



12-2000

Probabilistic fatigue crack growth analysis using response surface methodology

Glenn Chris Cox

Follow this and additional works at: https://trace.tennessee.edu/utk_gradthes

Recommended Citation

Cox, Glenn Chris, "Probabilistic fatigue crack growth analysis using response surface methodology. " Master's Thesis, University of Tennessee, 2000.
https://trace.tennessee.edu/utk_gradthes/9327

This Thesis is brought to you for free and open access by the Graduate School at TRACE: Tennessee Research and Creative Exchange. It has been accepted for inclusion in Masters Theses by an authorized administrator of TRACE: Tennessee Research and Creative Exchange. For more information, please contact trace@utk.edu.

To the Graduate Council:

I am submitting herewith a thesis written by Glenn Chris Cox entitled "Probabilistic fatigue crack growth analysis using response surface methodology." I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Civil Engineering.

Karen C. Chou, Major Professor

We have read this thesis and recommend its acceptance:

Richard M. Bennett, Edwin G. Burdette

Accepted for the Council:

Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

To the Graduate Council

I am submitting herewith a thesis written by Glenn Chris Cox entitled "Probabilistic Fatigue Crack Growth Analysis Using Response Surface Methodology" I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Civil Engineering

Karen C. Chou

Dr Karen C Chou, Major Professor

We have read this thesis
and recommend its acceptance

Richard M. Bennett

Edwin G. Burdette

Accepted for the Council

[Signature]

Associate Vice Chancellor and Dean of
The Graduate School

**PROBABILISTIC FATIGUE CRACK GROWTH ANALYSIS USING RESPONSE
SURFACE METHODOLOGY**

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

Glenn Chris Cox
December 2000

DEDICATION

I dedicate this thesis to my parents, Gordon and Janie, for the love and support they have always given me and for instilling in me the qualities that will continue to make me successful.

ACKNOWLEDGMENTS

All honor and glory to my Lord and Savior Jesus Christ for His everlasting love and guidance in my life.

During the process of obtaining my graduate degree there have been many people who have given support and assistance. I am grateful to the members of my thesis committee. Dr. Karen C. Chou, Dr. Richard M. Bennett, and Dr. Edwin G. Burdette. In particular, I would like to thank Dr. Chou, who served as my major professor, for the opportunity to work as a research assistant on this project. I would also like to thank her for all the activities she has assisted me with while a student at the University of Tennessee, Knoxville.

I would like to express my appreciation to The United States Navy who provided the funding for this project

I would also like to thank Allison Lockwood for her excellent assistance and encouragement in completing this project.

Lastly, I would like to thank my parents, Gordon and Janie, my brother, John, and fiancée, Ashley, as well as the rest of my family, for the love and encouragement they have given me during the process of obtaining my undergraduate and graduate degrees.

ABSTRACT

All aircraft operated by the United States Navy were designed to operate for a predetermined number of flight hours defined as the flight life expectancy (FLE). An aircraft is due to retire from service once the FLE is reached. Due to the end of the cold war and budget priority, it is anticipated that fleets of United States Naval aircraft will be used beyond their designed FLE. A study was performed for the US Navy to address the aging aircraft issue by examining crack growth at the logeron of the aircraft. The objective of the study was to develop a probability model for the number of load cycles (N) needed for a crack to grow from its initiation size to critical size. Fatigue crack growth due to constant stress intensity as well as random stress block was examined. The Paris equation was used to compute the fatigue crack growths. Response surface methodology was used to approximate the number of load cycles with respect to the geometric and material properties and stress intensity level. The most workable relationship was a linear function with all the input parameters and number of load cycles in their natural logarithmic base. The statistical analyses of the study showed that the mean number of load cycles (N) approximated from the response surface functions predicted the mean pseudo population load cycles at a 95 percent confidence level. The study also showed that the probability density function of the number of load cycles from the response surface functions, Gaussian distribution, fit the histogram of the pseudo population well.

TABLE OF CONTENTS

<u>Chapter</u>	<u>Page</u>
1. INTRODUCTION ..	1
2. ANALYSIS PROCEDURES..	6
2.1 Fatigue Crack Growth Model.....	6
2.2 Response Surface Methodology.....	9
2.3 Pseudo Population.....	11
2.4 Constant Stress and Random Stress Block Analyses.....	12
2.5 Statistical Analyses and Verifications.....	13
2.5.1 Statistical Moments of Response Surface Functions... 13	
2.5.2 Verification of the Response Surface Function Statistical Moments.....	15
2.5.3 Distribution of the Response Surface Functions and Pseudo Populations.....	16
3. DISCUSSION OF ANALYSIS RESULTS....	18
3.1 Constant Stress Analysis.....	18
3.1.1 Response Surface Functions and Pseudo Population.....	18
3.1.2 Verification of the Response Surface Function Statistical Moments.....	21
3.1.3 Statistical Analysis of the Pseudo Population.....	24
3.1.4 Comparison of the Response Surface Functions with the Pseudo Population.....	25
3.1.5 Optimal Sample Size and Range of Input Parameters.....	28
3.2 Random Stress Block Analysis.....	32
3.2.1 Response Surface Functions and Pseudo Population.....	32
3.2.2 Verification of the Response Surface Function Statistical Moments.....	38
3.2.3 Statistical Analysis of the Pseudo Population.....	47
3.2.4 Comparison of the Response Surface Functions with the Pseudo Population.....	47
3.2.5 Optimal Sample Size and Range of Input Parameters.....	49
4. CONCLUSIONS AND RECOMMENDATIONS.....	54
REFERENCES.....	58
VITA.....	61

LIST OF TABLES

<u>Table</u>	<u>Page</u>
2.1 Critical values of D_{cr} in the Kolmogorov-Smirnov test.....	17
3.1 Statistical data for input parameters	18
3.2 Sample sizes, input ranges, and coefficients from regression analysis for constant stress analysis.....	19
3.3 Summary of the sets of input values for crack growth computations.....	21
3.4 Means and standard deviations from point estimate method and results of the 95% confidence interval test for constant stress analysis...	22
3.5 Means and standard deviations from Taylor series expansion and results of the 95% confidence interval test for constant stress analysis...	23
3.6 Results of the Kolmogorov-Smirnov test for constant stress analysis.....	26
3.7 Sample sizes, input ranges, and coefficients from regression analysis for set 1 of random stress block analysis.....	33
3.8 Sample sizes, input ranges, and coefficients from regression analysis for set 2 of random stress block analysis.....	34
3.9 Sample sizes, input ranges, and coefficients from regression analysis for set 3 of random stress block analysis.....	35
3.10 Sample sizes, input ranges, and coefficients from regression analysis for set 4 of random stress block analysis.....	36
3.11 Means and standard deviations from point estimate method and results of the 95% confidence interval test for set 1 of random stress block analysis.....	39
3.12 Means and standard deviations from point estimate method and results of the 95% confidence interval test for set 2 of random stress block analysis.....	40

<u>Table</u>	<u>Page</u>
3.13 Means and standard deviations from point estimate method and results of the 95% confidence interval test for set 3 of random stress block analysis.....	41
3.14 Means and standard deviations from point estimate method and results of the 95% confidence interval test for set 4 of random stress block analysis.....	42
3.15 Means and standard deviations from Taylor series expansion and results of the 95% confidence interval test for set 1 of random stress block analysis.....	43
3.16 Means and standard deviations from Taylor series expansion and results of the 95% confidence interval test for set 2 of random stress block analysis.....	44
3.17 Means and standard deviations from Taylor series expansion and results of the 95% confidence interval test for set 3 of random stress block analysis.....	45
3.18 Means and standard deviations from Taylor series expansion and results of the 95% confidence interval test for set 4 of random stress block analysis.....	46
3.19 Results of the Kolmogorov-Smirnov test for all 4 sets of random stress block analyses.....	48

LIST OF FIGURES

<u>Figure</u>	<u>Page</u>
2.1 Typical crack growth curves.....	9
3.1 Typical comparison between the proposed probability model from a response surface function and histogram from the pseudo population.....	27
3.2 Typical comparison between a response surface function and the pseudo population cumulative distributions.....	27
3.3 Comparison of fraction of D_{cr} and maximum input range for a pseudo population size of 10,000.....	29
3.4 Comparison of fraction of D_{cr} and maximum input range for a pseudo population size of 50,000.....	30
3.5 Comparison of fraction of D_{cr} and maximum input range for a pseudo population size of 100,000.....	31
3.6 Comparison of fraction of D_{cr} and maximum input range for a pseudo population size of 10,000 (set 1).....	50
3.7 Comparison of fraction of D_{cr} and maximum input range for a pseudo population size of 10,000 (set 2).....	51
3.8 Comparison of fraction of D_{cr} and maximum input range for a pseudo population size of 10,000 (set 3).....	52
3.9 Comparison of fraction of D_{cr} and maximum input range for a pseudo population size of 10,000 (set 4).....	53

CHAPTER 1

INTRODUCTION

All aircraft operated by the United States Navy were designed to operate for a predetermined number of flight hours defined as flight life expectancy (FLE). An aircraft is due to retire from service once the FLE is reached. Since World War II, the FLE criterion has served the fleets of aircraft in the US Navy well. Due to the end of the cold war and budget priority, it is anticipated that fleets of aircraft will be used beyond their designed FLE. This undoubtedly raised the question of how safe an aircraft would be for Navel service beyond its designed FLE.

The US Navy has defined the FLE based on an undefined probability that a crack on the frame of an aircraft will reach a critical size (crack size is defined as the length of the crack) beyond FLE. However, once a crack, smaller than the critical size, is detected prior to its FLE, the component would be repaired or replaced and the aircraft would be allowed to continue to fly until it reaches the FLE. The University of Tennessee has been contracted by the Navy Air System to perform an assessment study on aging aircraft.

A realistic analysis of any structural system should consider the randomness inherent in the loads and resistance. For fatigue analysis, a stochastic crack growth model would be desired. Maymon (1998) gave three major reasons why stochastic crack growth models are preferred:

- 1) The geometry, dimensions, and material properties may differ slightly between specimens. Thus, the whole structure is not deterministic.
- 2) The external loading in practical engineering cases is usually random.
- 3) The structures or specimens properties are random even under well-controlled production conditions.

Maymon further stated that stochastic models can be incorporated in the calculation of the probability of structural failure due to crack propagation and that the engineering community should study and adopt such methods in practical design.

A 1983 NASA funded study (Wirsching, 1983) provided specific recommendations on how designers should model fatigue strength. The paper presented several engineering models used to describe fatigue behavior, one being the use of statistical models to analyze number of load cycles to failure data. The author stated that in order to make design decisions on the basis of a set of observations of a design factor, it was necessary to describe the distribution of that factor. Therefore, statistical models are usually employed. The author later developed a statistical model in another NASA funded study (Wu and Wirsching, 1983) The objective of that study was to determine the probability of having a certain crack size (or failure) given a service life of N cycles. While this study provided useful information at the service life, no information was known beyond the service life or the knowledge of how long a service life should be for a given crack size.

This type of study is typical of those focused in the area of crack size and load cycles. Despite the extensive studies performed by the researchers, their results cannot be directly applied to the aging aircraft with which the U.S. Navy is concerned. As stated earlier, Wu and Wirsching's study gave no knowledge of how long a service life should be for a given crack size. Furthermore, due to the time dependent relationship between crack size and load cycles, one cannot simply invert the relationship developed by Wu and Wirsching to yield the number of load cycles given crack size.

A 1998 internal report (Chou, 1998) and presentation (Hoffman, 1998) to the Structural Division at the Navy Air System showed that to assess aging aircraft was to evaluate the total life of the aircraft. The focus was narrowed to fatigue around boltholes at the longeron of an aircraft. A preliminary total life reliability model was developed (Chou, 1998). The total life was defined as the duration for crack initiation combined with the duration for crack growth (crack grows from initiation to critical size). In order to use the total life model, the probability models for the number of load cycles needed for crack initiation and for crack growth are required. The focus of this study is to develop a probability model for the number of load cycles for a crack to grow from initiation to critical size.

To begin the process of developing the probability model, a crack growth model must be established. The Paris equation, which was developed by Paris and Erdogan (Bannantine, et al., 1990) in the early 1960's, is still the most widely

used method used to model the crack growth rate. This equation was used in the study to determine the crack growth curves. However, due to the mathematical complexity in determining the crack growth curves, the probability model for the number of load cycles was developed numerically.

Researchers have demonstrated that response surface method is a viable tool in developing a relationship between a response and multiple parameters input (Myers et al., 1989, Bucher, 1990, Draper and Lin, 1990, Rajashekhar and Ellingwood, 1993, and Liu and Moses, 1994). Furthermore, the response surface can be determined using only a limited number of data points. Once a response surface is determined (it is usually represented by a polynomial function), basic probability can be applied to derive the probability model for the response value.

In this study, the response surface was determined using the number of load cycles computed by the Paris equation. An initiation crack size of 0.01 inches and a critical size of 0.3 inches were used in this study. These values were provided by the engineers at the Navy Air System. All material properties were assumed to be constant throughout the entire crack growth.

In Chapter 2, the analysis procedures used to develop the probability model are explained. This chapter presents the Paris equation for computing the fatigue crack growth model. It also discusses the response surface methodology and its application to the crack growth problem. Finally, it explains the statistical analyses used to verify and check the proposed probability model.

In Chapter 3, the results from analysis procedures outlined in Chapter 2 are presented. These include the results from all statistical testing performed, including the statistical moments and probability distribution. Finally, Chapter 4 presents some conclusions and recommendations.

CHAPTER 2

ANALYSIS PROCEDURES

2.1 FATIGUE CRACK GROWTH MODEL

The crack growth model used in this study was based on the Paris equation. The Paris equation (Bannantine et al., 1990) defines the fatigue crack growth rate as

$$\frac{da}{dN} = c(\Delta k)^m \quad (2.1)$$

where, a = crack size, N = number of load cycles, c = geometric parameter, m = material constant, and Δk = stress intensity factor which is defined as

$$\Delta k = \beta(\Delta\sigma)\sqrt{\pi a} \quad (2.2)$$

in which β = correction factor and $\Delta\sigma$ = stress intensity.

The Paris equation can be applied to any type or shape of metal, as long as one uses the appropriate geometric and material coefficients. This equation also has no regard for crack sizes in which it is not applicable. Theoretically, one could solve for the crack growth rate of a crack that is several feet in length. However, good engineering judgment would suggest that failure has probably long since occurred. Therefore, like many engineering equations, the user must be aware of where the formula is applicable.

The Paris equation provides one with the crack growth rate. However, to determine the number of cycles needed for this study, the Paris equation given

by Equation (2.1) needs to be integrated so that the number of cycles can be computed if the crack size and input parameters are known. If the equation is examined closely at its simplest form when the input parameters are constants, the crack size and number of cycles are the only changing variables. Equation (2.1) can be simplified to the following:

$$\frac{da}{dN} = \text{constant} (\sqrt{a})^m \quad (2.3)$$

However, even in this simple form, where m is not an even integer, the equation cannot be integrated to a closed form expression. Therefore, numerical integration has to be used instead.

Since the objective here is to examine the variation in the number of load cycles for a given crack size, the difference between the crack initiation size and critical size was divided into 100 equal intervals. The number of load cycles ΔN needed to achieve each increment of crack length Δa was computed as:

$$\frac{\Delta a}{\Delta N_j} = c (\Delta k_j)^m = c (\beta_j (\Delta \sigma) \sqrt{\pi a_j})^m \quad (2.4)$$

Rearranging Equation 2.4 to solve for ΔN yields:

$$\Delta N_j = \frac{\Delta a}{c (\Delta k_j)^m} = \frac{\Delta a}{c (\beta_j (\Delta \sigma) \sqrt{\pi a_j})^m} \quad (2.5)$$

where

$$\Delta a = \frac{a_f - a_i}{100} \quad (2.6)$$

and β_j and a_j are, respectively, the correction factor and crack size at the j^{th} increment and a_f and a_i are, respectively, the critical (0.3 inches) and the initiation (0.01 inches) crack sizes

The total number of load cycles needed for the crack to grow from its initiation size to critical size was simply computed as

$$N = \sum_{j=1}^{100} (\Delta N)_j \quad (2.7)$$

Typical crack growth curves, developed from the Paris equation, can be seen in Figure 2.1. The crack growth curves presented are for the crack sizes ranging from 0.01 inches to 0.3 inches considered in this study.

