports the conversion of ANG is $100 \%$ complete. There are 60 tree nodes in the ANG FORTRAN AST, and the new AST is saved to the Persistent Object Base (POB). The complete transcript from the conversion of the 3000 line FORTRAN- 77 legacy system is shown in Appendix E.

### 8.4 The Ballistic Missile Defense Simulation System

The legacy imperative system that was selected for the feasibility demonstration is the Ballistic Missile Defense Simulation (BMDSIM) system [36]. This system consists of 53 subprograms including the main program, BMDSIM1. This system was developed for use as a real production system and is not a toy system developed just for this research.

Table 5 and Table 6 show a summary of these subprograms. The type of subprogram (function or procedure), the subprogram identifier, the category ${ }^{1}$ into which this subprogram is classified, and the set of data items produced are presented in the table.

Some of the subprograms in BMDSIM were re-structured to eliminate unstructured uses of GOTO statements. There were no global variables in the system and none of the parameters were both input and output parameters. The RAND function had an output parameter, so the function was converted into a procedure with two output parameters.

After these changes, the total number of lines of code was 3109 .

### 8.5 Eliminating Category 4 and Category 5 subprograms

As described in Chapter VI, the first step in converting a legacy system is to eliminate the Category 4 and Category 5 subprograms by using program slicing. Figure 205 shows how the original 53 subprograms are classified according to the taxonomy of subprograms. After program slicing, the BMDSIM system consisted of 105 subprograms including the main program. Figure 205 also shows how each of these 105 subprograms are classified.

Tables 7, 8, and 9 show a summary of the new subprograms generated by program slicing. The tables show the name of each subprogram in BMDSIM. For any Category 4 or

[^6]| Type | Subprogram | Cat | Data Items Produced |
| :--- | :--- | :---: | :--- |
| Function | ANG | 2 |  |
| Function | ANGLE | 3 |  |
| Procedure | ASSIGN | 5 | DWELLA, IENG, IZTAB, LLAS, LMIRS, <br> NBPCR, RBENG, TLAS, TMIRS |
| Main Program | BMDSIM1 | 1 |  |
| Procedure | BOOSTR | 4 | R, V |
| Procedure | BOSTIT | 5 | AR, ITYPE, RBO, RL, RT, TBO, <br> TFBOT, VBO, VI |
| Function | BOUNCE | 3 |  |
| Function | CAPTURE | 2 |  |
| Procedure | CROSS | 2 |  |
| Procedure | CSP | 3 |  |
| Function | CUV | 2 |  |
| Procedure | DASET | 4 | DALASM, DWELLT, IASGN |
| Function | DOT | 2 |  |
| Procedure | KEP | 4 | R, V |
| Procedure | LASP | 4 | BFLU, BFLUX, TD |
| Procedure | LNKCAL | 5 | DALASM, DWELLT, IASGN, PHASE |
| Procedure | LNKCK | 4 | MIRF, RANGE |
| Procedure | LNKORD | 4 | DWELLA, DWELLT, LIASGN |
| Procedure | MAT | 2 |  |
| Procedure | MAXA | 4 | AMAX, NMAX |
| Procedure | MIRGEO | 3 |  |
| Procedure | MIRVIS | 5 | MIRF, MIRR |
| Procedure | MTM2 | 3 |  |
| Procedure | MTM3 | 3 |  |
| Procedure | MTPD | 2 |  |
| Procedure | MTRT | 2 |  |
| Procedure | ORBEL | 4 | A, E |
|  |  |  |  |

Table 5 Subprograms from BMDSIM

| Type | Subprogram | Cat | Data Items Produced |
| :--- | :--- | :---: | :--- |
| Function | PKILL | 2 |  |
| Procedure | POSVEC | 2 |  |
| Procedure | POSVECS | 2 |  |
| Function | PRDIV | 2 |  |
| Function | RADIUS | 2 |  |
| Procedure | RAND | 4 | RANDVAL, RSEED |
| Procedure | RELAY | 5 | AREA, DALAS, EBRITE, EFFNCY, PHASE |
| Function | RHO | 2 |  |
| Procedure | RRBVIS | 5 | RIANG, RRBM, SLANG |
| Procedure | RRPVIS | 5 | RPANG, RRPM |
| Function | RTAN | 2 |  |
| Procedure | SBMIT | 5 | DETAO, DRAO, ETAO, ORATE, |
|  | RAO, RSEED |  |  |$|$| Procedure | SBMLOC |
| :--- | :---: |
| Procedure | SBMPOS |
| Procedure | SELECL |
|  | 5 |
| ETA, R, RA |  |
| Function | SGN |
| Function | SUV |
| Procedure | TFBT |
| IDX, ILS, J1S, J2S, |  |
| K1S, K2S, TNGAGE |  |
| Procedure | TPANG |
| Procedure | TRAJ |
| Procedure | UNIT |
| Function | UPLREQ |
| Function | UPTRNS |
| Procedure | VADD |
| Procedure | VISCK |
| Function | VMAG |
|  | 2 |

Table 6 More Subprograms from BMDSIM

| Category | Before Slicing | After Slicing |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 22 | 52 |
| 3 | 8 | 52 |
| 4 | 9 | 0 |
| 5 | 13 | 0 |
| Total | 53 | 105 |

Figure 205 Subprograms Classified

Category 5 subprogram, the names of the new subprograms built from it and the category of the new subprogram are shown. The final column indicates on which of the slices the process of masking had to be used.

### 8.6 Implementing the PBOI

The next step in the conversion process is to transform the subprograms represented in the GIM into classes and methods represented in the GOM. This is done using the PBOI methodology described in Chapter V. Each of the PBOI transformations was implemented in the prototype using the REFINE programming language. Every attempt was made to match the names of the REFINE functions with the transformations they implement. The mappings from GIM AST entities to GOM AST entities are shown in Appendix F.

A Refine ${ }^{T M}$ workbench has been built that displays both the imperative subprogram (shown using GIL syntax) and the object-oriented class and method (shown using GOL syntax) extracted from this subprogram. Figure 206 shows an example of this workbench. The upper-left subwindow of the workbench shows the GIL representation of the GIM subprogram being transformed. The subprogram is transformed manually using the sigma option (not shown) from the Transformations menu. Once the subprogram is transformed, the upper-right subwindow displays the extracted class and method. The lower-left subwindow displays the object-oriented design (that has been extracted to this point) using class diagrams. The class diagram shows a class as a box displaying the name of the class, the name of its methods, and the name and type of its attributes. Finally, the lower-right subwindow displays the AST that represents the overall design. The GOM

| Subprogram | Cat | Slices | Cat | Masked |
| :---: | :---: | :---: | :---: | :---: |
| ANG | 2 |  |  |  |
| ANGLE | 3 |  |  |  |
| ASSIGN | 5 | ASSIGN-DWELLA | 3 | X |
|  |  | ASSIGN-IENG | 3 | X |
|  |  | ASSIGN-IZTAB | 3 | X |
|  |  | ASSIGN-LLAS | 3 | X |
|  |  | ASSIGN-LMIRS | 3 | X |
|  |  | ASSIGN-NBPCR | 3 | X |
|  |  | ASSIGN-RBENG | 3 | X |
|  |  | ASSIGN-TLAS | 3 | X |
|  |  | ASSIGN-TMIRS | 3 | X |
| BMDSIM1 | 1 |  |  |  |
| BOOSTR | 4 | BOOSTR-R | 2 |  |
|  |  | BOOSTR-V | 2 |  |
| BOSTIT | 5 | BOSTIT-AR | 3 | X |
|  |  | BOSTIT-ITYPE | 3 | X |
|  |  | BOSTIT-RBO | 3 | X |
|  |  | BOSTIT-RL | 3 |  |
|  |  | BOSTIT-RT | 3 | X |
|  |  | BOSTIT-TBO | 2 |  |
|  |  | BOSTIT-TFBOT | 3 | X |
|  |  | BOSTIT-VBO | 3 | X |
|  |  | BOSTIT-VI | 2 |  |
| BOUNCE | 3 |  |  |  |
| CAPTURE | 2 |  |  |  |
| CROSS | 2 |  |  |  |
| CSP | 3 |  |  |  |
| CUV | 2 |  |  |  |
| DASET | 4 | DASET-DALASM | 2 |  |
|  |  | DASET-DWELLT | 2 |  |
|  |  | DASET-IASGN | 2 |  |
| DOT | 2 |  |  |  |
| KEP | 4 | KEP-R | 2 |  |
|  |  | KEP-V | 2 |  |
| LASP | 4 | LASP-BFLU | 2 |  |
|  |  | LASP-BFLUX | 2 |  |
|  |  | LASP-TD | 2 | X |