2.2 RESPONSE SURFACE METHODOLOGY

Response surface method (RSM) uses a limited amount of data to approximate a system in which a large number of variables influence the response of the system. The history of RSM can be found in a report written by Myers and his colleagues (1989). The use of "response curves" has been dated back into the 1930's, but it wasn't introduced formally until an article (Box and Wilson, 1951) about the notion of composite designs was published in 1951. This article spurred the use of RSM as a powerful tool in design and data analysis.

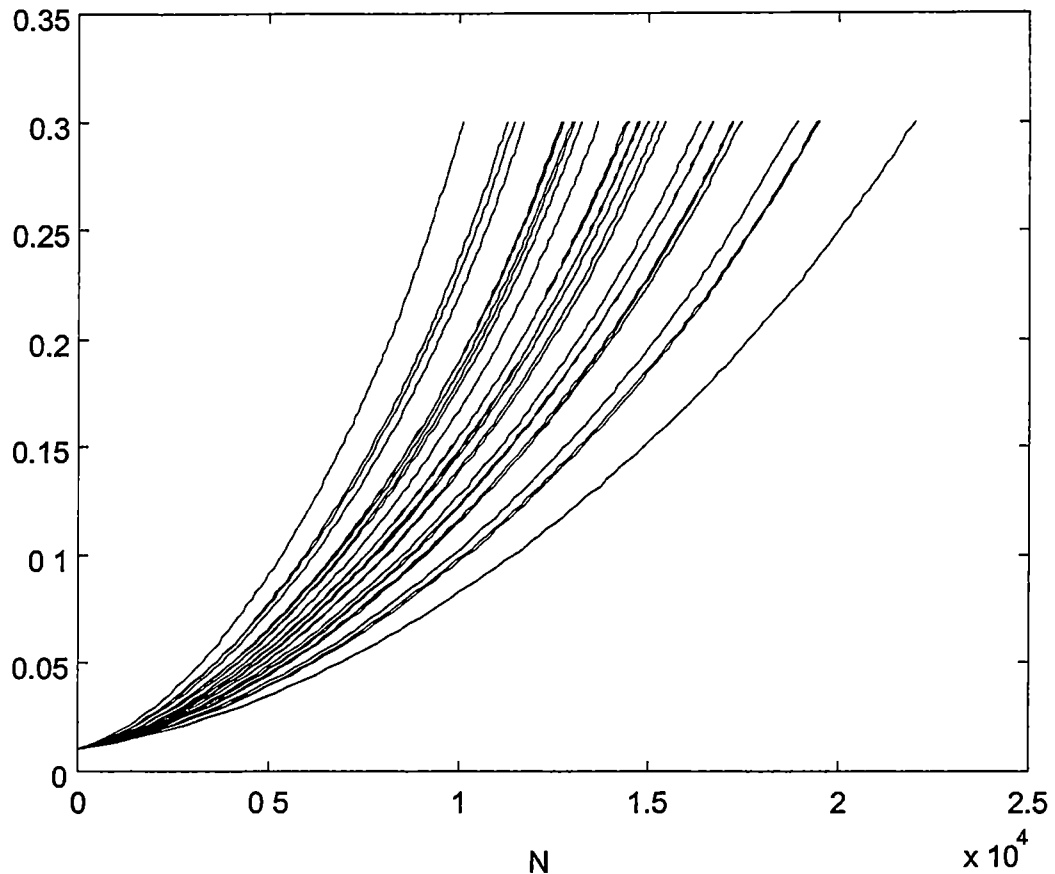


Figure 2.1. Typical crack growth curves

Rajashekhar and Ellingwood (1993) gave a concise description of how to apply RSM to engineering problems, and it is summarized here. Suppose a response variable Y depends on the input parameters $X_1, X_2, X_3, \dots, X_n$, and the design variables $X_1, X_2, X_3, \dots, X_n$ are obtained experimentally or simulated a sufficient number of times to define the response surface. Each experiment or simulation can be represented by a point with coordinates $x_{1j}, x_{2j}, x_{3j}, \dots, x_{nj}$ in an $(n+1)$ -dimensional space. At each point a response value of y_j is determined. Using the input parameters $(x_{1j}, x_{2j}, x_{3j}, \dots, x_{nj})$ and corresponding output (y_j), one

can establish a relationship between them. This relationship is referred to as the response surface (RS) function.

In most engineering systems, the relationship between the input parameters and the output value is very complex and almost impossible to derive mathematically. Response surface method offers a convenient approximation to develop such a relationship because only a limited number of input response data points are needed. Furthermore, these points can be obtained through experiments, engineering analyses (such as finite element method), or through simulation.

The study on hand fell naturally to the response surface method. The crack growth rate was well defined. However, the relationship between the number of load cycles and the input parameters could not be computed via a closed form expression. Therefore, RSM was a desirable choice to approximate the relationship for the purposes of establishing a probability model.

To develop the response surface functions, the response variable was the number of load cycles, N , which was a function of the input parameters (c , m , and $\Delta\sigma$). Multiple sets of input parameters were then defined and the corresponding output N for each set was solved using Equations (2.5-2.7). The input variables were defined rather than simulated, to ensure that a wide spectrum of input variables were used. With input parameters (c , m , and $\Delta\sigma$) and output (N) known, a relationship could be defined between the two using a regression analysis. Since N is a function of 3 variables, a typical linear

regression analysis (the simplest regression relationship) yields 4 coefficients. Hence, a minimum of 5 sets of (c, m, $\Delta\sigma$, and N) values are required for the regression analysis. MATLAB[®] statistical toolbox (by Mathwork, Inc.) was used to perform the regression analysis using the least-squared-error criteria. Both linear and nonlinear regression analyses were attempted to develop a response surface function for N as well as for natural log of N, lnN. The most workable regression function was a linear regression with all the variables transformed to the natural logarithmic values. Hence, the basic RS function became

$$\ln(N) = b_1 \ln(c) + b_2 \ln(m) + b_3 \ln(\Delta\sigma) + b_4 \quad (2.8)$$

where b_i = coefficients obtained from the regression analysis.

2.3 PSEUDO POPULATION

The pseudo population represents the actual population of the natural log of N, lnN. In theory, the pseudo population takes into account all possible values and combinations of c, m, and $\Delta\sigma$ and record the corresponding output N for each unique set. The pseudo population was used as the basis to check the validity of the response surface (RS) functions.

The pseudo population was created by randomly generating several thousand sets of the input variables (c, m, and $\Delta\sigma$). Equations (2.5-2.7) were then applied to determine the number of cycles N for the crack to grow from its initiation size to critical size. This pseudo population was generated in lieu of the

actual flight data because the sample size of the actual data was small and they were costly to produce.

2.4 CONSTANT AND RANDOM STRESS BLOCK ANALYSES

Two different types of stress intensity were analyzed, constant stress and random stress block. In both cases, the geometric parameter (c) and material constant (m) were assumed to be constant throughout the crack growth life. The constant stress analysis assumed that the stress intensity ($\Delta\sigma$) remained constant for a single crack growth curve, while the random stress block analysis assumed the stress intensity fluctuated from interval to interval throughout a single crack growth curve. The random stress block analysis best represents the actual loading to which an aircraft frame would be subjected, but the idea of constant stress intensity simplified the calculations and represents a deterministic crack growth.

Both analyses were easy to accomplish because the numerical integration of the Paris law required that the difference between the initiation and critical crack sizes be broken into several intervals to calculate the crack growth curves. The constant stress analysis was accomplished by holding the stress constant for each interval. Therefore, each curve was represented by a single set of c , m , $\Delta\sigma$, and N . To accomplish the random stress block analysis a stress value was randomly generated for each interval. Therefore, each curve was represented by a single set of (c , m , and N) along with 100 $\Delta\sigma$ values. However, the mean of the

100 stress values was used as $\Delta\sigma$ in each set, thus creating a single set of (c, m, $\Delta\sigma$, and N). The reason for using the mean was that the $\Delta\sigma$ values were centralized around that number.

2.5 STATISTICAL ANALYSIS AND VERIFICATIONS

2.5.1 Statistical Moments of Response Surface Functions

Any probability model based on experimental data requires statistical verifications. The statistical moments, such as the mean and standard deviation, were determined based on output (lnN) of the RS functions from both constant and random stress block analyses. Point estimate method and Taylor series expansion were used to calculate the mean and standard deviation of lnN for each RS function.

The point estimate method (Rosenbleuth, 1975, 1981; Harr, 1987) is simple and requires no continuity on the RS function. Hence, information on the derivatives of the RS function is not needed. The method requires eight points that represent the following combinations of c, m, and $\Delta\sigma$: ($\bar{c}+s_c$, $\bar{m}+s_m$, $\bar{\Delta\sigma}+s_{\Delta\sigma}$), ($\bar{c}+s_c$, $\bar{m}+s_m$, $\bar{\Delta\sigma}-s_{\Delta\sigma}$), ($\bar{c}+s_c$, $\bar{m}-s_m$, $\bar{\Delta\sigma}+s_{\Delta\sigma}$), ($\bar{c}-s_c$, $\bar{m}+s_m$, $\bar{\Delta\sigma}+s_{\Delta\sigma}$), ($\bar{c}-s_c$, $\bar{m}-s_m$, $\bar{\Delta\sigma}+s_{\Delta\sigma}$), ($\bar{c}-s_c$, $\bar{m}+s_m$, $\bar{\Delta\sigma}-s_{\Delta\sigma}$), ($\bar{c}+s_c$, $\bar{m}-s_m$, $\bar{\Delta\sigma}-s_{\Delta\sigma}$), ($\bar{c}-s_c$, $\bar{m}-s_m$, $\bar{\Delta\sigma}-s_{\Delta\sigma}$); in which \bar{c} , \bar{m} , and $\bar{\Delta\sigma}$ represent the mean of c, m and $\Delta\sigma$, respectively, and s_j represents the standard deviation of parameter j. Each set of (c, m, and $\Delta\sigma$) was substituted into its respective RS function (Equation

(2.8)) to solve for the $\ln N$ values, n_i . The mean and standard deviation of $\ln N$ became:

$$E[\ln N] = \frac{1}{8} \sum_{i=1}^8 n_i \quad (2.9)$$

$$s_{\ln N}^2 = \frac{1}{8} \sum_{i=1}^8 n_i^2 - (E[\ln N])^2 \quad (2.10)$$

The other method used to estimate the mean and standard deviation of $\ln N$ from the RS functions was Taylor series expansion. Taylor series expansion requires that the RS functions be continuous at least up to the second derivative and it is more accurate than the point estimate method. Second order Taylor series expansion was used to compute the mean and first order Taylor series expansion was used for the variance (Benjamin and Cornell, 1970). Based on the assumption of mutual independence among the parameters, the resulting mean and standard deviation were calculated as:

$$E[\ln N] = b_1 \ln(E[c]) + b_2 \ln(E[m]) + b_3 \ln(E[\Delta\sigma]) + b_4 - \frac{1}{2} [b_1 \zeta_c^2 + b_2 \zeta_m^2 + b_3 \zeta_{\Delta\sigma}^2] \quad (2.11)$$

$$s_{\ln N}^2 = b_1^2 \zeta_c^2 + b_2^2 \zeta_m^2 + b_3^2 \zeta_{\Delta\sigma}^2 \quad (2.12)$$

where $\zeta_*^2 = \ln[V_*^2 + 1]$ and $V_*^2 = \frac{s_*^2}{E[*]^2}$.

When $V_*^2 \ll 1$, then $\zeta_*^2 \approx V_*^2$. Therefore, ζ_*^2 in Equations (2.11) and (2.12) can be replaced by V_*^2 .

2.5.2 Verification of the Response Surface Function Statistical Moments

The means and standard deviations calculated from the two above methods were used to establish a confidence interval to compare with the mean of $\ln N$ from the pseudo population. If the pseudo population mean is within the range of the sample mean (based on the RS functions), then one can conclude that the sample mean of $\ln N$ is representative of the "population" mean within a certain confidence level. A ninety-five percent confidence interval of the population mean for each RS function was chosen. The lower and upper limit of the confidence interval were calculated as

$$\bar{x} - k_{\alpha/2} \frac{s}{\sqrt{n}} \quad \text{and} \quad \bar{x} + k_{\alpha/2} \frac{s}{\sqrt{n}} \quad (2.13)$$

where \bar{x} = mean of $\ln N$, s = standard deviation of $\ln N$, n = sample size, $\alpha = 0.05$ for a 95% confidence interval, and $k_{\alpha/2}$ is determined from a normal probability

plot and is defined as

$$k_{\alpha/2} = \Phi^{-1} \left(1 - \frac{\alpha}{2} \right) \quad (2.14)$$

in which Φ^{-1} = inverse cumulative distribution function of a standardized normal variant and $k_{\alpha/2} = 1.96$ for $\alpha = 0.05$.

2.5.3 Distribution of Response Surface Functions and Pseudo Population

Since the natural logarithm of the number of load cycles $\ln N$ was linearly related to the input parameters in the natural logarithm base, the probability

model of InN can be approximated by Gaussian distribution (Benjamin and Cornell, 1970). Therefore, it would be of interest to examine if the pseudo population possessed two fundamental characteristics of Gaussian distribution. These characteristics are the coefficient of skewness being zero and the coefficient of Kurtosis being three. If the coefficient of skewness of the pseudo population is zero and the coefficient of Kurtosis is three, then the probability model based on the RS function would have a higher probability to be a good fit to the pseudo population. The goodness of fit test, as presented subsequently, was a better method for assessing the quality of the probability model developed based on the limited data. However, the coefficients of skewness and Kurtosis provide a quick check for the case of Gaussian distribution. The coefficient of skewness was calculated as

$$\frac{1}{n} \left[\frac{\sum (x_i - \bar{x})^3}{s^3} \right] \quad (2.15)$$

and the coefficient of Kurtosis was calculated as

$$\frac{1}{n} \left[\frac{\sum (x_i - \bar{x})^4}{s^4} \right] \quad (2.16)$$

The last statistical verification performed was the Kolmogorov-Smirnov test. Kolmogorov-Smirnov (K-S) test is a common goodness-of-fit test used to verify the probability model of a population based on a limited number of samples. The test compared the difference in cumulative distribution between the model based on RS functions and the pseudo population. If the maximum

absolute difference in the cumulative distributions was within a critical value D_{cr} (tolerance) for a given significant level, then the proposed model was acceptable. The critical value D_{cr} depended on the number of “experimental” data (sample size or number of crack growth curves) that were used to develop the RS function (and hence the proposed probability model). Table 2.1 presents the critical values of D_{cr} in the K-S test for a significant level of five percent.

Table 2.1. Critical values of D_{cr} in the Kolmogorov-Smirnov test (values taken from Ang and Tang (1975))

Sample Size (n)	D_{cr}
5	0.56
10	0.41
15	0.34
20	0.29
25	0.27
30	0.24
35	0.23
40	0.21
45	0.2
50	0.19
>50	$1.36/\sqrt{n}$

CHAPTER 3

DISCUSSION OF ANALYSIS RESULTS

3.1 CONSTANT STRESS ANALYSIS

3.1.1 Response Surface Functions and Pseudo Population

To determine the constant stress response surface (RS) functions, all input parameters (c , m , and $\Delta\sigma$) were manually defined using the statistical data presented in Table 3.1. A sample size ranged from 5 to 729 sets of (c , m , $\Delta\sigma$, and N) values, as shown in Column 2 of Table 3.2, were used to create thirty-nine RS functions. Each set of (c , m , $\Delta\sigma$, and N) represents a single crack growth curve. Therefore, an RS function based on a sample size of 729 was actually based on 729 crack growth curves. It was found during the study that the number of samples (or curves) was not the only factor influencing the RS functions. The range of the input parameters (c , m , and $\Delta\sigma$) also influenced the "quality" of the RS functions. Hence, various values of (c , m , and $\Delta\sigma$) were considered as well.

Table 3.1. Statistical data for input parameters

Parameter	Mean	Standard Deviation	Coefficient of Variation	Probability Distribution
c	1.0 E-8	2.3 E-9	0.23	lognormal
m	3.0	0.1	0.033	normal
$\Delta\sigma$ (ksi)	13.9	1.39	0.1	normal

Table 3.2. Sample size, input ranges, and coefficients from regression analysis for constant stress analysis

RS Function No	Sample Size	No of Std. Dev. from Mean	Coefficients			
			b1	b2	b3	b4
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	5	0 01	-0 8383	-0 3668	-2 3913	0.8531
2	5	0 05	-1.2163	-6 5087	-2 8365	1 8106
3	5	0 1	-1.0955	-7 1310	-2 9251	4 9527
4	5	0 5	-1 0085	-7 5415	-3 0187	7 2528
5	5	1	-1.0173	-7 6537	-3.0375	7.2654
6	5	1 5	-1.0266	-7 7639	-3 0562	7.2680
7	5	2	-1 0362	-7.8722	-3 0750	7.2632
8	5	2 5	-1 0461	-7 9790	-3.0937	7 2526
9	5	3	-1.0563	-8 0842	-3.1125	7 2354
10	8	1	-1.0000	-7 4098	-3 0000	7 2106
11	8	1 5	-1 0000	-7.3873	-3 0000	7 1810
12	8	2	-1 0000	-7 3554	-3 0000	7 1396
13	8	1 5	-1 0000	-7 3138	-3 0000	7.0852
14	8	3	-1.0000	-7 2616	-3.0000	7 0170
15	27	0 5, 0	-1 0000	-7.4242	-3 0000	7.2297
16	27	1, 0	-1 0000	-7.4141	-3 0000	7 2167
17	27	2, 0	-1 0000	-7.3732	-3 0000	7 1644
18	27	2 5, 0	-1 0000	-7.3419	-3.0000	7.1239
19	27	3, 0	-1 0000	-7 3029	-3 0000	7.0742
20	27	3 5, 0	-1.0000	-7 2557	-3 0000	7 0141
21	27	4, 0	-1.0000	-7.1996	-3 0000	7 0742
22	125	1, 0 5, 0	-1 0000	-7 4171	-3 0000	7 2211
23	125	2, 1, 0	-1 0000	-7 3850	-3 0000	7 1799
24	125	2 5, 1 5, 0	-1 0000	-7 3573	-3 0000	7 1443
25	125	2 5, 1, 0	-1 0000	-7 3625	-3 0000	7 1518
26	125	2 5, 2, 0	-1 0000	-7 3472	-3 0000	7 1303
27	125	3, 1 5, 0	-1 0000	-7.3302	-3 0000	7 1101
28	125	3 5, 2, 0	-1.0000	-7 3152	-3 0000	7 0930
29	343	1 5, 1, 0 5, 0	-1 0000	-7 4062	-3 0000	7.2072
30	343	2, 1, 0 5, 0	-1.0000	-7 3924	-3.0000	7.1900
31	343	2 5, 2, 1, 0	-1.0000	-7 3610	-3.0000	7 1489
32	343	3, 2, 1, 0	-1.0000	-7 3403	-3.0000	7 1231
33	343	3 5, 2, 1, 0	-1.0000	-7 3139	-3 0000	7 0905
34	729	2, 1 5, 1, 0 5, 0	-1.0000	-7 3916	-3 0000	7 1885
35	729	2.5, 2, 1, 0.5, 0	-1 0000	-7.3704	-3 0000	7 1617
36	729	3, 2, 1, 0 5, 0	-1 0000	-7 3522	-3 0000	7 1394
37	729	3 5, 2.5, 1, 0.5, 0	-1.0000	-7 3217	-3 0000	7 1011
38	729	3 5, 2, 1, 0 5, 0	-1.0000	-7 3289	-3 0000	7 1110
39	729	4, 3, 2, 1, 0	-1.0000	-7 2788	-3 0000	7 0453

To create the sets of (c , m , $\Delta\sigma$, and N), combinations of the mean values along with the mean values plus or minus certain amounts of their standard deviation were used for each input parameter. Column 3 in Table 3.2 presents the range of the input parameters used to determine the sample size. For example, RS function number one from Table 3.2 has a sample size of 5 and a range of 0.01 standard deviations from the means of the input parameters. Thus, the values for c , m , and $\Delta\sigma$ for the five sets were $(\bar{c}, \bar{m}, \overline{\Delta\sigma})$, $(\bar{c} + 0.01s_c, \bar{m}, \overline{\Delta\sigma})$, $(\bar{c}, \bar{m} + 0.01s_m, \overline{\Delta\sigma})$, $(\bar{c}, \bar{m}, \overline{\Delta\sigma} + 0.01s_{\Delta\sigma})$, and $(\bar{c} + 0.01s_c, \bar{m} + 0.01s_m, \overline{\Delta\sigma} + 0.01s_{\Delta\sigma})$; in which \bar{c} , \bar{m} , and $\overline{\Delta\sigma}$ represent the mean of c , m and $\Delta\sigma$, respectively, and s_j represents the standard deviation of parameter j . Table 3.3 summarizes the combinations of the input values for crack growth curve computations using sets of 5, 8, and 27 curves. For sample sizes of 125, 343, and 729, the permutation sequence is similar to that of sample size 27.

Equations (2.5-2.7) were used to calculate the number of cycles N for each set of input parameters. A linear regression was then performed on each set of (c , m , $\Delta\sigma$, and N) to determine the coefficients for the RS functions (Equation 2.8). Recall that each value was transformed to its natural logarithmic base for the regression analysis. Columns 4-7 in Table 3.2 present the coefficients of each RS function.

Table 3.3. Summary of the sets of input values for crack growth computations (note: 0 = mean value; + = mean plus x standard deviations; - = mean minus x standard deviations; x given in third column of Table 3.2)

Sample Size	c	m	$\Delta\sigma$	c	m	$\Delta\sigma$	c	m	$\Delta\sigma$	c	m	$\Delta\sigma$
5	0	0	0	0	+	0	+	0	0	+	+	+
	0	0	+									
8	0	0	+	0	+	0	+	0	0	+	+	+
	0	0	-	0	-	0	-	0	0	-	-	-
27	0	0	+	0	+	-	+	+	+	-	+	0
	0	0	-	0	-	+	+	+	-	-	-	0
	0	0	0	+	0	+	+	-	+	-	+	+
	0	+	0	+	0	-	+	-	-	-	+	-
	0	-	0	+	0	0	-	0	+	-	-	+
	0	+	+	+	+	0	-	0	-	-	-	-
	0	-	-	+	-	0	-	0	0			

3.1.2 Verification of the Response Surface Function Statistical Moments

The point estimate method and Taylor series expansion were used to calculate the mean and standard deviation of $\ln N$ for each RS function. Columns 3 and 4 in Tables 3.4 and 3.5 summarize the results from both methods. Using two significant figures, the mean of $\ln N$ is 9.6 and the standard deviation is about 0.45. This corresponds to a mean number of load cycles of 16,340 cycles and a standard deviation of 7,740 cycles (Benjamin and Cornell, 1970).

One thousand sets of 10,000 crack growth curves were simulated to represent the pseudo population. The statistical data in Table 3.1 were used to randomly generate c, m, and $\Delta\sigma$ values for each curve. Columns 5 and 6 in Tables 3.4 and 3.5 present the 95 percent confidence interval of the population

Table 3.4. Means and standard deviations from point estimate method and results of the 95% confidence interval test for constant stress analysis

RS Function No	Sample Size	Average Value	Standard Deviation	95% Confidence Interval		Pseudo Population		Percent Acceptable
				Lower Limit	Upper Limit	Passed	Failed	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	5	9.63	0.310	9.36	9.91	1000	0	100%
2	5	9.65	0.457	9.25	10.1	1000	0	100%
3	5	9.65	0.457	9.25	10.0	1000	0	100%
4	5	9.65	0.459	9.24	10.0	1000	0	100%
5	5	9.65	0.463	9.24	10.1	1000	0	100%
6	5	9.65	0.468	9.24	10.1	1000	0	100%
7	5	9.66	0.472	9.24	10.1	1000	0	100%
8	5	9.66	0.477	9.25	10.1	1000	0	100%
9	5	9.67	0.481	9.25	10.1	1000	0	100%
10	8	9.64	0.454	9.33	9.96	1000	0	100%
11	8	9.64	0.454	9.32	9.95	1000	0	100%
12	8	9.63	0.453	9.32	9.94	1000	0	100%
13	8	9.62	0.453	9.31	9.94	1000	0	100%
14	8	9.61	0.452	9.30	9.92	1000	0	100%
15	27	9.64	0.455	9.47	9.82	1000	0	100%
16	27	9.64	0.455	9.47	9.81	1000	0	100%
17	27	9.64	0.454	9.46	9.81	1000	0	100%
18	27	9.63	0.453	9.46	9.80	1000	0	100%
19	27	9.62	0.452	9.45	9.79	1000	0	100%
20	27	9.61	0.452	9.44	9.78	1000	0	100%
21	27	9.74	0.451	9.57	9.91	1000	0	100%
22	125	9.64	0.455	9.56	9.72	1000	0	100%
23	125	9.64	0.454	9.56	9.72	1000	0	100%
24	125	9.63	0.453	9.55	9.71	1000	0	100%
25	125	9.63	0.454	9.56	9.71	1000	0	100%
26	125	9.63	0.453	9.55	9.71	1000	0	100%
27	125	9.63	0.453	9.55	9.71	1000	0	100%
28	125	9.63	0.453	9.55	9.71	1000	0	100%
29	343	9.64	0.454	9.59	9.69	1000	0	100%
30	343	9.64	0.454	9.59	9.69	1000	0	100%
31	343	9.63	0.454	9.59	9.68	1000	0	100%
32	343	9.63	0.453	9.58	9.68	1000	0	100%
33	343	9.63	0.453	9.58	9.67	1000	0	100%
34	729	9.64	0.454	9.61	9.67	964	36	96.4%
35	729	9.64	0.454	9.60	9.67	996	4	99.6%
36	729	9.63	0.453	9.60	9.67	1000	0	100%
37	729	9.63	0.453	9.60	9.66	1000	0	100%
38	729	9.63	0.453	9.60	9.66	1000	0	100%
39	729	9.62	0.452	9.59	9.65	1000	0	100%

Table 3.5. Means and standard deviations from Taylor series expansion and results of the 95% confidence interval test for constant stress analysis

RS Function No	Sample Size	Average Value	Standard Deviation	95% Confidence Interval		Pseudo Population		Percent Acceptable
				Lower Limit	Upper Limit	Passed	Failed	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	5	9.61	0.309	9.34	9.88	1000	0	100%
2	5	9.62	0.456	9.22	10.0	1000	0	100%
3	5	9.62	0.455	9.22	10.0	1000	0	100%
4	5	9.62	0.458	9.22	10.0	1000	0	100%
5	5	9.62	0.462	9.22	10.0	1000	0	100%
6	5	9.63	0.467	9.22	10.0	1000	0	100%
7	5	9.63	0.471	9.22	10.0	1000	0	100%
8	5	9.63	0.475	9.22	10.1	1000	0	100%
9	5	9.64	0.480	9.22	10.1	1000	0	100%
10	8	9.62	0.453	9.30	9.93	1000	0	100%
11	8	9.61	0.453	9.30	9.92	1000	0	100%
12	8	9.60	0.452	9.29	9.92	1000	0	100%
13	8	9.60	0.451	9.28	9.91	1000	0	100%
14	8	9.58	0.451	9.27	9.90	1000	0	100%
15	27	9.62	0.453	9.45	9.79	1000	0	100%
16	27	9.62	0.453	9.45	9.79	1000	0	100%
17	27	9.61	0.453	9.44	9.78	1000	0	100%
18	27	9.60	0.452	9.43	9.77	1000	0	100%
19	27	9.60	0.451	9.43	9.77	1000	0	100%
20	27	9.59	0.450	9.42	9.76	1000	0	100%
21	27	9.58	0.449	9.41	9.75	1000	0	100%
22	125	9.62	0.453	9.54	9.70	1000	0	100%
23	125	9.61	0.453	9.53	9.69	1000	0	100%
24	125	9.61	0.452	9.53	9.69	1000	0	100%
25	125	9.61	0.452	9.53	9.69	1000	0	100%
26	125	9.60	0.452	9.52	9.68	1000	0	100%
27	125	9.60	0.452	9.52	9.68	1000	0	100%
28	125	9.60	0.451	9.52	9.68	1000	0	100%
29	343	9.62	0.453	9.57	9.66	1000	0	100%
30	343	9.61	0.453	9.57	9.66	1000	0	100%
31	343	9.61	0.452	9.56	9.65	1000	0	100%
32	343	9.60	0.452	9.56	9.65	1000	0	100%
33	343	9.60	0.451	9.55	9.65	1000	0	100%
34	729	9.61	0.453	9.58	9.65	1000	0	100%
35	729	9.61	0.452	9.58	9.64	1000	0	100%
36	729	9.61	0.452	9.57	9.64	1000	0	100%
37	729	9.60	0.452	9.57	9.64	1000	0	100%
38	729	9.60	0.452	9.57	9.64	1000	0	100%
39	729	9.59	0.451	9.56	9.63	988	12	98.8%

mean using the point estimate and Taylor series approximations of the RS functions. For the point estimate method, all the pseudo population means fell within the 95 percent confidence interval determined from the response surface functions except for RS functions 34 and 35 (Table 3.4, columns 7-9) However, more than 95 percent of the pseudo population means still fell within the confidence interval for these functions. The results for the Taylor series expansion (Table 3.5, columns 7-9) were even better. All pseudo population means fell within 95 percent confidence intervals determined from the RS functions except for RS function no. 39, which 98.8 percent of the pseudo population means fell within that confidence interval.

3.1.3 Statistical Analysis of the Pseudo Population

To further assess the probability model of $\ln N$, the distribution of the pseudo population was examined. One hundred sets of 10,000 pseudo population crack growth curves were generated. The average coefficient of skewness from the 100 sets of data was 0.048 and the standard deviation of the coefficient of skewness was 0.026. The average coefficient of Kurtosis from the same 100 sets of data was 3.03 and the standard deviation was 0.049. The average coefficient of skewness indicates that the $\ln N$ values are skewed slightly to the right. Although the pseudo population indicates that the probability distribution of $\ln N$ is not exactly symmetric, the coefficient of skewness and the coefficient of Kurtosis are very close to the target values of zero and three,

respectively. Therefore, it was likely that InN could still be modeled by Gaussian distribution.

3.1.4 Comparison of the Response Surface Functions with the Pseudo Population

The Kolmogorov-Smirnov (K-S) test was used to check if the proposed probability model (based from the RS functions) was acceptable. For comparisons, three sets consisting of 10,000, 50,000, and 100,000 simulated crack growth curves were used. Columns 5, 7, and 9 in Table 3.6 present the maximum difference in cumulative distribution between the normal distribution developed based on the RS function and the pseudo population. As can be seen from the table, the maximum differences are much less than the critical difference of D_{cr} given in column 4. To better visualize how the maximum difference compared to D_{cr} , the fraction of D_{cr} was calculated by dividing D_{cr} into the difference. The resulting fractions are presented in Columns 6, 8, and 10. Most of the differences are less than 10 percent of D_{cr} and the greatest difference is 36 percent of the critical difference.