Table 7 Slices for Category 4 and Category 5 Subprograms

| Subprogram | Cat | Slices | Cat | Masked |
| :---: | :---: | :---: | :---: | :---: |
| LNKCAL | 5 | LNKCAL-DALASM | 3 | X |
|  |  | LNKCAL-DWELLT | 3 |  |
|  |  | LNKCAL-IASGN | 3 | X |
|  |  | LNKCAL-PHASE | 3 |  |
| LNKCK | 4 | LNKCK-MIRF | 2 |  |
|  |  | LNKCK-RANGE | ? |  |
| LNKORD | 4 | LNKORD-DWELLA | 2 | X |
|  |  | LNKORD-DWELLT | 2 |  |
|  |  | LNKORD-LIASGN | 2 | X |
| MAT | 2 |  |  |  |
| MAXA | 4 | MAXA-AMAX | 2 |  |
|  |  | MAXA-NMAX | 2 | X |
| MIRGEO | 3 |  |  |  |
| MIRVIS | 5 | MIRVIS-MIRF | ? |  |
|  |  | MIRVIS-MIRR | 3 |  |
| MTM2 | 3 |  |  |  |
| MTM3 | 3 |  |  |  |
| MTPD | 2 |  |  |  |
| MTRT | 2 |  |  |  |
| ORBEL | 4 | ORBEL-A | 2 | X |
|  |  | ORBEL-E | 2 | X |
| PKILL | 2 |  |  |  |
| POSVEC | 2 |  |  |  |
| POSVECS | 2 |  |  |  |
| PRDIV | 2 |  |  |  |
| RADIUS | 2 |  |  |  |
| RAND | 4 | RAND-RANDVAL | 2 | X |
|  |  | RAND-RSEED | 2 |  |
| RELAY | 5 | RELAY-AREA | 3 |  |
|  |  | RELAY-DALAS | 3 | X |
|  |  | RELAY-EBRITE | 3 |  |
|  |  | RELAY-EFFNCY | 3 |  |
|  |  | RELAY-PHASE | 3 |  |
| RHO | 2 |  |  |  |

Table 8 More Slices for Category 4 and Category 5 Subprograms

| Subprogram | Cat | Slices | Cat | Masked |
| :---: | :---: | :---: | :---: | :---: |
| RRBVIS | 5 | RRBVIS-RIANG | 3 |  |
|  |  | RRBVIS-RRBM | 3 |  |
|  |  | RRBVIS-SLANG | 3 |  |
| RRPVIS | 5 | RRPVIS-RPANG | 3 |  |
|  |  | RRPVIS-RRPM | 3 |  |
| RTAN | 2 |  |  |  |
| SBMIT | 5 | SBMIT-DETAO | 3 | X |
|  |  | SBMIT-DRAO | 3 | X |
|  |  | SBMIT-ETAO | 3 | X |
|  |  | SBMIT-ORATE | 2 |  |
|  |  | SBMIT-RAO | 3 | X |
|  |  | SBMIT-RSEED | 3 |  |
| SBMLOC | 2 |  |  |  |
| SBMPOS | 5 | SBMPOS-ETA | 3 |  |
|  |  | SBMPOS-R | 3 |  |
|  |  | SBMPOS-RA | 3 |  |
| SELECL | 4 | SELECL-IDX | 2 |  |
|  |  | SELECL-ILS | 2 |  |
|  |  | SELECL-J1S | 2 |  |
|  |  | SELECL-J2S | 2 |  |
|  |  | SELECL-K1S | 2 |  |
|  |  | SELECL-K2S | 2 |  |
|  |  | SELECL-TNGAGE | 2 |  |
| SGN | 2 |  |  |  |
| SUV | 2 |  |  |  |
| TFBT | 2 |  |  |  |
| TPANG | 5 | TPANG-ALP | 3 |  |
|  |  | TPANG-THE | 3 | X |
|  |  | TPANG-XINC | 3 |  |
| TRAJ | 5 | TRAJ-R | 3 |  |
|  |  | TRAJ-V | 3 |  |
| UNIT | 3 |  |  |  |
| UPLREQ | 2 |  |  |  |
| UPTRNS | 2 |  |  |  |
| VADD | 2 |  |  |  |
| VISCK | 5 | VISCK-MCK | 3 |  |
|  |  | VISCK-RRM | ? |  |
| VMAG | 2 |  |  |  |

Table 9 More Slices for Category 4 and Category 5 Subprograms


Figure 206 GOM Workbench

ASTs shown in this subwindow are hyper-linked with the GOL surface syntax shown in the upper-right subwindow.

Each of the 105 Category 2 and Category 3 subprograms from the BMDSIM system was converted to the object-oriented paradigm using the prototype. This resulted in an object-oriented design with 104 classes and 104 methods. The main program was also converted which added another class and method to the design.

During the conversion of the main program, 770 create messages were built for 511 distinct object instances. After removing duplicate objects, 109 create messages remained for 109 distinct object instances. After removing overlapping classes, the design consisted of 43 classes. Of these classes, 33 classes had only one method, 6 classes had between 2 and 11 methods, and 4 classes had between 29 and 47 methods.

In order to address the reasonableness of the objects being extracted, an informal semantic analysis was done. The comments in the BMDSIM system were examined to develop a rudimentary domain model. The extracted objects were compared against this domain model in order to find a correlation between the objects and a semantic entity from the domain. This analysis found that the extracted objects tended to contain attributes of a single domain entity, as opposed to multiple domain entities. The number of attributes in the extracted objects tended to be less than the number of attributes in the domain entity. That is, the extracted objects described pieces of the domain entities. This could be due to the rudimentary nature of the domain model developed. The entire methodology described to this point has been fully automated. To add semantics to the extracted objects may require a cooperative effort involving the analyst. Further analysis of the semantic nature of the extracted objects is left as future research.

### 8.7 Analysis

The prototype was built on a SUN SPARC 5 workstation with 128 MB of memory. Using the prototype, the conversion of BMDSIM to the GIM takes less than 20 minutes and the elimination of Category 4 and Category 5 subprograms takes approximately one hour. The prototype converts GIM subprograms to the GOM using the PBOI methodology
(except for merging overlapping classes) in approximately 24 hours. Merging overlapping classes using the prototype requires an additional 12 hours. Because of its inefficiency, using the prototype to convert millions of lines of imperative code is not feasible.

### 8.8 Summary

This chapter has presented the results of the feasibility demonstration. A summary of the BMDSIM system and its subprograms was provided along with the subprograms resulting from program slicing. The specific transformations used to transform FORTRAN to the GIM were described in this chapter. The object-oriented design extracted from BMDSIM was also described. This transformation of BMDSIM using the prototype implementation demonstrates the conversion of a FORTRAN system to an object-oriented design is technically feasible.

## IX. Contributions

### 9.1 Introduction

This chapter lists the contributions this research makes to the field of re-engineering.

### 9.2 Major Contributions

This research makes the following major contributions.

1. The Generic Imperative Model (GIM).
2. The Generic Object-Oriented Design Model (GOM).
3. The Parameter-Based Object Identification (PBOI) methodology.
4. The transformations that formalize the PBOI methodology.
5. A proof that the object-oriented design is a consistent implementation of the legacy code.

The GIM is significant because it provides a canonical form for imperative programming languages. This allows legacy code that does the same to look the same. There are several advantages to using the GIM in this research or other research involving imperative programming languages. First, the GIM is programming language independent, which means the representation is not tied to one specific programming language. By using the GIM in the prototype, legacy code in another language can be re-engineered once the transformations from that language to the GIM are developed. In this way, the GIM allows the prototype to be easily extended to other languages. Second, the GIM is program construct independent. This allows the GIM to represent imperative control flow structures in easily understood canonical forms. For example, there are many programming language implementations of the selective control flow construct from imperative languages, but the GIM is able to represent if-then statements, if-then-else statements, nested and embedded if-then-else statements, and case statements using a single control flow construct. This greatly simplifies the task of building formal transformations and prototypes based on the GIM constructs. Third, the GIM provides constructs for representing simulated control flow constructs so they can be transformed into a canonical form. For example, imperative iteration can be simulated with an if-then statement and a goto statement. This simulated iteration is converted into a GIM iteration construct, which simplifies the representation
of the imperative code. From these three advantages, it is clear that any research based on the GIM is simplified because the number of imperative programming language constructs to consider has been greatly reduced. Finally, the GIM provides a formal definition of each of the imperative programming language constructs based on the weakest precondition notation.

The GOM is an important contribution because it provides a canonical form for object-oriented programming languages. Much like the GIM, the GOM is programming language independent, program construct independent, and recognizes simulated control flow constructs. The GOM has been built primarily as a tool for forward engineering, so it provides a model from which an object-oriented program can be produced. By building transformations from the GOM to a specific object-oriented language, the programming constructs that best implement the canonical forms from the GOM can be used to generate the object-oriented program. As in the GIM, any research based on the GOM is simplified because the number of object-oriented constructs to consider is reduced. The GOM also provides formal definitions for object-oriented constructs including classes, objects, methods, and messages. Defining the formal semantics of these constructs using the weakest precondition notation is original research in this field.