It was also observed that on the average, the test results got better as the pseudo population size increased, that is the RS functions were "fitting" the pseudo population better as the pseudo population size increased. This is good considering that as the pseudo population size increases, it begins to approach the "actual" population. Figure 3.1 and 3.2 present typical comparisons between the proposed probability model and the pseudo population. Figure 3.1 presents

Table 3.6. Results of the Kolmogorov-Smirnov test for constant stress analysis

RS Function No	Sample Size	No of Std Dev from Mean	D _{cr}	Pseudo Population Size 10,000		Pseudo Population Size 50,000		Pseudo Population Size. 100,000	
				Difference	Fraction of D _{cr}	Difference	Fraction of D _{cr}	Difference	Fraction of D _{cr}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	5	0 01	0.560	0.096	0.171	0.094	0.168	0.093	0.166
2	5	0 05		0.020	0.035	0.011	0.020	0.008	0.014
3	5	0 1		0.020	0.037	0.012	0.021	0.008	0.015
4	5	0 5		0.022	0.039	0.013	0.023	0.010	0.018
5	5	1		0.024	0.042	0.016	0.029	0.013	0.022
6	5	1 5		0.027	0.049	0.020	0.036	0.016	0.029
7	5	2		0.032	0.057	0.025	0.044	0.021	0.037
8	5	2.5		0.037	0.066	0.030	0.054	0.026	0.046
9	5	3		0.043	0.077	0.036	0.064	0.032	0.057
10	8	1		0.017	0.036	0.009	0.019	0.005	0.010
11	8	1.5	0.470	0.013	0.027	0.005	0.010	0.003	0.006
12	8	1 5		0.012	0.025	0.012	0.026	0.016	0.033
13	8	2		0.009	0.020	0.007	0.016	0.008	0.017
14	8	3		0.019	0.039	0.021	0.045	0.025	0.053
15	27	0 5, 0	0.258	0.020	0.077	0.009	0.035	0.009	0.035
16	27	1, 0		0.018	0.071	0.007	0.028	0.007	0.029
17	27	2, 0		0.011	0.044	0.005	0.019	0.004	0.016
18	27	2 5, 0		0.010	0.037	0.010	0.038	0.007	0.027
19	27	3, 0		0.011	0.044	0.016	0.062	0.013	0.051
20	27	3 5, 0		0.016	0.061	0.023	0.090	0.021	0.080
21	27	4, 0		0.024	0.092	0.031	0.121	0.029	0.112
22	125	1, 0.5, 0	0.122	0.019	0.155	0.008	0.065	0.008	0.065
23	125	2, 1, 0		0.014	0.112	0.003	0.023	0.003	0.026
24	125	2 5, 1 5, 0		0.009	0.072	0.007	0.061	0.005	0.043
25	125	2 5, 1, 0		0.011	0.086	0.006	0.047	0.004	0.037
26	125	2 5, 2, 0		0.009	0.078	0.010	0.080	0.007	0.056
27	125	3, 1 5, 0		0.010	0.081	0.011	0.091	0.008	0.067
28	125	3 5, 2, 0		0.010	0.083	0.018	0.145	0.015	0.123
29	343	1.5, 1, 0 5, 0		0.073	0.017	0.233	0.006	0.083	0.006
30	343	2, 1, 0 5, 0	0.015		0.209	0.004	0.060	0.005	0.062
31	343	2 5, 2, 1, 0	0.009		0.128	0.007	0.092	0.005	0.067
32	343	3, 2, 1, 0	0.009		0.128	0.009	0.127	0.007	0.091
33	343	3 5, 2, 1, 0	0.010		0.141	0.012	0.169	0.010	0.132
34	729	2, 1 5, 1, 0 5, 0	0.050		0.015	0.293	0.004	0.077	0.004
35	729	2 5, 2, 1, 0 5, 0		0.012	0.232	0.005	0.091	0.004	0.077
36	729	3, 2, 1, 0 5, 0		0.010	0.190	0.007	0.131	0.005	0.098
37	729	3 5, 2.5, 1, 0 5, 0		0.010	0.196	0.011	0.211	0.008	0.156
38	729	3 5, 2, 1, 0 5, 0		0.009	0.187	0.009	0.178	0.007	0.130
39	729	4, 3, 2, 1, 0		0.012	0.241	0.018	0.361	0.016	0.308

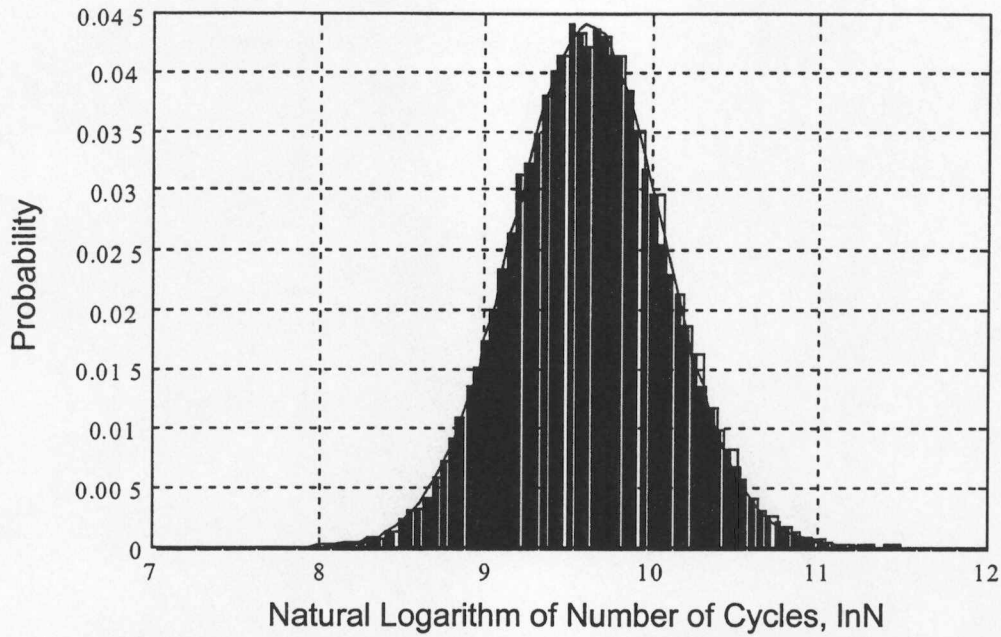


Figure 3.1. Typical comparison between the proposed probability model from a response surface function and histogram from the pseudo population

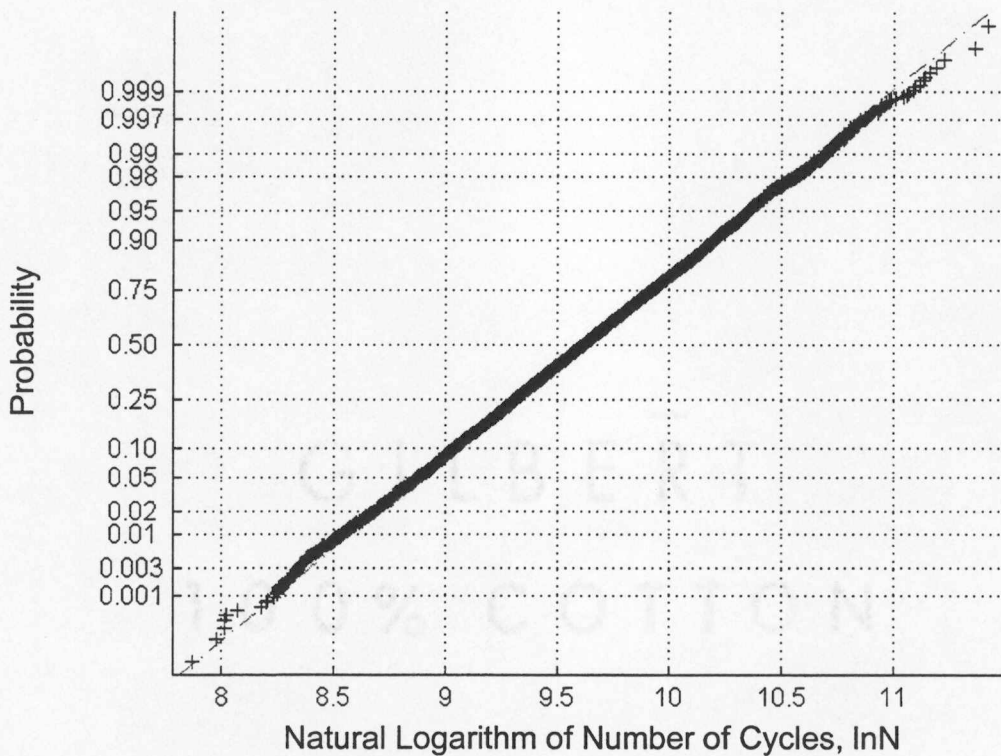


Figure 3.2. Typical comparison between a response surface function and the pseudo population cumulative distributions

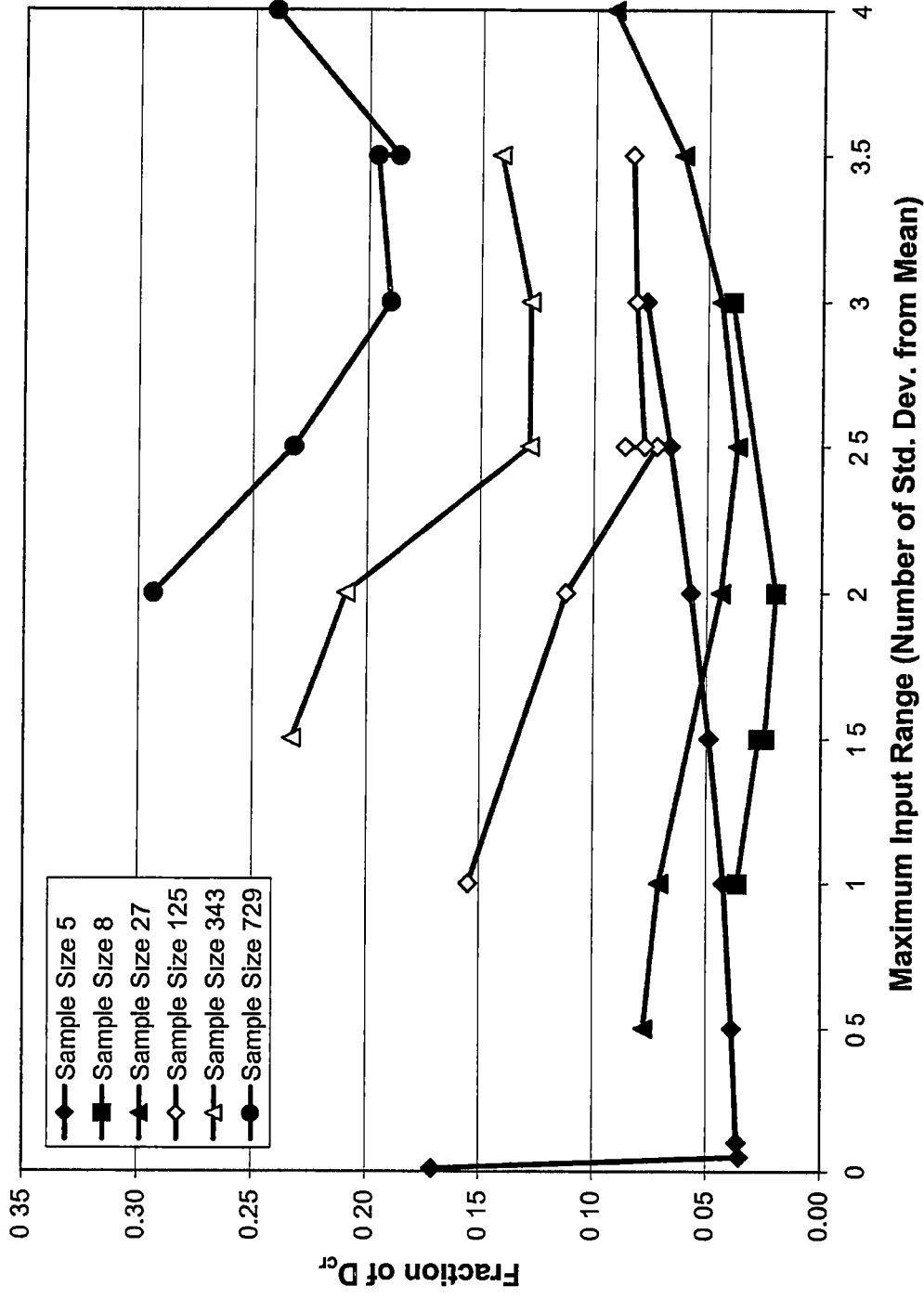
the fitting of the probability density function of an RS function (RS function no. 17 in Table 3.2) to the histogram compiled from the 10,000 pseudo population crack growth curves. Figure 3.2 presents the cumulative distribution function of the same RS function and 10,000 pseudo population crack growth curves.

3.1.5 Optimal Sample Size and Range of the Input Parameters

The previous sections have shown that all RS functions were acceptable to model the number of load cycles for a crack to grow from its initiation to critical size. The RS functions were developed using a limited number of crack growth curves that ranged from 5 to 729 as well as a wide range of input values (c , m , and $\Delta\sigma$) ranging from 0.01 standard deviations from the mean to 4 standard deviations from the mean. It was desirable to identify the optimal sample size and range of input values used to develop an RS function and eventually, a probability model would be derived from that function.

To determine these optimal values, Figures 3.3-3.5 were developed to create a graphical comparison between the maximum number of standard deviations from the mean and the fraction of D_{cr} for each RS function. The maximum number of standard deviations from the mean can be found in Table 3.6, column 3.

It was observed that as the sample size increases, the fraction of D_{cr} increases. However, this does not imply that the more samples one uses to determine a RS function, the worse the approximation. The reason for this



29 Figure 3.3. Comparison of fraction of D_{cr} and maximum input range for a pseudo population size of 10,000

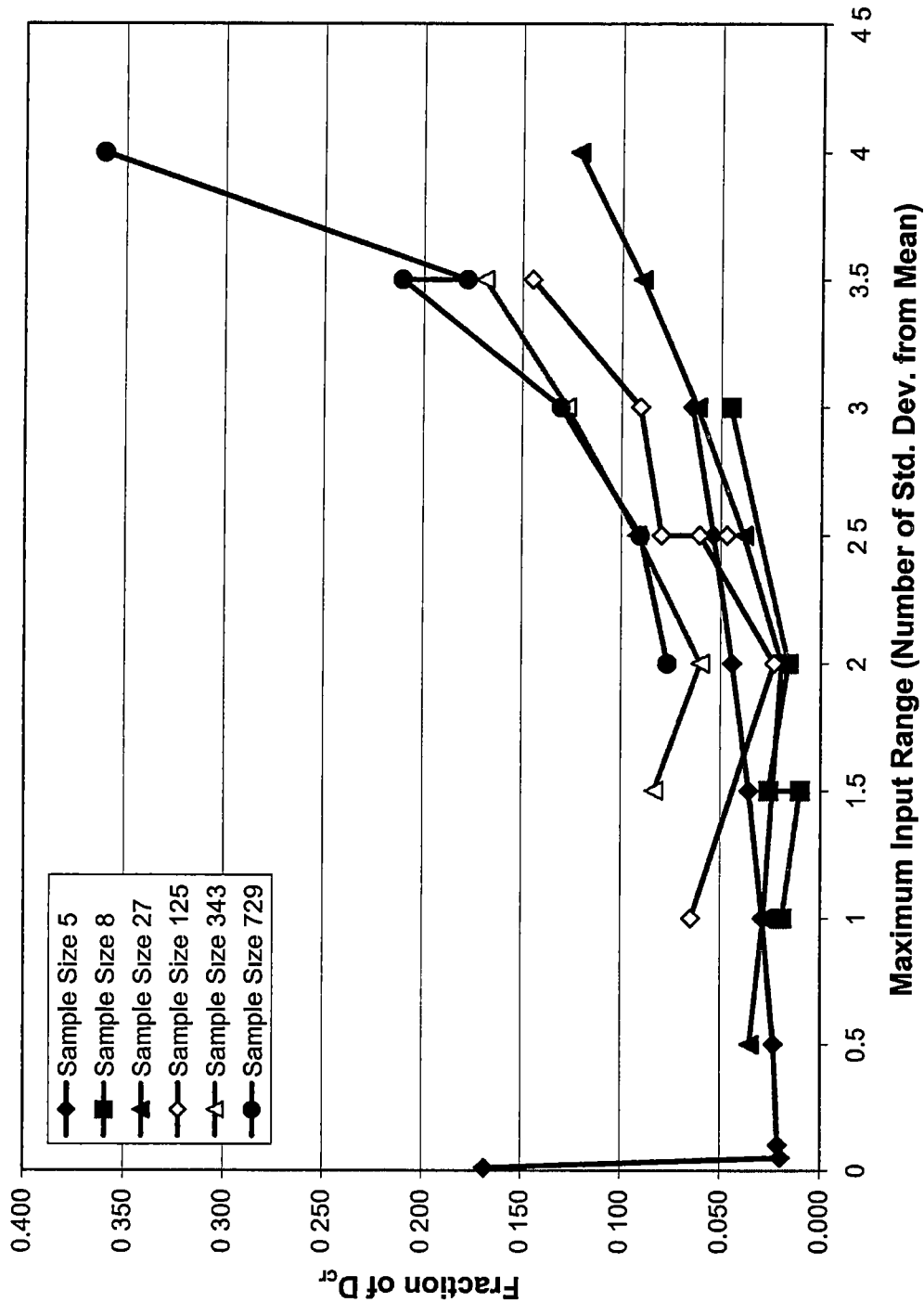


Figure 3.4. Comparison of fraction of Dcr and maximum input range for a pseudo population size of 50,000