The PBOI methodology is the most important contribution of this research because it provides a new way to extract objects from legacy imperative code. There have been other object identification methods presented in the literature (see Chapter I), but the PBOI methodology results in an object-oriented design that is a consistent implementation of the legacy imperative system. Other researchers have developed systems to extract functionally equivalent objects, but these researchers do not provide a proof of this claim. The objectoriented design extracted using the PBOI methodology is valuable because it produces the same output as the legacy system given the same input. The PBOI methodology is similar to the GBOI, TBOI, and RBOI methodologies because it extracts objects from legacy code. The PBOI methodology is significantly better because it extracts functionally equivalent objects, it has been formalized using mathematical transformations, a proof-of-concept prototype implementation has been developed, and it has been demonstrated on a (real world) $3000-$ line FORTRAN legacy system.

Using formal transformations to define the PBOI methodology is a significant contribution because it casts the methodology in a consistent, unambiguous, and provably correct form. The transformations demonstrate how an entire methodology for converting imperative subprograms can be expressed using formal methods. The transformations are based on the formal definitions of imperative subprograms, subprogram calls, and programming statements, which are minor research contributes as discussed in Section 9.3. Other researchers can use this work as an exemplar of how to express the transformation of imperative subprograms using the power of formal methods. These transformations were fully automated using the Refine ${ }^{T M}$ programming language, as described in Chapter VIII.

The proof that the PBOI methodology extracts an object-oriented design that is a consistent implementation of the legacy imperative code is a significant contribution in the area of re-engineering because it is the first such proof. Without defining any of the specific input or output values from the legacy imperative code, it has been proven that the extracted object-oriented design produces the same output as the legacy imperative code given the same input. This means it is not necessary to implement the design to prove it is consistent with the legacy code. The proofs of the individual theorems also provide exemplars of how to prove the consistency of formal transformations using weakest precondition arguments.

These major contributions combine to provide a significant advance in the area of re-engineering, viz. the definition, formalization, proof, and automation of a novel methodology for extracting a functionally equivalent object-oriented design from legacy imperative code. The minor contributions of this research are discussed in the following section.

### 9.3 Minor Contributions

This research also makes the following minor contributions.

1. A taxonomy of imperative subprograms.
2. The formal definitions of imperative subprograms, object-oriented classes, objectoriented methods, and object-oriented messages.
3. The Generic Imperative Language (GIL).
4. The Generic Object-Oriented Language (GOL).
5. The $\mu$ relation that links actual parameters to formal parameters.
6. The proof-of-concept prototype built for the FORTRAN language.

The taxonomy of imperative subprograms provides a meaningful way to classify any imperative subprogram. It focuses the job of re-engineering to six well defined categories. Other research on imperative subprograms can also use the taxonomy in the same way. The attributes that define the taxonomy could be modified to reflect different aspects of imperative subprograms.

The formal definitions of imperative subprograms, subprogram calls, statements, and designs presented in Chapter III are a consistent and unambiguous representation of imperative programming language constructs. They provide a foundation for formal transformations of imperative constructs that can be used to develop new transformations in other research. The formal definitions of object-oriented classes, methods, messages, statements, and designs provide a consistent and unambiguous representation of objectoriented programming language constructs. In this research, these definitions provide the representation of the target of the formal transformations. The definitions of imperative and object-oriented constructs are used extensively in the proof of functional equivalence. These representations simplify the proof because of their formal nature.

The GIL and the GOL provide a surface syntax for the GIM and the GOM, respectively. They have been implemented using a grammar that defines the surface syntax of GIM and GOM constructs. The GIL and the GOL demonstrate the use of a surface syntax as a convenient user interface to Abstract Syntax Trees. The surface syntax defined for the GIM and the GOM provide a user-friendly interface to the entities being transformed. In this way, the GIL and GOM are intended to be used to view an AST that has already been built. Since the GIL and the GOL have been built using a grammar, they can also be used to build ASTs using the syntax of the grammar. Implementing and testing this aspect of these languages is beyond the scope of this research, but the idea of defining a generic imperative language and a generic object-oriented language is appealing. Building the GIL and the GOL is a minor contribution that can be used as a stepping stone to future research.

The $\mu$ relation proved to an invaluable aid in the formalization of subprogram signatures. The transitive closure of $\mu$ is an effective way to determine which actual parameter is linked to a formal parameter that has been converted to an attribute of a class. It may be possible to apply this relation to other applications that update signatures of subprograms.

Finally, the proof-of-concept prototype demonstrates that legacy systems written in FORTRAN-77 can be converted to the GIM. It also shows that the entire PBOI methodology can be automated. The transformations that define the PBOI were easily implemented using the REFINE ${ }^{T M}$ programming language, which demonstrates the utility of such formal definitions and languages such as REFINE.

These minor contributions aided this research in achieving the overall goal of extracting functionally equivalent object-oriented designs from imperative legacy code. The items identified for future research are discussed in the following section.

### 9.4 Summary

This chapter has explained why the GIM, the GOM, the PBOI methodology, and the formal transformations are significant contributions to the field of re-engineering. The following chapter discusses future research and overall conclusions.

## X. Future Research and Conclusions

### 10.1 Introduction

This chapter summarizes several ideas for future research that could easily extend this research. Concluding remarks are included at the end of this chapter.

### 10.2 Future Research

The following sections discuss items that should be done as future research.
10.2.1 Data dependencies. The PBOI methodology may be too aggressive when determining which parameters should be attributes of classes. Specifically, PBOI Case 3 should be scrutinized and possibly extended as follows. It may be the case that a local variable is directly data dependent on an input parameter of a Category 3 subprogram. When this local variable is passed as a parameter to another subprogram, it will not remain as an attribute of a class. This is because it is a local variable.

However, it may be possible to extract more semantically meaningful objects by allowing the data item linked to this local variable to remain an attribute of a class. This is based on the local variable's data dependency to the formal parameter. There are three dependency cases to consider.
direct One or multiple assignment statements link the local variable back to the formal parameter.
indirect The local variable is produced from a subprogram call that includes the formal parameter as an input parameter.
hybrid A combination of both direct and indirect data dependencies.
An issue remains as to which data item should be built as the attribute of the class. Is the formal parameter of the called subprogram the attribute or a different view of the attribute? Is the formal parameter of the calling subprogram the attribute? Where should the code that changes the formal parameter into the local variable be placed as a method?
10.2.2 Arrays. Arrays present an interesting challenge. There are several things that have been left as future research. Specifically, a common method for storing data in the imperative paradigm is to build parallel arrays of data. It has been proposed that the attributes of a candidate object can be derived from the arrays because the values of the attributes of a single instance are stored at the same index in the arrays. Future research should convert these parallel arrays of attributes into a single array of instances with the appropriate attributes.

Another popular method for storing homogeneous attributes of candidate objects is to use a single array to represent all the attributes. A second array or another dimension of the array is used to store the instances of the objects. These arrays of attributes could be converted into parallel arrays of attributes and then converted into a single array of objects with attributes.
10.2.3 Modifying the Design. The process of merging overlapping classes based on the overlap of a single data item may be too aggressive. As described in the Feasibility Demonstration (see Chapter VIII), the design extracted for the BMDSIM legacy system included classes with large numbers of attributes and methods. Future research should explore different heuristics for the updates done to the design when the main program is being transformed.

One possibility is to merge two classes only when the attributes of one class are a subset of the attributes of the other class. This is a more restrictive heuristic for merging that may avoid building classes with such large numbers of attributes and methods. This may introduce duplication of system state variables, however, so a method for combining these classes without duplicating the imperative state variables must be developed. One possibility is to change attributes not in the intersection into parameters in the methods of the effected classes.

Another issue is the lack of analysis that is done on the subprograms that are called by the main program. The formal parameters of these subprograms are built as attributes of classes, but there is no filtering of these data items to see if they should remain as attributes.

It may be the case that changing one or more of these attributes into a parameter helps to avoid the problem of building classes with large numbers of attributes and methods.

During the conversion of the BMDSIM legacy system, several classes in the extracted design included no attributes. Future research could examine these classes and possibly merge them into one utility class. There appears to be a difference between a class where the attributes have been converted solely into attributes of other classes and a class where the attributes have been converted solely into parameters.

Currently, object instances are built only when the main program is being converted. It is possible to recognize that some collection of local variables in a subprogram actually represent an object and the object can be instantiated at the correct point in the subprogram. This might improve the prospects of identifying more "meaningful" objects from the GIM.

Finally, in order to add semantics to the extracted objects, some interaction with the user may be required. An extension to the prototype could include this interaction to determine if extracted objects should be combined to more closely resemble domain entities. If domain entities are expressed formally, it may be possible for the computer to compare the domain entities to the extracted objects.
10.2.4 Program Slices. The program slicing done to eliminate Category 4 and Category 5 subprograms produces inefficient slices. Future research could explore ways to optimize these slices and produce less conservative slices. Similarly, some of the program slices that are built during this process are repeated in whole as part of another program slice. Such slices could be built as object methods and called by the encompassing program slice. Finally, the same imperative output statement may appear in multiple slices which may cause spurious output when the design is finally implemented and executed. A more detailed analysis of the variables being output could be done as future research in order to ensure the same output statement does not appear in multiple slices.
10.2.5 The Generic Unstructured Model. It may be possible to add a front-end to the reverse engineering methodology that automatically re-structures imperative code.