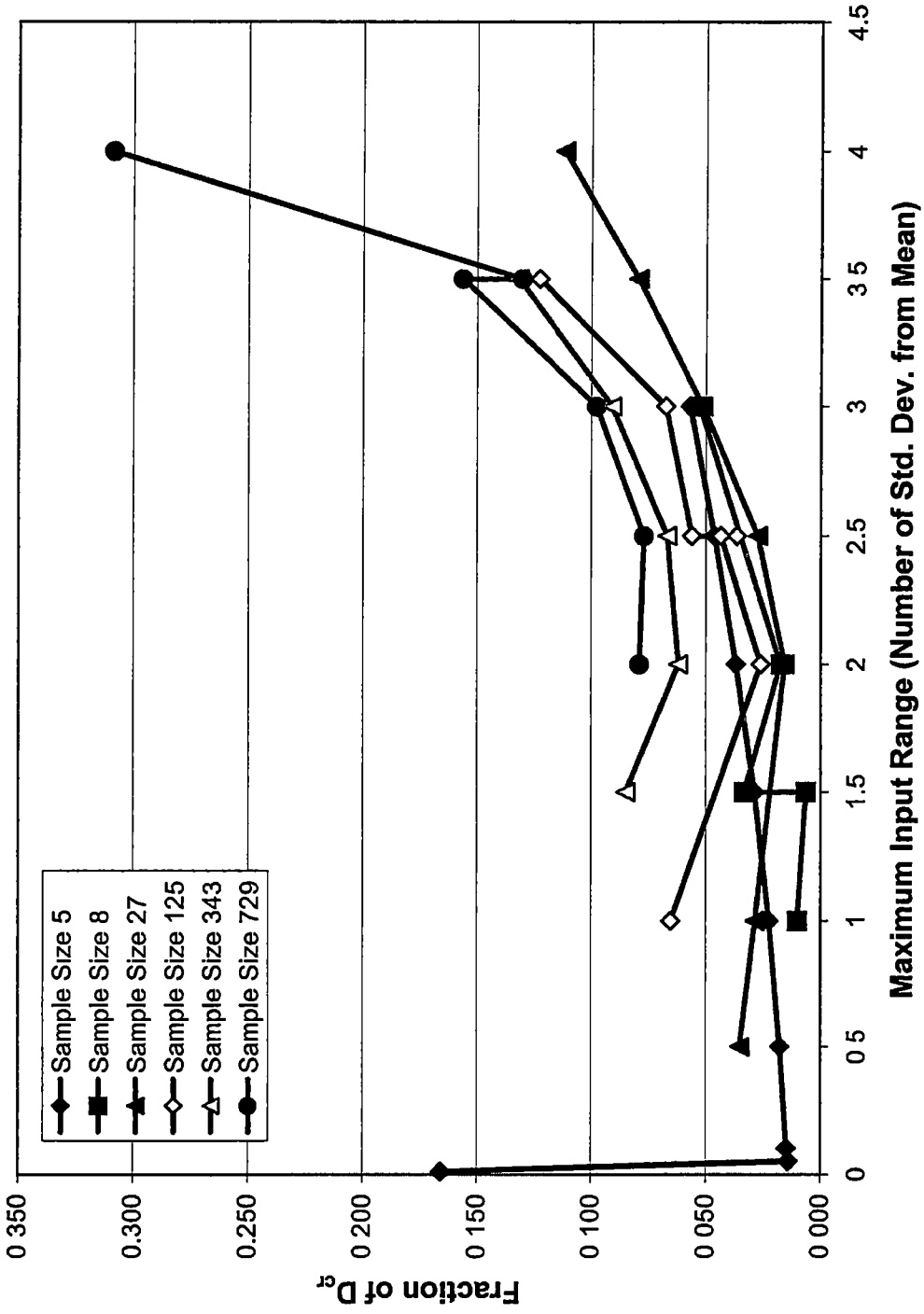


Figure 3.5. Comparison of fraction of D_{cr} and maximum input range for a pseudo population size of 100,000

observation is that D_{cr} decreases with the increasing sample size (Table 2.1). Therefore, as the sample size increases, D_{cr} decreases, which causes the fraction of D_{cr} to increase. Despite this observation, for a given sample size, there was a fairly distinct trend that if the range of input values were significantly close (zero to one standard deviations) or significantly far (three to four standard deviations) from the mean, the fraction of D_{cr} would be larger. Based on this trend, it is recommended that the optimal choice is one with a sample size of 27 and 2 to 2.5 standard deviations from the mean. Hence, RS function numbers 17 or 18 given in Table 3.2 would be the optimal RS functions for modeling the number of load cycles needed for a crack to grow from the initiation size of 0.01 inches to 0.3 inches for the parametric values given in Table 3.1.

3.2 RANDOM STRESS BLOCK ANALYSIS

3.2.1 Response Surface Functions and Pseudo Population

The random stress block response surface (RS) function was determined using a range of 9 to 81 sets of (c , m , $\Delta\sigma$, and N) values as shown in Column 2 of Tables 3.7-3.10. Each table represents one of four sets of 25 RS functions that were developed. More than one set was analyzed to ensure consistency among the RS functions because $\Delta\sigma$ was a random variable regenerated for each interval of each crack growth curve. As can be seen in Columns 2 and 3 of each table, all four sets had the same sample size and range of input variables. For

Table 3.7. Sample sizes, input ranges, and coefficients from regression analysis for set 1 of random stress block analysis

Set 1						
RS Function No	Sample Size	No. of Std Dev from Mean	Coefficients			
			b1	b2	b3	b4
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	9	0.5, 0	-1.0764	-7.6857	-2.9294	5.9954
2	9	1, 0	-0.9860	-7.7167	-1.8748	4.8982
3	9	2, 0	-0.9960	-7.1420	-1.7821	3.8418
4	9	2.5, 0	-1.0048	-7.1392	-3.6392	8.5623
5	9	3, 0	-0.9873	-7.0882	-2.5506	5.9580
6	9	3.5, 0	-0.9923	-7.1602	-2.5929	6.0526
7	9	4, 0	-0.9988	-7.2555	-3.7718	9.1179
8	25	1, 0.5, 0	-1.0274	-7.1966	-2.5699	5.4003
9	25	2, 1, 0	-0.9873	-7.3006	-2.8595	7.0110
10	25	2.5, 1.5, 0	-1.0049	-7.4089	-3.1474	7.5614
11	25	2.5, 1, 0	-0.9994	-7.3434	-2.6179	6.1931
12	25	2.5, 2, 0	-1.0141	-7.3210	-3.5680	8.3889
13	25	3, 1.5, 0	-1.0124	-7.1745	-3.7681	8.7967
14	25	3.5, 2, 0	-1.0068	-7.3509	-4.2062	10.2243
15	49	1.5, 1, 0.5, 0	-1.0249	-7.2475	-3.1146	6.9448
16	49	2, 1, 0.5, 0	-1.0033	-7.4570	-2.8443	6.8514
17	49	2.5, 2, 1, 0	-0.9954	-7.3256	-2.5492	6.0685
18	49	3, 2, 1, 0	-1.0058	-7.3341	-3.0202	7.1276
19	49	3.5, 2, 1, 0	-1.0043	-7.2638	-2.7447	6.3533
20	81	2, 1.5, 1, 0.5, 0	-1.0026	-7.1761	-2.8431	6.5500
21	81	2.5, 2, 1, 0.5, 0	-0.9945	-7.3736	-3.3822	8.3313
22	81	3, 2, 1, 0.5, 0	-0.9989	-7.2768	-3.1634	7.5694
23	81	3.5, 2.5, 1, 0.5, 0	-0.9989	-7.2129	-3.2588	7.7499
24	81	3.5, 2, 1, 0.5, 0	-0.9996	-7.3273	-2.5532	5.9963
25	81	4, 3, 2, 1, 0	-1.0030	-7.2492	-2.6345	6.0579

Table 3.8. Sample sizes, input ranges, and coefficients from regression analysis for set 2 of random stress block analysis

Set 2						
RS Function No	Sample Size	No of Std Dev from Mean	Coefficients			
			b1	b2	b3	b4
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	9	0 5, 0	-1 0903	-7 2182	-4 3285	8.8938
2	9	1, 0	-1 0305	-7 9127	-2 9133	7 0445
3	9	2, 0	-0 9893	-7.3553	-2.3365	5 6562
4	9	2 5, 0	-1 0026	-7 3374	-4 1131	10 0554
5	9	3, 0	-1 0018	-7 1871	-3.3523	7 9199
6	9	3 5, 0	-1 0012	-7 3008	-1 2521	2 4884
7	9	4, 0	-0 9883	-7 3910	-4 3472	10 9922
8	25	1, 0.5, 0	-1 0124	-6 9337	-3 0539	6 6626
9	25	2, 1, 0	-1 0198	-7.0277	-3 7205	8 3837
10	25	2 5, 1 5, 0	-1 0150	-7 3195	-3 8196	9 0499
11	25	2 5, 1, 0	-1 0004	-7 3741	-3 5403	8 6402
12	25	2 5, 2, 0	-0 9993	-7 3499	-3 6070	8 8129
13	25	3, 1 5, 0	-1 0076	-7.2343	-2 4311	5 4234
14	25	3 5, 2, 0	-1 0075	-7 3045	-1 8996	4 0963
15	49	1 5, 1, 0 5, 0	-1 0130	-7 2645	-3 0419	6 9866
16	49	2, 1, 0 5, 0	-1 0162	-7 3791	-3.4716	8 1800
17	49	2 5, 2, 1, 0	-1 0008	-7 3182	-3 3983	8 1953
18	49	3, 2, 1, 0	-1 0031	-7.1956	-2 2746	5 0656
19	49	3 5, 2, 1, 0	-1 0030	-7.3001	-3 1874	7 5724
20	81	2, 1 5, 1, 0 5, 0	-0 9962	-7 2027	-3 2615	7 8015
21	81	2 5, 2, 1, 0 5, 0	-1 0019	-7 3676	-2 7263	6 4613
22	81	3, 2, 1, 0 5, 0	-1 0020	-7 2118	-2 8603	6 6415
23	81	3 5, 2 5, 1, 0 5, 0	-0 9913	-7 2996	-3 4559	8 5008
24	81	3 5, 2, 1, 0 5, 0	-1 0040	-7 2278	-3 6212	8 6184
25	81	4, 3, 2, 1, 0	-1 0006	-7 2536	-3 4110	8 1480

Table 3.9. Sample sizes, input ranges, and coefficients from regression analysis for set 3 of random stress block analysis

Set 3						
RS Function No.	Sample Size	No of Std Dev from Mean	Coefficients			
			b1	b2	b3	b4
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	9	0 5, 0	-1 0492	-6 5216	-3.7205	7.2973
2	9	1, 0	-0 9950	-7.1331	-2.7995	6 5443
3	9	2, 0	-0 9941	-7.1689	-2.2940	5 2474
4	9	2 5, 0	-0 9891	-7.4936	-4.0574	10 3406
5	9	3, 0	-0 9904	-7 1027	-3 4679	8 3217
6	9	3 5, 0	-1 0121	-7 1153	-2 1386	4 4325
7	9	4, 0	-0.9932	-7.4008	-3.9267	9 7996
8	25	1, 0.5, 0	-1 0329	-7.6381	-4 3541	10 4839
9	25	2, 1, 0	-1 0073	-7.3088	-2 7039	6 2448
10	25	2 5, 1.5, 0	-1 0008	-7 3121	-3 6646	8 8946
11	25	2 5, 1, 0	-0 9872	-7 1995	-2 4429	5.8118
12	25	2 5, 2, 0	-1 0265	-7 2084	-3.3279	7 4120
13	25	3, 1.5, 0	-0 9918	-7.3485	-2 8174	6 8646
14	25	3.5, 2, 0	-0 9968	-7 3619	-2 1012	4 8932
15	49	1 5, 1, 0 5, 0	-0 9818	-7 4112	-3 3303	8 4816
16	49	2, 1, 0 5, 0	-0 9951	-7 4759	-3 1385	7 7941
17	49	2 5, 2, 1, 0	-1 0058	-7 2171	-3 7047	8 7987
18	49	3, 2, 1, 0	-0 9995	-7 2302	-3 1968	7 5948
19	49	3 5, 2, 1, 0	-0 9949	-7.2767	-2.8075	6 6929
20	81	2, 1 5, 1, 0.5, 0	-0 9967	-7.2365	-3.1169	7 4486
21	81	2 5, 2, 1, 0 5, 0	-0.9999	-7.3189	-2 7477	6 5023
22	81	3, 2, 1, 0 5, 0	-1.0003	-7 3065	-3 1746	7 6034
23	81	3 5, 2 5, 1, 0 5, 0	-0.9939	-7 2895	-3.8231	9 4051
24	81	3.5, 2, 1, 0 5, 0	-1 0077	-7.2009	-2.9253	6.7001
25	81	4, 3, 2, 1, 0	-0.9974	-7 2843	-2.8318	6 7188

Table 3.10. Sample sizes, input ranges, and coefficients from regression analysis for set 4 of random stress block analysis

Set 4						
RS Function No	Sample Size	No of Std Dev from Mean	Coefficients			
			b1	b2	b3	b4
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	9	0 5, 0	-1 0750	-8 1408	-3 6738	8 4646
2	9	1, 0	-0 9939	-7.4898	-3.2190	8.0532
3	9	2, 0	-0 9656	-7 3683	-0 0324	0 0476
4	9	2.5, 0	-0 9901	-7 3413	-4 8926	12 3527
5	9	3, 0	-1 0084	-7 2923	-4 1942	10 1094
6	9	3 5, 0	-1 0021	-7 1151	-2 1318	4.5943
7	9	4, 0	-1 0114	-7 0179	-2 9335	6 4335
8	25	1, 0 5, 0	-0 9741	-7 3712	-2 9379	7 5503
9	25	2, 1, 0	-1 0049	-7 3967	-3 2364	7 7854
10	25	2 5, 1.5, 0	-1 0195	-7.2885	-2 7473	6 1010
11	25	2 5, 1, 0	-0 9808	-7 2012	-3 6670	9.1468
12	25	2 5, 2, 0	-1 0004	-7 2894	-3 7440	9 0758
13	25	3, 1 5, 0	-1 0091	-7 3565	-2 6733	6 1746
14	25	3 5, 2, 0	-1 0066	-7 3050	-2 9651	6 9151
15	49	1 5, 1, 0 5, 0	-0 9915	-7.4343	-2 7740	6 8646
16	49	2, 1, 0 5, 0	-1 0056	-7 2345	-3 0817	7 1953
17	49	2 5, 2, 1, 0	-0 9942	-7 3280	-2 7692	6 6716
18	49	3, 2, 1, 0	-0 9970	-7 2928	-2 6429	6 2497
19	49	3 5, 2, 1, 0	-1 0032	-7 3241	-3 1683	7 5473
20	81	2, 1 5, 1, 0 5, 0	-1 0010	-7 3905	-2 9921	7 2140
21	81	2.5, 2, 1, 0.5, 0	-1 0015	-7 3395	-2 8170	6 6754
22	81	3, 2, 1, 0 5, 0	-1 0052	-7 3448	-2 9197	6 8874
23	81	3 5, 2 5, 1, 0 5, 0	-0 9998	-7 3145	-3 1287	7 4950
24	81	3 5, 2, 1, 0 5, 0	-0 9962	-7 3095	-2 8594	6 8461
25	81	4, 3, 2, 1, 0	-0 9960	-7 2676	-3 1442	7 5479

example, RS function number 1 from Tables 3.7-3.10 have a sample size of nine and 0.5 standard deviations from the means of the input parameters. Thus the values for c , m , and $\Delta\sigma$ for the nine sets were $(\bar{c}, \bar{m}, \Delta\sigma)$, $(\bar{c} + 0.5s_c, \bar{m}, \Delta\sigma)$, $(\bar{c}, \bar{m} + 0.5s_m, \Delta\sigma)$, $(\bar{c} - 0.5s_c, \bar{m}, \Delta\sigma)$, $(\bar{c}, \bar{m} - 0.5s_m, \Delta\sigma)$, $(\bar{c} + 0.5s_c, \bar{m} + 0.5s_m, \Delta\sigma)$, $(\bar{c} - 0.5s_c, \bar{m} + 0.5s_m, \Delta\sigma)$, $(\bar{c} + 0.5s_c, \bar{m} - 0.5s_m, \Delta\sigma)$, and $(\bar{c} - 0.5s_c, \bar{m} - 0.5s_m, \Delta\sigma)$, in which \bar{c} and \bar{m} represent the mean of c and m , respectively, $\Delta\sigma$ is the average of the 100 stress values for each curve and s_j represents the standard deviation of parameter j . As one can see from these sets of input parameters, $\Delta\sigma$ was not altered by its standard deviation and was left as the average of the 100 stress values (which can be different from curve to curve) for each curve in every set.

Equations (2.5-2.7) were used to calculate the number of cycles N for each set of input parameters. A linear regression was then performed on each set of $(c, m, \Delta\sigma, \text{ and } N)$ to determine the coefficients for the RS functions. Recall that each value was transformed to its logarithmic base for the regression analysis. Columns 4-7 in Table 3.7-3.10 present the coefficients of each RS function.

3.2.2 Verification of the Response Surface Function Statistical Moments

The point estimate method and Taylor series expansion were used to calculate the mean and standard deviation of $\ln N$ for each RS function. Columns 3 and 4 in Tables 3.11-3.18 summarize the results from both methods.

Using two significant figures, the mean of $\ln N$ is 9.7 and the standard deviation is about 0.34. This corresponds to a mean number of load cycles of 17,290 cycles and a standard deviation of 6,050 cycles (Benjamin and Cornell, 1970). It is of interest to note that the standard deviations using random stress blocks were smaller than those using constant stresses.

One hundred sets of 10,000 crack growth curves were simulated to represent the pseudo population. The statistical data in Table 3.1 were used to randomly generate c and m , for each crack growth curve while $\Delta\sigma$ was randomly generated for each of the 100 increments for each curve.

Columns 5 and 6 in Tables 3 11-3.18 present the 95 percent confidence interval of the population mean using the point estimate and Taylor series approximations of the RS functions. As can be seen columns 7-9 from the tables, all pseudo population means fell within the confidence interval determined from the RS functions.

Table 3.11. Means and standard deviations from point estimate method and results of the 95% confidence interval test for set 1 of random stress block analysis

Set 1								
RS Function No	Sample Size	Average Value	Standard Deviation	95% Confidence Interval		Pseudo Population		Percent Acceptable
				Lower Limit	Upper Limit	Passed	Failed	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	9	9.70	0.359	9.47	9.94	100	0	100%
2	9	9.68	0.346	9.45	9.91	100	0	100%
3	9	9.68	0.333	9.47	9.90	100	0	100%
4	9	9.68	0.335	9.46	9.90	100	0	100%
5	9	9.68	0.331	9.46	9.89	100	0	100%
6	9	9.67	0.333	9.45	9.89	100	0	100%
7	9	9.65	0.337	9.43	9.87	100	0	100%
8	25	9.69	0.340	9.55	9.82	100	0	100%
9	25	9.68	0.336	9.55	9.81	100	0	100%
10	25	9.68	0.341	9.55	9.81	100	0	100%
11	25	9.68	0.339	9.54	9.81	100	0	100%
12	25	9.67	0.341	9.53	9.80	100	0	100%
13	25	9.68	0.337	9.55	9.81	100	0	100%
14	25	9.66	0.340	9.52	9.79	100	0	100%
15	49	9.70	0.341	9.60	9.79	100	0	100%
16	49	9.69	0.342	9.59	9.78	100	0	100%
17	49	9.68	0.338	9.58	9.77	100	0	100%
18	49	9.68	0.340	9.59	9.78	100	0	100%
19	49	9.68	0.338	9.59	9.78	100	0	100%
20	81	9.68	0.335	9.61	9.76	100	0	100%
21	81	9.68	0.339	9.61	9.75	100	0	100%
22	81	9.68	0.337	9.61	9.75	100	0	100%
23	81	9.68	0.336	9.61	9.75	100	0	100%
24	81	9.67	0.338	9.60	9.74	100	0	100%
25	81	9.67	0.337	9.59	9.74	100	0	100%

Table 3.12. Means and standard deviations from point estimate method and results of the 95% confidence interval test for set 2 of random stress block analysis

Set 2								
RS Function No	Sample Size	Average Value	Standard Deviation	95% Confidence Interval		Pseudo Population		Percent Acceptable
				Lower Limit	Upper Limit	Passed	Failed	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	9	9.69	0.351	9.46	9.92	100	0	100%
2	9	9.70	0.358	9.47	9.93	100	0	100%
3	9	9.68	0.337	9.46	9.90	100	0	100%
4	9	9.67	0.339	9.45	9.89	100	0	100%
5	9	9.69	0.335	9.47	9.91	100	0	100%
6	9	9.65	0.338	9.43	9.87	100	0	100%
7	9	9.67	0.338	9.45	9.89	100	0	100%
8	25	9.69	0.331	9.56	9.82	100	0	100%
9	25	9.69	0.335	9.56	9.82	100	0	100%
10	25	9.68	0.341	9.55	9.82	100	0	100%
11	25	9.68	0.340	9.55	9.81	100	0	100%
12	25	9.68	0.339	9.55	9.82	100	0	100%
13	25	9.67	0.337	9.54	9.80	100	0	100%
14	25	9.66	0.339	9.53	9.80	100	0	100%
15	49	9.69	0.339	9.60	9.79	100	0	100%
16	49	9.69	0.342	9.59	9.78	100	0	100%
17	49	9.68	0.338	9.58	9.77	100	0	100%
18	49	9.68	0.336	9.59	9.78	100	0	100%
19	49	9.67	0.338	9.58	9.77	100	0	100%
20	81	9.69	0.335	9.61	9.76	100	0	100%
21	81	9.68	0.340	9.60	9.75	100	0	100%
22	81	9.68	0.336	9.61	9.75	100	0	100%
23	81	9.68	0.336	9.60	9.75	100	0	100%
24	81	9.67	0.337	9.60	9.75	100	0	100%
25	81	9.66	0.337	9.59	9.74	100	0	100%

Table 3.13. Means and standard deviations from point estimate method and results of the 95% confidence interval test for set 3 of random stress block analysis

Set 3								
RS Function No	Sample Size	Average Value	Standard Deviation	95% Confidence Interval		Pseudo Population		Percent Acceptable
				Lower Limit	Upper Limit	Passed	Failed	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	9	9.70	0.328	9.49	9.91	100	0	100%
2	9	9.70	0.333	9.48	9.92	100	0	100%
3	9	9.68	0.334	9.46	9.90	100	0	100%
4	9	9.68	0.341	9.46	9.90	100	0	100%
5	9	9.67	0.331	9.45	9.88	100	0	100%
6	9	9.66	0.335	9.44	9.88	100	0	100%
7	9	9.66	0.339	9.44	9.88	100	0	100%
8	25	9.69	0.351	9.55	9.83	100	0	100%
9	25	9.69	0.339	9.55	9.82	100	0	100%
10	25	9.68	0.338	9.55	9.82	100	0	100%
11	25	9.69	0.333	9.56	9.82	100	0	100%
12	25	9.67	0.340	9.54	9.81	100	0	100%
13	25	9.68	0.338	9.54	9.81	100	0	100%
14	25	9.67	0.339	9.54	9.80	100	0	100%
15	49	9.69	0.338	9.60	9.79	100	0	100%
16	49	9.68	0.341	9.59	9.78	100	0	100%
17	49	9.68	0.337	9.58	9.77	100	0	100%
18	49	9.68	0.336	9.59	9.77	100	0	100%
19	49	9.67	0.336	9.57	9.76	100	0	100%
20	81	9.69	0.336	9.61	9.76	100	0	100%
21	81	9.68	0.338	9.61	9.75	100	0	100%
22	81	9.68	0.338	9.61	9.75	100	0	100%
23	81	9.67	0.337	9.60	9.75	100	0	100%
24	81	9.68	0.337	9.61	9.76	100	0	100%
25	81	9.67	0.337	9.59	9.74	100	0	100%

Table 3.14. Means and standard deviations from point estimate method and results of the 95% confidence interval test for set 4 of random stress block analysis

Set 4								
RS Function No	Sample Size	Average Value	Standard Deviation	95% Confidence Interval		Pseudo Population		Percent Acceptable
				Lower Limit	Upper Limit	Passed	Failed	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	9	9.69	0.370	9.45	9.93	100	0	100%
2	9	9.69	0.341	9.47	9.92	100	0	100%
3	9	9.68	0.334	9.47	9.90	100	0	100%
4	9	9.68	0.337	9.46	9.90	100	0	100%
5	9	9.67	0.339	9.44	9.89	100	0	100%
6	9	9.66	0.334	9.44	9.88	100	0	100%
7	9	9.66	0.333	9.45	9.88	100	0	100%
8	25	9.69	0.335	9.56	9.83	100	0	100%
9	25	9.68	0.341	9.55	9.82	100	0	100%
10	25	9.67	0.341	9.54	9.81	100	0	100%
11	25	9.68	0.332	9.55	9.81	100	0	100%
12	25	9.67	0.338	9.54	9.81	100	0	100%
13	25	9.68	0.341	9.54	9.81	100	0	100%
14	25	9.66	0.339	9.53	9.79	100	0	100%
15	49	9.69	0.340	9.60	9.79	100	0	100%
16	49	9.69	0.337	9.60	9.79	100	0	100%
17	49	9.68	0.338	9.58	9.77	100	0	100%
18	49	9.68	0.337	9.58	9.77	100	0	100%
19	49	9.67	0.339	9.58	9.77	100	0	100%
20	81	9.69	0.340	9.62	9.76	100	0	100%
21	81	9.68	0.339	9.60	9.75	100	0	100%
22	81	9.68	0.340	9.61	9.76	100	0	100%
23	81	9.67	0.338	9.60	9.75	100	0	100%
24	81	9.67	0.337	9.60	9.75	100	0	100%
25	81	9.67	0.336	9.59	9.74	100	0	100%

Table 3.15. Means and standard deviations from Taylor series expansion and results of the 95% confidence interval test for set 1 of random stress block analysis

Set 1								
RS Function No.	Sample Size	Average Value	Standard Deviation	95% Confidence Interval		Pseudo Population		Percent Acceptable
				Lower Limit	Upper Limit	Passed	Failed	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	9	9 69	0 359	9 45	9 92	100	0	100%
2	9	9 66	0.345	9 44	9 89	100	0	100%
3	9	9 67	0 333	9.45	9 88	100	0	100%
4	9	9 67	0 334	9 45	9 89	100	0	100%
5	9	9 66	0 330	9 45	9 88	100	0	100%
6	9	9 66	0 332	9 44	9 88	100	0	100%
7	9	9.64	0 336	9 42	9 86	100	0	100%
8	25	9 67	0 339	9 54	9 81	100	0	100%
9	25	9 67	0 335	9 54	9 80	100	0	100%
10	25	9 67	0 340	9 54	9 80	100	0	100%
11	25	9 66	0 338	9 53	9 80	100	0	100%
12	25	9 66	0 340	9.52	9 79	100	0	100%
13	25	9 67	0 336	9 54	9 80	100	0	100%
14	25	9 65	0 339	9 52	9 78	100	0	100%
15	49	9 68	0 340	9 59	9 78	100	0	100%
16	49	9 67	0 341	9 58	9 77	100	0	100%
17	49	9.66	0 337	9 57	9 76	100	0	100%
18	49	9 67	0 339	9 57	9 76	100	0	100%
19	49	9 66	0 337	9 57	9 76	100	0	100%
20	81	9 67	0 334	9 60	9 74	100	0	100%
21	81	9 67	0 338	9 60	9 74	100	0	100%
22	81	9 67	0 336	9 60	9 74	100	0	100%
23	81	9 67	0 335	9 60	9 74	100	0	100%
24	81	9 66	0 338	9 58	9 73	100	0	100%
25	81	9 65	0 336	9 58	9 73	100	0	100%

Table 3.16. Means and standard deviations from Taylor series expansion and results of the 95% confidence interval test for set 2 of random stress block analysis

Set 2								
RS Function No	Sample Size	Average Value	Standard Deviation	95% Confidence Interval		Pseudo Population		Percent Acceptable
				Lower Limit	Upper Limit	Passed	Failed	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	9	9.68	0.350	9.45	9.91	100	0	100%
2	9	9.69	0.357	9.45	9.92	100	0	100%
3	9	9.67	0.337	9.45	9.89	100	0	100%
4	9	9.66	0.338	9.44	9.88	100	0	100%
5	9	9.68	0.335	9.46	9.90	100	0	100%
6	9	9.63	0.337	9.41	9.85	100	0	100%
7	9	9.66	0.337	9.44	9.88	100	0	100%
8	25	9.68	0.330	9.55	9.81	100	0	100%
9	25	9.68	0.334	9.55	9.81	100	0	100%
10	25	9.68	0.340	9.54	9.81	100	0	100%
11	25	9.67	0.339	9.54	9.81	100	0	100%
12	25	9.68	0.338	9.54	9.81	100	0	100%
13	25	9.66	0.337	9.52	9.79	100	0	100%
14	25	9.65	0.338	9.51	9.78	100	0	100%
15	49	9.68	0.338	9.59	9.78	100	0	100%
16	49	9.68	0.342	9.58	9.77	100	0	100%
17	49	9.67	0.338	9.57	9.76	100	0	100%
18	49	9.67	0.335	9.57	9.76	100	0	100%
19	49	9.66	0.337	9.57	9.76	100	0	100%
20	81	9.68	0.334	9.60	9.75	100	0	100%
21	81	9.67	0.339	9.59	9.74	100	0	100%
22	81	9.67	0.335	9.59	9.74	100	0	100%
23	81	9.67	0.336	9.59	9.74	100	0	100%
24	81	9.66	0.336	9.59	9.74	100	0	100%
25	81	9.66	0.336	9.58	9.73	100	0	100%

Table 3.17. Means and standard deviations from Taylor series expansion and results of the 95% confidence interval test for set 3 of random stress block analysis