Unstructured legacy code could be transformed into a canonical form, (termed the Generic Unstructured Model (GUM)) that models the unstructured use of goto statements. The transformation of GUM entities to GIM entities would provide a canonical means for restructuring legacy imperative systems.
10.2.6 Extensions. Currently, the prototype transforms only FORTRAN-77 to the GIM. Future research should explore the conversion of other imperative languages to the GIM. Reasoning Systems ${ }^{T M}$ currently provides language tools only for FORTRAN, C, Cobol, and Ada, which restricts the extension of the prototype to these languages until parsers for other languages can be built. Some prototype transformations have been done for Ada 83. Specifically, four different forms of iteration in Ada have been canonicalized into one representation in the GIM. To extend the GIM further, other imperative entities such as heterogeneous data types and pointers could be added to the GIM. If larger systems with millions of lines of imperative code are to be transformed, the efficiency of the prototype must be improved.

### 10.3 Conclusions

It is clear that this research has been completed successfully. The objectives defined in the Problem Statement (see Chapter II) have been met. The GIM provides the desired canonical form for re-engineering legacy imperative code, as presented in Chapter III. The GOM provides the desired canonical form for the object-oriented target system, as presented in Chapter IV. Several in-depth examples of how the PBOI methodology extracts objects from GIM subprograms were presented in Chapter V. The PBOI formal transformations presented in Chapter VI define the PBOI methodology in an unambiguous, consistent, and provably correct manner. Using the PBOI methodology is desirable because the extracted object-oriented design is functionally equivalent to the legacy imperative code. The proof of this claim was presented in Chapter VII. It is feasible to automate this methodology as demonstrated in Chapter VIII. It should be clear to the reader that this research makes significant contributions to the field of re-engineering.

## Appendix A. The Generic Imperative Language

## A. 1 Introduction

This appendix includes the surface syntax developed for the Generic Imperative Language (GIL ${ }^{1}$ ). The surface syntax provides an easy way to view the GIM representation of an imperative program. The GIL is not intended to be used as a forward-engineering programming language.

The notation used to present the surface syntax is taken from the Software Refinery ${ }^{T M}$ Dialect tool [53]. Each production for producing surface syntax is presented using the following format:

```
<GIM object> ::= [ <surface syntax constructs> ]
```

The left-hand-side of this production gives the GIM object for which this surface syntax is defined, and the right-hand-side shows the sequence of constructs used to define the surface syntax. This sequence can include attributes of the object or literal syntax tokens. Optional constructs are enclosed in braces, i.e. \{ \}. Sequences of constructs are indicated by listing an attribute of the object followed by a star token $\left(^{*}\right)$ or plus token ( + ) followed by the delimiter token. The star token indicates zero or more members of the sequence and plus token indicates one or more members. For example, the surface syntax for GIM imperative-name objects is shown below.

```
imperative-name : := [imp-identifier !! imp-has-indices
        "(" imp-indices + "," ")"]
    builds imperative-name,
```

This production means the surface syntax for an imperative-name is the value of the imp-identifier attribute followed by any indices in the imp-indices attribute separated by commas. The imp-has-indices attribute is a boolean attribute used to determine if the imperative-name is an array being accessed.

## A. 2 The Generic Imperative Language

The productions that define the GIL grammar are presented below.

[^7]```
!! in-package("RU")
!! in-grammar('syntax)
grammar Generic-Imperative-Language
    file-classes imperative-subprogram
    productions
%----------------------------------------------------------------------------------------
% imperative design
%----------------------------------------------------------------------------------------
    imperative-design ::= [imperative-programs + ""]
        builds imperative-design,
%--------------------------------------------------------------------------------------
% data types
%---------------------------------------------------------------------------------------
    imperative-integer ::= ["integer"]
    builds imperative-integer,
imperative-real ::= ["real"]
    builds imperative-real,
imperative-boolean ::= ["boolean"]
    builds imperative-boolean,
imperative-character ::= ["character"]
    builds imperative-character,
imperative-string ::= ["string"]
    builds imperative-string,
imperative-array ::= ["array" "(" imp-array-dimensions * "," ")"
                    "of" imp-array-element-type ]
    builds imperative-array,
imperative-index-type ::= [imp-index-lower-bound ".." imp-index-upper-bound]
    builds imperative-index-type,
%----------------------------------------------------------------------------------------
% variables and names
%------------------------------------------------------------------------------------
    imperative-name ::= [imp-identifier "!! imp-has-indices]
    builds imperative-name,
    imperative-name ::= [imp-identifier !! imp-has-indices
        "(" imp-indices + "," ")"]
        builds imperative-name,
%----------------------------------------------------------------------------------------
% binary expressions
%-------------------------------------------------------------------------------------------
```

```
    imperative-addition ::= [imp-bin-exp-seq ++ "+"]
    builds imperative-addition,
imperative-and ::= [imp-bin-exp-seq ++ "and"]
    builds imperative-and,
imperative-concat ::= [imp-bin-exp-seq ++ "&"]
    builds imperative-concat,
imperative-division ::= [imp-bin-exp-seq ++ "/"]
    builds imperative-division,
imperative-equal ::= [imp-bin-exp-operand-1 "=" imp-bin-exp-operand-2]
    builds imperative-equal,
imperative-exponent ::= [imp-bin-exp-operand-1 "a" imp-bin-exp-operand-2]
    builds imperative-exponent,
imperative-greater-than-or-equal ::= [imp-bin-exp-operand-1 ">=" imp-bin-exp-operand-2]
    builds imperative-greater-than-or-equal,
imperative-greater-than ::= [imp-bin-exp-operand-1 ">" imp-bin-exp-operand-2]
    builds imperative-greater-than,
imperative-less-than-or-equal ::= [imp-bin-exp-operand-1 "<=" imp-bin-exp-operand-2]
    builds imperative-less-than-or-equal,
imperative-less-than ::= [imp-bin-exp-operand-1 "<" imp-bin-exp-operand-2]
    builds imperative-less-than,
imperative-multiplication ::= [imp-bin-exp-seq ++ "*"]
    builds imperative-multiplication,
imperative-not-equal ::= [imp-bin-exp-operand-1 "/=" imp-bin-exp-operand-2]
    builds imperative-not-equal,
imperative-or ::= [imp-bin-exp-seq ++ "or"]
    builds imperative-or,
imperative-subtraction ::= [imp-bin-exp-seq ++ "-"]
    builds imperative-subtraction,
%-------------------------------------------------------------------------------------------
% unary expressions
%--------------------------------------------------------------------------------------------
imperative-negate ::= ["-" imp-unary-operand]
    builds imperative-negate,
imperative-not ::= ["not" imp-unary-operand]
    builds imperative-not,
imperative-null ::= [" "]
    builds imperative-null,
```

```
%------------------------------------------------------------------------------------------
% literals
%----------------------------------------------------------------------------------------
    imperative-literal-true ::= "true"
    builds imperative-literal-true,
    imperative-literal-false ::= "false"
    builds imperative-literal-false,
    imperative-literal-integer ::= imperative-literal-value
        builds imperative-literal-integer,
    imperative-literal-real ::= imperative-literal-value
    builds imperative-literal-real,
    imperative-literal-charstring ::= imperative-literal-value
        builds imperative-literal-charstring,
    imperative-literal-newline ::= "~"
    builds imperative-literal-newline,
%-----------------------------------------------------------------------------------------
% assignment
%------------------------------------------------------------------------------------
    imperative-assignment ::= [imp-assign-lhs ":=" imp-assign-rhs]
        builds imperative-assignment,
%------------------------------------------------------------------------------------------
% selection
%-------------------------------------------------------------------------------------------
    imperative-selection ::= ["if" imperative-exp "then"
                imperative-then-part + ";"
                        "else"
                            imperative-else-part + ";"
                            "endif"]
        builds imperative-selection,
%------------------------------------------------------------------------------------------
% iteration
%----------------------------------------------------------------------------------------
    imperative-iteration ::= ["while" iter-exp "do"
                            ["begin"
                                    iter-body + ";"
                                    "end"]
]
    builds imperative-iteration,
%---------------------------------------------------------------------------------------------
% imperative subprograms
%-------------------------------------------------------------------------------------
    imperative-procedure ::= ["procedure" imp-proc-identifier
```

```
    "(" imp-proc-formals * "," ")"
    ["begin"
    imp-proc-statements + ";"
        "end"]
    ]
    builds imperative-procedure,
imp-procedure-call ::= [ imp-proc-call-identifier
                                    "(" imp-proc-call-actuals * "," ")" ]
    builds imp-procedure-call,
imperative-function ::= [imp-function-return-type "function" imp-func-identifier
                    "(" imp-func-formals * "," ")"
                    ["begin"
                    imp-func-statements + ";"
                        "end"]
        I
    builds imperative-function,
imperative-function-call ::= [ imp-fun-call-identifier
                            "(" imp-fun-call-actuals * "," ")" ]
    builds imperative-function-call,
%--------------------------------------------------------------------------------------------------
% output statements
%------------------------------------------------------------------------------------------------
imperative-input ::= ["read" "(" imp-in-logical-file ","
                                    imp-input-list + "," ")"]
    builds imperative-input,
imperative-output ::= ["write" "(" imp-out-logical-file ","
                                    imp-output-list + "," ")"]
    builds imperative-output,
imperative-format-item ::= [imp-fmt-item]
    builds imperative-format-item,
imperative-format-item ::= [imp-fmt-item imp-fmi-format]
    builds imperative-format-item,
imperative-format-integer ::= ["~i" imp-format-width ]
    builds imperative-format-integer,
imperative-format-integer ::= ["~I" imp-format-width ]
    print-only,
imperative-format-decimal ::= ["~ d" imp-format-width "." imp-decimal-part-width]
    builds imperative-format-decimal,
imperative-format-decimal ::= ["~D" imp-format-width "." imp-decimal-part-width]
    print-only,
imperative-format-scientific ::= ["~e" imp-format-width ]
    builds imperative-format-scientific,
```

```
    imperative-format-scientific ::= [""E" imp-format-width ]
    print-only,
    imperative-format-string ::= [""s" imp-format-width ]
    builds imperative-format-string,
    imperative-format-string ::= ["~S" imp-format-width ]
    print-only
start-classes imperative-ast, imperative-subprogram
no-patterns
precedence
    for imperative-expression brackets "(" matching ")"
    (same-level "or" associativity left),
    (same-level "and" associativity left),
    (same-level "not" associativity right),
    (same-level "<", "<=", "=", ">=", ">", "/=" associativity none),
    (same-level "+", "-" associativity left),
    (same-level "*", "/" associativity left),
    (same-level "^" associativity right)
```

end

## Appendix B. The Generic Object-Oriented Language

## B. 1 Introduction

This appendix includes the surface syntax defined for the Generic Object-Oriented Language ( $\mathrm{GOL}^{1}$ ). This surface syntax is not intended for use as a forward-engineering programming language, but as a user-friendly interface to the Generic Object-Oriented Model (GOM) ASTs. Instead of printing each GOM AST and comparing the values of the attributes of the AST, the GOL provides a short-hand view that is used to quickly and easily view the attributes of a GOM AST.

The notation used to present the surface syntax is taken from the Software Refinery ${ }^{T M}$ Dialect tool [53]. Each production for producing surface syntax is presented using the following format:

```
<GOM object> ::= [ <surface syntax constructs> ]
```

The left-hand-side of this production gives the GOM object for which this surface syntax is defined, and the right-hand-side shows the sequence of constructs used to define the surface syntax. This sequence can include attributes of the object or literal syntax tokens. Optional constructs are enclosed in braces, i.e. \{ \}. Sequences of constructs are indicated by listing an attribute of the object followed by a star token $\left(^{*}\right)$ or plus token ( + ) followed by the delimiter token. The star token indicates zero or more members of the sequence and plus token indicates one or more members. For example, the surface syntax for GOM GOM-Variable objects is shown below.

```
GOM-Variable ::= [gom-name !! gom-has-indices
    "(" gom-indices + "," ")" ]
    builds GOM-Variable,
```

This production means the surface syntax for an GOM-Variable is the value of the gom-name attribute followed by any indices in the gom-indices attribute separated by commas. The gom-has-indices attribute indicates whether or not this variable access includes indices into an array.

[^8]
## B. 2 The Generic Object-Oriented Language

The productions that define the GOL grammar are presented below.

```
!! in-package("RU")
!! in-grammar('syntax)
grammar Generic-00-Language
    file-classes GOM-Design
    productions
%----------------------------------------------------------------------------------------
% overall design class
%--------------------------------------------------------------------------------------
    GOM-Design ::= [GOM-classes + ""]
        builds GOM-design,
%----------------------------------------------------------------------------------------
% DO objects
%--------------------------------------------------------------------------------------
    GOM-Class ::= ["class" gom-name
                            ["attributes" gom-attrs * ","]
                    [gom-opers * ""]
                    ["superclass" gom-super]]
        builds GOM-Class,
    GOM-Instantiate ::= ["new" "(" gom-inst-class ")"]
        builds GOM-Instantiate,
    GOM-Method ::= ["method" gom-name "(" [gom-params * ","] ")"
                                    "!! gom-rtns-val
                                    "begin"
                                    [gom-stmts * ";"]
                                    "end"]
        builds GOM-Method,
    GOM-Method ::= ["method" gom-name "(" [gom-params * ","] ")"
                                !! gom-rtns-val ":" gom-return-type
                        "begin"
                        [gom-stmts * ";"]
                        "end"]
        builds GOM-Method,
    GOM-Message ::= [ gom-call "(" gom-actuals * "," ")" ]
        builds GOM-Message,
%------------------------------------------------------------------------------------------
% variables, attributes, and parameters
%----------------------------------------------------------------------------------------
```

```
        GOM-Variable ::= [gom-name ~!! gom-has-indices]
        builds GOM-Variable,
GOM-Variable ::= [gom-name !! gom-has-indices
            "(" gom-indices + "," ")" ]
        builds GOM-Variable,
GOM-Attribute ::= [gom-name *!! gom-has-indices]
    builds GOM-Attribute,
GOM-Attribute ::= [gom-name !! gom-has-indices
            "(" gom-indices + "," ")" ]
    builds GOM-Attribute,
GOM-Attr-Access : := [ gom-tar-object "." gom-attrib ]
    builds GOM-Attr-Access,
GOM-Parameter ::= [gom-name ~!! gom-has-indices]
    builds GOM-Parameter,
GOM-Parameter ::= [gom-name !! gom-has-indices
            "(" gom-indices + "," ")" ]
    builds GOM-Parameter,
%--------------------------------------------------------------------------------------------------
% data types
%----------------------------------------------------------------------------------------
    GOM-integer ::= ["integer"]
        builds GOM-integer,
GOM-real ::= ["real"]
    builds GOM-real,
GOM-boolean ::= ["boolean"]
    builds GOM-boolean,
GOM-character ::= ["character"]
    builds GOM-character,
GOM-string ::= ["string"]
    builds GOM-string,
% arrays
    GOM-array ::= ["array" "(" gom-array-dimensions * "," ")"
                "of" gom-array-element-type ]
        builds GOM-array,
    GOM-index-type ::= [gom-index-lower-bound ".." gom-index-upper-bound]
        builds GOM-index-type,
% instances
    GOM-instance ::= [ "a" gom-instance-of ]
        builds GOM-instance,
```

```
%--------------------------------------------------------------------------------------------
% binary expressions
%------------------------------------------------------------------------------------------
    GOM-addition ::= [GOM-bin-exp-seq ++ "+"]
    builds GOM-addition,
    GOM-and ::= [GOM-bin-exp-seq ++ "and"]
    builds GOM-and,
GOM-concat ::= [GOM-bin-exp-seq ++ "&"]
    builds GOM-concat,
    GOM-division ::= [GOM-bin-exp-seq ++ "/"]
    builds GOM-division,
GOM-equal ::= [GOM-bin-exp-operand-1 "=" GOM-bin-exp-operand-2]
    builds GOM-equal,
GOM-exponent ::= [GOM-bin-exp-operand-1 "~" GOM-bin-exp-operand-2]
    builds GOM-exponent,
GOM-greater-than-or-equal ::= [GOM-bin-exp-operand-1 ">="
                                    GOM-bin-exp-operand-2]
    builds GOM-greater-than-or-equal,
    GOM-greater-than ::= [GOM-bin-exp-operand-1 ">" GOM-bin-exp-operand-2]
    builds GOM-greater-than,
GOM-less-than-or-equal ::= [GOM-bin-exp-operand-1 "<="
                                    GOM-bin-exp-operand-2]
    builds GOM-less-than-or-equal,
GOM-less-than ::= [GOM-bin-exp-operand-1 "<" GOM-bin-exp-operand-2]
    builds GOM-less-than,
GOM-multiplication ::= [GOM-bin-exp-seq ++ "*"]
    builds GOM-multiplication,
    GOM-not-equal ::= [GOM-bin-exp-operand-1 "/=" GOM-bin-exp-operand-2]
    builds GOM-not-equal,
    GOM-or ::= [GOM-bin-exp-seq ++ "or"]
    builds GOM-or,
GOM-subtraction ::= [GOM-bin-exp-seq ++ "-"]
    builds GOM-subtraction,
%--
% unary expressions
%------------------------------------------------------------------------------------------------
GOM-negate ::= ["-" GOM-unary-operand]
    builds GOM-negate,
```