Set 3								
RS Function No	Sample Size	Average Value	Standard Deviation	95% Confidence Interval		Pseudo Population		Percent Acceptable
				Lower Limit	Upper Limit	Passed	Failed	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	9	9 69	0.327	9 48	9 90	100	0	100%
2	9	9 69	0 332	9 47	9.90	100	0	100%
3	9	9 66	0.333	9 44	9 88	100	0	100%
4	9	9 67	0 340	9 45	9 90	100	0	100%
5	9	9 66	0 331	9 44	9 87	100	0	100%
6	9	9 65	0 335	9 43	9 86	100	0	100%
7	9	9 65	0 338	9 43	9 87	100	0	100%
8	25	9 69	0 350	9 55	9.82	100	0	100%
9	25	9 67	0 338	9 54	9 80	100	0	100%
10	25	9 67	0 337	9.54	9 81	100	0	100%
11	25	9 67	0.333	9 54	9 80	100	0	100%
12	25	9 66	0 339	9 53	9 80	100	0	100%
13	25	9 67	0 337	9 53	9 80	100	0	100%
14	25	9 65	0 338	9 52	9 78	100	0	100%
15	49	9 68	0 337	9 59	9 78	100	0	100%
16	49	9 67	0 340	9 58	9 77	100	0	100%
17	49	9 67	0 336	9 58	9 76	100	0	100%
18	49	9 67	0 335	9.58	9 76	100	0	100%
19	49	9 65	0 336	9 56	9 75	100	0	100%
20	81	9 68	0 335	9.60	9 75	100	0	100%
21	81	9 67	0.337	9 59	9 74	100	0	100%
22	81	9 67	0.337	9 60	9 74	100	0	100%
23	81	9 67	0 336	9 59	9 74	100	0	100%
24	81	9 67	0 336	9 60	9 74	100	0	100%
25	81	9 65	0 336	9 58	9 73	100	0	100%

Table 3.18. Means and standard deviations from Taylor series expansion and results of the 95% confidence interval test for set 4 of random stress block analysis

Set 4								
RS Function No	Sample Size	Average Value	Standard Deviation	95% Confidence Interval		Pseudo Population		Percent Acceptable
				Lower Limit	Upper Limit	Passed	Failed	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	9	9.68	0.369	9.44	9.92	100	0	100%
2	9	9.68	0.341	9.46	9.90	100	0	100%
3	9	9.66	0.333	9.44	9.88	100	0	100%
4	9	9.68	0.336	9.46	9.90	100	0	100%
5	9	9.66	0.338	9.44	9.88	100	0	100%
6	9	9.64	0.333	9.42	9.86	100	0	100%
7	9	9.65	0.332	9.44	9.87	100	0	100%
8	25	9.68	0.335	9.55	9.81	100	0	100%
9	25	9.67	0.340	9.54	9.81	100	0	100%
10	25	9.66	0.340	9.53	9.79	100	0	100%
11	25	9.68	0.332	9.55	9.81	100	0	100%
12	25	9.67	0.337	9.53	9.80	100	0	100%
13	25	9.66	0.340	9.53	9.80	100	0	100%
14	25	9.65	0.338	9.52	9.78	100	0	100%
15	49	9.68	0.339	9.58	9.77	100	0	100%
16	49	9.68	0.336	9.59	9.77	100	0	100%
17	49	9.67	0.337	9.57	9.76	100	0	100%
18	49	9.66	0.336	9.57	9.76	100	0	100%
19	49	9.66	0.338	9.57	9.76	100	0	100%
20	81	9.68	0.339	9.60	9.75	100	0	100%
21	81	9.67	0.338	9.59	9.74	100	0	100%
22	81	9.67	0.339	9.60	9.74	100	0	100%
23	81	9.66	0.337	9.59	9.73	100	0	100%
24	81	9.66	0.337	9.59	9.73	100	0	100%
25	81	9.66	0.336	9.58	9.73	100	0	100%

3.2.3 Statistical Testing of the Pseudo Population

The coefficients of skewness and coefficient of Kurtosis were calculated to examine the distribution of the pseudo population. One hundred sets of 10,000 pseudo population crack growth curves were generated. The average coefficient of skewness from the 100 sets of data was 0.046 and the standard deviation was 0.026. The average coefficient of Kurtosis from the same 100 sets of data was 3.03 and the standard deviation was 0.054. The average coefficient of skewness indicates that the $\ln N$ values are skewed slightly to the right. However, as the case in the constant stress analysis, both coefficients are very close to their target values of zero and three, respectively. Therefore, it was likely that $\ln N$ could still be modeled by Gaussian distribution

3.2.4 Comparison of the Response Surface Functions with the Pseudo Population

The Kolmogorov-Smirnov (K-S) test was used to check if the proposed probability model (based on the RS functions) was acceptable. For comparisons, one set of 10,000 simulated growth curves was used. Table 3 19 presents the results for all four sets of RS functions. Columns 5, 7, 9, and 11 present the maximum difference in cumulative distribution between the normal distribution developed based on the RS function and the pseudo population. As can be seen from the table, the maximum differences are much less than the critical difference of D_{cr} (column 4). The fraction of D_{cr} is presented in Columns 6, 8, 10,

Table 3.19. Results of the Kolmogorov-Smirnov test for all 4 sets of random stress block analysis

RS Function No	Sample Size	No of Std Dev from Mean	D _{cr}	Pseudo Population Size Used for all Comparisons 10,000											
				Set 1		Set 2		Set 3		Set 4					
				Difference	Fraction of D _{cr}	Difference	Fraction of D _{cr}	Difference	Fraction of D _{cr}	Difference	Fraction of D _{cr}				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)				
1	9	0.5, 0	0.440	0.038	0.086	0.028	0.063	0.037	0.084	0.034	0.077				
2	9	1, 0		0.007	0.016	0.034	0.076	0.031	0.071	0.024	0.055				
3	9	2, 0		0.008	0.019	0.006	0.013	0.005	0.011	0.006	0.014				
4	9	2.5, 0		0.015	0.034	0.003	0.008	0.016	0.036	0.019	0.043				
5	9	3, 0		0.007	0.015	0.019	0.044	0.010	0.022	0.004	0.008				
6	9	3.5, 0		0.008	0.019	0.043	0.099	0.021	0.048	0.025	0.057				
7	9	4, 0		0.026	0.058	0.003	0.006	0.011	0.026	0.015	0.033				
8	25	1, 0.5, 0	0.270	0.014	0.052	0.021	0.077	0.032	0.118	0.025	0.093				
9	25	2, 1, 0		0.012	0.044	0.024	0.087	0.013	0.049	0.014	0.053				
10	25	2.5, 1.5, 0		0.010	0.035	0.019	0.071	0.016	0.058	0.005	0.018				
11	25	2.5, 1, 0		0.003	0.013	0.014	0.051	0.017	0.061	0.018	0.068				
12	25	2.5, 2, 0		0.007	0.026	0.017	0.061	0.004	0.014	0.006	0.021				
13	25	3, 1.5, 0		0.010	0.038	0.009	0.033	0.005	0.018	0.003	0.013				
14	25	3.5, 2, 0		0.016	0.060	0.021	0.078	0.014	0.053	0.017	0.064				
15	49	1.5, 1, 0.5, 0	0.192	0.028	0.144	0.023	0.119	0.024	0.124	0.020	0.106				
16	49	2, 1, 0.5, 0		0.016	0.084	0.019	0.100	0.013	0.070	0.022	0.116				
17	49	2.5, 2, 1, 0		0.004	0.021	0.009	0.045	0.011	0.058	0.005	0.028				
18	49	3, 2, 1, 0		0.010	0.051	0.009	0.046	0.012	0.060	0.005	0.026				
19	49	3.5, 2, 1, 0		0.003	0.016	0.003	0.017	0.010	0.054	0.003	0.016				
20	81	2, 1.5, 1, 0.5, 0		0.151	0.012	0.081	0.019	0.128	0.017	0.112	0.021	0.136			
21	81	2.5, 2, 1, 0.5, 0			0.011	0.070	0.006	0.042	0.008	0.053	0.005	0.034			
22	81	3, 2, 1, 0.5, 0	0.012		0.080	0.008	0.055	0.009	0.060	0.010	0.068				
23	81	3.5, 2.5, 1, 0.5, 0	0.012		0.079	0.008	0.054	0.007	0.048	0.003	0.019				
24	81	3.5, 2, 1, 0.5, 0	0.007		0.048	0.005	0.034	0.012	0.079	0.004	0.024				
25	81	4, 3, 2, 1, 0	0.011		0.074	0.009	0.061	0.010	0.069	0.009	0.061				

and 12. Most of the differences are less than 10 percent of D_{cr} and the greatest difference is only 14 percent of the critical difference.

3.2.5 Optimal Sample Size and Range of the Input Parameters

Figures 3.6-3.9 present a graphical comparison between the maximum number of standard deviations from the mean and the fraction of D_{cr} for each RS function. The maximum number of standard deviations from the mean can be found in Table 3 19, column 3.

Unlike the constant stress case where the fraction of D_{cr} showed a distinct trend with respect to sample size and maximum number of standard deviations from the mean, the trend is not nearly as clear for the random stress situation. In particular, almost all sample sizes showed a high fraction of D_{cr} at low values (0.5 and 1) of the maximum number of standard deviations from the mean. Hence, the choice of optimal sample size and range of input parameters was not quite as obvious. Examination of Figures 3.6-3.9 suggested that a sample size of 9 to 25 with a standard deviation of 2.0 to 2.5 would perhaps yield the best RS function.

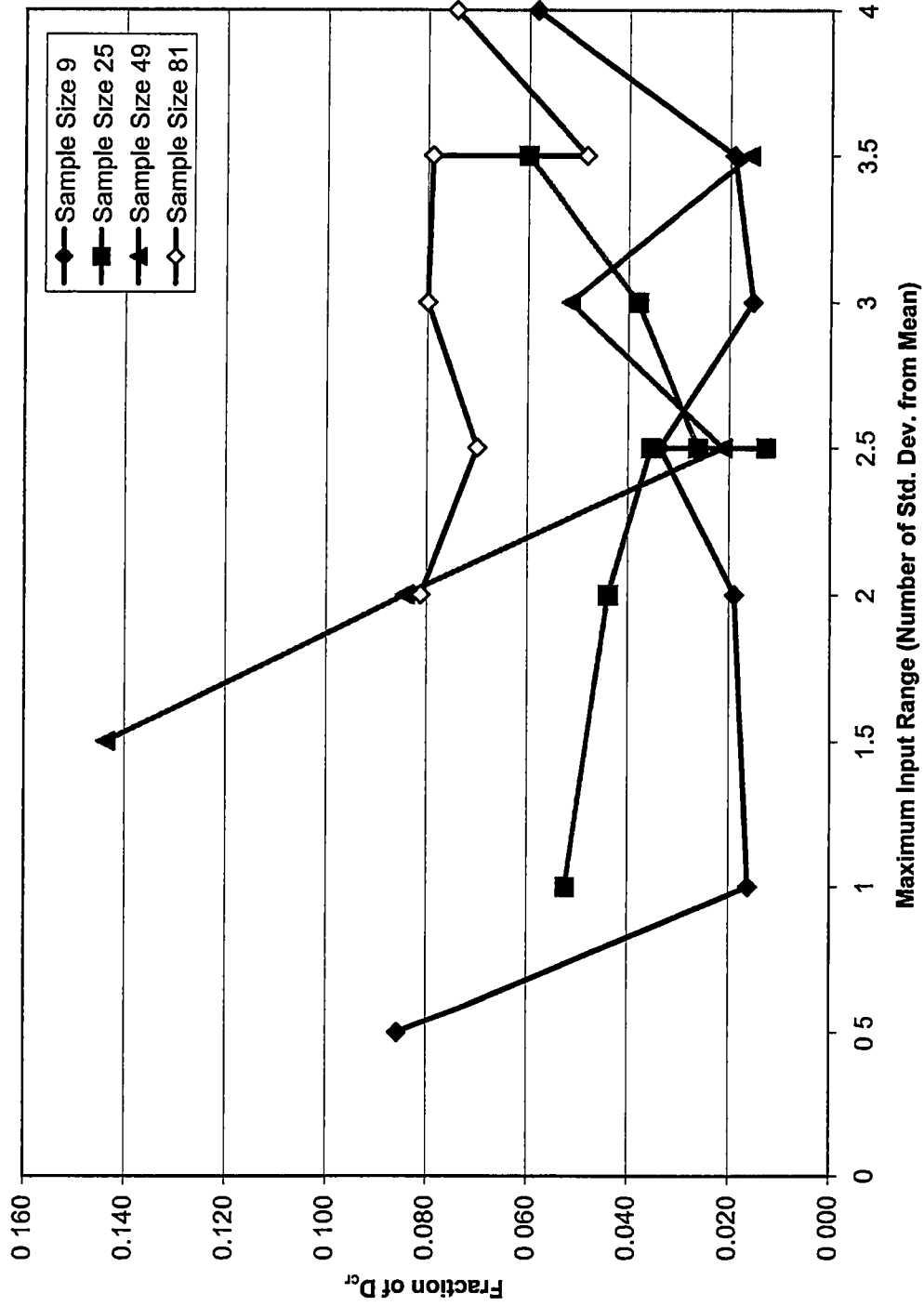


Figure 3.6. Comparison of fraction of D_{cr} and maximum input range for a pseudo population size of 10,000 (set 1)

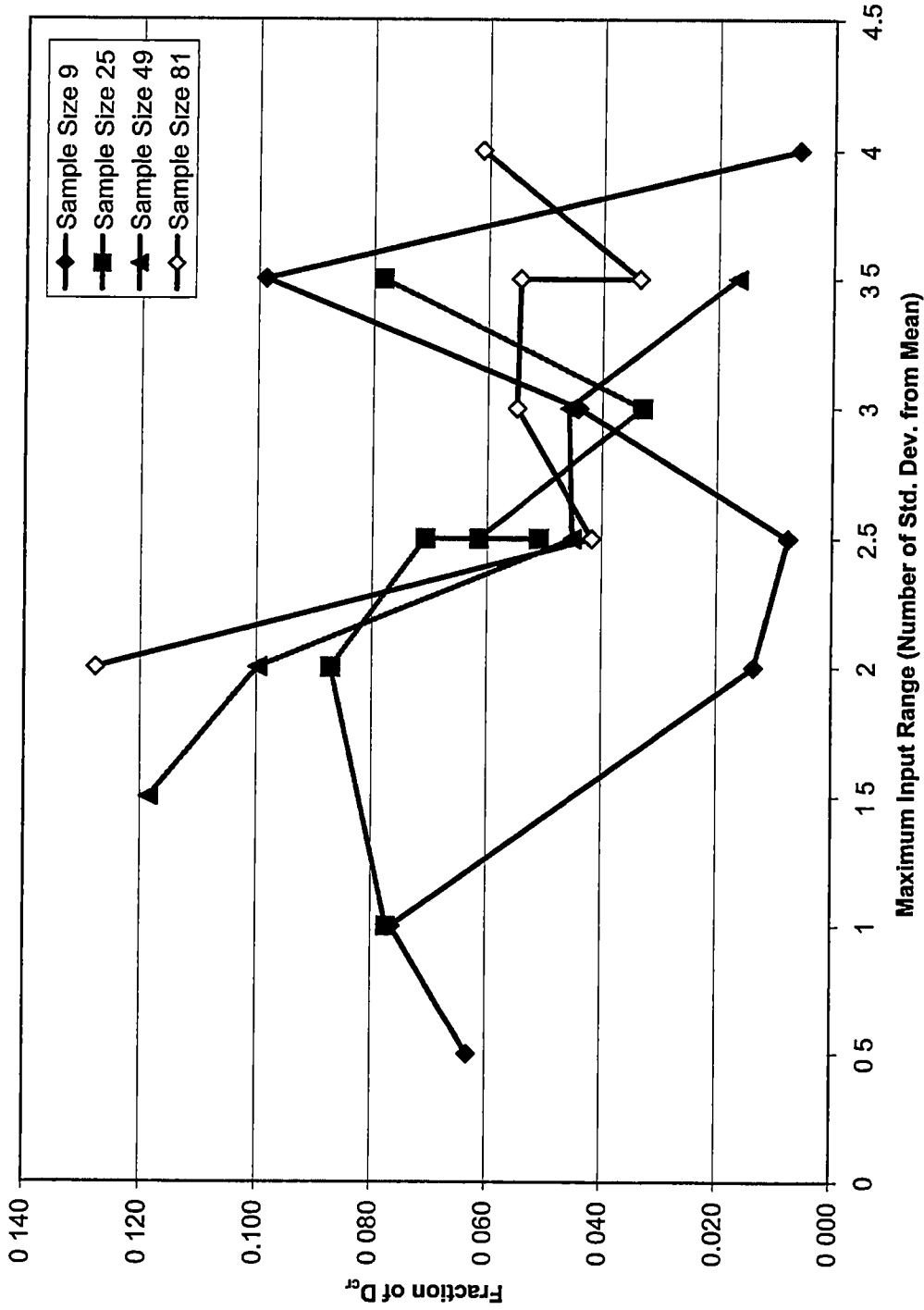


Figure 3.7. Comparison of fraction of Dcr and maximum input range for a pseudo population size of 10,000 (set 2)