```
    GOM-not ::= ["not" GOM-unary-operand]
    builds GOM-not,
    GOM-null ::= [" "]
    builds GOM-null,
%----------------------------------------------------------------------------------------
% literals
%-----------------------------------------------------------------------------------------
    GOM-literal-true ::= "true"
    builds GOM-literal-true,
    GOM-literal-false ::= "false"
    builds GOM-literal-false,
    GOM-literal-integer ::= GOM-literal-value
    builds GOM-literal-integer,
    GOM-literal-real ::= GOM-literal-value
    builds GOM-literal-real,
    GOM-literal-charstring ::= GOM-literal-value
    builds GOM-literal-charstring,
    GOM-literal-newline ::= "-%"
    builds GOM-literal-newline,
%-------------------------------------------------------------------------------------
% assignment
%---------------------------------------------------------------------------------------
    GOM-assignment ::= [gom-assign-lhs ":=" gom-assign-rhs]
        builds GOM-assignment,
%-----------------------------------------------------------------------------------------
% selection
%----------------------------------------------------------------------------------------
    GOM-selection ::= ["if" gom-exp "then"
                            gom-then-part + ";"
                            "else"
                            gom-else-part + ";"
                            "endif"]
        builds GOM-selection,
%-----------------------------------------------------------------------------------------
% iteration
%----------------------------------------------------------------------------------------
    GOM-iteration ::= ["while" gom-iter-exp "do"
                            ["begin"
                            gom-iter-body + ";"
                            "end"]
    ]
    builds GOM-iteration,
```

```
%-
% non-user-defined subprogram calls
%---------------------------------------------------------------------------------------
    GOM-function-call ::= [ gom-fun-call-identifier
                                    "(" gom-fun-call-actuals * "," ")" ]
    builds GOM-function-call,
    GOM-procedure-call ::= [ gom-proc-call-identifier
                                    "(" gom-proc-call-actuals * "," ")" ]
    builds GOM-procedure-call,
%-
% output statements
%-------------------------------------------------------------------------------------
    GOM-input ::= ["read" "(" gom-in-logical-file ","
        gom-input-list + "," ")"]
        builds GOM-input,
    GOM-output ::= ["write" "(" gom-out-logical-file ","
                        gom-output-list + "," ")"]
        builds GOM-output,
    GOM-format-item ::= [gom-fmt-item]
        builds GOM-format-item,
    GOM-format-item ::= [gom-fmt-item gom-fmt-format]
        builds GOM-format-item,
    GOM-format-integer ::= [""i" gom-format-width ]
        builds GOM-format-integer,
    GOM-format-integer ::= ["~I" gom-format-width ]
        print-only,
    GOM-format-decimal ::= ["~d" gom-format-width "." gom-decimal-part-width]
        builds GOM-format-decimal,
    GOM-format-decimal ::= ["~D" gom-format-width "." gom-decimal-part-width]
        print-only,
    GOM-format-scientific ::= ["~ e" gom-format-width ]
        builds GOM-format-scientific,
    GOM-format-scientific ::= [""E" gom-format-width ]
        print-only,
    GOM-format-string ::= ["~s" gom-format-width ]
        builds GOM-format-string,
    GOM-format-string ::= ["`S" gom-format-width ]
        print-only
start-classes GOM-Design, GOM-Class
```

no-patterns
precedence
for GOM-expression brackets "(" matching ")"
(same-level "or" associativity left),
(same-level "and" associativity left),
(same-level "not" associativity right),
(same-level "\&" associativity right),
(same-level "<", "<=", "=", ">=", ">", "/=" associativity none),
(same-level "+", "-" associativity left), (same-level "*", "/" associativity left), (same-level "~" associativity left)
end

## Appendix C. Nested If Proof

## Prove:

wp(if $B_{1}$ then $S_{1}$ elsif $B_{2}$ then $S_{2}$ else $\left.S_{3}, \mathrm{R}\right) \Leftrightarrow$ wp(if $B_{1}$ then $S_{1}$ else (if $B_{2}$ then $S_{2}$ else $S_{3}$ ), R)

## Proof:

wp(if $B_{1}$ then $S_{1}$ elsif $B_{2}$ then $S_{2}$ else $\left.S_{3}, \mathrm{R}\right) \Leftrightarrow$

$$
\begin{aligned}
& \left(B_{1} \Longrightarrow \operatorname{wp}\left(S_{1}, \mathrm{R}\right)\right) \wedge \\
& \left(\left(\neg B_{1} \wedge B_{2}\right) \Longrightarrow \mathrm{wp}\left(S_{2}, \mathrm{R}\right)\right) \wedge \\
& \left(\left(\neg B_{1} \wedge \neg B_{2}\right) \Longrightarrow \operatorname{wp}\left(S_{3}, \mathrm{R}\right)\right) \Leftrightarrow \\
& \left(B_{1} \Longrightarrow \operatorname{wp}\left(S_{1}, \mathrm{R}\right)\right) \wedge \\
& \left(\neg\left(B_{1} \vee \neg B_{2}\right) \Longrightarrow w p\left(S_{2}, \mathrm{R}\right)\right) \wedge \\
& \left(\neg\left(B_{1} \vee B_{2}\right) \Longrightarrow \operatorname{wp}\left(S_{3}, \mathrm{R}\right)\right) \Leftrightarrow \\
& \left(B_{1} \Longrightarrow \operatorname{wp}\left(S_{1}, \mathrm{R}\right)\right) \wedge \\
& \left(\left(B_{1} \vee \neg B_{2}\right) \vee \mathrm{wp}\left(S_{2}, \mathrm{R}\right)\right) \wedge \\
& \left(\left(B_{1} \vee B_{2}\right) \vee \mathrm{wp}\left(S_{3}, \mathrm{R}\right)\right) \Leftrightarrow \\
& \left(B_{1} \Longrightarrow \operatorname{wp}\left(S_{1}, \mathrm{R}\right)\right) \wedge \\
& \left(B_{1} \vee\left(\neg B_{2} \vee \mathrm{wp}\left(S_{2}, \mathrm{R}\right)\right)\right) \wedge \\
& \left(B_{1} \vee\left(B_{2} \vee \mathrm{wp}\left(S_{3}, \mathrm{R}\right)\right)\right) \Leftrightarrow \\
& \left(B_{1} \Longrightarrow \operatorname{wp}\left(S_{1}, \mathrm{R}\right)\right) \wedge \\
& \left(B_{1} \vee\left(\left(\neg B_{2} \vee \mathrm{wp}\left(S_{2}, \mathrm{R}\right)\right) \wedge\left(B_{2} \vee \mathrm{wp}\left(S_{3}, \mathrm{R}\right)\right)\right)\right) \Leftrightarrow \\
& \left(B_{1} \Longrightarrow \operatorname{wp}\left(S_{1}, \mathrm{R}\right)\right) \wedge \\
& \left(B_{1} \vee\left(\left(B_{2} \Longrightarrow \mathrm{wp}\left(S_{2}, \mathrm{R}\right)\right) \wedge\left(\neg B_{2} \Longrightarrow \mathrm{wp}\left(S_{3}, \mathrm{R}\right)\right)\right)\right) \Leftrightarrow \\
& \left(B_{1} \Longrightarrow \operatorname{wp}\left(S_{1}, \mathrm{R}\right)\right) \wedge \\
& \left(\neg B_{1} \Longrightarrow\left(\left(B_{2} \Longrightarrow \operatorname{wp}\left(S_{2}, \mathrm{R}\right)\right) \wedge\left(\neg B_{2} \Longrightarrow \operatorname{wp}\left(S_{3}, \mathrm{R}\right)\right)\right)\right) \Leftrightarrow \\
& \left(B_{1} \Longrightarrow \operatorname{wp}\left(S_{1}, \mathrm{R}\right)\right) \wedge \\
& \left(\neg B_{1} \Longrightarrow \operatorname{wp}\left(\text { if } B_{2} \text { then } S_{2} \text { else } S_{3}, \mathrm{R}\right)\right) \Leftrightarrow
\end{aligned}
$$

$\operatorname{wp}\left(\right.$ if $B_{1}$ then $S_{1}$ else (if $B_{2}$ then $S_{2}$ else $S_{3}$ ), R) $\Leftrightarrow$
Q.E.D.

## Appendix D. Parameter Transformation Proof

## D. 1 Introduction

This appendix explains how to convert a procedure that has at least one parameter that is both an input parameter and an output parameter into a procedure with parameters that are either input only or output only.

## D. 2 New Procedure Definition

Because of the importance of the distinction between input and output parameters when converting the GIM to the GOM, parameters that are both input and output parameters require special processing. A new vector of output parameters is used to represent the "output" aspect of the input/output parameters. The original parameters are assumed to be input only and the new output parameters hold the new values returned from a procedure call. These new values are stored in the original data items by using an assignment statement after the call to the procedure.

Let $\bar{y}_{o u t}$ represent the vector of output parameters needed to represent the "output" aspect of the input/output parameters in $\bar{y}$. Let $P_{\bar{y}_{\text {out }}}^{\bar{y}}$ represent the precondition $P$ with all occurrences of the parameters in $\bar{y}$ replaced by the parameters in $\bar{y}_{\text {out }}$. Let body $y_{\bar{y}_{\text {out }}}^{\bar{y}}$ represent the body of procedure $p$, with all occurrences of the parameters in $\bar{y}$ replaced by the parameters in $\bar{y}_{\text {out }}$. Using these definitions, a new procedure $p^{\prime}$ is built that accepts the new output parameter $\bar{y}_{\text {out }}$.

```
procedure \(p^{\prime}\left(\bar{x}, \bar{y}, \bar{y}_{\text {out }}, \bar{z}\right)\)
    \(\{P\}\)
    \(\bar{y}_{o u t}:=\bar{y} ;\)
    \(\left\{P_{\bar{y}_{\text {out }}}^{\bar{y}}\right\}\)
    body \(y_{\bar{y} \text { out }}^{\bar{y}}\)
    \(\{R\}\)
```

The body of $p^{\prime}$ includes as its first statement an assignment statement to assign all the output parameters in $\bar{y}_{\text {out }}$ the initial values of $\bar{y}$. All references to $\bar{y}$ in the original
body of procedure $p$ (and the precondition $P$ ) have been replaced by references to $\bar{y}_{\text {out }}$ to create the rest of the body of the procedure $p^{\prime}$.

If the precondition and the postcondition for $p^{\prime}$ are the same as the precondition and the postcondition for $p$, then the semantics of $p^{\prime}$ are the same as that of $p$, i.e. the two procedure definitions are equivalent. The proof of this is shown below.

Proof. 1. The postcondition $R$ is given as the postcondition for both $p$ and $p^{\prime}$.
2. All references to $\bar{y}$ are replaced by references to $\bar{y}_{\text {out }}$ in both body and $P$ for $p^{\prime}$, thus $P_{\bar{y}_{\text {out }}}^{\bar{y}}$ is the precondition for body $y_{\bar{y}_{\text {out }}}^{\bar{y}}$.
3. $w p\left(\bar{y}_{\text {out }}:=\bar{y}, P_{\bar{y}_{\text {out }}}^{\bar{y}}\right)=\left(P_{\bar{y}_{\text {out }}}^{\bar{y}} \bar{y}_{\bar{y}}^{\bar{y}_{\text {out }}}=P\right.$, thus the precondition for $p^{\prime}$ is $P$.
4. Since the precondition and postcondition are the same for $p$ and $p^{\prime}$, the semantics for these two procedure definitions are the same.

## D. 3 New Procedure Call

Let $\bar{b}_{\text {out }}$ represent a vector of output parameters that represent the "output" aspect of the input/output parameters in $\bar{b}$. The invocation of the new procedure $p^{\prime}$ has the form

$$
\begin{array}{r}
p^{\prime}\left(\bar{a}, \bar{b}, \bar{b}_{\text {out }}, \bar{c}\right) ; \\
\bar{b}:=\bar{b}_{\text {out }}
\end{array}
$$

The original $\bar{b}$ parameters are used as input only parameters. The new $\bar{b}_{\text {out }}$ vector holds the output parameters and these new values are stored back into the $\bar{b}$ parameters after the call to $p^{\prime}$ using an assignment statement. Each call to $p^{\prime}$ is now followed by such an assignment statement.

Let body' represent the body of statements from procedure $p^{\prime}$. The execution of a call to procedure $p^{\prime}$ is equivalent to

$$
\begin{array}{rll}
\bar{x} & := & \bar{a} ; \\
\bar{y} & := & \bar{b} ; \\
& \text { bod } y^{\prime} ; \\
\bar{b}_{\text {out }} & := & \bar{y}_{\text {out }} ; \\
\bar{c} & := & \bar{z} ;
\end{array}
$$

This sequence clearly shows how the new output parameters in $\bar{b}_{\text {out }}$ are set to the values in the parameters $\bar{y}_{\text {out }}$ returned from the procedure $p^{\prime}$.

Let $\beta^{\prime}$ represent the execution of $p^{\prime}$. The semantics for this procedure call are defined as

$$
w p\left(p^{\prime}\left(\bar{a}, \bar{b}, \bar{b}_{o u t}, \bar{c}\right), R\right)=w p\left(\beta^{\prime}, R\right)
$$

The assignment statement following the procedure call is required in order to maintain the semantics of the original procedure call. Thus, it is more proper to define the semantics for the new form of procedure call as

$$
w p\left(p^{\prime}\left(\bar{a}, \bar{b}_{,}, \bar{b}_{o u t}, \bar{c}\right) ; \bar{b}:=\bar{b}_{o u t}, R\right)=w p\left(\beta^{\prime}, w p\left(\bar{b}:=\bar{b}_{o u t}, R\right)\right)
$$

In order to prove that the addition of the new vectors $\bar{y}_{\text {out }}$ and $\bar{b}_{\text {out }}$ is correct, it must be proven that the new form of the procedure call maintains the semantics of the original procedure call. Specifically, it must be proven that the following is true.

$$
w p(p(\bar{a}, \bar{b}, \bar{c}), R) \Leftrightarrow w p\left(p^{\prime}\left(\bar{a}, \bar{b}, \bar{b}_{o u t}, \bar{c}\right) ; \bar{b}:=\bar{b}_{o u t}, R\right)
$$

The proof is shown below.

## Proof.

$w p(p(\bar{a}, \bar{b}, \bar{c}), R)$
$\Leftrightarrow w p(\bar{x}:=\bar{a} ; \bar{y}:=\bar{b} ;$ body $; \bar{b}:=\bar{y} ; \bar{c}:=\bar{z}, R)$
$\Leftrightarrow w p(w p(\bar{x}:=\bar{a}, w p(\bar{y}:=\bar{b}, w p(b o d y, w p(\bar{b}:=\bar{y}, w p(\bar{c}:=\bar{z}, R))))))$
$\Leftrightarrow w p\left(\bar{x}:=\bar{a} ; \bar{y}:=\bar{b} ; \bar{y}_{\text {out }}:=\bar{y} ;\right.$ body $\left.\bar{y}_{\text {out }} ; \bar{b}_{\text {out }}:=\bar{y}_{\text {out }} ; \bar{b}:=\bar{b}_{\text {out }} ; \bar{c}:=\bar{z}, R\right)$
$\Leftrightarrow w p\left(\bar{x}:=\bar{a} ; \bar{y}:=\bar{b} ;\right.$ body' $\left.; \bar{b}_{\text {out }}:=\bar{y}_{\text {out }} ; \bar{c}:=\bar{z}, R_{\bar{b}_{\text {out }}}^{\bar{b}}\right)$
$\Leftrightarrow w p\left(\bar{x}:=\bar{a} ; \bar{y}:=\bar{b} ;\right.$ body $\left.{ }^{\prime} ; \bar{b}_{\text {out }}:=\bar{y}_{\text {out }} ; \bar{c}:=\bar{z}, w p\left(\bar{b}:=\bar{b}_{\text {out }}, R\right)\right)$
$\Leftrightarrow w p\left(p^{\prime}\left(\bar{a}, \bar{b}, \bar{b}_{\text {out }}, \bar{c}\right) ; \bar{b}:=\bar{b}_{\text {out }}, R\right)$

## Appendix E. BMDSIM Conversion

Transcript
This appendix shows the complete transcript of the conversion of the 53 BMDSIM subprograms from FORTRAN to the GIM.
.> (rfu::test-conversion)
Not re-loading analysis...
Converting "ANG" to the GIM...
No side effects in this function. Checking transformation completeness...

The transformation was complete.
Total number of tree nodes: 60
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "ANGLE" to the GIM...
No side effects in this function.
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 92
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "ASSIGN" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 609
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file

Converting "BMDSIM1" to the GIM...
Warning: Make sure STOP statement is last statement in main program

Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 2084
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "BOOSTR" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 338
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "BOSTIT" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 559
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "BOUNCE" to the GIM...
No side effects in this function.
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 71
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file

Converting "CAPTURE" to the GIM...
No side effects in this function. Checking transformation completeness...

The transformation was complete.
Total number of tree nodes: 82
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file Saving surface syntax in file

Converting "CROSS" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 86
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "CSP" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 529
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "CUV" to the GIM...
No side effects in this function.
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 85
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file

Converting "DASET" to the GIM... Checking transformation completeness...

The transformation was complete.
Total number of tree nodes: 169
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "DOT" to the GIM...
No side effects in this function.
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 48
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "KEP" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 102
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "LASP" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 64
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file

Converting "LNKCAL" to the GIM... Checking transformation completeness...

The transformation was complete.
Total number of tree nodes: 703
Number of untransformed nodes: 0 Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "LNKCK" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 178
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "LNKORD" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 316
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file Saving surface syntax in file

Converting "MAT" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 149
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$ Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file

Converting "MAXA" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 62
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "MIRGEO" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 348
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "MIRVIS" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 357
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$ Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "MTM2" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 65
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file Saving surface syntax in file

Converting "MTM3" to the GIM... Checking transformation completeness...

The transformation was complete.
Total number of tree nodes: 75
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file Saving surface syntax in file

Converting "MTPD" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 84
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "MTRT" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 71
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "ORBEL" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 377
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file

Converting "PKILL" to the GIM...
No side effects in this function.
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 63
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "POSVEC" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 85
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "POSVECS" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 85
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "PRDIV" to the GIM...
No side effects in this function.
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 58
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file

Converting "RADIUS" to the GIM...