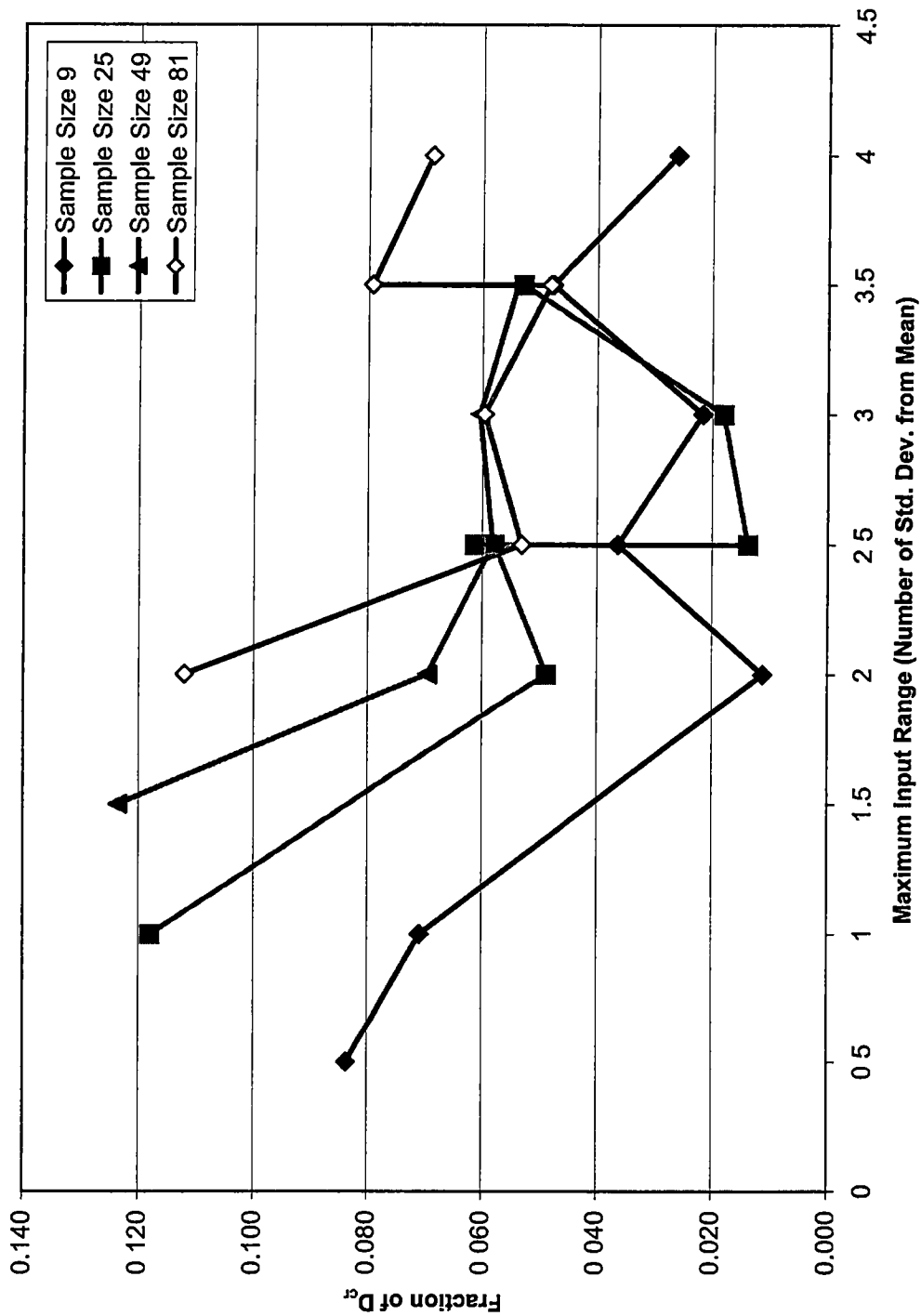


Figure 3.8. Comparison of fraction of Dcr and maximum input range for a pseudo population size of 10,000 (set 3)

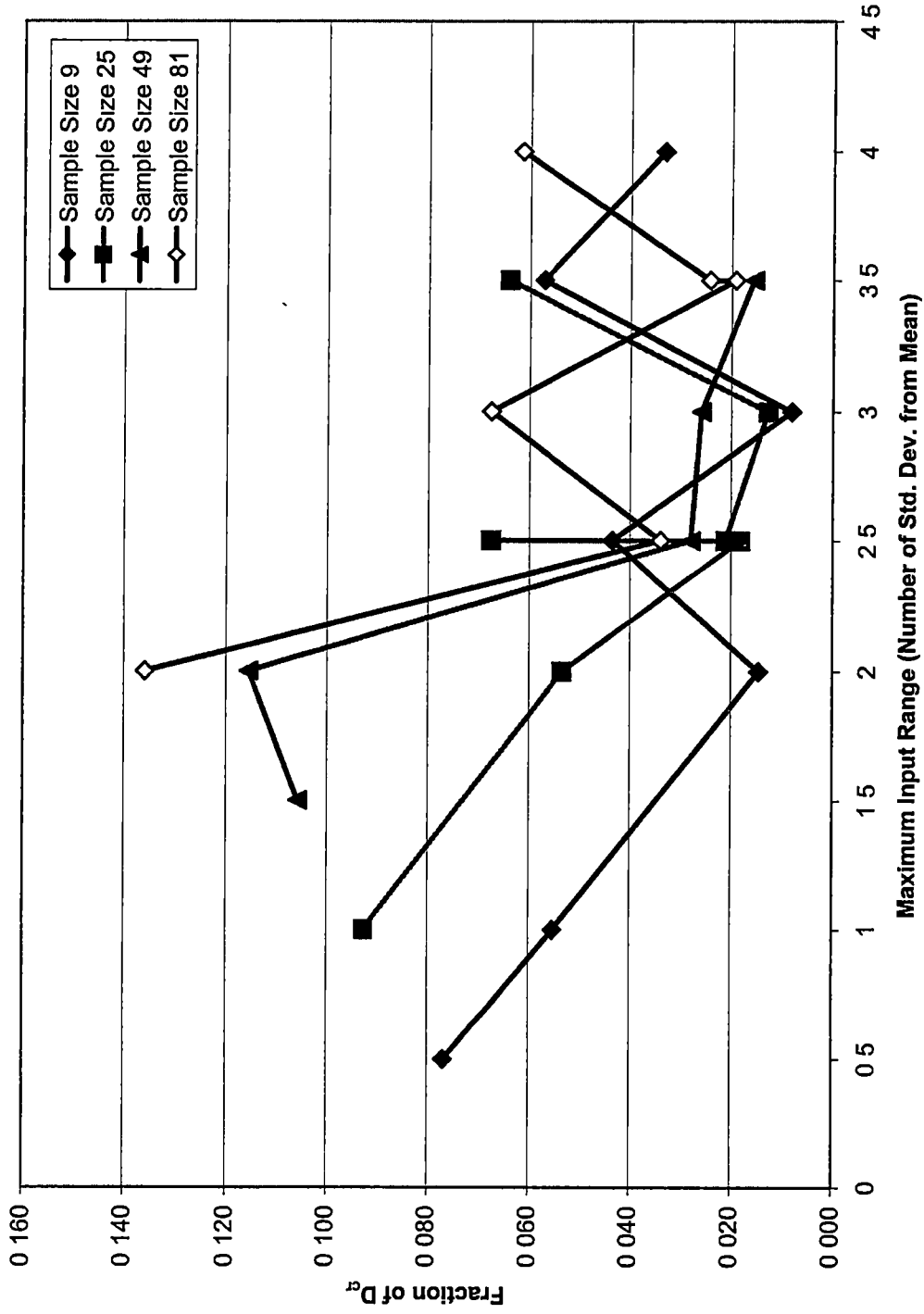


Figure 3.9. Comparison of fraction of Dcr and maximum input range for a pseudo population size of 10,000 (set 4)

CHAPTER 4

CONCLUSIONS AND RECOMMENDATIONS

A study was performed to assess aging aircraft by examining crack growth at the logeron of the aircraft. The objective of the study was to develop a probability model for the number of load cycles N needed for a crack to grow from its initiation size to critical size. Two different types of stress intensity were analyzed, constant stress and random stress block.

A fatigue crack growth model was developed based on the Paris equation. Response surface method (RSM) was used to develop relationships (RS functions) between the load cycles N (calculated from the Paris equation) and the input variables. A wide spectrum of sample sizes and ranges of the input variables were considered. The most workable regression function to approximate N was a linear regression with all variables transformed to their natural logarithmic values. Point estimate method and Taylor series approximation were used to determine the means and standard deviations of $\ln N$ from the RS functions. By central limit theorem, $\ln N$ from the RS functions was assumed to be Gaussian distributed.

Comparing the means of $\ln N$ from the pseudo populations to the 95 percent confidence interval computed from the RS functions indicated the RS function provides an acceptable mean value for the pseudo population. The results of the Kolmogorov-Smirnov (K-S) test showed that the Gaussian

distribution was an acceptable model for both the constant stress and random stress block despite the observation that the coefficient of skewness and coefficient of Kurtosis of the pseudo population were slightly greater than their target values of zero and three, respectively, for a Gaussian distribution.

Once the probability model for the number of load cycles is known, a decision can be made on the flight hours where a tolerable risk is defined. For instance, assume the response surface function using 27 samples with a maximum input range of 2.0 standard deviations from the mean (RS function no. 17 in Table 3.2) for the constant stress was to be used to determine the number cycles the aircraft should fly. Table 3.5 gives the mean and standard deviation of $\ln N$ as 9.61 and 0.453, respectively. If an acceptable risk is defined as five percent, then the maximum number of cycles one should fly the aircraft after a crack has been initiated is

$$\begin{aligned} \ln N &= \bar{x} - \Phi^{-1}(0.05)(s) \\ \ln N &= 9.61 - 1.645(0.453) \\ \ln N &= 8.865 \\ n &= 7078 \text{ cycles} \end{aligned}$$

During the study, it was found that the RS function not only depended upon the sample size, but also the range of input values. From the spectrum of sample sizes and ranges of input values examined, the optimal sample size and range of the input parameters, for the constant stress analysis, were respectively 27 and 2 to 2.5 standard deviations from the mean. Likewise, for the random

stress block, the optimal choice was 9 to 25 samples and 2 to 2.5 standard deviations from the mean.

The optimal sample size and range of input parameters information will help future development of crack growth models. For example, when a different type of material, geometry, stress level, or even critical crack size are considered, one only needs to have 27 sample crack growth curves and a range of input parameters with 2 standard deviations from the mean to develop a probabilistic crack growth model when constant stress is used.

While the probability model presented here for crack growth compared well with the pseudo population, a couple of issues could be addressed to enhance the overall goal of assessing aging aircraft. The crack size interval should be examined more closely. As stated in the introduction, most studies of fatigue crack growth consider the crack size as a changing variant, while N is a given integer. However, in this study the crack size was divided into equal increments, which were held constant. The constant crack size could affect the accuracy of the crack growth model. This is especially true where the crack growth is making a transition from slow growth to rapid growth. Continuing studies should consider a variable crack size increment. Consideration should also be given to dividing the crack size range of interest into smaller crack size increments.

In this study, the input parameters were known and the crack growth curve was well behaved. One may argue that Monte Carlo simulations is a more

efficient method for developing the probability model of N. However, this study demonstrated that the response surface method is a viable method even in the situation where the statistics of the input parameters is not available. Furthermore, response surface method will be extended to study crack initiation, where few studies have been performed and the statistics of the input parameters were not well developed

It is also suggested that the probability model be compared to any existing data that are available. Although the existing data may be small and limited, a comparison is vital before the probability model is used in the total life model of an aircraft.

REFERENCES

REFERENCES

- Ang, A.H-S. and Tang, W.H. (1975) *Probability Concepts in Engineering Planning and Design*, Vol. 1, John Wiley & Sons, New York.
- Bannantine, J.A , Comer, J.J. and Handrock, J.L. (1990) *Fundamentals of Metal Fatigue Analysis*, Prentice Hall, New York.
- Benjamin, J.R. and Cornell, C.A. (1970) *Probability, Statistics and Decision for Civil Engineers*, McGraw Hill, New York.
- Box, G E.P. and Wilson, K.B. (1951) "On the Experimental Attainment of Optimum Conditions," *Journal of the Royal Statistical Society*, Ser. B, 13, 1-45.
- Bucher, C.G. and Bourgund, U. (1990) "A Fast and Efficient Response Surface Approach for Structural Reliability Problems," *Structural Safety*, Vol. 7, pp. 57-66.
- Chou, K.C. (1998) "Reliability Assessment Model for Aging Aircraft," preliminary report to Structures Division, Navy Air Station, Patuxent River, MD, part of Navy-ASEE Summer Faculty Program.
- Draper, N.R. and Lin, D.K.J. (1990) "Small Response-Surface Designs," *Technometrics*, Vol 32, No. 2, May, pp. 187-194.
- Harr, M.E. (1987) *Reliability Based Design in Civil Engineering*, McGraw Hill, New York.
- Hoffman, M.E. (1998) "STIP Project: Reliability Assessment Method," Status Report: 03FY98, August 5, presented to the Structures Division, Navy Air Station, Patuxent River, MD.
- Liu, Y.W and Moses, F. (1994) "A Sequential Response Surface Method and its Application in the Reliability Analysis of Aircraft Structural Systems," *Structural Safety*, Vol. 16, pp. 39-46.
- Maymon, G. (1998) "Some Engineering Applications in Random Vibrations and Random Structures," *Progress in Astronautics and Aeronautics*, Vol. 178.
- Myers, R.H., Khuri, A.I. and Carter, W.H. Jr. (1989) "Response Surface Methodology: 1966-1988," *Technometrics*, Vol. 31, No. 2, May, pp. 137-157.

Rajashekhar, M.R. and Ellingwood, B.R. (1993) "A New Look at the Response Surface Approach for Reliability Analysis," *Structural Safety*, December, pp. 205-220.

Rosenblueth, E. (1975) "Point Estimates for Probability Moments," *Proceedings of National Academy of Science, U.S.A.*, Vol. 72, No. 10, October, pp. 3812-3814.

Rosenblueth, E. (1981) "Two Point Estimate in Probability," *Applied Math. Modelling*, Vol. 5, October, pp. 329-335.

Wirsching, P.H. (1983) *Statistical Summaries of Fatigue Data for Design Purposes*, NASA Contractor Report 3697, National Aeronautics and Space Administration, Lewis Research Center.

Wu, T-T. and Wirsching, P.H. (1983) *Application of Advanced Reliability Methods to Local Strain Fatigue Analysis*, NASA Contractor Report 168198, National Aeronautics and Space Administration, Lewis Research Center.

VITA

Glenn Chris Cox was born in Knoxville, Tennessee on October 8, 1976 to Gordon and Janie Carpenter Cox and was raised in nearby Lake City, Tennessee. He attended public school in Anderson County and graduated from Anderson County High School in June, 1994. Upon graduation he enrolled at the University of Tennessee, Knoxville (UTK). As an undergraduate at UTK he was active in the student chapter of American Society of Civil Engineers (ASCE), serving as secretary from June 1997 to May 1998 and vice president from June 1998 to May 1999. He became a member of Chi Epsilon, a national civil engineering honor society, during the fall term of 1998. He served as president of the Chi Epsilon student chapter at UTK from June 1999 to May 2000. He received his Bachelor of Science in Civil Engineering in August, 1999. He enrolled in graduate school at UTK during the summer term of 1999 and graduated with a Master of Science in Civil Engineering in December, 2000. He became engaged to the love of his life, Ashley Hamby, on June 23, 2000. Their wedding is planned for July 7, 2001. He began employment September, 2000 as an engineer assistant with Lockwood Greene, Inc., in Knoxville, Tennessee.