No side effects in this function. Checking transformation completeness...

The transformation was complete. Total number of tree nodes: 84 Number of untransformed nodes: 0 Percentage of AST transformed: $100.0 \%$ Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "RAND" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 55
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "RELAY" to the GIM... Checking transformation completeness...

The transformation was complete.
Total number of tree nodes: 459
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "RHO" to the GIM...
No side effects in this function.
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 22
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file

Converting "RRBVIS" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 419
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "RRPVIS" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 201
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "RTAN" to the GIM...
No side effects in this function.
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 41
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "SBMIT" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 195
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file

Converting "SBMLOC" to the GIM...
Checking transformation completeness...
The transformation was complete. Total number of tree nodes: 77 Number of untransformed nodes: 0 Percentage of AST transformed: $100.0 \%$ Saving to pob...
Saving AST and Symbol Table in file Saving surface syntax in file

Converting "SBMPOS" to the GIM... Checking transformation completeness...

The transformation was complete. Total number of tree nodes: 167 Number of untransformed nodes: 0 Percentage of AST transformed: $\mathbf{1 0 0 . 0 \%}$ Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "SELECL" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 325
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "SGN" to the GIM...
No side effects in this function.
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 19
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file

Converting "SUV" to the GIM...
No side effects in this function.
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 80
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "TFBT" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 94
Number of untransformed nodes: 0
Percentage of AST transformed: 100.0\%
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file
Converting "TPANG" to the GIM... Checking transformation completeness...

The transformation was complete.
Total number of tree nodes: 162
Number of untransformed nodes: 0
Percentage of AST transformed: 100.0\%
Saving to pob...
Saving AST and Symbol Table in file Saving surface syntax in file

Converting "TRAJ" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 217
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file

Converting "UNIT" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 47
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file Saving surface syntax in file

Converting "UPLREQ" to the GIM... No side effects in this function. Checking transformation completeness...

The transformation was complete.
Total number of tree nodes: 67
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$ Saving to pob... Saving AST and Symbol Table in file Saving surface syntax in file

Converting "UPTRNS" to the GIM... No side effects in this function. Checking transformation completeness...

The transformation was complete.
Total number of tree nodes: 384
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file Saving surface syntax in file

Converting "VADD" to the GIM...
Checking transformation completeness...
The transformation was complete.
Total number of tree nodes: 81
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file
Saving surface syntax in file

Converting "VISCK" to the GIM... Checking transformation completeness...

The transformation was complete. Total number of tree nodes: 164
Number of untransformed nodes: 0
Percentage of AST transformed: $100.0 \%$
Saving to pob...
Saving AST and Symbol Table in file Saving surface syntax in file

Converting "VMAG" to the GIM... No side effects in this function. Checking transformation completeness...

The transformation was complete. Total number of tree nodes: 37 Number of untransformed nodes: 0 Percentage of AST transformed: $100.0 \%$ Saving to pob...
Saving AST and Symbol Table in file Saving surface syntax in file

Overall number of nodes in analysis: 11451.0 Overall number of untransformed nodes: 0.0 Percentage of analysis transformed: $100.0 \%$

## Appendix F. Mapping GIM Entities to the GOM

Figures 207 through 214 show the mappings between GIM entities and GOM entities. The transformations are shown in an abbreviated format where the GIM entity on the left-hand-side of the transformation is converted into the GOM entity on the right-hand-side. These transformations are straightforward given the description of the GIM in Chapter III and the description of the GOM in Chapter IV.

| imperative-assignment $\longrightarrow$ | gom-assignment |
| ---: | ---: |
| imperative-selection $\longrightarrow$ | gom-selection |
| imperative-iteration $\longrightarrow$ | gom-iteration |

Figure 207 Control Flow Constructs

| imp-subprogram-call $\longrightarrow$ | gom-procedure-call |
| ---: | ---: |
| imperative-function-call $\longrightarrow$ | gom-function-call |
| imp-subprogram-call $\xrightarrow{u}$ | gom-message |
| imperative-function-call $\xrightarrow{u}$ | gom-message |

Figure 208 Subprogram Calls

| imperative-variable $\longrightarrow$ | gom-variable |
| ---: | ---: |
| imperative-name $\longrightarrow$ | gom-variable |

Figure 209 Data Storage Constructs

| imperative-integer $\longrightarrow$ | gom-integer |
| ---: | ---: |
| imperative-real $\longrightarrow$ | gom-real |
| imperative-boolean $\longrightarrow$ | gom-boolean |
| imperative-character $\longrightarrow$ | gom-character |
| imperative-string $\longrightarrow$ | gom-string |
| imperative-array $\longrightarrow$ | gom-array |
| imperative-index-type $\longrightarrow$ | gom-index-type |

Figure 210 Data Type Classes
imperative-literal-boolean $\longrightarrow$
imperative-literal-integer $\longrightarrow$ imperative-literal-real $\longrightarrow$ imperative-literal-charstring $\longrightarrow$ imperative-literal-newline $\longrightarrow$
gom-literal-boolean
gom-literal-integer gom-literal-real gom-literal-charstring
gom-literal-newline

Figure 211 Literals

| imperative-addition $\longrightarrow$ |  |
| ---: | ---: |
| imperative-and $\longrightarrow$ | gom-addition <br> imperative-concat $\longrightarrow$ |
| gom-and |  |
| imperative-division $\longrightarrow$ | gom-concat |
| imperative-equal $\longrightarrow$ | gom-division |
| imperative-exponent $\longrightarrow$ | gom-equal |
| imperative-greater-than-or-equal $\longrightarrow$ | gom-greater-than-or-equal |
| imperative-greater-than $\longrightarrow$ | gom-greater-than |
| imperative-less-than-or-equal $\longrightarrow$ | gom-less-than-or-equal |
| imperative-less-than $\longrightarrow$ | gom-less-than |
| imperative-multiplication $\longrightarrow$ | gom-multiplication |
| imperative-not-equal $\longrightarrow$ | gom-not-equal |
| imperative-or $\longrightarrow$ | gom-or |
| imperative-subtraction $\longrightarrow$ |  |

Figure 212 Binary Expressions

| imperative-negate $\longrightarrow$ | gom-negate |
| ---: | ---: |
| imperative-not $\longrightarrow$ | gom-not |
| imperative-null $\longrightarrow$ | gom-null |

Figure 213 Unary Expressions

| imperative-file $\longrightarrow$ | gom-file |
| ---: | ---: |
| imperative-input $\longrightarrow$ | gom-input |
| imperative-output $\longrightarrow$ | gom-output |
| imperative-format $\longrightarrow$ | gom-format |

Figure 214 Input and Output

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| 13. ABSTRACT (Maximum 200 words) <br> This research defines a methodology for automatically extracting functionally equivalent object-oriented designs from legacy imperative programs. The Parameter-Based Object Identification (PBOI) methodology is based on fundamental ideas that relate programs written in imperative languages such as C or COBOL to objects and classes written in object-oriented languages such as Ada 95 or $\mathrm{C}++$. Transformations have been developed that formalize the PBOI methodology and a formal proof is provided showing the extracted object-oriented design is functionally equivalent to the legacy imperative system. To focus the task of re-engineering, generic models of imperative programming languages and object-oriented programming languages have been developed. The formal transformations convert imperative subprograms represented in the Generic Imperative Model (GIM) into classes and objects represented in the Generic Object-Oriented Design Model (GOM). A taxonomy of imperative subprograms has also been developed which classifies all imperative subprograms into one of six categories. A proof-of-concept prototype has been developed and a 3000 -line FORTRAN- 77 system has been converted to an object-oriented design as a feasibility demonstration. |  |  |  |
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[^0]:    ${ }^{1}$ This aspect of parameter passing will be restricted by the re-engineering methodology, as described in Section 3.11.

[^1]:    ${ }^{2}$ Here, $p$ represents both the procedure entity and the identifier that names the procedure.

[^2]:    ${ }^{1}$ For simplicity, the fact that C-5 is an instance of CLASS-5 is represented in this diagram by using a symbol.

[^3]:    ${ }^{2}$ For simplicity, this instance of CLASS- 5 is represented in this figure as the symbol CLASS- 5

[^4]:    ${ }^{1}$ Recognizing differently-named attributes that represent the same data item is done using the transitive closure of the $\mu$ relation, as described in Section 6.8.2

[^5]:    ${ }^{1}$ There is no surface syntax defined for the skip statement.

[^6]:    ${ }^{1}$ One of the six categories from the taxonomy of imperative subprograms presented in Chapter V.

[^7]:    ${ }^{1}$ Pronounced "Jill", as in Jack and Jill.

[^8]:    ${ }^{1}$ Pronounced "gôl" as in "golf".

